Relaxed hypermultiplet in four dimensional $\mathcal{N} = 2$ conformal supergravity

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Superconformal matter multiplets play a crucial role in the construction of Poincaré supergravity invariants. Off-shell multiplets allow for construction of general matter couplings in supergravity. In [P. S. Howe, K. S. Stelle, and P. K. Townsend, *Nucl. Phys.* **B214**, 519 (1983)], relaxed hypermultiplet was constructed in rigid supersymmetry which on coupling with the real scalar multiplet allowed for an off-shell formulation of the rigid hypermultiplet. In this paper, we extend the relaxed hypermultiplet to conformal supergravity. For consistency with the superconformal algebra, we find that the fields have to be allowed to transform in a noncanonical way under $SU(2)$ R -symmetry. We find suitable field redefinitions to obtain fields which are irreducible representations of $SU(2)$ R -symmetry and present the full nonlinear transformation rule.

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I. INTRODUCTION

Conformal supergravity plays an important role in constructing physical supergravity theories with higher derivative corrections. The higher degree of symmetries in conformal supergravity allows for the superconformal symmetries to be realized off-shell for certain multiplets and this plays a crucial role in the construction of higher derivative corrections to the physical supergravity theories with lesser number of symmetries than conformal supergravity. The higher derivative corrections are important for several reasons. They can be studied as supersymmetric counterterms for removing divergences as well as anomalies of some tree level global symmetries at the loop level. They can also be used to study the higher derivative as well as quantum corrections to the entropy of black holes using techniques such as localization [1] and entropy function [2,3] and use the results to take the comparison with the microstate counting in string theory beyond the leading order Bekenstein-Hawking entropy.

One of the crucial ingredients of conformal supergravity is the Weyl multiplet which is a multiplet of fields that contains the graviton and its superpartner gravitino. The superconformal algebra closes on the multiplet without using any equations of motion while the structure constants

of the algebra become field dependent. Such an algebra is known as a soft algebra. Further, one finds various representations of the soft algebra, which are known as matter multiplets, and consider their coupling to conformal supergravity using a set of procedures which goes by the name of superconformal multiplet calculus. Upon using some of these matter multiplets as compensators to gauge fix the additional symmetries in conformal supergravity, one gets the physical supergravity theory with symmetries belonging to the super-Poincaré group. In the case of extended supergravity, one has to add auxiliary fields to the Weyl multiplet so that the off-shell degrees of freedom for bosons and fermions match. There is a standard choice of the auxiliary fields which lead to the standard Weyl multiplet (see [4] for the $\mathcal{N} = 2$ and $\mathcal{N} = 4$ standard Weyl multiplet in four dimensions). There is a different choice of auxiliary fields that contains a scalar field of nonvanishing mass dimension and is known as the dilaton Weyl multiplet (see [5] for the $\mathcal{N} = 2$ dilaton Weyl multiplet in four dimensions).

In $\mathcal{N} = 2$ supergravity there are basically two matter multiplets, with spin less than 2, which has distinct propagating degrees of freedom. The multiplet which has one propagating spin-1 degree of freedom with helicity $= \pm 1$, two propagating spin-0 degrees of freedom and two propagating spin-1/2 degree of freedom with helicity $= \pm 1/2$ is known as the vector multiplet. Another multiplet which has four propagating spin-0 degrees of freedom and two propagating spin-1/2 degrees of freedom with helicity $= \pm 1/2$ is known as the scalar multiplet or the hypermultiplet. One can add auxiliary fields to the vector multiplet to obtain an off-shell $\mathcal{N} = 2$ vector multiplet with

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$8 + 8$ (bosonic + fermionic) off-shell degrees of freedom. This has been very well studied along with its coupling to $\mathcal{N} = 2$ conformal supergravity and has also been used as a compensator to get $\mathcal{N} = 2$ Poincaré supergravity. However, to have an off-shell completion of hypermultiplet with a finite number of auxiliary fields is not very straightforward. One of the ways to get an off-shell completion of the $\mathcal{N} = 2$ hypermultiplet is to introduce a nontrivial central charge in the supersymmetry algebra [6]. Another way to get an off-shell completion of the $\mathcal{N} = 2$ hypermultiplet without a central charge is to consider a different multiplet where one of the propagating spin-0 degree of freedom is represented by a dual 2-form gauge field. Such a multiplet is known as a linear multiplet or a tensor multiplet [7,8]. Its off-shell completion has $8 + 8$ degrees of freedom. It is also very well studied along with its coupling to $\mathcal{N} = 2$ conformal supergravity [9] and has also been used as a compensator to get $\mathcal{N} = 2$ Poincaré supergravity. However, the dualization of a 2-form gauge field to a scalar will generally not work if we have higher number of derivatives. At the level of two derivatives, the couplings of the tensor multiplet are very restrictive in the sense that they do not couple to a spin-1 gauge field and do not have self coupling. These drawbacks of the tensor multiplet necessitates the formulation of an off-shell hypermultiplet without dualizing any spin-0 degree of freedom. Such an attempt was made in [10] for supersymmetric field theory, where the authors introduced two new multiplets: the $32 + 32$ components relaxed hypermultiplet and the $24 + 24$ components real scalar multiplet. They further considered a supersymmetric invariant action for the relaxed hypermultiplet and the coupling between the relaxed hypermultiplet and the real scalar multiplet. Upon using the equations of motion they showed that the combined system of the relaxed hypermultiplet and the real scalar multiplet describes the physical degrees of freedom of a hypermultiplet.

In [11], the extension of the $24 + 24$ components real scalar multiplet to conformal supergravity was studied. At the linearized level, the components of this multiplet along with the $24 + 24$ components of an $\mathcal{N} = 2$ standard Weyl multiplet coupled with the $48 + 48$ components current multiplet of a rigid $\mathcal{N} = 2$ tensor multiplet. It was also shown that one can impose a consistent set of constraints on the real scalar multiplet to reduce its components to an $8 + 8$ components restricted real scalar multiplet which can be mapped to the $8 + 8$ components of a $\mathcal{N} = 2$ tensor multiplet coupled to conformal supergravity. Further, in [12] an invariant action for the real scalar multiplet was constructed using a new density formula based on a fermionic multiplet which was used to find new higher derivative couplings of the tensor multiplet in $\mathcal{N} = 2$ conformal supergravity.

In this paper, we would like to extend the $32 + 32$ components relaxed hypermultiplet to $\mathcal{N} = 2$ conformal supergravity. This will pave the way for an off-shell treatment

of the hypermultiplet in $\mathcal{N} = 2$ conformal supergravity. In [13], extension of relaxed hypermultiplet to conformal supergravity was studied in harmonic superspace by relating it to a well-known rigid supersymmetry multiplet albeit with additional global $SU(2)$ indices. In this paper, we explicitly couple the relaxed hypermultiplet to conformal supergravity. In Appendix B, we will comment on the relation between our results and the harmonic superspace discussion in [13].

The plan of the paper is as follows. In Sec. II, we will review the relaxed hypermultiplet and real scalar multiplet in supersymmetric field theory as done in [10]. We will also discuss how the two multiplets were used to describe the physical degrees of freedom of a hypermultiplet. In Sec. III, we will discuss the extension of the relaxed hypermultiplet to conformal supergravity. In Sec. IV we will end with some conclusions and future directions. We will also present the details of an $\mathcal{N} = 2$ standard Weyl multiplet, that are relevant for the paper, in Appendix A.

II. REVIEW OF THE RIGID RELAXED HYPERMULTIPLY

As we discussed in the previous section, while considering specific actions the tensor/linear multiplet provides an off-shell description of the hypermultiplet degrees of freedom using dualization, this mapping between the 2-form gauge field and the 0-form scalar field does not hold true for more general actions.

In [10], the authors tried to evade the use of 2-form gauge fields by generalizing the superspace constraints corresponding to the linear multiplet so that there is no conserved vector that can be identified as the field strength of a 2-form gauge field. The resulting $32 + 32$ multiplet was dubbed the relaxed hypermultiplet. In this section, we will elaborate on this construction and explain how when coupled with another $24 + 24$ multiplet, this gives the precise on-shell degrees of freedom as the hypermultiplet. As this construction was carried out in $\mathcal{N} = 2$ flat space supersymmetry, components of various multiplets under consideration can be organized as irreducible representations of global $SU(2)$ symmetry. In this section, this global $SU(2)$ symmetry is simply referred to as $SU(2)$.

The superspace constraint for the linear multiplet can be written as:

$$D_\alpha^{(i} \mathbf{L}^{jk)} = 0, \quad (1)$$

where $\mathbf{L}^{ij}$ is a triplet of real superfields. Here the superspace covariant derivative D_{ai} is given as:

$$D_{ai} = \frac{\partial}{\partial \theta^{ai}} + i \sigma_{aa}^\mu \bar{\theta}_i^\alpha \frac{\partial}{\partial x^\mu}, \quad (2)$$

where $(x^\mu, \theta^{ai}, \bar{\theta}_{\dot{a}i})$ are the superspace coordinates written in two component Weyl spinors notation.

The above constraint in component form means that there is no field that transforms in the **4** representation of

$SU(2)$ in the transformation of L^{ij} which is the lowest mass dimension field of the linear multiplet. That $\mathbf{L}^{ij}$ is real triplet of scalar fields means that the field L^{ij} satisfies the pseudoreality constraint,

$$L^{ij} = (L_{ij})^* = \epsilon^{ik}\epsilon^{jl}L_{kl}. \quad (3)$$

By an investigation of the possible $SU(2)$ representations that appear in the transformation of L^{ij} and higher mass dimension fields that appear in its transformation rules, consistent with the above constraint and the supersymmetry algebra, one finds that the linear multiplet fields are L^{ij} , G , φ^i , and H^a . Here, G is a complex scalar field and φ^i is an $SU(2)$ doublet of Majorana fermions. Supersymmetry algebra implies that H^a is a real conserved vector, which can also be seen by trying to match the bosonic and fermionic off-shell degrees of freedom. The resulting multiplet has $8 + 8$ off-shell degrees of freedom.

