Theorem on vanishing contributions to $\sin^2\theta_W$ and the intermediate mass scale in grand unified theories with trinification symmetry

Chandini Dash^{1,*}, Snigdha Mishra^{1,†} and Sudhanwa Patra^{2,‡}

¹Department of Physics, Berhampur University, Odisha-760007, India

²Department of Physics, Indian Institute of Technology Bhilai, Raipur-492015, India

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We prove that the values of the electroweak mixing angle $\sin^2\theta_W$ and the intermediate mass scale M_I have vanishing contributions due to one-loop, two-loop, and gravitational corrections in grand unified theories which accommodate an intermediate trinification symmetry (G_{333D}) invoked with spontaneous D-parity mechanism operative at mass scale greater than M_I . The proof of the theorem is robust and we verify the results numerically using supersymmetric as well as nonsupersymmetric versions of E_6 -GUT.

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I. INTRODUCTION

The profound discovery of Higgs boson at Large Hadron Collider glorifies the success of the Standard Model (SM) and the importance of spontaneous symmetry breaking. The final step of spontaneous symmetry breaking for all grand unified theories (GUTs) have to proceed through the SM gauge group and to reproduce all SM observables. The theoretical and experimental determination of known SM observables including the electroweak mixing angle $\sin^2\theta_W$ is more meaningful if the theoretical predictions from GUT models are consistent with these experimentally measured values. Toward achieving this goal, many theoretical GUTs models—like $SU(5)$ [1], $SO(10)$ [2–10], E_6 [11–15] etc. with or without supersymmetry (SUSY), have been proposed as prospective theories of nature awaiting experimental feasibility. With the development of string theories [16–18] the attention is now more pointing toward reinvestigating the exceptional group E_6 as a more plausible choice, as it accommodates most of the nice features of the well known GUT groups $SU(5)$, $SU(6)$, and $SO(10)$ through its embedding. The E_6 GUT, started in the late 1970s (just after $SO(10)$ GUT models) is primarily introduced to explain the hierarchy problem, the mysterious anomaly cancellations of the Standard Model and the quantization of electric charge etc. Many of the pressing issues of SM can be adequately addressed here, with key

predictions on achieving gauge unification, stability of proton, strong CP problem, predictions of dark matter candidature, nonzero neutrino masses and mixing etc. Some of the above phenomenological aspects have been discussed for the E_6 gauge model via its maximal subgroups $SO(10) \otimes U(1)$ [19] and $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ [20–27].

The usual GUT predictions on these observables using standard formalism can be derived by solving renormalization group equations (RGEs) by taking into account one-loop [28], two-loop effects [29,30], as well as gravitational [31–34] and GUT-threshold corrections [35–37]. The gravitational and threshold corrections can arise due to the presence of higher dimensional operators or due to the presence of heavy particles contained in large representations sitting at/around GUT scale. Here it is noteworthy to mention that the stability of the intermediate mass scale M_I and $\sin^2\theta_W$ is essential for neutrino mass predictions through seesaw mechanism and other relevant phenomenological observables. In this context, we aim to demonstrate that there is cancellation of all corrections due to one-loop, two-loop and gravitational effects for the intermediate mass scale M_I and the electroweak mixing angle $\sin^2\theta_W$ in a class of GUT models accommodating intermediate trinification symmetry $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ invoked with D-parity (G_{333D}). It has been shown earlier the vanishing contributions [35,38–40] due to one-loop, two-loop, gravitational as well as threshold in case of intermediate Pati-Salam symmetry [2,4,6] with D-parity [41,42]. However, all these corrections survive for the unification mass scale M_U as well as GUT coupling constant.

This paper is organized as follows. In Sec. II, we propose a general theorem along with its proof for vanishing corrections due to one-loop, two-loop and gravitational effect for the intermediate mass scale M_I and the electroweak mixing angle $\sin^2\theta_W$ in a class of GUT models with

*dash25chandini@gmail.com

†mishrasnigdha60@gmail.com

‡sudhanwa@iitbhilai.ac.in

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intermediate G_{333D} symmetry. We verify the theorem numerically in E_6 GUT model with and without supersymmetry in the consequent Secs. III and IV respectively. The last section is devoted to the concluding remarks along with a comment on threshold corrections.

