Reconciling the $X(2240)$ with the $Y(2175)$

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In the present work, we reanalyzed the cross sections for $e^+e^- \rightarrow K^+K^-$, where a new structure $X(2240)$ was reported by the BES III collaboration. By including the interference between the direct coupling and vector meson intermediate processes, we find the mass and width of $X(2240)$ are 2197.4 ± 4.4 and 75.6 ± 7.2 MeV, respectively, which are well consistent with the Particle Data Group average values of the resonance parameters for $Y(2175)$, thus, we conclude that the $X(2240)$ should be the same state as the $Y(2175)$.

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I. INTRODUCTION

In the past decades, an increasing number of new hadron states have been observed in charm and bottom sectors (see Refs. [1–10] for recent reviews of experimental and theoretical status). These experimental observations stimulate theorists' great interests in hunting QCD exotic states in the forest of heavy-quarkonium-like states. However, in the light flavor sector, the researches of the new hadron states in both experimental and theoretical sides are not as prosperous as those in heavy flavor sectors. As one of the few interesting new hadron states in the light sector, the strangeonium-like state $Y(2175)$ was first reported by the BABAR collaboration in $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi f_0(980)$ in 2006 [11]. Then it was confirmed by the BESII collaboration in the $J/\psi \rightarrow \eta\phi f_0(980)$ [12] and by the Belle collaboration in the $e^+e^- \rightarrow \phi f_0(980)$ and $\phi\pi^+\pi^-$ processes [13].

Since its discovery, $Y(2175)$ has been searched in $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi\pi^+\pi^-$, $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi\eta$, $e^+e^- \rightarrow \eta\phi f_0(980)$, $e^+e^- \rightarrow \gamma_{\text{ISR}}\phi f_0(980)$, $e^+e^- \rightarrow \gamma_{\text{ISR}}K^+K^-f_0(980)$, $J/\psi \rightarrow \eta\phi f_0(980)$, $e^+e^- \rightarrow K^+K^-K^+K^-$, $e^+e^- \rightarrow K^+K^-\pi^0\pi^0$ and $e^+e^- \rightarrow \phi\eta'$ by BABAR [11,14–16], Belle [13] and BES [12,17–20] collaborations. The resonance parameters reported by different collaborations are presented in Fig. 1. From the figure, one can find the reported mass of $Y(2175)$ is mostly located in the vicinity of 2180 MeV, while its width is about 80 MeV. The Particle Data Group (PDG) averages of its resonance parameters are [21]

$$m_{Y(2175)} = (2188 \pm 10) \text{ MeV},$$

$$\Gamma_{Y(2175)} = (83 \pm 12) \text{ MeV}, \quad (1)$$

respectively.

At the early stage, the strangeonium-like state, $Y(2175)$, was only observed in the hidden-strange process, which is similar to the case of $Y(4260)$ in $\pi^+\pi^-J/\psi$ [23], $Y(4360)$ in $\pi^+\pi^-\psi(2S)$ [24] and $Y_b(10890)$ in $\Upsilon(nS)\pi^+\pi^-$, ($n = 1, 2$) processes [25]. The lack of the observation in the

FIG. 1. The resonance parameters of $Y(2175)$ reported by BABAR [11,14–16], Belle [13] and BES [12,17] collaborations, as well as the PDG average [21]. For comparison, we also present the resonance parameters of $X(2240)$ reported by the BES III collaboration [22].

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open-strange channels makes this state particularly interesting. In Ref. [26], by using the QCD sum rule approach and take a more phenomenological analysis, the mass of $Y(2175)$ can be reproduced in a tetraquark frame. As indicated in Ref. [27], the strangeonium-like state $Y(2175)$ can also be assigned as an excited tetraquark state with spin 1 and the estimation in the flux-tube model supported that $Y(2175)$ is a tetraquark state with diquark-antidiquark configuration [28]. Considering the particular production processes of $Y(2175)$, a hybrid interpretation was proposed in Refs. [29,30], and the estimations in both the flux-tube model and the constituent gluon model supported such an interpretation. But the nonperturbative lattice estimation indicated that $Y(2175)$ could not be a $s\bar{s}g$ hybrid due to its decay behavior [31]. The recent estimations with QCD Gaussian sum rule also find that $Y(2175)$ can hardly be assigned as a hybrid due to the small relative strength [32]. Moreover, in the vicinity of $Y(2175)$, there are some thresholds of a pair of hadrons, such as $\Lambda\bar{\Lambda}$, $\phi f_0(980)$, thus, in Ref. [1], $Y(2175)$ was interpreted as a $\Lambda\bar{\Lambda}$ baryonium and in Ref. [33], $Y(2175)$ was considered as a resonance of $\phi K\bar{K}$. Besides these exotic interpretations, $Y(2175)$ was also tried to be categorized as a conventional strangeonium, such as 3^3S_1 [34] or 2^3D_1 [29,35] states.