In [10], a $24 + 24$ multiplet was presented with the superfield $\mathbf{L}^{ijkl}$ satisfying the constraint that the lowest mass dimension field L^{ijkl} does not contain in its transformation any fermion that transforms in the **6** representation of $SU(2)$. This can be analogously written as the constraint,

$$D_\alpha^{(i}\mathbf{L}^{jklm)} = 0. \quad (4)$$

Analogous to L^{ij} , the field L^{ijkl} satisfies the pseudoreality condition,

$$L^{ijkl} = (L_{ijkl})^* = \epsilon^{ip}\epsilon^{jq}\epsilon^{kr}\epsilon^{ls}L_{pqrs}. \quad (5)$$

We will not review this multiplet here, but relevant to us is the fact that this multiplet contains a fermion ψ^{ijk} that transforms in the **4** representation of $SU(2)$. To extend the linear multiplet to a multiplet with no conserved vectors, the authors relaxed the constraint (1) such that a fermion in **4** representation appears in the transformation of L^{ij} and it is identified with the fermion ψ^{ijk} from the multiplet that corresponds to $\mathbf{L}^{ijkl}$, while keeping the constraint (4) intact. Relaxation of the constraint (1) allows for a rearrangement of the off-shell degrees of freedom and H^a need not be conserved. The two multiplets coupled this way form a multiplet which contains $32 + 32$ off-shell degrees of freedom, with no conserved vectors. This multiplet is referred to as the relaxed hypermultiplet. Transformation rule for the components of the multiplet is given as:

$$\begin{aligned} \delta L^{ij} &= \frac{2\sqrt{2}}{3}\bar{\epsilon}^{(i}\lambda^{j)} - \sqrt{2}\epsilon_{km}\bar{\epsilon}^k\psi^{mij} + \frac{2\sqrt{2}}{3}\epsilon^{jk}\epsilon^{im}\bar{\epsilon}_{(m}\lambda_{k)} - \sqrt{2}\epsilon^{jk}\epsilon^{im}\epsilon^{lp}\bar{\epsilon}_l\psi_{pmk}, \\ \delta L^{ijkl} &= \frac{4\sqrt{2}}{5}\bar{\epsilon}^{(i}\psi^{jkl)} + \frac{4\sqrt{2}}{5}\epsilon^{iq}\epsilon^{jr}\epsilon^{ks}\epsilon^{lt}\bar{\epsilon}_{(q}\psi_{rst)}, \\ \delta\psi^{ijk} &= \frac{3\sqrt{2}}{8}\epsilon^{(i}M^{jk)} + \frac{3\sqrt{2}}{8}\gamma^\mu\epsilon_m\epsilon^{m(i}V_\mu^{jk)} + \frac{5\sqrt{2}}{4}\gamma^\mu\epsilon_m\partial_\mu L^{mijk}, \\ \delta\lambda^i &= \frac{3\sqrt{2}}{4}\epsilon_{kj}\epsilon^jM^{ki} - \frac{1}{4}\sqrt{2}\epsilon^iN + \frac{3\sqrt{2}}{4}\gamma^\mu\epsilon_jV_\mu^{ij} - \frac{1}{4}\sqrt{2}\epsilon^{ij}\gamma^\mu\epsilon_jG_\mu + \frac{3\sqrt{2}}{2}\gamma^\mu\epsilon_j\partial_\mu L^{ij}, \\ \delta M^{ij} &= \frac{2\sqrt{2}}{3}\epsilon^{k(i}\epsilon^{j)m}\bar{\xi}_m\epsilon_k - \frac{8\sqrt{2}}{3}\partial_\mu\bar{\psi}^{ijk}\gamma^\mu\epsilon_k, \\ \delta N &= -3\sqrt{2}\epsilon^{im}\bar{\xi}_m\epsilon_i + 4\sqrt{2}\partial_\mu\bar{\lambda}^i\gamma^\mu\epsilon_i, \\ \delta V_\mu^{ij} &= -\frac{\sqrt{2}}{3}\epsilon^{m(j}\bar{\epsilon}^{i)}\gamma_\mu\xi_m + \frac{4\sqrt{2}}{3}\epsilon_{mn}\bar{\epsilon}^n\gamma_{\mu\nu}\partial^\nu\psi^{mij} - \frac{2\sqrt{2}}{3}\epsilon_{mn}\bar{\epsilon}^n\partial_\mu\psi^{mij} \\ &\quad + \epsilon^{im}\epsilon^{jn}\left\{-\frac{\sqrt{2}}{3}\epsilon_{k(n}\bar{\epsilon}_{m)}\gamma_\mu\xi^k + \frac{4\sqrt{2}}{3}\epsilon^{kl}\bar{\epsilon}_l\gamma_{\mu\nu}\partial^\nu\psi_{mnk} - \frac{2\sqrt{2}}{3}\epsilon^{kl}\bar{\epsilon}_l\partial_\mu\psi_{mnk}\right\}, \\ \delta G_\mu &= \frac{3\sqrt{2}}{2}\bar{\epsilon}^i\gamma_\mu\xi_i + 2\sqrt{2}\epsilon_{mi}\bar{\epsilon}^i\gamma_{\mu\nu}\partial^\nu\lambda^m + \text{H.c.} \\ \delta\xi^i &= -\sqrt{2}\epsilon^{ji}\epsilon^{mk}\gamma^\mu\epsilon_k\partial_\mu M_{mj} + 2\sqrt{2}\epsilon^{ji}\epsilon^k\partial_\mu V_{\mu kj} + \frac{\sqrt{2}}{3}\epsilon^i\partial_\mu G^\mu - \sqrt{2}\epsilon^{ji}\gamma^{\mu\nu}\epsilon^k\partial_\mu V_{\nu kj}, \end{aligned} \quad (6)$$

where we have given the results of [10] in four component formalism. The $SU(2)$ representation of the various fields of the relaxed hypermultiplet is obvious from the index structure in the above transformation rule. M^{ij} and N are complex triplet and singlets under $SU(2)$. V_μ^{ij} and G_μ are real triplet and singlet vector fields respectively. We have chosen the normalization of the fields in the relaxed hypermultiplet such that $[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]X = (2\epsilon_2^i\gamma^\mu\epsilon_{1i} + \text{H.c.})\partial_\mu X$, where X is any field in the relaxed hypermultiplet.

The above $32 + 32$ multiplet when coupled to a $24 + 24$ multiplet corresponding to a superfield contragradient to the $\mathbf{L}^{ijkl}$, was shown to contain the appropriate on-shell degrees of freedom as the hypermultiplet. This $24 + 24$ multiplet is based on a real scalar superfield $\mathbf{V}$ satisfying the constraint:

$$D_{\alpha\beta}\mathbf{V} = [D_{\alpha}^i, \bar{D}_{\dot{\alpha}i}]\mathbf{V} = 0. \quad (7)$$

The detailed transformation rule for the components of the multiplet in flat space was presented in [10] and was generalized to conformal supergravity recently in [11] following which we will refer to this multiplet as the real scalar multiplet.

The supersymmetric invariant action with coupling between the real scalar multiplet and relaxed hypermultiplet in flat space was presented in [10] and is given as:

$$\begin{aligned} I_1 = & \frac{1}{2} \int d^4x \left[L_{ij} \square L^{ij} - \frac{25}{6} L_{ijkl} \square L^{ijkl} + 3 L_{ij} \partial^\mu V_\mu^{ij} - \frac{3}{4} M^{ij} M_{ij} + \frac{1}{18} N \bar{N} - \frac{3}{4} V^{\mu ij} V_{\mu ij} + \frac{1}{18} G^\mu G_\mu \right. \\ & \left. - \frac{8}{9} \bar{\lambda}^i \not{\partial} \lambda_i + \frac{16}{3} \bar{\psi}^{ijk} \not{\partial} \psi_{ijk} - \frac{4}{3} \bar{\lambda}^i \xi_i + \text{H.c.} \right], \\ I_2 = & \int d^4x \left[-\frac{1}{6} V \partial^\mu G_\mu + \frac{3}{4} K^{ij} M_{ij} + \frac{3}{4} A^{\mu ij} V_{\mu ij} + \frac{5}{8} C^{ijkl} L_{ijkl} + \bar{\psi}^i \xi_i - 2 \bar{\psi}^{ijk} \xi_{ijk} + \text{H.c.} \right], \end{aligned} \quad (8)$$

where I_1 is the kinetic action for relaxed hypermultiplet and I_2 is the coupling of the relaxed hypermultiplet with the real scalar multiplet. $V, K^{ij}, A^{\mu ij}, C^{ijkl}, \psi^i$ and ψ^{ijk} are the field components of the real scalar multiplet. V is a real scalar field, K^{ij} is a complex triplet of scalars. $A^{\mu ij}$ and C^{ijkl} have the same pseudoreality properties as $V^{\mu ij}$ and L^{ijkl} .

Equation of motion for L^{ij} leads to

$$\square L^{ij} = 0. \quad (9)$$

Thus L^{ij} correspond to three scalar on-shell degrees of freedom. Equations of motion for G^μ and $V^{\mu ij}$ lead to

$$\square V = 0. \quad (10)$$

Thus V corresponds to one scalar on-shell degree of freedom. The field ψ_i is identified with λ_i from using ξ^i equations of motion. Equation of motion for λ_i gives:

$$\not{\partial} \lambda_i = 0. \quad (11)$$

Thus λ_i correspond to four on-shell spin-1/2 degrees of freedom. The remaining equations of motion set $L^{ijkl}, C^{ijkl}, \psi^{ijk}, \xi^{ijk}$ and ξ_i to zero. Thus the bosonic on-shell degrees of freedom are contained in V, L^{ij} as four scalar degrees of freedom and fermionic on-shell degrees of freedom are contained in λ_i (or ψ_i) as four spin-1/2 degrees of freedom. Thus the $56 + 56$ multiplet with the above action has the appropriate on-shell degrees of freedom for the $4 + 4$ hypermultiplet.