II. THEOREM AND ITS PROOF

Theorem 1. In a class of grand unified theories (GUTs) having trinification symmetry $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \otimes D$ as the highest intermediate symmetry, the electroweak mixing angle $\sin^2 \theta_W$ and the intermediate trinification symmetry breaking scale (M_I) have vanishing contributions due to one-loop, two-loop and gravitational corrections arising from mass scales greater than the intermediate scale (i.e., $\mu > M_I$).

Proof.—In order to prove the theorem, we consider symmetry breaking chain of a GUT model with intermediate trinification symmetry as

$$\begin{aligned} \mathbb{G}_{\text{GUT}} &\xrightarrow{M_U} SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \otimes D(G_{333D}) \\ &\xrightarrow{M_I} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y(G_{321}) \\ &\xrightarrow{M_Z} SU(3)_C \otimes U(1)_Q(G_{31}) \end{aligned} \quad (1)$$

Here, D stands for D-parity known as discrete left-right symmetry. We mainly use $E_6 \equiv \mathbb{G}_{\text{GUT}}$ in proving the theorem numerically, whereas $\mathbb{G}_{\text{GUT}}$ can be any GUT model which can accommodate trinification gauge symmetry as highest intermediate symmetry.

Now to evaluate the loop-effects, we follow the standard procedure by using the renormalization group equations (RGEs)[43]. The evolution of gauge coupling constant $g_i(\mu)$ of an intermediate gauge symmetry $\mathbb{G}_i$ occurring in $\mathbb{G}_{\text{GUT}} \rightarrow G_1 \rightarrow \mathbb{G}_{\text{SM}}$ is given by,

$$\mu \frac{\partial g_i}{\partial \mu} = \frac{b_i}{16\pi^2} g_i^3 + \frac{1}{(16\pi^2)^2} \sum_j B_{ij} g_i^3 g_j^2. \quad (2)$$

After simplification, the known analytic formula derived for one loop as well as two loop RGEs for inverse coupling constant valid from μ to the intermediate scale M (M can be of any scale $> \mu$ where new theory appears) as,

$$\begin{aligned} \frac{1}{\alpha_i(\mu)} &= \frac{1}{\alpha_i(M)} + \frac{b_i}{2\pi} \ln\left(\frac{M}{\mu}\right) \\ &+ \frac{1}{8\pi^2} \sum_j B_{ij} \int_{\mu}^M \alpha_j(\mu) \frac{d\mu}{\mu}. \end{aligned} \quad (3)$$

Here, $\alpha_i = g_i^2/(4\pi)$, where g_i being the coupling constant for the i th gauge group. b_i (B_{ij}) is the one (two)-loop beta coefficients in the mass range $\mu - M$.

The RGEs for evolution of gauge coupling constant in between the mass scale M_Z and M_I are given by

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M_I) + \frac{b_i}{2\pi} \ln\left(\frac{M_I}{M_Z}\right) + \Theta_i \quad (4)$$

with $i = 1Y, 2L, 3C$. However, there is an additional gravitational corrections from M_I and M_U scale emerging from nonrenormalizable higher dimensional operators. The corresponding RGEs for evolution of gauge coupling constants are,

$$\alpha_i^{-1}(M_I) = \alpha_i^{-1}(M_U) + \frac{b'_i}{2\pi} \ln\left(\frac{M_U}{M_I}\right) + \Theta'_i - \Delta_i^{\text{NRO}} \quad (5)$$

where, $i = 3C, 3L, 3R$. The second and third term in the RHS of both Eqs. (4) and (5) represent the one-loop effects and two-loop effects respectively while the fourth term of Eq. (5) is for gravitational contributions. Here b_i (b'_i) is the one-loop beta coefficients derived for particle spectrum within mass range $M_Z - M_I$ ($M_I - M_U$). In the above mass range, the two-loop contributions are Θ_i and Θ'_i , respectively, as given by

$$\begin{aligned} \Theta_i &= \frac{1}{4\pi} \sum_j B_{ij} \ln \frac{\alpha_j(M_I)}{\alpha_j(M_Z)}, \quad (\text{for } M_Z - M_I) \\ \Theta'_i &= \frac{1}{4\pi} \sum_j B'_{ij} \ln \frac{\alpha_j(M_U)}{\alpha_j(M_I)}, \quad (\text{for } M_I - M_U) \end{aligned} \quad (6)$$

with,

$$B_{ij} = \frac{b_{ij}}{b_j}, \quad B'_{ij} = \frac{b'_{ij}}{b'_j}. \quad (7)$$

Here, b_{ij} (b'_{ij}) is the two-loop beta coefficients derived for particle spectrum within mass range $M_Z - M_I$ ($M_I - M_U$). The role of Δ_i^{NRO} in Eq. (5) is to modify the GUT scale boundary condition at $\mu = M_U$ as,

$$\alpha_i(M_U)(1 + \epsilon_i) = \alpha_G \quad \text{with } i = 3C, 3L, 3R. \quad (8)$$