Among these interpretations, the open strange decays are important criteria, thus searching the signal of $Y(2175)$ experimentally in the open strange channels is crucial. In 2018, the BES III collaboration measured the cross sections for $e^+e^- \rightarrow K^+K^-$ at $\sqrt{s} = 2.0\text{--}3.08$ GeV with the best precision achieved so far; no signal of $Y(2175)$ was found but a new structure, named $X(2240)$, was reported with the resonance parameters to be [22]

$$\begin{aligned} m_{X(2240)} &= 2239.2 \pm 7.1 \pm 11.3 \text{ MeV}, \\ \Gamma_{X(2240)} &= 139.8 \pm 12.3 \pm 20.6 \text{ MeV}, \end{aligned} \quad (2)$$

respectively. In Fig. 1, we also present the resonance parameters of $X(2240)$ for comparison, which differs from PDG average values of $Y(2175)$ resonance parameters by more than 3σ in mass and more than 2σ in width. After the observation of $X(2400)$, it has been investigated in a different scenario, such as tetraquark [36,37], $\Lambda\bar{\Lambda}$ baryonium [38], $\omega(3^3D_1)$ state [39].

If carefully checking the observation of $Y(2175)$ and $X(2240)$, one can find that all the two-body discovery channels of $Y(2175)$ are neutral, such as $\phi\eta$, $\phi f_0(980)$. Even the three-body channel of $Y(2175)$, such as $\phi\pi^+\pi^-$ and $K^+K^-f_0(980)$, the charged $\pi^+\pi^-$ and K^+K^- can form a neutral bloc. However, as for $X(2240)$, it was discovered in a charged K^+K^- channel, which indicates that the virtual photon from the e^+e^- annihilation can couple with the K^+K^- pair directly, or the virtual photon couples to the vector resonance and the vector resonance decays into K^+K^- . These two kinds of mechanisms working together

in the $e^+e^- \rightarrow K^+K^-$ process and their interferences can change the peak of the resonance. Such phenomena have been applied in the investigations of the cross sections for some processes involved charmonium-like states [40–43]. Moreover, besides $Y(2175)$, there is another state, $\rho(2150)$ in the vicinity of 2.2 GeV. Its mass was reported from 1990 ± 80 [44] to 2254 ± 22 MeV [45], but almost all the measurements indicate that $\rho(2150)$ are much broader with the width to be several hundreds MeV [21,44]. The experimental data from the BES III collaboration indicate that there is one peak in the vicinity of 2.2 GeV, thus in the present work, we tried to reproduce the cross sections for $e^+e^- \rightarrow K^+K^-$ with the interference between the direct and the resonance contributions. In the fit, we treat the resonance parameters as free parameters, which can be determined by fitting the cross sections for $e^+e^- \rightarrow K^+K^-$. By comparing the fitted resonance parameters with those of $Y(2175)$ and $\rho(2150)$, one can conclude the source of the peak near 2.2 GeV.

This work is organized as follows. After introduction, we present the concrete formula of the interference mechanism in the following section. In Sec. III, we present our fit results and the related discussions and the last section is devoted to a short summary.

II. INTERFERENCE PICTURE OF $e^+e^- \rightarrow K^+K^-$

As indicated in the Introduction, there exist two kinds of mechanisms working in the $e^+e^- \rightarrow K^+K^-$ process as shown in Fig. 2, which are the direct coupling process as shown in diagram (a) and the vector resonance intermediate process as shown in diagram (b). In the present work, we estimate these diagrams in hadron level, and the interaction vertexes are depicted by effective Lagrangians. The involved effective Lagrangians are [46–49]

$$\begin{aligned} \mathcal{L}_{KK\gamma} &= ieA^\mu(\bar{K}\partial_\mu K - \partial_\mu \bar{K}K) \\ \mathcal{L}_{VKK} &= ig_{VKK}V^\mu(\bar{K}\partial_\mu K - \partial_\mu \bar{K}K), \\ \mathcal{L}_{\gamma V} &= -e\frac{m_V^2}{f_V}V^\mu A_\mu. \end{aligned} \quad (3)$$