As mentioned earlier, the $24 + 24$ real scalar multiplet was generalized to $\mathcal{N} = 2$ conformal supergravity in [11]. In the next section, we will generalize the $32 + 32$ relaxed hypermultiplet to $\mathcal{N} = 2$ conformal supergravity.

III. EXTENSION OF THE $\mathcal{N} = 2$ RELAXED HYPERMULTIPLY TO CONFORMAL SUPERGRAVITY

In $\mathcal{N} = 2$ conformal supergravity, multiplets form representations of the following soft algebra [14]:

$$\begin{aligned} [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] &= \delta^{(cov)}(\xi) + \delta_M(\epsilon) + \delta_K(\Lambda_K) + \delta_S(\eta) \\ &\quad + \delta_{\text{gauge}}, \\ [\delta_S(\eta), \delta_Q(\epsilon)] &= \delta_M(\bar{\eta}^i \gamma^{ab} \epsilon_i + \text{H.c.}) + \delta_D(\bar{\eta}_i \epsilon^i + \text{H.c.}) \\ &\quad + \delta_A(i \bar{\eta}_i \epsilon^i + \text{H.c.}) \\ &\quad + \delta_V(-2 \bar{\eta}^i \epsilon_i - (\text{H.c.; traceless})), \\ [\delta_S(\eta_1), \delta_S(\eta_2)] &= \delta_K(\bar{\eta}_{2i} \gamma^a \eta_1^i + \text{H.c.}), \end{aligned} \quad (12)$$

where $\delta_Q, \delta_S, \delta_M, \delta_K, \delta_D, \delta_A$, and δ_V are respectively the infinitesimal transformations corresponding to Q-supersymmetry, S-supersymmetry, local Lorentz transformation, special conformal transformation, dilatation, $U(1)$ and $SU(2)$ R-symmetry. The infinitesimal transformations δ_{gauge} on the right-hand side of $[\delta_Q, \delta_Q]$ correspond to the additional gauge symmetries that a multiplet may possess. The infinitesimal transformation $\delta^{(cov)}(\xi)$, is the covariant general coordinate transformation defined as:

$$\delta^{(cov)}(\xi) = \delta_{\text{gct}}(\xi) - \sum_T \delta_T(-\xi^\mu h_\mu(T)), \quad (13)$$

where δ_{gct} is the general coordinate transformation. In the summation, T runs over all the superconformal transformations (including the additional gauge symmetries of the multiplet) except local translation, and $h_\mu(T)$ is the corresponding gauge field with appropriate factors as listed in Table III (see Appendix A). The parameters for the transformations listed on the right-hand side (RHS) of first of the equations (12) are given as:

$$\begin{aligned}
\xi^\mu &= 2\bar{\epsilon}_2^i \gamma^\mu \epsilon_{1i} + \text{H.c.}, \\
\epsilon^{ab} &= \bar{\epsilon}_1^i \epsilon_2^j T_{ij}^{ab} + \text{H.c.}, \\
\Lambda_K^a &= \bar{\epsilon}_1^i \epsilon_2^j D_b T_{ij}^{ba} - \frac{3}{2} \bar{\epsilon}_2^i \gamma^a \epsilon_{1i} D + \text{H.c.}, \\
\eta^i &= 6\bar{\epsilon}_1^i \epsilon_2^j \chi_j.
\end{aligned} \tag{14}$$

We will obtain the superconformal transformation rule for the relaxed hypermultiplet, by demanding the consistent closure of the superconformal algebra (12) on the relaxed hypermultiplet.

The fields L^{ij} , L^{ijkl} , V_a^{ij} , and G_a are real. Therefore, the chiral weights of these fields are 0. Let us assign an arbitrary Weyl weight w to the field L^{ij} . From the supersymmetry transformation rule (6) and from the knowledge of the weights of the supersymmetry parameters [see Eq. (A7) in Appendix A], the chiral weights of the remaining fields get completely fixed, and the Weyl weights get determined in terms of w . The fields L^{ij} and

L^{ijkl} are the lowest Weyl weight components in the multiplet and therefore must be invariant under S-transformations. For the S-transformation of ψ^{ijk} and λ^i we will consider terms allowed by the Weyl weights, chiral weights, and the global $SU(2)$ index structure as follows:

$$\begin{aligned}
\delta_S \lambda^i &= x L^{ij} \eta_j, \\
\delta_S \psi^{ijk} &= y L^{ijkl} \eta_l + z \epsilon^{l(k} L^{ij)} \eta_l,
\end{aligned} \tag{15}$$

where x , y , and z are arbitrary coefficients which will be determined using the Q-S commutation relation. To do this we need to know how L^{ij} and L^{ijkl} transform under V-transformation. At the level of flat space supersymmetry, we had discussed in the previous section that the fields in the relaxed hypermultiplet are irreducible representations of the global $SU(2)$ symmetry. Assuming this to be true even for $SU(2)$ R-symmetry in conformal supergravity, we evaluate the S-Q commutator on L^{ij} :

$$\begin{aligned}
[\delta_S, \delta_Q] L^{ij} &= x \frac{2\sqrt{2}}{3} \bar{\epsilon}^{(i} L^{j)k} \eta_k - y \sqrt{2} \epsilon_{km} \bar{\epsilon}^k L^{mijl} \eta_l - z \sqrt{2} \epsilon^{(l} L^{ij)} \eta_l + x \frac{2\sqrt{2}}{3} \epsilon^{jk} \epsilon^{im} \bar{\epsilon}_{(m} L_{k)l} \eta^l - y \sqrt{2} \epsilon^{jk} \epsilon^{im} \epsilon^{lp} \bar{\epsilon}_l L_{pmkr} \eta^r \\
&\quad - z \sqrt{2} \epsilon^{jk} \epsilon^{im} \bar{\epsilon}_{(r} L_{mk)} \eta^r.
\end{aligned} \tag{16}$$

We also calculate the expected result of the commutator:

$$\begin{aligned}
&\delta_M(\bar{\eta}^i \gamma^{ab} \epsilon_i + \text{H.c.}) L^{ij} + \delta_D(\bar{\eta}_i \epsilon^i + \text{H.c.}) L^{ij} + \delta_A(i \bar{\eta}_i \epsilon^i + \text{H.c.}) L^{ij} + \delta_V(-2 \bar{\eta}^i \epsilon_j - (\text{H.c.}; \text{traceless})) L^{ij} \\
&= (w-2) \bar{\eta}_k \epsilon^k L^{ij} + (w+2) \bar{\eta}^k \epsilon_k L^{ij} - 4 \bar{\eta}^{(i} \epsilon_k L^{j)k} + 4 \bar{\eta}_k \epsilon^{(i} L^{j)k}.
\end{aligned} \tag{17}$$

For the superconformal algebra to close consistently Eqs. (16) and (17) should match. But we clearly see that there is mismatch between the equations due to the presence of L^{ijkl} terms in Eq. (16). Setting the coefficient $y=0$ might seem to resolve this mismatch; however, it leads to similar inconsistency in the $[\delta_S, \delta_Q] L^{ijkl}$ equation. These mismatches that arise are rooted in the way in which relaxed hypermultiplet was constructed. The superspace constraint (1) was relaxed which led to the intertwining of the $24+24$ multiplet and the $8+8$ linear multiplet through the supersymmetry transformation. This suggests that for consistent coupling of the relaxed hypermultiplet to conformal supergravity, we should allow the two multiplets to get intertwined not just through Q-SUSY, but also through other superconformal transformations. Thus we come up with the following prescription to accommodate the required changes in the superconformal transformation:

- (I) We allow any field in the relaxed hypermultiplet to mix with any other field in the multiplet (or even the derivatives of the fields) under V-transformation, given that such a mixing is consistent with Weyl weights, chiral weights and Lorentz structure.

- (II) We also allow for covariant derivative terms consistent with the weights to appear in the S-transformation of the fields.
- (III) In addition to the soft algebra relations (12), we make use of the following commutation relations involving V-transformations to fix the coefficients in the transformations.

$$\begin{aligned}
[\delta_V(\lambda_1), \delta_V(\lambda_2)] &= \delta_V(\lambda_3), \\
[\delta_Q(\epsilon), \delta_V(\lambda)] &= \delta_Q(\lambda_k^j \epsilon^k + \text{H.c.}), \\
[\delta_S(\epsilon), \delta_V(\lambda)] &= \delta_S(-\lambda_k^j \eta_j + \text{H.c.}),
\end{aligned} \tag{18}$$

where $\lambda_{3j}^i = \lambda_{2k}^i \lambda_{1j}^k - \lambda_{1k}^i \lambda_{2j}^k$.

We will now illustrate our prescription. Consider the fields L^{ij} and L^{ijkl} . They have the same Weyl weight, chiral weight and are the lowest Weyl weight components of the multiplet with no Lorentz indices. Hence we propose the following ansatz for their V-transformation:

$$\delta_V L^{ij} = \alpha \Lambda_{V_k}^{(i} L^{j)k} + \beta \epsilon_{kl} \Lambda_{V_m}^k L^{lmij} \tag{19}$$

$$\delta_V L^{ijkl} = \mu \epsilon^{m(l} \Lambda_{V_m}^k L^{ij)} + \theta \Lambda_{V_m}^{(i} L^{jkl)m} \quad (20)$$

where $\alpha, \beta, \mu, \theta$ are arbitrary coefficients which need to be fixed by using the soft algebra (12) and the commutation relations (18). On evaluating the V-V commutator on L^{ij} and L^{ijkl} , we find the following two possibilities for the coefficients:

- (i) $\alpha = \frac{5}{2}, \mu\beta = \frac{3}{4}$ and $\theta = 3$.
- (ii) $\alpha = -\frac{1}{2}, \mu\beta = \frac{3}{4}$ and $\theta = 1$.

