Phenomenology of the new light Higgs bosons in Gildener-Weinberg models

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Gildener-Weinberg (GW) models of electroweak symmetry breaking are especially interesting because the low mass and nearly Standard Model couplings of the 125 GeV Higgs boson H are protected by approximate scale symmetry. Another important but so far underappreciated feature of these models is that a sum rule bounds the masses of the new charged and neutral Higgs bosons appearing in *all* these models to be below about 500 GeV. Therefore, they are within reach of LHC data currently or soon to be in hand. Also so far unnoticed of these models, certain cubic and quartic Higgs scalar couplings vanish at the classical level. This is due to spontaneous breaking of the scale symmetry. These couplings become nonzero from explicit scale breaking in the Coleman-Weinberg loop expansion of the effective potential. In a two-Higgs doublet GW model, we calculate $\lambda_{HHH} \simeq 2(\lambda_{HHH})_{\text{SM}} = 64$ GeV. This corresponds to $\sigma(pp \rightarrow HH) \simeq 15\text{--}20$ fb, its *minimum* value for $\sqrt{s} = 13\text{--}14$ TeV at the LHC. It will require at least the 27 TeV HE-LHC to observe this cross section. We also find $\lambda_{HHHH} \simeq 4(\lambda_{HHHH})_{\text{SM}} = 0.129$, whose observation in $pp \rightarrow HHH$ requires a 100 TeV collider. Because of the above-mentioned sum rule, these results apply to *all* GW models. In view of this unpromising forecast, we stress that LHC searches for the new relatively light Higgs bosons of GW models are by far the surest way to test them in this decade.

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I. SYNOPSIS

Section II of this paper reviews the Gildener-Weinberg (GW) mechanism for producing a model of a naturally light and aligned Higgs boson H in multi-Higgs-scalar models of electroweak symmetry breaking [1]. This is done in the context of a two-Higgs doublet model (2HDM) due to Lee and Pilaftsis [2]. The tree-level Higgs potential in GW models is scale invariant, but that symmetry can be spontaneously broken, resulting in H as a massless dilaton with exactly Standard Model (SM) couplings to gauge bosons and fermions. This scale symmetry is explicitly broken in one-loop order of the Coleman-Weinberg effective potential [3], resulting in $M_H^2 > 0$ but only small deviations from its exact SM couplings. An important corollary of the formula for M_H^2 is a sum rule for the masses of the additional Higgs scalars, generically $\mathcal{H}$. In *any* GW model of electroweak breaking in which the only weak

bosons are $W^\pm$ and Z^0 and the only heavy fermion is the top quark, the sum rule in the first-order loop-perturbation theory is [2,4,5]

$$\left(\sum_{\mathcal{H}} M_{\mathcal{H}}^4 \right)^{1/4} = 540 \text{ GeV}. \quad (1)$$

In the GW-2HDM model, the additional Higgs bosons are a charged pair, $H^\pm$, and one CP -even and one CP -odd scalar, which we call H_2 and A . This sum rule has profound consequences for the phenomenology of GW models that this paper emphasizes. For example, in a search for these new Higgses, care must be taken in using the sum rule to estimate the light scalar's mass when the other scalar masses are assumed to exceed 400–500 GeV.

In Sec. III, we discuss features of the cubic and quartic Higgs boson self-couplings peculiar to GW models. As a consequence of unbroken scale invariance in the classical Higgs potential, certain of them vanish. These couplings do become nonzero once the scale symmetry is explicitly broken. We calculate the most important of these, finding that the experimentally most relevant ones, λ_{HHH} and λ_{HHHH} , imply $\sigma(pp \rightarrow HH)$ and $\sigma(pp \rightarrow HHH)$ too small to detect at even the High-Luminosity (HL) LHC [6]. Again, because of the sum rule (1), this conclusion is

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true in *all* GW models of electroweak symmetry breaking, regardless of their Higgs sector.

This leads to Sec. IV, where we refocus on direct searches at the LHC for the new light Higgs bosons of GW models. We briefly summarize these Higgses' main search channels and the status of these searches. Substantial progress is in reach of data in hand or to be collected in the near future. There is nothing exotic about these searches; what is required for discovery or exclusion is greater sensitivity at relatively low masses.