Here ϵ_i is the parameter which induces the gravitational correction. Thus the gravitational contributions Δ_i^{NRO} can be put in simplified form,

$$\Delta_i^{\text{NRO}} = -\frac{\epsilon_i}{\alpha_G} \quad \text{with } i = 3C, 3L, 3R \quad (9)$$

where α_G is the GUT-coupling constant. Here it may be noted that the gravitational contribution is due to the nonrenormalizable term [44] given by

$$\mathbb{L}_{\text{NRO}} = -\frac{\eta}{4M_G} \text{Tr}(F_{\mu\nu} \Phi F^{\mu\nu}) \quad (10)$$

where Φ is the Higgs field responsible for breaking of the GUT symmetry G to G_{333D} at the mass scale M_U . The

gravitational correction parameter ϵ_i depends on η , VEV of ϕ and M_G . Here it is noteworthy to mention that the nonrenormalizable term in the Lagrangian may arise as effects of quantum gravity [32,44] or as a result of compactification of extra dimensions [31]. Thus in Eq. (10), the scale $M_G \simeq M_{\text{Pl}} \simeq 10^{19}$ GeV, if it is due to quantum gravity effect. However, if it emerges as a result of compactification of extra dimensions then $M_G \leq M_{\text{Pl}}$ [31].

Analytical proof for $\sin^2 \theta_W$ Using the standard procedure, the analytic formula for $\sin^2 \theta_W$ due to contributions arising from one-loop, two-loop, and gravitational correction is given by

$$\begin{aligned} \sin^2 \theta_W = & \frac{1}{A_U} \left[\frac{3}{8} A_U + \left(\frac{\alpha_{\text{em}}}{\alpha_s} - \frac{3}{8} \right) B_U \right. \\ & + \frac{\alpha_{\text{em}}(A_U B_I - A_I B_U)}{16\pi} \ln \left(\frac{M_I}{M_Z} \right) \\ & + \frac{1}{A_U} \left[\frac{\alpha_{\text{em}}(A_U K_{\Theta\Theta'} - B_U J_{\Theta\Theta'})}{16\pi} \right] \\ & \left. + \frac{1}{A_U} \left[\frac{\alpha_{\text{em}}(A_U E_0 - B_U E_1)}{8\alpha_G} \right] \right] \quad (11) \end{aligned}$$

The first, second and third squared-bracketed terms in (11) are due to one-loop, two-loop, and gravitational effects respectively. Now in order to prove the theorem, we need to focus on each term separately.

- (i) The one-loop contributions to the electroweak mixing angle $\sin^2 \theta_W$ is given by

$$\begin{aligned} (\sin^2 \theta_W)_{1\text{-loop}} = & \frac{1}{A_U} \left[\frac{3}{8} A_U + \left(\frac{\alpha_{\text{em}}}{\alpha_s} - \frac{3}{8} \right) B_U \right. \\ & \left. + \frac{\alpha_{\text{em}}(A_U B_I - A_I B_U)}{16\pi} \ln \left(\frac{M_I}{M_Z} \right) \right]. \end{aligned}$$

Here the parameters A_I , B_I , A_U , and B_U (as can be inferred from Appendix) contains the one-loop effect at $\mu > M_1$. However in the first term A_U canceled out in both numerator as well as in denominator which leads to 3/8 only. The second term completely vanishes as $B_U = 4b'_{3L} - 4b'_{3R} = 0$ since $b'_{3L} = b'_{3R}$ in all class of GUT models with intermediate trinification symmetry invoked with D-parity (G_{333D}). With $B_U = 0$ the third term contains the parameter $B_I = 5b_{2L} - 5b_Y$ (as has been shown in the Appendix), here we may note that B_I has only one-loop effect at $\mu < M_1$. Thus the modified expression for $\sin^2 \theta_W$ is given by

$$(\sin^2 \theta_W)_{1\text{-loop}} = \frac{3}{8} + \frac{\alpha_{\text{em}}(5b_{2L} - 5b_Y)}{16\pi} \ln \left(\frac{M_I}{M_Z} \right).$$

which shows that $\sin^2 \theta_W$ has vanishing contributions from one-loop effects emerging from $\mu > M_1$.