On operating Q-V commutator on L^{ij} , we find the second possibility to be ruled out. Further by making use of the S-Q commutator on L^{ij} and L^{ijkl} , the coefficients β, μ and x, y, z get fixed. In addition the lowest Weyl weight value w also gets determined.

$$\begin{aligned} \delta_V L^{ij} &= \frac{5}{2} \Lambda_{V_k}^{(i} L^{j)k} - \frac{15}{4} \epsilon_{kl} \Lambda_{V_m}^k L^{lmij} & \delta_V L^{ijkl} &= -\frac{1}{5} \epsilon^{m(l} \Lambda_{V_m}^k L^{ij)} + 3 \Lambda_{V_m}^{(i} L^{jkl)m} \\ \delta_S \psi^{ijk} &= \frac{15}{2\sqrt{2}} L^{ijkl} \eta_l - \frac{1}{2\sqrt{2}} \epsilon^{l(k} L^{ij)} \eta_l & \delta_S \lambda^i &= \frac{8}{\sqrt{2}} L^{ij} \eta_j \quad w = 3. \end{aligned} \quad (21)$$

Proceeding analogously, using the aforementioned prescription and algebraic relations we determine the S- and V-transformation of all the fields in the multiplet. Having determined the S-transformation of all the fields in the multiplet, we make use of the S-S commutation relation from (12) to determine the K-transformation for fields in the multiplet. The linearized superconformal transformation rule for the relaxed hypermultiplet is found to be

$$\begin{aligned} \delta L^{ij} &= \frac{2\sqrt{2}}{3} \bar{\epsilon}^{(i} \lambda^{j)} - \sqrt{2} \epsilon_{km} \bar{\epsilon}^k \psi^{mij} + \frac{2\sqrt{2}}{3} \epsilon^{jk} \epsilon^{im} \bar{\epsilon}_{(m} \lambda_{k)} - \sqrt{2} \epsilon^{jk} \epsilon^{im} \epsilon^{lp} \bar{\epsilon}_l \psi_{pmk} + 3 \Lambda_D L^{ij} + \frac{5}{2} \Lambda_{V_k}^{(i} L^{j)k} - \frac{15}{4} \epsilon_{kl} \Lambda_{V_m}^k L^{lmij}, \\ \delta L^{ijkl} &= \frac{4\sqrt{2}}{5} \bar{\epsilon}^{(i} \psi^{jkl)} + \frac{4\sqrt{2}}{5} \epsilon^{iq} \epsilon^{jr} \epsilon^{ks} \epsilon^{lt} \bar{\epsilon}_{(q} \psi_{rst)} + 3 \Lambda_D L^{ijkl} - \frac{1}{5} \epsilon^{m(l} \Lambda_{V_m}^k L^{ij)} + 3 \Lambda_{V_m}^{(i} L^{jkl)m}, \\ \delta \psi^{ijk} &= \frac{3\sqrt{2}}{8} \epsilon^{(i} M^{jk)} + \frac{3\sqrt{2}}{8} \gamma^a \epsilon_m \epsilon^{m(i} V_a^{jk)} + \frac{5\sqrt{2}}{4} \gamma^a \epsilon_m \mathcal{D}_a L^{mijk} + \frac{7}{2} \Lambda_D \psi^{ijk} + \frac{i}{2} \Lambda_A \psi^{ijk} \\ &\quad + \frac{15}{2\sqrt{2}} L^{ijkl} \eta_l - \frac{1}{2\sqrt{2}} \epsilon^{l(k} L^{ij)} \eta_l + 2 \Lambda_{V_m}^{(i} \psi^{jk)m} - \frac{1}{6} \epsilon^{m(i} \Lambda_{V_m}^j \lambda^k) + \frac{1}{4} \Lambda_M^{ab} \gamma_{ab} \psi^{ijk}, \\ \delta \lambda^i &= \frac{3\sqrt{2}}{4} \epsilon_{kj} \epsilon^j M^{ki} - \frac{1}{4} \sqrt{2} \epsilon^i N + \frac{3\sqrt{2}}{4} \gamma^a \epsilon_j V_a^{ij} - \frac{1}{4} \sqrt{2} \epsilon^{ij} \gamma^a \epsilon_j G_a + \frac{3\sqrt{2}}{2} \gamma^a \epsilon_j \mathcal{D}_a L^{ij} \\ &\quad + \frac{7}{2} \Lambda_D \lambda^i + \frac{i}{2} \Lambda_A \lambda^i + \frac{8}{\sqrt{2}} L^{ij} \eta_j + \frac{4}{3} \Lambda_{V_j}^i \lambda^j + 4 \epsilon_{jm} \Lambda_{V_k}^m \psi^{ijk} + \frac{1}{4} \Lambda_M^{ab} \gamma_{ab} \lambda^i, \\ \delta M^{ij} &= \frac{2\sqrt{2}}{3} \epsilon^{k(i} \epsilon^{j)m} \bar{\xi}_m \epsilon_k - \frac{8\sqrt{2}}{3} \mathcal{D}_a \bar{\psi}^{ijk} \gamma^a \epsilon_k + 4 \Lambda_D M^{ij} + i \Lambda_A M^{ij} - \frac{16\sqrt{2}}{3} \bar{\psi}^{ijk} \eta_k - \frac{4\sqrt{2}}{9} \bar{\lambda}^{(i} \epsilon^{j)k} \eta_k + \Lambda_{V_k}^{(i} M^{j)k} - \frac{1}{9} \Lambda_{V_k}^{(i} \epsilon^{j)k} N, \\ \delta N &= -3\sqrt{2} \epsilon^{im} \bar{\xi}_m \epsilon_i + 4\sqrt{2} \mathcal{D}_a \bar{\lambda}^i \gamma^a \epsilon_i + 4 \Lambda_D N + i \Lambda_A N + \frac{12}{\sqrt{2}} \bar{\lambda}^i \eta_i - \frac{9}{2} \Lambda_{V_j}^i \epsilon_{ki} M^{jk}, \\ \delta V_a^{ij} &= -\frac{\sqrt{2}}{3} \epsilon^{m(j} \bar{\epsilon}^{i)} \gamma_a \xi_m + \frac{4\sqrt{2}}{3} \epsilon_{mn} \bar{\epsilon}^n \gamma_{ab} \mathcal{D}^b \psi^{mij} - \frac{2\sqrt{2}}{3} \epsilon_{mn} \bar{\epsilon}^n \mathcal{D}_a \psi^{mij} + \epsilon^{im} \epsilon^{jn} \left\{ -\frac{\sqrt{2}}{3} \epsilon_{k(n} \bar{\epsilon}_{m)} \gamma_a \xi^k + \frac{4\sqrt{2}}{3} \epsilon^{kl} \bar{\epsilon}_l \gamma_{ab} \mathcal{D}^b \psi_{mnk} \right. \\ &\quad \left. - \frac{2\sqrt{2}}{3} \epsilon^{kl} \bar{\epsilon}_l \mathcal{D}_a \psi_{mnk} \right\} + \frac{2\sqrt{2}}{9} (\bar{\lambda}^{(i} \gamma_a \eta^{j)} + \epsilon^{ik} \epsilon^{jm} \bar{\lambda}_{(k} \gamma_a \eta_{m)}) - \frac{11\sqrt{2}}{3} (\bar{\psi}^{ijk} \gamma_a \eta^l \epsilon_{kl} + \epsilon^{iq} \epsilon^{jr} \epsilon^{kl} \bar{\psi}_{qrk} \gamma_a \eta_l) + 4 \Lambda_D V_a^{ij} \\ &\quad + \Lambda_{V_k}^{(i} V_a^{j)k} + \frac{1}{9} \Lambda_{V_k}^{(i} \epsilon^{j)k} G_a - \frac{1}{3} \Lambda_{V_k}^{(i} \mathcal{D}_a L^{j)k} + \frac{5}{6} \epsilon_{kl} \Lambda_{V_m}^k \mathcal{D}_a L^{lmij} - \Lambda_{Ma}^b V_b^{ij} + \frac{2}{3} \Lambda_{Ka} L^{ij}, \\ \delta G_a &= \frac{3\sqrt{2}}{2} \bar{\epsilon}^i \gamma_a \xi_i + 2\sqrt{2} \epsilon_{mi} \bar{\epsilon}^i \gamma_{ab} \mathcal{D}^b \lambda^m - 4\sqrt{2} \bar{\lambda}^i \gamma_a \eta^j \epsilon_{ij} + \text{H.c.} + 4 \Lambda_D G_a - \frac{9}{2} \epsilon_{kl} \Lambda_{V_m}^k V_a^{lm} - \epsilon_{kl} \Lambda_{V_m}^k \mathcal{D}_a L^{lm} - \Lambda_{Ma}^b G_b, \\ \delta \xi_i &= -\sqrt{2} \epsilon_{ji} \epsilon_{mk} \gamma^a \epsilon^k \mathcal{D}_a M^{mj} + 2\sqrt{2} \epsilon_{ji} \epsilon_k \mathcal{D}_a V_a^{kj} + \frac{\sqrt{2}}{3} \epsilon_i \mathcal{D}_a G^a - \sqrt{2} \epsilon_{ji} \gamma^{ab} \epsilon_k \mathcal{D}_a V_b^{kj} + \frac{5}{\sqrt{2}} \gamma^a V_a^{kj} \eta_j \epsilon_{ki} \\ &\quad + \frac{1}{3\sqrt{2}} \gamma^a G_a \eta_i - \frac{7}{\sqrt{2}} M^{jk} \eta^m \epsilon_{km} \epsilon_{ji} - \frac{1}{3\sqrt{2}} N \eta^j \epsilon_{ji} - \frac{\sqrt{2}}{3} \gamma^a \mathcal{D}_a L^{kj} \eta_j \epsilon_{ki} + \frac{9}{2} \Lambda_D \xi_i + \frac{i}{2} \Lambda_A \xi_i + \frac{1}{4} \Lambda_M^{ab} \gamma_{ab} \xi_i \\ &\quad - \frac{4}{9} \Lambda_{V_m}^k \gamma^a \mathcal{D}_a \lambda^m \epsilon_{ki} + \frac{4}{3} \Lambda_{V_m}^k \gamma^a \mathcal{D}_a \psi^{mln} \epsilon_{kl} \epsilon_{ni} + \frac{2}{3} \Lambda_{Ka} \gamma^a \lambda^m \epsilon_{mi}, \end{aligned} \quad (22)$$

TABLE I. Field content of the relaxed hypermultiplet in the new basis.