II. THE TWO-HIGGS DOUBLET MODEL

In 1976, Gildener and Weinberg proposed a scheme, based on broken scale symmetry, to generate a light Higgs boson in multiscalar models of electroweak symmetry breaking. In essence, their motivation was to generalize the work of Coleman and Weinberg [3] to completely general electroweak models, with arbitrary gauge groups and representations of the fermions and scalars. What GW did not appreciate then—there was no reason for them to—was that their Higgs boson was also aligned [7]. That is, of all the scalars, its couplings to gauge bosons and fermions were exactly those of the single Higgs boson of the SM [8]. Like the Higgs boson's mass, its alignment is protected by the approximate scale symmetry [5].

GW assumed an electroweak Lagrangian whose Higgs potential V_0 has only quartic interactions. With no quadratic nor cubic Higgs couplings and, assuming that gauge boson and fermion masses arise entirely from their couplings to Higgs scalars, the GW theory is scale invariant at the classical level. This Lagrangian may, however, have a nontrivial extremum. If it does, it is along a ray in scalar-field space and it is a flat minimum if the quartic couplings satisfy certain positivity conditions. Thus, scale symmetry is spontaneously broken at tree level, and there is a massless (Goldstone) dilaton, H , which GW called the “scalón.” Higgs alignment is a simple consequence of the linear combination of fields composing H having the *same form* as the Goldstone bosons $w^\pm$ and z that become the longitudinal components of the $W^\pm$ and Z bosons; see Eqs. (8) below.

Importantly, scale symmetry is explicitly broken by the first-order term V_1 in the Coleman-Weinberg loop expansion of the effective scalar potential [3]: $V_0 + V_1$ can have a deeper minimum than the trivial one at zero fields. If it does, it occurs at a specific vacuum expectation value (VEV) $\langle H \rangle = v$, explicitly breaking scale invariance. Then M_H and all other masses in the theory are proportional to v . The GW scheme is the only one we know in which the entire breaking of scale and electroweak symmetries is caused by the same electroweak operator, namely, $\langle H \rangle$. Hence, the dilaton decay constant $f = v$ [9], which we take to be 246 GeV.

In 2012, Lee and Pilaftsis (LP) proposed a simple 2HDM model of the GW mechanism employing the Higgs doublets [2]:

$$\Phi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_i^+ \\ \rho_i + ia_i \end{pmatrix}, \quad i = 1, 2. \quad (2)$$

Here, ρ_i and a_i are neutral CP -even and -odd fields. Their potential is

$$\begin{aligned} V_0(\Phi_1, \Phi_2) = & \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2}\lambda_5((\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2). \end{aligned} \quad (3)$$

All five quartic couplings are real so that V_0 is CP invariant as well. This potential is consistent with a $\mathcal{Z}_2$ symmetry that prevents tree-level flavor-changing interactions among fermions, ψ , induced by neutral scalar exchange [10]:

$$\begin{aligned} \Phi_1 &\rightarrow -\Phi_1, & \Phi_2 &\rightarrow \Phi_2, \\ \psi_L &\rightarrow -\psi_L, & \psi_{uR} &\rightarrow \psi_{uR}, & \psi_{dR} &\rightarrow \psi_{dR}. \end{aligned} \quad (4)$$

This is the usual type-I 2HDM [11], but with Φ_1 and Φ_2 interchanged; we refer henceforth to this version of the model as the GW-2HDM. This choice of Higgs couplings differs from LP's choice of type II [2]. It was made to remain consistent with limits from CMS [12] and ATLAS [13] on charged Higgs decay into $t\bar{b}$. The limits from these papers are consistent with $\tan\beta \lesssim 0.5$ for $M_{H^\pm} \lesssim 500$ GeV. This range of $\tan\beta$ also suppresses $gg \rightarrow A(H') \rightarrow \bar{b}b, \bar{t}t$, where $A(H')$ is a CP -odd (-even) Higgs, relative to a heavy Higgs boson H with SM couplings. See the discussion and references in Ref. [5].