- (ii) The two-loop contributions to the electroweak mixing angle is given by

$$(\sin^2 \theta_W)_{2\text{-loop}} = \frac{1}{A_U} \left[\frac{\alpha_{\text{em}}(A_U K_{\Theta\Theta'} - B_U J_{\Theta\Theta'})}{16\pi} \right]$$

Now using the similar analysis as in case of one-loop, we can see that the above expression reduces to $\frac{\alpha_{\text{em}} K_{\Theta\Theta'}}{16\pi}$, where $K_{\Theta\Theta'}$ has been defined in the Eq. (A12). However $K_{\Theta\Theta'}$ is independent of one-loop contribution emerging from mass scale $\mu > M_1$ since $\Theta'_{3L} = \Theta'_{3R}$. This is quite natural in all GUT models with conserved D-parity. Thus $\sin^2 \theta_W$ has vanishing contributions from two-loop effects emerging from $\mu > M_1$.

- (iii) The important gravitational contribution to $\sin^2 \theta_W$ is as follows,

$$(\sin^2 \theta_W)_{NRO} = \frac{1}{A_U} \left[\frac{\alpha_{\text{em}}(A_U E_0 - B_U E_1)}{8\alpha_G} \right].$$

Here we may note that for $B_U = 0$, the second term vanishes identically. In the first term A_U canceled out from both numerator and denominator. Again $E_0(= 4\epsilon_{3L} - 4\epsilon_{3R})$ vanishes due to D-parity conservation. As a result of this, there is no effect of gravitational corrections at all to the electroweak mixing angle $\sin^2 \theta_W$ i.e., $(\sin^2 \theta_W)_{NRO} = 0$.

Analytical proof for intermediate mass scale M_1 The analytic formula for intermediate mass scale M_1 due to one-loop, two-loop, and gravitational corrections is read as,

$$\begin{aligned} \ln \left(\frac{M_I}{M_Z} \right) = & \frac{B_U D_S - A_U D_W}{B_U A_I - B_I A_U} + \frac{A_U K_{\Theta\Theta'} - B_U J_{\Theta\Theta'}}{B_U A_I - B_I A_U} \\ & - \frac{2\pi(B_U E_1 - A_U E_0)\alpha_G^{-1}}{B_U A_I - B_I A_U}. \quad (12) \end{aligned}$$

Now in order to prove the theorem for intermediate mass scale, we study the expression term by term.

- (i) One loop contribution to intermediate symmetry breaking scale read as,

$$\left(\ln \frac{M_I}{M_Z} \right)_{1\text{-loop}} = \frac{B_U D_S - A_U D_W}{B_U A_I - B_I A_U}$$

Using the same logic as has been discussed in case of $\sin^2 \theta_W$, for $B_U = 0$, the above expression reduces to

$$\left(\ln \frac{M_I}{M_Z} \right)_{1\text{-loop}} = \frac{D_W}{B_I}.$$

Here the parameter D_W contains experimentally measured values like $\sin^2 \theta_W$ and α_{em} . B_I has been

already shown to be independent of one-loop effects at $\mu > M_I$, hence the proof of the theorem is natural to show the intermediate mass scale M_I has vanishing contributions due to one-loop effect emerging from mass scale $\mu > M_I$.

- (ii) Similarly the two loop effect on M_I is exhibited in the 2nd term of Eq. (12) given as

$$\left(\ln \frac{M_I}{M_Z}\right)_{2\text{-loop}} = \frac{A_U K_{\Theta\Theta'} - B_U J_{\Theta\Theta'}}{B_U A_I - B_I A_U}.$$

Here for $B_U = 0$ the expression reduces to

$$\left(\ln \frac{M_I}{M_Z}\right)_{2\text{-loop}} = -\frac{K_{\Theta\Theta'}}{B_I}.$$

It has already been demonstrated that $K_{\Theta\Theta'}$ and B_I are independent of two-loop contributions emerging from mass scale $\mu > M_I$.

- (iii) The gravitational correction for M_I as noted in the 3rd term of Eq. (12) is as follows,

$$\left(\ln \frac{M_I}{M_Z}\right)_{NRO} = -\frac{2\pi(B_U E_1 - A_U E_0)}{\alpha_G(B_U A_I - B_I A_U)}.$$

Here again for $B_U = 0$ and $E_0 = 0$, we see that

$$\left(\ln \frac{M_I}{M_Z}\right)_{NRO} = 0.$$

Thus the gravitational corrections has no effect on the intermediate mass scale.