Field	$SU(2)$ Irreps	Properties	Weyl weight (w)	Chiral weight (c)
A^{ijk}	4	Complex	3	0
ψ^{ij}	3	$\gamma_5 \psi^{ij} = \psi^{ij}$	7/2	1/2
λ^{ij}	3	$\gamma_5 \lambda^{ij} = \lambda^{ij}$	7/2	1/2
M^i	2	Complex	4	1
N^i	2	Complex	4	1
V_a^i	2	Complex, Lorentz vector	4	0
ξ	1	$\gamma_5 \xi = -\xi$	9/2	1/2
Ω	1	$\gamma_5 \Omega = -\Omega$	9/2	1/2

where the derivative $\mathcal{D}_a$ that appears in the above transformations is the superconformal covariant derivative (defined by Eq. (A6) in Appendix A). The transformation parameters Λ_D , Λ_M , Λ_{Ka} , Λ_A , and Λ_{Vj}^i correspond to dilatation, local Lorentz transformation, special conformal transformation, $U(1)$ and $SU(2)$ R -symmetry respectively. Notice that the S -transformation of ξ_i and the V -transformations of V_a^{ij} , G_a , and ξ_i involves covariant derivative terms.

As was shown above, the fields in the relaxed hypermultiplet had to be allowed to transform in a noncanonical way under $SU(2)$, when we extend the multiplet to conformal supergravity. Moreover, we note in the above transformation rules that there are fields in the relaxed hypermultiplet which are not K -invariant as contrary to other known matter multiplets in the literature. But, we will show that by appropriate redefinition of the relaxed hypermultiplet fields we can switch to a new $SU(2)$ basis wherein all the fields transform in a way that $SU(2)$ irreps do. In addition, the fields will also be K -invariant in the new basis. Consider the V -transformation of ξ_i . We note that it transforms into a linear combination of $\gamma^a \mathcal{D}_a \lambda^i$ and $\gamma^a \mathcal{D}_a \psi^{ijk}$, and the diagonal elements (i.e., terms containing ξ_i) are absent in the transformation. Thus by taking appropriate linear combinations (which we list out in Appendix B) of the components of the fields ξ_i , $\gamma^a \mathcal{D}_a \lambda^i$, and $\gamma^a \mathcal{D}_a \psi^{ijk}$ we define two new fields Ω and ξ which are K -invariant and transform as $SU(2)$ scalars. Along the same lines we carry out the redefinition of the other fields as well.

Table I summarizes the field content and properties of the relaxed hypermultiplet in the new basis. The linearized transformation of the relaxed hypermultiplet in terms of the redefined fields is given as follows¹:

$$\begin{aligned}
\delta A^{ijk} &= -\bar{\epsilon}^{(i} \lambda^{jk)} + \epsilon^{il} \epsilon^{jm} \epsilon^{kn} \bar{\epsilon}_{(l} \psi_{mn)} + 3\Lambda_D A^{ijk} + 3\Lambda_{V_m}^{(i} A^{jk)m}, \\
\delta \psi^{ij} &= \epsilon^{(i} M^{j)} + \epsilon^{(i} N^{j)} + \gamma^a \epsilon_m \epsilon^{m(i} V_a^{j)} - 2\gamma^a \epsilon_m \mathcal{D}_a \bar{A}^{mij} + \frac{7}{2} \Lambda_D \psi^{ij} + \frac{i}{2} \Lambda_A \psi^{ij} - 6\bar{A}^{ijk} \eta_k + 2\Lambda_{V_k}^{(i} \psi^{j)k} + \frac{1}{4} \Lambda_M^{ab} \gamma_{ab} \psi^{ij}, \\
\delta \lambda^{ij} &= \epsilon^{(i} M^{j)} - \epsilon^{(i} N^{j)} + \gamma^a \epsilon_m \epsilon^{m(i} \epsilon^{k|j)} V_{ak} - 2\gamma^a \epsilon_m \mathcal{D}_a A^{mij} + \frac{7}{2} \Lambda_D \lambda^{ij} + \frac{i}{2} \Lambda_A \lambda^{ij} - 6A^{ijk} \eta_k + 2\Lambda_{V_k}^{(i} \lambda^{j)k} + \frac{1}{4} \Lambda_M^{ab} \gamma_{ab} \lambda^{ij}, \\
\delta M^i &= \epsilon^{ik} \bar{\epsilon}_k \Omega + \bar{\epsilon}_k \gamma^a \mathcal{D}_a \psi^{ik} + \bar{\epsilon}_k \gamma^a \mathcal{D}_a \lambda^{ik} - 2\bar{\eta}_j \psi^{ij} - 2\bar{\eta}_j \lambda^{ij} + 4\Lambda_D M^i + i\Lambda_A M^i + \Lambda_{V_k}^i M^k, \\
\delta N^i &= -\epsilon^{ik} \bar{\epsilon}_k \xi + \bar{\epsilon}_k \gamma^a \mathcal{D}_a \psi^{ik} - \bar{\epsilon}_k \gamma^a \mathcal{D}_a \lambda^{ik} - 2\bar{\eta}_j \psi^{ij} + 2\bar{\eta}_j \lambda^{ij} + 4\Lambda_D N^i + i\Lambda_A N^i + \Lambda_{V_k}^i N^k, \\
\delta V_a^i &= \frac{1}{2} \bar{\epsilon}^i \gamma_a \xi - \frac{1}{2} \bar{\epsilon}^i \gamma_a \Omega + \epsilon_{mn} \bar{\epsilon}^n \gamma_{ab} \mathcal{D}^b \psi^{mi} - \frac{1}{3} \epsilon_{mn} \bar{\epsilon}^n \mathcal{D}_a \psi^{mi} - \frac{1}{2} \epsilon^{ik} \bar{\epsilon}_k \gamma_a \xi_L - \frac{1}{2} \epsilon^{ik} \bar{\epsilon}_k \gamma_a \Omega_L + \epsilon^{tk} \epsilon^{mn} \bar{\epsilon}_n \gamma_{ab} \mathcal{D}^b \lambda_{mk} \\
&\quad - \frac{1}{3} \epsilon^{ik} \epsilon^{mn} \bar{\epsilon}_n \mathcal{D}_a \lambda_{mk} + \frac{8}{3} \bar{\eta}^n \gamma_a \psi^{im} \epsilon_{mn} + \frac{8}{3} \bar{\eta}_n \gamma_a \lambda_{jm} \epsilon^{mn} \epsilon^{ij} + 4\Lambda_D V_a^i + \Lambda_{V_k}^i V_a^k - \Lambda_{Ma}^b V_b^i, \\
\delta \xi &= \gamma^a \epsilon^i \mathcal{D}_a N^j \epsilon_{ij} + \frac{3}{2} \epsilon_k \mathcal{D}^a V_a^k - \frac{3}{2} \epsilon_k \mathcal{D}_a V_m^m \epsilon^{mk} - \frac{1}{2} \gamma^{ab} \epsilon_k \mathcal{D}_a V_b^k + \frac{1}{2} \gamma_{ab} \epsilon_k \mathcal{D}^a V_l^b \epsilon^{lk} - 4N^i \eta^j \epsilon_{ij} + \frac{3}{2} \gamma^a V_a^i \eta_i \\
&\quad - \frac{3}{2} \gamma_a V_k^a \eta_i \epsilon^{ki} + \frac{9}{2} \Lambda_D \xi + \frac{i}{2} \Lambda_A \xi + \frac{1}{4} \Lambda_M^{ab} \gamma_{ab} \xi, \\
\delta \Omega &= -\gamma^a \epsilon^i \mathcal{D}_a M^j \epsilon_{ij} - \frac{3}{2} \epsilon_k \mathcal{D}^a V_a^k - \frac{3}{2} \epsilon_k \mathcal{D}_a V_l^l \epsilon^{lk} + \frac{1}{2} \gamma^{ab} \epsilon_k \mathcal{D}_a V_b^k + \frac{1}{2} \gamma_{ab} \epsilon_k \mathcal{D}^a V_l^b \epsilon^{lk} + 4M^i \eta^j \epsilon_{ij} - \frac{3}{2} \gamma^a V_a^i \eta_i \\
&\quad - \frac{3}{2} \gamma_a V_l^a \eta_i \epsilon^{li} + \frac{9}{2} \Lambda_D \Omega + \frac{i}{2} \Lambda_A \Omega + \frac{1}{4} \Lambda_M^{ab} \gamma_{ab} \Omega,
\end{aligned} \tag{23}$$

¹See Appendix B for the details of the field redefinition.

where $\bar{A}^{ijk} = \epsilon^{il}\epsilon^{jm}\epsilon^{kn}A_{lmn}$. The subscript L in ξ_L and Ω_L denotes the left chirality, and the fields ξ and Ω have been defined with right chirality.