The potential V_0 can have a flat minimum along the ray

$$\Phi_{1\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi c_\beta \end{pmatrix}, \quad \Phi_{2\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi s_\beta \end{pmatrix}. \quad (5)$$

Here $\phi > 0$ is any real mass scale, $c_\beta = \cos\beta$ and $s_\beta = \sin\beta$. The nontrivial tree-level extremal conditions are (for $\beta \neq 0, \pi/2$)

$$\lambda_1 c_\beta^2 + \frac{1}{2}\lambda_{345} s_\beta^2 = 0, \quad \lambda_2 s_\beta^2 + \frac{1}{2}\lambda_{345} c_\beta^2 = 0, \quad (6)$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$. Scale symmetry is spontaneously, but not yet explicitly, broken. Note that $V_{0\beta} = V_0(\Phi_{1\beta}, \Phi_{2\beta}) = 0$, degenerate with the trivial vacuum. As explained in Sec. III, $V_{0\beta} = 0$ is a consequence of the scale-invariance of the classical potential V_0 . The squared “mass” matrices of the CP -odd, charged, and CP -even scalars are given by

$$\mathcal{M}_S^2 = -\lambda_S \phi^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix}, \quad (7)$$

where the subscript $S = H_{0^-}, H^\pm$, and H_{0^+} . In terms of the quartic couplings in the Higgs potential, they are $\lambda_{H_{0^-}} = \lambda_5$; $\lambda_{H^\pm} = \frac{1}{2}(\lambda_4 + \lambda_5) = \frac{1}{2}\lambda_{45}$; and $\lambda_{H_{0^+}} = \lambda_{345}$. All λ_S are negative to ensure non-negative eigenvalues of the matrices. The respective eigenvectors and eigenvalues are

$$\begin{aligned} \begin{pmatrix} z \\ A \end{pmatrix} &= \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad M_z^2 = 0, \quad M_A^2 = -\lambda_5 \phi^2; \\ \begin{pmatrix} w^\pm \\ H^\pm \end{pmatrix} &= \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \quad M_{w^\pm}^2 = 0, \quad M_{H^\pm}^2 = -\frac{1}{2}\lambda_{45}\phi^2; \\ \begin{pmatrix} H \\ H' \end{pmatrix} &= \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}, \quad M_H^2 = 0, \quad M_{H'}^2 = -\lambda_{345}\phi^2. \end{aligned} \quad (8)$$

The one-loop effective potential, presented in Ref. [2], is given by

$$\begin{aligned} V_1 = \frac{1}{64\pi^2} \left[6M_W^4 \left(-\frac{5}{6} + \ln \frac{M_W^2}{\Lambda_{\text{GW}}^2} \right) + 3M_Z^4 \left(-\frac{5}{6} + \ln \frac{M_Z^2}{\Lambda_{\text{GW}}^2} \right) \right. \\ \left. + M_{H'}^4 \left(-\frac{3}{2} + \ln \frac{M_{H'}^2}{\Lambda_{\text{GW}}^2} \right) + M_A^4 \left(-\frac{3}{2} + \ln \frac{M_A^2}{\Lambda_{\text{GW}}^2} \right) \right. \\ \left. + 2M_{H^\pm}^4 \left(-\frac{3}{2} + \ln \frac{M_{H^\pm}^2}{\Lambda_{\text{GW}}^2} \right) - 12m_t^4 \left(-1 + \ln \frac{m_t^2}{\Lambda_{\text{GW}}^2} \right) \right], \end{aligned} \quad (9)$$

where Λ_{GW} is the GW renormalization scale [related to the Higgs VEV v by Eqs. (40) and (41) in LP]. The background field-dependent masses in V_1 are

$$\begin{aligned} M_W^2 &= \frac{1}{2}g^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2), \\ M_Z^2 &= \frac{1}{2}(g^2 + g'^2)(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2), \\ M_A^2 &= -2\lambda_5(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2), \\ M_{H^\pm}^2 &= -\lambda_{45}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2), \\ M_{H'}^2 &= -2\lambda_{345}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2), \\ m_t^2 &= \Gamma_t^2\Phi_1^\dagger\Phi_1, \end{aligned} \quad (10)$$

where g, g' are the electroweak $SU(2)$ and $U(1)$ gauge couplings and $\Gamma_t = \sqrt{2}m_t/v_1 = \sqrt{2}m_t/v \cos \beta$ is the Higgs-Yukawa coupling of the top quark.