The proof of the theorem is independent of the choice of particle content and thus, the proof is robust. It can be generalized to both supersymmetric as well as nonsupersymmetric version of grand unified theories that accommodates intermediate trinification symmetry (G_{333D}).

We shall now show the stability of electroweak mixing angle $\sin^2 \theta_W$ and intermediate mass scale M_I in specific E_6 GUT models with or without supersymmetry by numerical analysis.

III. PREDICTIONS IN SUSY E_6 GUT

We consider first the supersymmetric version of E_6 GUT with intermediate trinification symmetry as,

$$E_6 \otimes SUSY \xrightarrow{M_U} G_{333D} \otimes SUSY \xrightarrow{M_I} G_{SM} \otimes SUSY \xrightarrow{M_Z} SU(3)_C \otimes U(1)_Q \quad (13)$$

At first stage, SUSY- E_6 is broken down to the trinification gauge symmetry G_{333D} at unification scale M_U by assigning nonzero vacuum expectation value (VEV) in the singlet direction $(1, 1, 1) \subset 650_H$ which transforms

evenly under D-parity. This singlet scalar preserves the symmetry between left- and right-handed Higgs field. The next stage of symmetry breaking from G_{333D} to G_{SM} is done by nonzero VEV of $(1, \bar{3}, 3)_{27} \oplus (1, 3, \bar{3})_{\bar{27}}$. The last stage of symmetry breaking G_{SM} to G_{31} is done by the weak doublets $(1, 2, 1)_{27} \oplus (1, 2, -1)_{\bar{27}}$ at M_Z scale reproducing all known SM fermion masses. Here the supersymmetry is broken at the M_Z scale.

The usual MSSM particle spectrum are used for derivation of one-loop, two-loop beta coefficients in the mass range $M_Z - M_I$. The fermions contained in fundamental representation of E_6 i.e., 27_F plus scalar sectors including $\Phi(1, \bar{3}, 3) + \bar{\Phi}(1, 3, \bar{3})$ are considered in mass range $M_I - M_U$. The derived values of one-loop beta coefficients are, $b_{3C} = -3$, $b_{2L} = 1$, $b_Y = 6.66$ in the mass range $M_Z - M_I$ while $b'_{3C} = 0$, $b'_{3L} = b'_{3R} = 3$ in the mass range $M_I - M_U$. Similarly, the two-loop beta-coefficients derived for mass range $M_Z - M_I$ and $M_I - M_U$, respectively, are as follows

$$b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}, \quad b'_{ij} = \begin{pmatrix} 82 & 40 & 24 \\ 40 & 82 & 24 \\ 24 & 24 & 48 \end{pmatrix}.$$

In the present case, the gravitational contribution will arise from the nonzero VEV of the Higgs $\phi(1, 1, 1) \subset 650_H \subset E_6$.

$$\langle \Phi_{650} \rangle = \frac{\langle \phi^0 \rangle}{3\sqrt{2}} \text{Diag} \left\{ \underbrace{2, \dots, 2}_9, \underbrace{-1, \dots, -1}_9, \underbrace{-1, \dots, -1}_9 \right\}$$

From above equation, we have $\epsilon_{3C} = 2\epsilon$, $\epsilon_{3L} = \epsilon_{3R} = -\epsilon$, with $\epsilon = \frac{1}{3\sqrt{2}} \frac{\eta \langle \phi^0 \rangle}{M_G}$, which obviously conserves D-parity. Now using the above values of one-loop beta coefficients, we estimated numerically the values of M_I , M_U , α_G^{-1} , and $\sin^2 \theta_W$ for different values of gravitational corrections in terms of ϵ (we skipped the two loop effects for simplicity) as presented in Table I.

The above results show that the intermediate mass scale M_I as well as $\sin^2 \theta_W$ are not affected by the gravitational correction.

TABLE I. Numerical results for $\sin^2 \theta_W$, M_I , M_U with different values of gravitational correction within SUSY- E_6 GUT with intermediate trinification symmetry.

ϵ	M_I (GeV)	M_U (GeV)	α_G^{-1}	$\sin^2 \theta_W$
0	1.691×10^{16}	5.044×10^{16}	24.0851	0.2329
0.01	1.691×10^{16}	2.225×10^{17}	23.6125	0.2329
0.025	1.691×10^{16}	1.853×10^{18}	22.9375	0.2329

TABLE II. Proof of theorem via numerical results within non-SUSY- E_6 GUT with intermediate trinification symmetry.