Finally we obtain the complete nonlinear superconformal transformation by introducing nonlinear covariant terms into the Q- and S-transformation rules. The nonlinear terms are constructed by taking simple products of standard Weyl multiplet fields, curvatures and relaxed hypermultiplet

fields. On careful analysis, we find that there is no such nonlinear term consistent with Weyl weight, chiral weight and the representations, which can be added to the S-transformation. Hence, the S-transformation rules remain unchanged. However, for the Q-transformation there are nonlinear terms consistent with weights and representations. We add all such possible terms and fix the coefficients using the algebra (12). We give the final result below:

$$\begin{aligned}
\delta A^{ijk} &= -\bar{\epsilon}^{(i}\lambda^{jk)} + \epsilon^{il}\epsilon^{jm}\epsilon^{kn}\bar{\epsilon}_{(l}\psi_{mn)} + 3\Lambda_D A^{ijk} + 3\Lambda_{V_m}^{(i}A^{jk)m}, \\
\delta\psi^{ij} &= \epsilon^{(i}M^{j)} + \epsilon^{(i}N^{j)} + \gamma^a\epsilon_m\epsilon^{m(i}V_a^{j)} - 2\gamma^a\epsilon_m\mathcal{D}_a\bar{A}^{mij} + \frac{7}{2}\Lambda_D\psi^{ij} + \frac{i}{2}\Lambda_A\psi^{ij} - 6\bar{A}^{ijk}\eta_k + 2\Lambda_{V_k}^{(i}\psi^{j)k} + \frac{1}{4}\Lambda_M^{ab}\gamma_{ab}\psi^{ij}, \\
\delta\lambda^{ij} &= \epsilon^{(i}M^{j)} - \epsilon^{(i}N^{j)} + \gamma^a\epsilon_m\epsilon^{m(i}\epsilon^{k|j)}V_{ak} - 2\gamma^a\epsilon_m\mathcal{D}_aA^{mij} + \frac{7}{2}\Lambda_D\lambda^{ij} + \frac{i}{2}\Lambda_A\lambda^{ij} - 6A^{ijk}\eta_k + 2\Lambda_{V_k}^{(i}\lambda^{j)k} + \frac{1}{4}\Lambda_M^{ab}\gamma_{ab}\lambda^{ij}, \\
\delta M^i &= \epsilon^{ik}\bar{\epsilon}_k\Omega + \bar{\epsilon}_k\gamma^a\mathcal{D}_a\psi^{ik} + \bar{\epsilon}_k\gamma^a\mathcal{D}_a\lambda^{ik} - 2\bar{\eta}_j\psi^{ij} - 2\bar{\eta}_j\lambda^{ij} + 4\Lambda_DM^i + i\Lambda_AM^i + \Lambda_{V_k}^iM^k \\
&\quad + \frac{1}{16}\bar{\lambda}_{lj}\gamma^{ab}\epsilon_k\epsilon^{kj}\epsilon^{il}T_{ab}^+ - \frac{1}{16}\bar{\psi}_{lj}\gamma^{ab}\epsilon_k\epsilon^{kj}\epsilon^{il}T_{ab}^+ - 9\bar{\epsilon}_j\chi_kA^{ijk} - 9\bar{\epsilon}_j\chi_k\bar{A}^{ijk}, \\
\delta N^i &= -\epsilon^{ik}\bar{\epsilon}_k\xi + \bar{\epsilon}_k\gamma^a\mathcal{D}_a\psi^{ik} - \bar{\epsilon}_k\gamma^a\mathcal{D}_a\lambda^{ik} - 2\bar{\eta}_j\psi^{ij} + 2\bar{\eta}_j\lambda^{ij} + 4\Lambda_DN^i + i\Lambda_AN^i + \Lambda_{V_k}^iN^k \\
&\quad + \frac{1}{16}\bar{\lambda}_{lj}\gamma^{ab}\epsilon_k\epsilon^{kj}\epsilon^{il}T_{ab}^+ + \frac{1}{16}\bar{\psi}_{lj}\gamma^{ab}\epsilon_k\epsilon^{kj}\epsilon^{il}T_{ab}^+ + 9\bar{\epsilon}_j\chi_kA^{ijk} - 9\bar{\epsilon}_j\chi_k\bar{A}^{ijk}, \\
\delta V_a^i &= \frac{1}{2}\bar{\epsilon}^i\gamma_a\xi - \frac{1}{2}\bar{\epsilon}^i\gamma_a\Omega + \epsilon_{mn}\bar{\epsilon}^n\gamma_{ab}\mathcal{D}^b\psi^{mi} - \frac{1}{3}\epsilon_{mn}\bar{\epsilon}^n\mathcal{D}_a\psi^{mi} - \frac{1}{2}\epsilon^{ik}\bar{\epsilon}_k\gamma_a\xi_L - \frac{1}{2}\epsilon^{ik}\bar{\epsilon}_k\gamma_a\Omega_L + \epsilon^{ik}\epsilon^{mn}\bar{\epsilon}_n\gamma_{ab}\mathcal{D}^b\lambda_{mk} \\
&\quad - \frac{1}{3}\epsilon^{ik}\epsilon^{mn}\bar{\epsilon}_n\mathcal{D}_a\lambda_{mk} + \frac{8}{3}\bar{\eta}^n\gamma_a\psi^{im}\epsilon_{mn} + \frac{8}{3}\bar{\eta}_n\gamma_a\lambda_{jm}\epsilon^{mn}\epsilon^{ij} + 4\Lambda_DV_a^i + \Lambda_{V_k}^iV_a^k - \Lambda_{Ma}^bV_b^i \\
&\quad - \frac{1}{12}\bar{\epsilon}^k\gamma^b\lambda_{kl}T_{ab}^+\epsilon^{il} - \frac{1}{12}\bar{\epsilon}_k\gamma^b\psi^{ki}T_{ab}^- - 3\bar{\epsilon}^l\gamma_a\chi_j\bar{A}^{ijm}\epsilon_{ml} + 3\bar{\epsilon}_j\gamma_a\chi^l\bar{A}^{ijm}\epsilon_{ml}, \\
\delta\xi &= \gamma^a\epsilon^i\mathcal{D}_aN^j\epsilon_{ij} + \frac{3}{2}\epsilon_k\mathcal{D}^aV_a^k - \frac{3}{2}\epsilon_k\mathcal{D}_aV_m^a\epsilon^{mk} - \frac{1}{2}\gamma^{ab}\epsilon_k\mathcal{D}_aV_b^k + \frac{1}{2}\gamma_{ab}\epsilon_k\mathcal{D}^aV_l^b\epsilon^{lk} - 4N^i\eta^j\epsilon_{ij} + \frac{3}{2}\gamma^aV_a^i\eta_i \\
&\quad - \frac{3}{2}\gamma_aV_k^a\eta_i\epsilon^{ki} + \frac{9}{2}\Lambda_D\xi + \frac{i}{2}\Lambda_A\xi + \frac{1}{4}\Lambda_M^{ab}\gamma_{ab}\xi + \frac{1}{16}\epsilon^{ij}M_i\gamma^{ab}\epsilon_jT_{ab}^+ - 3\bar{\chi}^i\lambda^{jk}\epsilon_k\epsilon_{ij} + 3\bar{\chi}^i\psi^{jk}\epsilon_k\epsilon_{ij} \\
&\quad + \frac{1}{2}\gamma^{ab}\epsilon_l\hat{R}_{ab}(V)_j^m A^{ijkl}\epsilon_{mk} - \frac{1}{2}\gamma^{ab}\epsilon_l\hat{R}_{ab}(V)_j^m \bar{A}^{ijkl}\epsilon_{mk}, \\
\delta\Omega &= -\gamma^a\epsilon^i\mathcal{D}_aM^j\epsilon_{ij} - \frac{3}{2}\epsilon_k\mathcal{D}^aV_a^k - \frac{3}{2}\epsilon_k\mathcal{D}_aV_l^a\epsilon^{lk} + \frac{1}{2}\gamma^{ab}\epsilon_k\mathcal{D}_aV_b^k + \frac{1}{2}\gamma_{ab}\epsilon_k\mathcal{D}^aV_l^b\epsilon^{lk} + 4M^i\eta^j\epsilon_{ij} - \frac{3}{2}\gamma^aV_a^i\eta_i \\
&\quad - \frac{3}{2}\gamma_aV_l^a\eta_i\epsilon^{li} + \frac{9}{2}\Lambda_D\Omega + \frac{i}{2}\Lambda_A\Omega + \frac{1}{4}\Lambda_M^{ab}\gamma_{ab}\Omega + \frac{1}{16}\epsilon^{ij}N_i\gamma^{ab}\epsilon_jT_{ab}^+ - 3\bar{\chi}^i\lambda^{jk}\epsilon_k\epsilon_{ij} - 3\bar{\chi}^i\psi^{jk}\epsilon_k\epsilon_{ij} \\
&\quad + \frac{1}{2}\gamma^{ab}\epsilon_l\hat{R}_{ab}(V)_j^m A^{ijkl}\epsilon_{mk} + \frac{1}{2}\gamma^{ab}\epsilon_l\hat{R}_{ab}(V)_j^m \bar{A}^{ijkl}\epsilon_{mk}.
\end{aligned} \tag{24}$$

IV. CONCLUSIONS AND FUTURE DIRECTIONS

Matter multiplets in conformal supergravity play a crucial role in construction of physical Poincaré supergravity theories. Off-shell multiplets, in particular, allow the construction of general matter couplings and higher derivative invariants in supergravity which are of interest in various contexts such as higher derivative corrections to black hole entropy.

In this paper we have extended the $32 + 32$ relaxed hypermultiplet constructed in [10] for flat space to $\mathcal{N} = 2$ conformal supergravity. In doing this, we find the peculiar

feature that the fields which are irreducible representations of the global $SU(2)$ symmetry must be allowed to transform in a noncanonical way under the local $SU(2)$ R -symmetry in order to be consistent with the superconformal algebra. We perform suitable field redefinitions so that the redefined fields are irreducible representations of the $SU(2)$ R -symmetry as well as invariant under K -symmetry. In Appendix B, we present the precise field redefinitions as well as the relation of our results to the constructions in harmonic superspace [13] and projective superspace [15].

In [10], in flat space, the relaxed hypermultiplet was coupled to a $24 + 24$ real scalar multiplet to obtain an off-shell formulation for the hypermultiplet. The real scalar multiplet was extended to conformal supergravity in [11]. Further, it was shown to be reducible to an $8 + 8$ restricted real scalar multiplet by imposing $16 + 16$ constraints. This restricted real scalar multiplet admitted a tensor multiplet embedding. In [12], a new density formula in $\mathcal{N} = 2$ conformal supergravity was presented with application to real scalar multiplet. This allowed for a new higher derivative invariant for the tensor multiplet in supergravity.