With the above effective Lagrangians, we can get the amplitudes corresponding to diagrams in Fig. 2, which are

FIG. 2. The mechanisms working in the $e^+e^- \rightarrow K^+K^-$ process. Diagram (a) is the direct coupling process, where the cyan square indicates the form factor of kaon. Diagram (b) is the resonance intermediate process, where the red line indicates the intermediate vector meson.

$$\begin{aligned}
\mathcal{M}_{\text{Dir}} &= [\bar{v}(p_2, m_e)(ie\gamma^\mu)u(p_2, m_e)] \\
&\quad \times \frac{-g_{\mu\nu}}{q^2} [ie(p_4^\nu - p_3^\nu)F_K(q^2)], \\
\mathcal{M}_{\text{Res}} &= [\bar{v}(p_2, m_e)(ie\gamma_\mu)u(p_2, m_e)] \frac{-g^{\mu\rho}}{q^2} \left(-e \frac{m_V^2}{f_V} \right) \\
&\quad \times \frac{-g_{\rho\nu} + q_\rho q_\nu / m_V^2}{q^2 - m_V^2 + im_V \Gamma_V} [ig_{VKK}(p_4^\nu - p_3^\nu)], \quad (4)
\end{aligned}$$

where $q = p_1 + p_2$, F_K is the timelike form factor of charged kaon. The total amplitude of $e^+e^- \rightarrow K^+K^-$ is

$$\begin{aligned}
\mathcal{M}_{\text{Tot}} &= \mathcal{M}_{\text{Dir}} + e^{i\phi} \mathcal{M}_{\text{Res}} \\
&= [\bar{v}(p_2, m_e)(ie\gamma^\mu)u(p_2, m_e)] \frac{1}{q^2} [ie(p_4^\nu - p_3^\nu)] \\
&\quad \times \left[-F_K(q^2)g_{\mu\nu} - e^{i\phi}g_V \frac{-g_{\mu\nu} + q_\mu q_\nu / m_V^2}{q^2 - m_V^2 + im_V \Gamma_V} \right] \quad (5)
\end{aligned}$$

where ϕ is the phase angle between two amplitudes and $g_V = g_{VKK}m_V^2/f_V$. With the above amplitude, we can get the cross sections for $e^+e^- \rightarrow K^+K^-$.

In the above amplitude, we can treat ϕ , g_V , m_V and Γ_V as free parameters, which can be determined by fitting the cross sections for $e^+e^- \rightarrow K^+K^-$. Besides these parameters, the form factor of charge kaon is not determined. In principle, the form factor in the timelike region is a complex function [51,52]. Generally, the argument of the form factor changes slowly when q^2 is far away from the threshold. So we can suppose that the argument of the form factor is approximately to be a constant in the considered center-of-mass energy. Thus, the argument of the form factor can be absorbed by the phase angle between two amplitudes and the form factor is treated as a real function of s . In the present work, we assume the form factor in the form

$$F_K(s) = as^b e^{-cs}, \quad (6)$$

where a , b , c are considered as free parameters.

III. NUMERICAL RESULTS

A. The cross sections for $e^+e^- \rightarrow K^+K^-$

With the above preparation, we can fit the cross sections for $e^+e^- \rightarrow K^+K^-$ reported by the BES III collaboration

[22], where seven free parameters, a , b , c , ϕ , g_V , m_V and Γ_V are determined. It should be noticed that the experimental data from the BES III collaboration have very small uncertainties [22], but the continuity of the data is not as good as its precision. In particular, there are only two almost degenerated data from 2.31 to 2.5 GeV, which cannot reflect more detail of the cross sections in this energy range. Thus, in the present work, we set the error to be 10 pb for every data. With this assumption, we fit the cross sections for $e^+e^- \rightarrow K^+K^-$ reported by the BES III collaboration [22]. The determined parameter values are listed in Table I. The errors on the parameters are determined by the contour in the parameter space defined by $\chi^2(\theta_i) = \chi_{\min}^2 + 1$ with θ_i to be the undetermined parameters. With the center values of the fitted parameters, the $\chi^2/\text{n.d.f}$ is estimated to be 16.9/15. From the present fit, we find the resonance parameters of the vector resonance are