ϵ	M_I (GeV)	M_U (GeV)	α_G^{-1}	$\sin^2 \theta_W$
-0.0365	1.18712×10^{13}	5.35202×10^{16}	47.0441	0.2314
-0.034	1.18712×10^{13}	5.18769×10^{17}	48.732	0.2314
-0.0315	1.18712×10^{13}	5.95424×10^{18}	50.5449	0.2314

IV. PREDICTIONS IN NON-SUSY E_6 GUT

The non-SUSY version of E_6 GUT with intermediate trinification symmetry is given by

$$E_6 \xrightarrow{M_U} \mathbb{G}_{333D} \xrightarrow{M_I} \mathbb{G}_{SM} \xrightarrow{M_Z} \mathbb{G}_{31}. \quad (14)$$

We have used 650_H as well as 27_H for spontaneous symmetry breaking while all the fermions are contained in fundamental representation 27_F of E_6 which accommodates all 15-SM component per generation. All the particle content of SM with Higgs boson have been used in the mass range $M_Z - M_I$ while all the fermions belonging to 27_F and the scalar $\Phi(1, \bar{3}, 3)$ are used in the mass range of $M_I - M_U$. The derived values of one-loop beta coefficients are $b_{3C} = -7, b_{2L} = -19/6, b_Y = 41/10$ in the mass range $M_Z - M_I$ while $b'_{3C} = -5, b'_{3L} = b'_{3R} = -9/2$ in the mass range $M_I - M_U$. The two-loop beta-coefficients derived for mass range $M_Z - M_I$ and $M_I - M_U$, respectively, as follows

$$b_{ij} = \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix}, \quad b'_{ij} = \begin{pmatrix} 23 & 20 & 12 \\ 20 & 23 & 12 \\ 12 & 12 & 12 \end{pmatrix} \quad (15)$$

The numerically estimated values of $M_I, M_U, \sin^2 \theta_W$, and α_G^{-1} for different values of ϵ have been presented in Table II. We see that the intermediate mass scale M_I as well as $\sin^2 \theta_W$ are not affected by the gravitational correction. Here the unification mass scale M_U and the GUT gauge coupling constant α_G changes with different values of ϵ .

In the present model the minimal particle content of E_6 is not enough for proper gauge coupling unification [15] at $\epsilon = 0$, which comes out to be the scale beyond Planck energy. However, if we introduce $(1, 8, 8) \subset 650_H$, we achieve the unification at 10^{14} GeV. In order to achieve for a long proton lifetime the unification mass scale has to be enhanced with finite value of ϵ as well as with threshold effects [45].

V. CONCLUSION

In summary, we have proved a theorem on vanishing contributions to the electroweak mixing angle $\sin^2 \theta_W$ and

intermediate mass scale M_I (equivalent to the scale at which the spontaneous breaking of the trinification gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \otimes D$ occurs), due to one-loop, two-loop, and gravitational corrections emerging from higher mass scales ($\mu > M_I$) in a class of grand unified theories which accommodates trinification symmetry invoked with spontaneous D-parity mechanism. We have established the robustness of the proof by considering supersymmetric as well as nonsupersymmetric version of E_6 GUT for demonstration purpose. This proof can be generalized to show that there is no effect of GUT-threshold correction on these parameters arising from the mass scale $\mu > M_I$. In order to prove this, the $SO(10)$ GUT has to be replaced by E_6 GUT and the intermediate Pati-Salam symmetry $\mathbb{G}_{224D}$ has to be replaced by trinification symmetry ($\mathbb{G}_{333D}$). The detailed proof is beyond the scope of this paper which is planned for a separate work.

We, finally, conclude that the origin behind these vanishing contributions to $\sin^2 \theta_W$ and intermediate mass scale M_I are primarily because of: (a) grand unified theories like E_6 GUT that accommodates trinification symmetry as an intermediate breaking symmetry. (b) Due to presence of discrete left-right symmetry (D-parity) and the implications of spontaneous D-parity breaking mechanism [41,42] thereby resulted simplified relations for the proof, (c) Due to key matching condition between gauge couplings, $\alpha_Y^{-1}(M_I) = \frac{1}{5} \alpha_{3L}^{-1}(M_I) + \frac{4}{5} \alpha_{3R}^{-1}(M_I)$. It can also be explained in the GUT models like $SU(9)$ and $SO(18)$ with the trinification symmetry at the intermediate breaking symmetry [46].