With the relaxed hypermultiplet in $\mathcal{N} = 2$ conformal supergravity presented in this paper, it would be interesting to construct the analogue of the action (8) in conformal supergravity. The generalization is not straight forward for two reasons. The action (8) does not have the appropriate Weyl weight to be invariant in the superconformal setting. This is akin to tensor multiplet in four dimensions, where the rigid tensor multiplet action does not carry the appropriate Weyl weight to be superconformal invariant and hence one needs to consider an improved tensor multiplet action in conformal supergravity. In addition to this, the rigid supersymmetry action in (8) is not invariant with respect to the rigid part of the local $SU(2)$ R-symmetry but is invariant with respect to a different rigid $SU(2)$ symmetry. Such a generalization is important to obtain an off-shell description of a single hypermultiplet in conformal supergravity. In the superconformal approach to construct Poincaré supergravity theories, the role of a hypermultiplet is confined to being used as a compensator. Therefore generalization of the off-shell description of a single hypermultiplet to conformal supergravity is of interest and is sufficient for the purposes of superconformal tensor calculus. However, one may want to study the coupling of one or more hypermultiplets as matter couplings to Poincaré supergravity. In that case, a generalization of the study presented in this paper to more than one hypermultiplet may be desirable.

In [16], an extension of the analysis from [10] was generalized to six dimensional $\mathcal{N} = (1, 0)$ curved superspace to construct the off-shell representation for the hypermultiplet in six dimensional supergravity. This extension is different from that of this paper in the following sense. In [16], an additional real superfield $\mathbf{T}$ appeared in the superspace constraints along with the $\mathbf{L}^{ij}$ and $\mathbf{L}^{ijkl}$ superfields to define the relaxed hypermultiplet. The construction of [16] in six dimensions may have a straightforward generalization in four dimensions and one can have an alternate formulation of relaxed hypermultiplet with more number of components.

It will also be interesting to investigate if there exist constraints to reduce the relaxed hypermultiplet to a restricted multiplet. Whether the multiplet can be embedded into a density formula or can be mapped to other multiplets are the questions central to the techniques

of superconformal tensor calculus, which need to be addressed for the case of relaxed hypermultiplet in conformal supergravity.

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APPENDIX A: $\mathcal{N} = 2$ STANDARD WEYL MULTIPLY

The $\mathcal{N} = 2$ standard Weyl multiplet is a superconformal gauge multiplet which has the field content $(e_\mu^a, \psi_\mu^i, b_\mu, A_\mu, \mathcal{V}_{\mu j}^i, T_{ab}^{ij}, \chi^i, D)$, with $24 + 24$ off-shell degrees of freedom [17, 18]. The fields $e_\mu^a, \psi_\mu^i, b_\mu, A_\mu, \mathcal{V}_{\mu j}^i$ are identified as the gauge fields of local translation (P), Q- supersymmetry (Q), dilatation (D), $U(1)_R$ symmetry, (A) and $SU(2)_R$ symmetry (V), respectively. The fields T_{ab}^{ij} which is an antiself-dual antisymmetric Lorentz tensor, χ^i an $SU(2)$ doublet of Majorana spinors and D a real scalar field, are auxiliary fields which are essential for the off-shell bosonic and fermionic degrees of freedom to match in the multiplet.

Table II summarizes the properties of the standard Weyl multiplet. We follow the chiral convention wherein spinors with upper $SU(2)_R$ index have positive (left) chirality and those with lower $SU(2)_R$ index have negative (right) chirality. The chiral convention allows us to raise and lower the $SU(2)$ indices by complex conjugation. Thus for fermions complex conjugation switches the chirality. Chiral weight (c) and Weyl weight (w) of a field X are real numbers defined by the transformations:

$$\begin{aligned}\delta_A X &= ic \Lambda_A X, \\ \delta_D X &= w \Lambda_D X.\end{aligned}\tag{A1}$$

From Eqs. (A1) we can deduce that the chiral weight of a field and that of its complex conjugate differ by a factor

TABLE II. Field content for the $\mathcal{N} = 2$ standard Weyl multiplet.

Field	$SU(2)$ Irreps	Weyl weight (w)	Chiral weight (c)	Chirality for fermions
e_μ^a	1	-1	0	
ψ_μ^i	2	-1/2	-1/2	+
b_μ	1	0	0	
A_μ	1	0	0	
$\mathcal{V}_{\mu j}^i$	3	0	0	
T_{ab}^{ij}	1	1	-1	
χ^i	2	3/2	-1/2	+
D	1	2	0	

of -1 , while the Weyl weights are unaffected by complex conjugation.

The field T_{ab}^- listed in the Table II is related to the field T_{ab}^{ij} by:

$$T_{ab}^{ij} = \frac{1}{2} T_{ab}^- \epsilon^{ij}. \quad (\text{A2})$$

The dual of an antisymmetric Lorentz tensor is defined as:

$$\tilde{G}_{ab} = \frac{1}{2} \epsilon_{abcd} G^{cd}. \quad (\text{A3})$$

One can show in Minkowski space that self-dual and anti-self-dual parts of an antisymmetric Lorentz tensor are related by complex conjugation. Therefore, we have the following expressions relating the anti-self-dual tensor T_{ab}^{ij} with its self-dual counterpart:

$$\begin{aligned} T_{abij} &= (T_{ab}^{ij})^*, \\ &= \frac{1}{2} (T_{ab}^- \epsilon^{ij})^* = \frac{1}{2} T_{ab}^+ \epsilon_{ij}. \end{aligned} \quad (\text{A4})$$

In addition to the gauge fields listed in the Table II, conformal supergravity also consists of the dependent gauge fields ω_{μ}^{ab} corresponding to local Lorentz transformation, f_{μ}^a corresponding to special conformal transformation and ϕ_{μ}^i corresponding to S-supersymmetry. These fields are determined in terms of the independent fields in the Weyl multiplet by the following set of conventional constraints:

$$\begin{aligned} \hat{R}_{\mu\nu}(P) &= 0, \\ \gamma^{\mu} \left(\hat{R}_{\mu\nu}(Q)^i + \frac{1}{2} \gamma_{\mu\nu} \chi^i \right) &= 0, \\ e_b^{\nu} \hat{R}_{\mu\nu}(M)_a^b - i \tilde{\hat{R}}_{\mu a}(A) + \frac{1}{8} T_{abij} T_{\mu b}^{ij} - \frac{3}{2} D e_{\mu a} &= 0, \end{aligned} \quad (\text{A5})$$

where $\hat{R}(P)$, $\hat{R}(Q)$, $\hat{R}(M)$ and $\hat{R}(A)$ are respectively the super-covariant curvatures associated with local translation, Q-supersymmetry, local Lorentz transformation and $U(1)$ R-symmetry, and $\tilde{\hat{R}}(A)$ is the dual of $\hat{R}(A)$. The curvature $\hat{R}(V)$ that appears in the superconformal transformation of the relaxed hypermultiplet (24) is the super-covariant curvature associated with $SU(2)$ R-symmetry. On imposing the above constraints, the soft algebra relations (12) are satisfied on the fields. Finally we end this section by defining the superconformal derivative and listing out the Weyl weights and chiral weights of supersymmetry parameters:

$$\begin{aligned} \mathcal{D}_{\mu} &= \partial_{\mu} - \sum_T \delta(h_{\mu}(T)), \\ \mathcal{D}_a &= e_a^{\mu} \mathcal{D}_{\mu}. \end{aligned} \quad (\text{A6})$$

TABLE III. Gauge fields for superconformal transformations.

Superconformal transformation	Generator T	Normalized gauge field $h_{\mu}(T)$	Parameter
Local translation	P^a	e_{μ}^a	ξ^a
Q-SUSY	Q^i	$\frac{1}{2} \psi_{\mu}^i$	ϵ^i
Dilatation	D	b_{μ}	Λ_D
$U(1)_R$	A	$-i A_{\mu}$	Λ_A
$SU(2)_R$	$(V_{\Lambda})_j^i$	$-\frac{1}{2} \mathcal{V}_{\mu j}^i$	Λ_V^i
S-SUSY	S^i	$\frac{1}{2} \phi_{\mu}^i$	η^i
Local Lorentz	M^{ab}	ω_{μ}^{ab}	$\epsilon^{ab} / \Lambda_M^{ab}$
Special conformal	K^a	f_{μ}^a	Λ_K^a

where the summation runs over all the superconformal generators T except local translation and $h_{\mu}(T)$ are the corresponding gauge fields as listed in Table III.