The nontrivial extremal conditions for $V_0 + V_1$ are [2]

$$\begin{aligned} \left. \frac{\partial(V_0 + V_1)}{\partial \rho_1} \right|_\langle &\propto \lambda_1 c_\beta^2 + \frac{1}{2}\lambda_{345} s_\beta^2 + \Delta \hat{t}_1 / 64\pi^2 = 0, \\ \left. \frac{\partial(V_0 + V_1)}{\partial \rho_2} \right|_\langle &\propto \lambda_2 s_\beta^2 + \frac{1}{2}\lambda_{345} c_\beta^2 + \Delta \hat{t}_2 / 64\pi^2 = 0, \end{aligned} \quad (11)$$

where $\langle \rangle$ means that the derivatives of $V_0 + V_1$ are evaluated at the vacuum expectation values of the fields, and

$$\begin{aligned} \Delta \hat{t}_i = \frac{4}{v^4} \left[2M_W^4 \left(3 \ln \frac{M_W^2}{\Lambda_{\text{GW}}^2} - 1 \right) + M_Z^4 \left(3 \ln \frac{M_Z^2}{\Lambda_{\text{GW}}^2} - 1 \right) \right. \\ \left. + M_{H'}^4 \left(\ln \frac{M_{H'}^2}{\Lambda_{\text{GW}}^2} - 1 \right) + M_A^4 \left(\ln \frac{M_A^2}{\Lambda_{\text{GW}}^2} - 1 \right) \right. \\ \left. + 2M_{H^\pm}^4 \left(\ln \frac{M_{H^\pm}^2}{\Lambda_{\text{GW}}^2} - 1 \right) - 12m_t^4 \left(\ln \frac{m_t^2}{\Lambda_{\text{GW}}^2} - \frac{1}{2} \right) \delta_{i1} \right]_\langle. \end{aligned} \quad (12)$$

For nontrivial extrema with $\beta \neq 0, \pi/2$, these conditions lead to a deeper minimum than the zeroth-order ones, $(V_0 + V_1)_{\min} < V_{0\beta} = V_0(0) + V_1(0) = 0$. This minimum occurs at a particular value v of the scale ϕ which, as we have said, is identified as the electroweak breaking scale, $v = 246$ GeV. The VEVs of Φ_1 and Φ_2 are $v_1 = v \cos \beta$ and $v_2 = v \sin \beta$, with $\tan \beta = v_2/v_1$ as usual in 2HDM.

The CP -odd and charged Higgs bosons' masses receive no contribution from V_1 and, so, they are given by Eqs. (8) with $\phi = v$. The CP -even masses, however, receive important corrections from V_1 . The eigenvectors H_1 and H_2 are

$$\begin{aligned} H_1 &= c_\delta H - s_\delta H' = c_{\beta'} \rho_1 + s_{\beta'} \rho_2, \\ H_2 &= s_\delta H + c_\delta H' = -s_{\beta'} \rho_1 + c_{\beta'} \rho_2, \end{aligned} \quad (13)$$

where $\beta' = \beta - \delta$, $c_{\beta'} = \cos \beta'$, etc. The angle δ measures the departure of the Higgs boson H_1 from perfect alignment, and it should be small. Furthermore, the accuracy of first-order perturbation theory requires $|\delta/\beta| \ll 1$. Both these criteria are met in calculations with a wide range of input parameters; they are illustrated in Fig. 1. From now on we refer interchangeably to the 125 GeV Higgs boson as H_1 or H , as clarity requires. Its mass is given by [1,2,5]

$$\begin{aligned} M_{H_1}^2 &\cong M_H^2 \\ &= \frac{1}{8\pi^2 v^2} (6M_W^4 + 3M_Z^4 + M_{H'}^4 + M_A^4 + 2M_{H^\pm}^4 - 12m_t^4). \end{aligned} \quad (14)$$

In accord with first-order perturbation theory, all the masses on the right side of this formula are obtained from zeroth-order perturbation theory, i.e., from V_0 plus gauge and Yukawa interactions, with $\phi = v$. As we see in Fig. 1, the Higgs masses M_H and M_{H_1} derived from Eq. (14) and from diagonalizing the one-loop mass matrix $\mathcal{M}_{H_{0^+}}$, respectively, are extremely close, as they should be.