With reference to the cosmological issues, it is noteworthy to mention that the present model does not have the domain-wall problem as the intermediate mass scale M_I is high [34] with 10^{13} GeV for non-Susy and 10^{16} GeV for SUSY cases (as shown in Table I, II) [47]. As far as proton decay is concerned, the model with high M_U allows a stable proton.

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APPENDIX: FORMALISM FOR RGES OF GAUGE COUPLINGS IN GUTS WITH TRINIFICATION SYMMETRY

We already proved the theorem showing the remarkable property of E_6 GUT or all possible grand unified theories with $\mathbb{G}_{333D}$ trinification intermediate symmetry on vanishing contributions to the electroweak mixing angle $\sin^2 \theta_W$ and intermediate symmetry breaking scale M_I due to one-loop, two-loop and gravitational corrections (even can be

true for GUT-threshold corrections). The attempts have also been made to show that GUT threshold corrections arising out of super heavy masses or higher dimensional operators identically vanish on $\sin^2 \theta_W$ or the G_{224D} breaking scale within $SO(10)$ models with $\mathcal{G}_{224D}$ Pati-Salam intermediate symmetry [2,4,6] and we rather carried out our analysis in E_6 GUT with $\mathcal{G}_{333D}$ intermediate trinification symmetry. We aim to derive all the necessary analytic formulas which have been used in the text for proof of the theorem. The simple symmetry breaking chain considered here with intermediate trinification symmetry is given by

$$\begin{aligned} \mathbb{G}_{\text{GUT}} &\xrightarrow{M_U} SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \otimes D(G_{333D}) \\ &\xrightarrow{M_I} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y (G_{321}) \\ &\xrightarrow{M_Z} SU(3)_C \otimes U(1)_Q (G_{31}) \end{aligned} \quad (\text{A1})$$

It is also worth it to mention here that b'_i ($i = 3C, 3L, 3R$) are one-loop beta coefficients from mass range $M_I - M_U$, b_i ($i = 3C, 2L, 1Y$) are one-loop beta coefficients from mass range $M_Z - M_I$ while b'_{ij} ($i, j = 3C, 3L, 3R$), b_{ij} ($i, j = 3C, 2L, 3Y$) are two-loop beta coefficients from mass range $[M_I - M_U]$ [$M_Z - M_I$], respectively. We can deduce three key relations including one-loop, two-loop, and gravitational corrections using Eqs. (3), (4) and (5) as,

$$\begin{aligned} \alpha_{3C}^{-1}(M_Z) &= \alpha_G^{-1}(1 + \epsilon_{3C}) + \frac{b_{3C}}{2\pi} \ln\left(\frac{M_I}{M_Z}\right) \\ &+ \frac{b'_{3C}}{2\pi} \ln\left(\frac{M_U}{M_I}\right) + \Theta_{3C} + \Theta'_{3C}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \alpha_{2L}^{-1}(M_Z) &= \alpha_G^{-1}(1 + \epsilon_{3L}) + \frac{b_{2L}}{2\pi} \ln\left(\frac{M_I}{M_Z}\right) \\ &+ \frac{b'_{3L}}{2\pi} \ln\left(\frac{M_U}{M_I}\right) + \Theta_{2L} + \Theta'_{3L}, \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} \alpha_Y^{-1}(M_Z) &= \frac{1}{5} \alpha_G^{-1}(1 + \epsilon_{3L}) + \frac{4}{5} \alpha_G^{-1}(1 + \epsilon_{3R}) \\ &+ \frac{b_Y}{2\pi} \ln\left(\frac{M_I}{M_Z}\right) + \frac{\frac{1}{5} b'_{3L} + \frac{4}{5} b'_{3R}}{2\pi} \ln\left(\frac{M_U}{M_I}\right) \\ &+ \Theta_Y + \frac{\Theta'_{3L} + 4\Theta'_{3R}}{5}, \end{aligned} \quad (\text{A4})$$

We used two more relations derived based on RGEs for the gauge coupling constants as,

$$\begin{aligned} \alpha_{em}^{-1}\left(\sin^2 \theta_W - \frac{3}{8}\right) &= \frac{5}{8} (\alpha_{2L}^{-1}(M_Z) - \alpha_Y^{-1}(M_Z)) \\ 8\left(\alpha_s^{-1} - \frac{3}{8} \alpha_{em}^{-1}\right) &= 8\alpha_{3C}^{-1} - 3\alpha_{2L}^{-1} - 5\alpha_Y^{-1}. \end{aligned} \quad (\text{A5})$$