The chiral weight and Weyl weight of the Q- and S-SUSY parameters are

$$\begin{aligned} \epsilon^i : c &= -\frac{1}{2}, & w &= -\frac{1}{2}, \\ \eta^i : c &= -\frac{1}{2}, & w &= \frac{1}{2}. \end{aligned} \quad (\text{A7})$$

APPENDIX B: REDEFINITION OF THE RELAXED HYPERMULTIPLY FIELD

Here we give the necessary details of redefinition of the relaxed hypermultiplet fields. Consider the V- and K-transformation of the field ξ_i :

$$\begin{aligned} \delta_V \xi_i &= -\frac{4}{9} \Lambda_{V m}^k \gamma^a \mathcal{D}_a \lambda^m \epsilon_{ki} + \frac{4}{3} \Lambda_{V m}^k \gamma^a \mathcal{D}_a \psi^{m l n} \epsilon_{kl} \epsilon_{ni}, \\ \delta_K \xi_i &= \frac{2}{3} \Lambda_{K a} \gamma^a \lambda^m \epsilon_{mi}. \end{aligned} \quad (\text{B1})$$

As discussed in Sec. III, ξ_i has a nontrivial K-transformation. Also, diagonal elements (i.e., ξ_i terms) are absent in the V-transformation. The absence of diagonal term in the V-transformation suggests that it can be decomposed into two $SU(2)$ scalars say $\tilde{\xi}_1, \tilde{\xi}_2$. We also expect the resultant $SU(2)$ scalars to be invariant under SCT. So, we consider the following ansatz:

$$\begin{aligned} \tilde{\xi}_{1/2} &= x_1 \xi_1 + x_2 \xi_2 + x_3 \gamma^a \mathcal{D}_a \lambda^1 + x_4 \gamma^a \mathcal{D}_a \lambda^2 \\ &\quad + x_5 \gamma^a \mathcal{D}_a \psi^{111} + x_6 \gamma^a \mathcal{D}_a \psi^{222} + x_7 \gamma^a \mathcal{D}_a \psi^{121} \\ &\quad + x_8 \gamma^a \mathcal{D}_a \psi^{122}. \end{aligned} \quad (\text{B2})$$

By considering the K-transformation of the Eq. (B2) and demanding $\delta_K \tilde{\xi}_1 = \delta_K \tilde{\xi}_2 = 0$, we get:

$$\tilde{\xi}_{1/2} = x_1 \xi_1 + x_2 \xi_2 + \frac{x_2}{3} \gamma^a \mathcal{D}_a \lambda^1 - \frac{x_1}{3} \gamma^a \mathcal{D}_a \lambda^2. \quad (\text{B3})$$

By taking the V- transformations on the RHS of (B3), one can see that the new fields are already V-invariant, i.e., the K-invariance by itself has made sure that the new fields are $SU(2)$ scalars. We have the freedom to choose the coefficients x_1 and x_2 . Let us choose $x_1 = x_2 = 1$ for $\tilde{\xi}_1$, and $x_1 = 1, x_2 = -1$ for $\tilde{\xi}_2$:

$$\tilde{\xi}_1 = \xi_1 + \xi_2 + \frac{1}{3} \gamma^a \mathcal{D}_a \lambda^1 - \frac{1}{3} \gamma^a \mathcal{D}_a \lambda^2, \quad (\text{B4})$$

$$\tilde{\xi}_2 = \xi_1 - \xi_2 - \frac{1}{3} \gamma^a \mathcal{D}_a \lambda^1 - \frac{1}{3} \gamma^a \mathcal{D}_a \lambda^2. \quad (\text{B5})$$

So, in effect we have decomposed the two $SU(2)$ components of ξ_i into $1 \oplus 1$ of $SU(2)$.

Next we look at M^{ij} and N . These fields mix with each other under $SU(2)$ and have a total of 4 $SU(2)$ components. So, with these four components, the possible irreps of $SU(2)$ that we can form are 4, $3 \oplus 1$ and $2 \oplus 2$. 4 of $SU(2)$ has 3 fundamental $SU(2)$ indices, 3 of $SU(2)$ has 2, 2 of

TABLE IV. Field redefinitions to obtain the components as $SU(2)_R$ irreps.

Old fields	No. of $SU(2)$ components	Decomposition	New fields
ξ_i	2	$1 \oplus 1$	$\tilde{\xi}_1, \tilde{\xi}_2$
M^{ij}, N	4	$2 \oplus 2$	$\tilde{M}^i, \tilde{N}^i$
V_a^{ij}, G_a	4	$2 \oplus 2$	$\tilde{V}_a^i, \tilde{G}_a^i$
λ^i, ψ^{ijk}	6	$3 \oplus 3$	$\tilde{\lambda}^{ij}, \tilde{\psi}^{ij}$
L^{ij}, L^{ijkl}	8	$4 \oplus 4$	$\tilde{A}^{ijk}, \tilde{B}^{ijk}$

$SU(2)$ has 1 and 1 of $SU(2)$ has 0 $SU(2)$ indices. We know that Q-SUSY transformations of M^{ij} and N contain ξ_i terms. Therefore, the new irreps that we form out of M^{ij} and N should contain $\tilde{\xi}_{1/2}$ terms in the Q- transformations. For these transformations to be consistent with the index structure, one has to decompose M^{ij} and N as $2 \oplus 2$. Similar arguments follow for the remaining fields. Table IV summarizes the decomposition of all the fields. We list out the remaining field redefinitions below:

$$\begin{aligned}
(\tilde{M}^1, \tilde{M}^2) &= \left(M^{11} + M^{12} + \frac{N}{9}, M^{12} + M^{22} - \frac{N}{9} \right), \\
(\tilde{N}^1, \tilde{N}^2) &= \left(M^{11} - M^{12} - \frac{N}{9}, M^{12} - M^{22} - \frac{N}{9} \right), \\
(\tilde{V}_a^1, \tilde{V}_a^2) &= \left(V_a^{11} + \frac{2}{9} \mathcal{D}_a L^{11}, V_a^{12} + \frac{1}{9} G_a + \frac{2}{9} \mathcal{D}_a L^{12} \right), \\
(\tilde{G}_a^1, \tilde{G}_a^2) &= \left(V_a^{12} - \frac{1}{9} G_a + \frac{2}{9} \mathcal{D}_a L^{12}, V_a^{22} + \frac{2}{9} \mathcal{D}_a L^{22} \right), \\
(\tilde{\psi}^{11}, \tilde{\psi}^{12}, \tilde{\psi}^{22}) &= \left(\psi^{111}, \psi^{112} + \frac{1}{12} \lambda^1, \psi^{122} + \frac{1}{6} \lambda^2 \right), \\
(\tilde{\lambda}^{11}, \tilde{\lambda}^{12}, \tilde{\lambda}^{22}) &= \left(\psi^{112} - \frac{1}{6} \lambda^1, \psi^{122} - \frac{1}{12} \lambda^2, \psi^{222} \right), \\
(\tilde{A}^{111}, \tilde{A}^{112}, \tilde{A}^{122}, \tilde{A}^{222}) &= \left(L^{11} - 5L^{112}, \frac{2}{3} L^{12} - 5L^{122}, \frac{1}{3} L^{22} - 5L^{1222}, -5L^{2222} \right), \\
(\tilde{B}^{111}, \tilde{B}^{112}, \tilde{B}^{122}, \tilde{B}^{222}) &= \left(L^{1111}, \frac{1}{15} L^{11} + L^{1112}, \frac{2}{15} L^{12} + L^{1122}, \frac{1}{5} L^{22} + L^{1222} \right). \quad (\text{B6})
\end{aligned}$$

In the new basis, the reality constraints of $L^{ij}, L^{ijkl}, V_a^{ij}$, and G_a translate into:

$$\begin{aligned}
\tilde{B}^{ijk} &= -\frac{1}{5} \epsilon^{il} \epsilon^{jm} \epsilon^{kn} (\tilde{A}^{lmn})^*, \\
\tilde{G}_a^i &= \epsilon^{ji} (\tilde{V}_a^j)^*. \quad (\text{B7})
\end{aligned}$$

Further to tidy up things, we make the following normalization choice for the fields:

$$\begin{aligned}
A^{ijk} &= \tilde{A}^{ijk}, (B^{ijk} = \tilde{B}^{ijk}), \quad \lambda^{ij} = 4\sqrt{2}\tilde{\lambda}^{ij}, \\
\psi^{ij} &= 4\sqrt{2}\tilde{\psi}^{ij}, \quad M^i = \frac{3}{2}\tilde{M}^i, \quad N^i = \frac{3}{2}\tilde{N}^i, \\
V_a^i &= 3\tilde{V}_a^i, \quad \xi = \sqrt{2}\tilde{\xi}_1, \quad \Omega = \sqrt{2}\tilde{\xi}_2. \quad (\text{B8})
\end{aligned}$$

Also we define $\tilde{A}^{ijk} = \epsilon^{il} \epsilon^{jm} \epsilon^{kn} A_{lmn}$, which implies $B^{ijk} = -\frac{1}{5} \tilde{A}^{ijk}$.

We will now discuss the relation between our results and the discussion in [13].² In Table IV, we see that for each mass dimension we have two sets of fields which transform under the same representation of $SU(2)$. At the lowest mass dimension level, we have A^{ijk} and B^{ijk} fields that transform under **4** representation of $SU(2)$ R -symmetry. In (24), we see that in the transformation rule for A^{ijk} there is no fermionic field that transforms under **5** representation of $SU(2)$. We can write this constraint in superspace, analogous to the discussion in Sec. II, as follows:

$$D_\alpha^{(i} \mathbf{L}^{jkl)a} = 0, \quad (\text{B9})$$

where a runs over 1,2 and is an additional global $SU(2)$ index carried by the superfield $\mathbf{L}^{ijka}$ whose lowest mass dimension components are $L^{ijka} = (A^{ijk}, B^{ijk})$. At each mass dimension, the two fields with the same representation under $SU(2)$ R -symmetry are now placed inside a **2** representation of the additional global $SU(2)$ using the index a , up to field redefinitions. Clearly, the field

²We thank Daniel Butter for useful correspondence regarding this.

redefinitions carried out in the present section are valid even in the case of rigid supersymmetry. Therefore at the level of rigid supersymmetry, relaxed hypermultiplet can be identified as the superfield $\mathbf{L}^{ijka}$. Such an identification was made at the rigid supersymmetry level in [13]. The superfield $\mathbf{L}^{ijka}$ was identified as the well known $O(3)$ superfield with an additional flavor index a . The $O(3)$ superfield falls under the general class of $O(n)$ superfields defined by the constraint:

$$D_\alpha^{(i_0} \mathbf{L}^{i_1 i_2 \dots i_n)} = 0. \quad (\text{B10})$$

In this language, the linear multiplet defined by the constraint (1) is referred to as the $O(2)$ multiplet. Coupling of $O(n)$ multiplets to conformal supergravity was discussed in [13] in harmonic superspace and in [15] in projective superspace.

Thus in [13], it was argued that extension of relaxed hypermultiplet to conformal supergravity would be the same as the $O(3)$ superfield. In this paper, we explicitly coupled the relaxed hypermultiplet to conformal supergravity and indeed, we find that the only way to extend it to conformal supergravity is through an $O(3)$ superfield with an additional flavor $SU(2)$ index.

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