This formula can be used in two related ways. First, assuming that there are no other heavy fermions and weak bosons, it implies a sum rule on all the new scalar masses in this GW-2HDM [2,4,5]:

$$(M_{H'}^4 + M_A^4 + 2M_{H^\pm}^4)^{1/4} = 540 \text{ GeV}. \quad (15)$$

The sum rule is illustrated in Fig. 2 for $M_H \cong M_{H_1} = 125$ GeV and $M_{H^\pm} = M_{S_1}$, where $M_{S_1} = M_A$ or $M_{H'}$; the

FIG. 1. Left: The CP -even Higgs one-loop mass eigenvalues M_{H_1} and M_{H_2} , the tree-level mass $M_{H'} = \sqrt{-\lambda_{345}}v$ and the one-loop mass M_H from Eq. (14) as functions of $\lambda_3 = (2M_{H^\pm}^2 - M_{H'}^2)/v^2$. Here, $\tan\beta = 0.50$ and $M_{H^\pm} = M_A = 390$ GeV corresponding to $\lambda_4 = \lambda_5 = -2.513$. The input $H \cong H_1$ mass is $M_H = 125.0$ GeV, the corresponding initial $M_{H'} = 353$ GeV and $\lambda_3 = 2.966$. $M_{H'}$ vanishes at $\lambda_3 = 2M_{H^\pm}^2/v^2 = 5.027$. Right: The angle $\delta = \beta - \beta'$ measuring the deviation from perfect alignment of H_1 and the ratio δ/β for $\beta = 0.4637$. The procedure used in creating these figures is spelled out in the Appendix of Ref. [5].

FIG. 2. The mass of the neutral Higgs $S_2 = H'$ ($S_2 = M_A$) as a function of the common mass of $H^\pm$ and the other neutral Higgs, $S_1 = A$ ($S_1 = H'$), from Eqs. (14) and (15) with $M_H = 125$ GeV. Note the considerable sensitivity of M_{S_2} to small changes in $M_{H^\pm} = M_{S_1}$ when it is large. From Ref. [5].

mass of the other neutral scalar, M_{S_2} , is plotted against $M_{H^\pm} = M_{S_1}$. The smallness of δ in Fig. 1 and the magnitude of Higgs couplings we obtain in Sec. III give us confidence that the one-loop approximation (14) is reliable. Still, we would not be surprised if higher-order corrections change the right side of Eq. (15) by $\mathcal{O}(100 \text{ GeV})$. The important point is that the sum rule tells us that new Higgs bosons should be found at surprisingly low masses. To repeat, this sum rule holds in *any* GW model of electroweak breaking in which the only weak bosons are W and Z and the only heavy fermion is the top quark. Thus, the larger the Higgs sector, the lighter will be the masses of at least some of the new Higgs bosons expected in a GW model.

FIG. 3. The tree-approximation mass $M_{H'}$ of the CP -even Higgs calculated from the sum rule (15) and the larger eigenvalue M_{H_2} of the one-loop corrected CP -even mass matrix $\mathcal{M}_{H_0^+}$. Both are calculated as a function of $M_{H^\pm} = M_A$. $M_{H'}$ starts to dive to zero at $M_{H^\pm} \cong 370$ GeV and becomes zero at $M_{H^\pm} \cong 410.22$ GeV.

Second, as an instructive example in the present model, we assume that $M_{H^\pm} = M_A$ and imagine searching for $H_2 \simeq H'$. (The assumption $M_{H^\pm} = M_A$ is motivated by the fact that it makes the contribution to the T parameter from the scalars vanish identically [14,15].) Due to the sum-rule constraint in Eq. (15), the mass of $M_{H'}$ is very sensitive to small changes in $M_{H^\pm}$ when it is large. Figure 2 suggests we can use the sum rule until $M_{H'}$ starts to dive to zero. To be quantitative about this, Fig. 3 shows $M_{H'}$ and M_{H_2} as a function of $M_{H^\pm} = M_A$ over the range allowed by the sum rule.¹ The two masses are very nearly equal up to $M_{H^\pm} = 370$ GeV. At that mass,

¹The only model parameter that enters this calculation is $\tan\beta$; Fig. 3 is practically independent of $\tan\beta$.

$M_{H'} \cong M_{H_2} = 412$ GeV and, beyond it, $M_{H'}$ starts its dive. The most sensible thing to do, in our opinion, is to use the large CP -even mass eigenvalue, M_{H_2} , over the entire considered range of $M_{H^\pm} = M_A$.

We will do this for our estimates of the scalars' production cross sections and decay branching ratios in Sec. IV. We recommend this approach for searches by ATLAS and CMS. For example, in a search involving the three GW Higgs bosons (say, $pp \rightarrow A \rightarrow ZH_2$ and $pp \rightarrow H^\pm \rightarrow W^\pm H_2$, with $H_2 \rightarrow b\bar{b}$), one could use ellipsoidal search regions in $(M_{H^\pm}, M_A, M_{H_2})$ space roughly consistent with $M_{H^\pm} = M_A$ and the sum rule and calculate the model's predicted $\sigma \cdot BR$'s accordingly. Therefore, as with H and H_1 , we refer henceforth to the heavier CP -even scalar as H' or H_2 , as clarity or the situation requires.