$$\begin{aligned}
m_V &= (2197.4 \pm 4.4) \text{ MeV}, \\
\Gamma_V &= (75.6 \pm 7.2) \text{ MeV}, \quad (7)
\end{aligned}$$

respectively, which are well consistent with the PDG average of resonance parameters for $Y(2175)$. In Fig. 1, we also present the resonance parameters of this vector state obtained from the present fit. One can find the resonance parameters of this vector state are also consistent with those of $Y(2175)$ reported from *BABAR* and BES collaborations [11,12,14–17]. Thus, we conclude that the $X(2240)$ reported by the BES III collaboration should be the same state as the $Y(2175)$.

From the effective Lagrangians listed in Eq. (3), one can get the dilepton and K^+K^- decay widths of $Y(2175)$, which are

$$\begin{aligned}
\Gamma_{e^+e^-} &= \frac{e^4 m_Y}{12\pi f_Y^2}, \\
\Gamma_{K^+K^-} &= \frac{g_{VKK}^2 (m_Y^2 - 4m_K^2)^{3/2}}{48\pi m_Y^2}, \quad (8)
\end{aligned}$$

respectively. Thus, one can get the product of two widths, which is

$$\Gamma_{e^+e^-} \times \Gamma_{K^+K^-} = \frac{e^4 g_Y^2 (m_Y^2 - 4m_K^2)^{3/2}}{576\pi^2 m_Y^5}. \quad (9)$$

TABLE I. The parameter values obtained by fitting the cross sections for $e^+e^- \rightarrow K^+K^-$. The parameter a is in unit of GeV^{-2b} , which makes the form factor F_K dimensionless.

Parameter	Value	Parameter	Value
g_V	$(1.33 \pm 0.13) \times 10^{-2} \text{ GeV}^2$	a	2.49 ± 0.06
ϕ	$(3.06 \pm 0.11) \text{ rad}$	b	-2.79 ± 0.07
m_V	$(2197.4 \pm 4.4) \text{ MeV}$	c	$(2.28 \pm 0.71) \times 10^{-2} \text{ GeV}^{-2}$
Γ_V	$(75.6 \pm 7.2) \text{ MeV}$		

TABLE II. A comparison of $\Gamma_{e^+e^-}\mathcal{B}(Y \rightarrow f)$ with $f = (K^+K^-, \phi f_0(980), \phi\eta, \phi\pi^+\pi^-)$. The one for K^+K^- is obtained by Eq. (9) with the center values of the fitting parameters.

Process	Value (eV)
$\Gamma_{e^+e^-}\mathcal{B}(Y \rightarrow \phi f_0(980))$	$2.5 \pm 0.8 \pm 0.4$ [16] $2.3 \pm 0.3 \pm 0.3$ [14]
$\Gamma_{e^+e^-}\mathcal{B}(Y \rightarrow \phi\eta)$	$1.7 \pm 0.7 \pm 1.3$ [15]
$\Gamma_{e^+e^-}\mathcal{B}(Y \rightarrow \phi\pi^+\pi^-)$	2.90^a [14] 18.1 ± 1.8^b [53]
$\Gamma_{e^+e^-}\mathcal{B}(Y \rightarrow K^+K^-)$	0.51

^aIn Ref. [14], *BABAR* collaboration reported the cross sections for $e^+e^- \rightarrow \phi\pi^+\pi^-$ and the fitted mass and width of $Y(2175)$ in this channel is $2176 \pm 14 \pm 4$ and $90 \pm 22 \pm 10$ MeV, respectively. The cross sections resulted from $Y(2175)$ is fitted to be $0.082 \pm 0.024 \pm 0.010$ nb. The product of the dilepton width and branching fraction to $\phi\pi^+\pi^-$ is $\Gamma_{e^+e^-}\mathcal{B}(\phi\pi^+\pi^-) = \Gamma_Y \sigma m_Y^2 / (12\pi C_0)$ with $C_0 = 0.389 \text{ mb}(\text{GeV})^2$. With the center values of the mass, width and cross sections, we roughly estimate the product $\Gamma_{e^+e^-}\mathcal{B}(\phi\pi^+\pi^-) = 2.90$ eV.

^bIn Ref. [53], the authors performed a combined fit to *BABAR* and Belle data on $e^+e^- \rightarrow \phi\pi^+\pi^-$.