Now using Eqs. (3), (4), and (5) along with above relation (A5), we obtain the following analytic formulas for intermediate mass scale, unification scale and $\sin^2 \theta_W$ as,

$$\begin{aligned} \ln\left(\frac{M_I}{M_Z}\right) &= \frac{B_U D_S - A_U D_W}{B_U A_I - B_I A_U} + \frac{A_U K_{\Theta\Theta'} - B_U J_{\Theta\Theta'}}{B_U A_I - B_I A_U} \\ &- \frac{2\pi(B_U E_1 - A_U E_0) \alpha_G^{-1}}{B_U A_I - B_I A_U} \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} \ln\left(\frac{M_U}{M_Z}\right) &= \frac{A_I D_W - B_I D_S}{B_U A_I - B_I A_U} + \frac{B_I J_{\Theta\Theta'} - A_I K_{\Theta\Theta'}}{B_U A_I - B_I A_U} \\ &- \frac{2\pi(A_I E_0 - B_I E_1) \alpha_G^{-1}}{B_U A_I - B_I A_U} \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \sin^2 \theta_W &= \frac{1}{A_U} \left[\frac{3}{8} A_U + \left(\frac{\alpha_{em}}{\alpha_s} - \frac{3}{8} \right) B_U \right. \\ &+ \frac{\alpha_{em}(A_U B_I - A_I B_U)}{16\pi} \ln\left(\frac{M_I}{M_Z}\right) \\ &+ \frac{1}{A_U} \left[\frac{\alpha_{em}(A_U K_{\Theta\Theta'} - B_U J_{\Theta\Theta'})}{16\pi} \right] \\ &+ \left. \frac{1}{A_U} \left[\frac{\alpha_{em}(A_U E_0 - B_U E_1)}{8\alpha_G} \right] \right] \end{aligned} \quad (\text{A8})$$

where the relevant parameters used in deriving all these key analytic formulas are as follows,

$$D_S = 16\pi \left[\alpha_s^{-1}(M_Z) - \frac{3}{8} \alpha_{em}^{-1}(M_Z) \right] \quad (\text{A9})$$

$$D_W = 16\pi \alpha_{em}^{-1}(M_Z) \left[\sin^2 \theta_W - \frac{3}{8} \right] \quad (\text{A10})$$

$$\begin{aligned} J_{\Theta\Theta'} &= 2\pi[(8\Theta_{3C} - 3\Theta_{2L} - 5\Theta_Y) \\ &+ (8\Theta'_{3C} - 4\Theta'_{3L} - 4\Theta'_{3R})] \end{aligned} \quad (\text{A11})$$

$$K_{\Theta\Theta'} = 2\pi[(5\Theta_{2L} - 5\Theta_Y) + (4\Theta'_{3L} - 4\Theta'_{3R})] \quad (\text{A12})$$

$$E_I = (8\epsilon_{3C} - 4\epsilon_{3L} - 4\epsilon_{3R}) \quad (\text{A13})$$

$$E_0 = (4\epsilon_{3L} - 4\epsilon_{3R}) \quad (\text{A14})$$

$$A_I = [(8b_{3C} - 3b_{2L} - 5b_Y) - (8b'_{3C} - 4b'_{3L} - 4b'_{3R})] \quad (\text{A15})$$

$$A_U = (8b'_{3C} - 4b'_{3L} - 4b'_{3R}) \quad (\text{A16})$$

$$B_I = [(5b_{2L} - 5b_Y) - (4b'_{3L} - 4b'_{3R})] \quad (\text{A17})$$

$$B_U = (4b'_{3L} - 4b'_{3R}). \quad (\text{A18})$$

These parameters are characteristics of one loop, two loop, and gravitational corrections to the electroweak mixing angle $\sin^2 \theta_W$ and the intermediate mass scale while we focus on two of the parameters B_I and B_U , in most of times, in the analysis. With $b'_{3L} = b'_{3R}$, the factor B_U vanishes

exactly and B_I is independent of one-loop effects operative at mass scale $\mu > M_I$. Similarly, $K_{\Theta\Theta}$ can be shown to be independent of two-loop effects emerging from mass scale

$\mu > M_I$. However, it is found that the unification scale and the GUT gauge coupling constant are fully dependent on these corrections.

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