III. TRIPLE AND QUARTIC HIGGS COUPLINGS

In GW models of electroweak symmetry breaking, the tree-level triple-scalar couplings involving two or three of the Goldstone bosons $H, z, w^\pm$ vanish, as do the quartic couplings involving three or four of them. This is unlike any other multi-Higgs model. The reason for this, of course, must be scale invariance of the tree-level Lagrangian, in particular, that the potential V_0 contains only quartic couplings. But how does it work? We show how in this section. Then we calculate at one-loop order the

triple-scalar couplings involving at least one $H \cong H_1$ and the quartic coupling $\lambda_{H_1 H_1 H_1 H_1}$.

The way to see simply why certain scalar couplings vanish is to write V_0 in the “aligned basis”:

$$\Phi = \Phi_1 c_\beta + \Phi_2 s_\beta, \quad \Phi' = -\Phi_1 s_\beta + \Phi_2 c_\beta. \quad (16)$$

On the ray Eq. (5) on which V_0 has nontrivial extrema, these fields are

$$\Phi_\beta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad \Phi'_\beta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (17)$$

where $\phi \in (0, \infty)$ is a constant mass scale. Then, in terms of the tree-level mass-eigenstate scalars, the fields Φ and Φ' are (after shifting $H \rightarrow H + \phi$)

$$\begin{aligned} \Phi &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}w^+ \\ \phi + H + iz \end{pmatrix}, \\ \Phi' &= \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H' + iA \end{pmatrix}. \end{aligned} \quad (18)$$

Rewritten in terms of quartic polynomials in Φ and Φ' , Eq. (3) becomes (with $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$, etc.)

$$\begin{aligned} V_0 &= [\lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \lambda_{345} s_\beta^2 c_\beta^2](\Phi^\dagger \Phi)^2 + [(2\lambda_2 s_\beta^2 + \lambda_{345} c_\beta^2) - (2\lambda_1 c_\beta^2 + \lambda_{345} s_\beta^2)]s_\beta c_\beta (\Phi^\dagger \Phi)(\Phi^\dagger \Phi' + \Phi'^\dagger \Phi) \\ &\quad + [2(\lambda_1 + \lambda_2 - \lambda_{345})s_\beta^2 c_\beta^2 + \lambda_3](\Phi^\dagger \Phi)(\Phi'^\dagger \Phi') + [2(\lambda_1 + \lambda_2 - \lambda_{345})s_\beta^2 c_\beta^2 + \lambda_4](\Phi'^\dagger \Phi')(\Phi^\dagger \Phi) \\ &\quad + \frac{1}{2}[2(\lambda_1 + \lambda_2 - \lambda_{345})s_\beta^2 c_\beta^2 + \lambda_5][(\Phi^\dagger \Phi')^2 + (\Phi'^\dagger \Phi)^2] + [(2\lambda_2 c_\beta^2 + \lambda_{345} s_\beta^2) - (2\lambda_1 s_\beta^2 + \lambda_{345} c_\beta^2)]s_\beta c_\beta (\Phi'^\dagger \Phi')(\Phi^\dagger \Phi' + \Phi'^\dagger \Phi) \\ &\quad + [\lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \lambda_{345} s_\beta^2 c_\beta^2](\Phi'^\dagger \Phi')^2. \end{aligned} \quad (19)$$

By virtue of its scale invariance, V_0 is a homogeneous polynomial of degree four:

$$\begin{aligned} V_0 &= \frac{1}{4} \sum_{i=1}^2 \left[\Phi_i^\dagger \frac{\partial V_0}{\partial \Phi_i^\dagger} + \frac{\partial V_0}{\partial \Phi_i} \Phi_i \right] \\ &= \frac{1}{4} \left[\Phi^\dagger \frac{\partial V_0}{\partial \Phi^\dagger} + \frac{\partial V_0}{\partial \Phi} \Phi + \Phi'^\dagger \frac{\partial V_0}{\partial \Phi'^\dagger} + \frac{\partial V_0}{\partial \Phi'} \Phi' \right]. \end{aligned} \quad (20)$$