With this formula and the fitting data, one can get $\Gamma_{e^+e^-}\mathcal{B}(Y \rightarrow K^+K^-)$, which is listed in Table II. As a comparison, we also collect the measured data for other processes. From the table, one can find the branching ratios of $Y(2175) \rightarrow K^+K^-$ are at least several times smaller than those of $\phi\eta, \phi f_0(980)$ and $\phi\pi^+\pi^-$.

With the central values of fitted parameters, we can obtain the cross sections for $e^+e^- \rightarrow K^+K^-$, which is shown as the red curve in Fig. 3. The fitted curve can

FIG. 3. The fitted cross sections for $e^+e^- \rightarrow K^+K^-$ depending on the center-of-mass energy. For comparison, we also present the measured data from BES III [22] and *BABAR* collaborations [50].

FIG. 4. The individual contributions to the cross sections for $e^+e^- \rightarrow K^+K^-$. The green dashed and dotted curves indicate the contributions from $Y(2175)$ intermediate process and direct coupling process, respectively. The blue dot-dashed curve is the interference between two different contributions.

well reproduce the experimental data of the cross sections for $e^+e^- \rightarrow K^+K^-$. However, the fitted curve is a bit larger than the experimental data around 2.4 GeV. As we indicated at the beginning of this section, there are not enough experimental data around this area, we cannot get the detail of the line shape in the vicinity of 2.4 GeV, especially, some measurements have indicated that there may be a new state around 2.45 GeV [12,13,16,17,53]. We expect the BES III collaboration could provide more precise data around 2.4 GeV, which could help us to identify the existence of a new structure in this area.

We present the individual contributions to the cross sections in Fig. 4. From the figure, one can find the dominant contribution comes from the direct coupling process and the contribution from $Y(2175)$ is small. However, the interference between these two mechanisms is very important to reproduce the experimental data, which shifts the peak of $Y(2175)$ to 2240 MeV.

B. The cross sections for $e^+e^- \rightarrow \pi^+\pi^-$

In the above fit, we find the interferences are crucial in understanding the source of the structure near 2.2 GeV. Such kind of interferences should also appear in the similar process, such as $e^+e^- \rightarrow \pi^+\pi^-$. But unfortunately, the cross sections for this process are not as precise as those for $e^+e^- \rightarrow K^+K^-$. In Refs. [45,54], the *BABAR* collaboration reported their measurements for $e^+e^- \rightarrow \pi^+\pi^-$ up to 3 GeV. Recent analysis indicates there exists a resonance in the cross sections for $e^+e^- \rightarrow \pi^+\pi^-$ near 2.2 GeV with the resonance parameters to be

TABLE III. The same as Table I but for the cross sections for $e^+e^- \rightarrow \pi^+\pi^-$ with different fitting modes.

Parameter	Mode I	Mode II	Mode III
a	$(7.71 \pm 2.88) \times 10^{-2}$	$(1.66 \pm 0.96) \times 10^{-1}$	$(2.11 \pm 1.10) \times 10^{-2}$
b	-2.68 ± 0.93	-3.89 ± 1.59	$(8.97 \pm 20.1) \times 10^{-1}$
$c(\text{GeV}^{-2})$	$(9.56 \pm 13.7) \times 10^{-2}$	$(5.54 \pm 1.65) \times 10^{-2}$	$(3.96 \pm 1.54) \times 10^{-1}$
$g_V(\text{GeV}^2)$	$(4.56 \pm 2.73) \times 10^{-4}$	$(9.18 \pm 2.79) \times 10^{-4}$	$(4.94 \pm 1.97) \times 10^{-4}$
ϕ (rad)	3.48 ± 0.68	3.42 ± 0.27	4.20 ± 0.31
m_V (MeV)	2230.1 ± 8.8	2232 (fixed)	2188 (fixed)
Γ_V (MeV)	34.3 ± 72.1	133 (fixed)	83 (fixed)
χ^2	2.11	2.78	4.55

$$\begin{aligned}
m_Y &= (2232 \pm 8 \pm 9) \text{ MeV} \\
\Gamma_Y &= (133 \pm 14 \pm 4) \text{ MeV},
\end{aligned} \tag{10}$$

which are well consistent with those of $X(2240)$ [22].

Here, we adopt the same interference scenario to fit the cross sections for $e^+e^- \rightarrow \pi^+\pi^-$, where the resonance parameters are considered as free parameters. The fitted parameters are collected in Table III as mode I. From our fit, we find the mass of the vector resonance is similar to the one of *BABAR* fit as shown in Eq. (10) but the width is much smaller, which is $\Gamma = (34.3 \pm 72.1) \text{ MeV}$. The χ^2 is estimated to be 2.11. In mode I, the uncertainty of the vector meson resonance is particularly large and its center value is much different with the *BABAR* analysis. With the center values of the fitted parameters, we can obtain the fitted curve, which is present in Fig. 5 and we find the fitted curve can well reproduce the experimental data.

FIG. 5. The fitted cross sections of $e^+e^- \rightarrow \pi^+\pi^-$ depending on the center-of-mass energy.

To check the effect of the resonance widths in our fit, we fixed the resonance parameters of the vector meson as those of the *BABAR* analysis and fit the cross sections for $e^+e^- \rightarrow \pi^+\pi^-$. The fitted parameters are present in Table III as mode II and the χ^2 is estimated to be 2.78, which is a bit larger than the one of mode I. From the dashed curve in Fig. 5, one can find mode II can also reproduce the experimental data. Comparing to mode I, one can find the fitted curve can only reach up the minimum of the experimental data at $\sqrt{s} = 2.275 \text{ GeV}$. From our fit, we find the $\chi^2/\text{n.d.f}$ is much smaller than one due to the large uncertainties of the experimental data.

To further check the relation of the structures in $e^+e^- \rightarrow K^+K^-$ and $e^+e^- \rightarrow \pi^+\pi^-$, we further fixed the resonance parameters to be those of $Y(2175)$ as shown in Eq. (1). The fitted parameter values are presented in Table III as mode III and the χ^2 is estimated to be 4.55, which is a bit larger than those of mode I and mode II. However, the $\chi^2/\text{n.d.f}$ is $4.55/6$, which indicates such a fit is also acceptable. The fitted curve is also present in Fig. 5 for comparison. From the figure, one can find $Y(2175)$ can also produce a structure near 2.2 GeV, which is similar to the one of mode II.

In the present interference scenario, we find the cross sections for $e^+e^- \rightarrow \pi^+\pi^-$ can also be reproduced with an acceptable χ^2 , but the resonance parameter can be different with different fitting assumptions. The resonance $Y(2175)$ can also produce a structure near 2.2 GeV. Due to the large uncertainties of the experimental data, we cannot draw any solid conclusion from the present fit. We expect the BES III and Belle II collaborations can provide more precise data for this process in the future, which can help us to reveal the source of the structure near 2.2 GeV in the cross sections for $e^+e^- \rightarrow \pi^+\pi^-$.

IV. SUMMARY

The strangeonium-like state $Y(2175)$ has been observed in various hidden strange channels. Its nature is a long-standing puzzle. There are some different interpretations to the inner structure of $Y(2175)$, such as

tetraquark, molecular, hybrid and conventional strangeonium. The open strange decays are important criteria of different interpretations. Thus, searching $Y(2175)$ in the open strange channels is crucial to understanding the nature of $Y(2175)$.

In 2018, the BES III collaboration reported their precise measurements of the cross sections for $e^+e^- \rightarrow K^+K^-$. No evidence of $Y(2175)$ was found, but a new structure around 2240 MeV, named $X(2240)$, was observed. In the present work, we consider the interference between the direct coupling and the vector resonance intermediate contributions to fit the cross sections for $e^+e^- \rightarrow K^+K^-$. We find the structure around 2240 MeV is resulted from $Y(2175)$ and the interference between two mechanisms is very important, which shifts the peak of $Y(2175)$ to 2240 MeV.

In our fit, we find that there are only two almost degenerate data from 2.31 to 2.5 GeV. The experimental data in this center-of-mass energy rang are important to

determine the line shape of the cross sections. We expect the BES III collaboration could provide more precise data in this energy range, which can help us to better understand the cross sections for $e^+e^- \rightarrow K^+K^-$.

We also extend the present interference scenario to investigate the cross sections for $e^+e^- \rightarrow \pi^+\pi^-$, where a structure near 2.2 GeV was also reported by the *BABAR* collaboration. We find that $Y(2175)$ can also produce a structure near 2.2 GeV, but we cannot draw any solid conclusion due to the large uncertainties of the experimental data.

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