Thus, V_0 vanishes at *any* extremum, in particular for $\Phi_\beta = (0, \phi)/\sqrt{2}$ and $\Phi'_\beta = (0, 0)$, the flat direction associated with spontaneous scale symmetry breaking. We know that the conditions for the nontrivial extrema of V_0 are those in Eq. (6). It follows that the coefficients of $(\Phi^\dagger \Phi)^2$ and $(\Phi^\dagger \Phi)(\Phi^\dagger \Phi' + \Phi'^\dagger \Phi)$ terms in V_0 vanish. It is easy to see why these coefficients, C_1 and C_2 , had to vanish. On the ray Φ_β, Φ'_β ,

$$\left. \frac{\partial V_0}{\partial \Phi} \right|_{\Phi_\beta, \Phi'_\beta} = 2C_1 \Phi_\beta^\dagger (\Phi_\beta^\dagger \Phi_\beta), \quad (21)$$

$$\left. \frac{\partial V_0}{\partial \Phi'} \right|_{\Phi_\beta, \Phi'_\beta} = C_2 (\Phi_\beta^\dagger \Phi_\beta) \Phi_\beta^\dagger. \quad (22)$$

Neither operator vanishes; hence, their coefficients must.² This would not have happened had V_0 also contained polynomials of degree less than four. That is, spontaneously broken scale invariance is the reason for the vanishing Goldstone boson couplings at tree level. And it is obvious that this analysis using homogeneous polynomials of fourth degree generalizes to *any* GW model of the electroweak interactions.

²Of course, $C_1 = C_2 = 0$ implies the conditions of Eq. (6).

Using the tree-level extremal conditions, the nonzero coefficients in V_0 are simplified by using

$$2(\lambda_1 + \lambda_2 - \lambda_{345})s_\beta^2 c_\beta^2 = -\lambda_{345}, \quad (23)$$

$$[(2\lambda_2 c_\beta^2 + \lambda_{345} s_\beta^2) - (2\lambda_1 s_\beta^2 + \lambda_{345} c_\beta^2)]s_\beta c_\beta = -2\lambda_{345} \cot 2\beta, \quad (24)$$

$$\lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \lambda_{345} s_\beta^2 c_\beta^2 = -2\lambda_{345} \cot^2 2\beta. \quad (25)$$

Then,

$$\begin{aligned} V_0(\text{cubic}) = & -\frac{1}{2}\lambda_{45}vH[(H')^2 + A^2 + 2H^+H^-] \\ & -\frac{1}{2}\lambda_{35}v[H'(HH' + zA + w^+H^- + H^+w^-) + A(HA - zH') + iA(w^+H^- - H^+w^-)] \\ & -\frac{1}{2}\lambda_{34}v[H'(HH' + zA + w^+H^- + H^+w^-) - A(HA - zH') - iA(w^+H^- - H^+w^-)] \\ & -\lambda_{345} \cot 2\beta vH'[(H')^2 + A^2 + 2H^+H^-]. \end{aligned} \quad (27)$$

The quartic terms are

$$\begin{aligned} V_0(\text{quartic}) = & -\frac{1}{4}\lambda_{45}(H^2 + z^2 + 2w^+w^-)[(H')^2 + A^2 + 2H^+H^-] \\ & -\frac{1}{4}\lambda_{35}[(HH' + zA + 2w^+H^-)(HH' + zA + 2H^+w^-) + (HA - zH')^2 + 2i(HA - zH')(w^+H^- - H^+w^-)] \\ & -\frac{1}{4}\lambda_{34}\left[\frac{1}{2}(HH' + zA + 2w^+H^-)^2 + \frac{1}{2}(HH' + zA + 2H^+w^-)^2 - (HA - zH')^2 - 2i(HA - zH')(w^+H^- - H^+w^-)\right] \\ & -\lambda_{345} \cot 2\beta[(H')^2 + A^2 + 2H^+H^-][HH' + zA + w^+H^- + H^+w^-] \\ & -\frac{1}{2}\lambda_{345} \cot^2 2\beta[(H')^2 + A^2 + 2H^+H^-]^2. \end{aligned} \quad (28)$$

Recall from Eq. (7) that $-\lambda_{345} = M_{H'}^2/v^2$, $-\lambda_{45} = 2M_{H^\pm}^2/v^2$ and $-\lambda_5 = M_A^2/v^2$.

We turn to the one-loop corrections, focusing on the triple-scalar couplings involving the 125 GeV Higgs boson, $H \cong H_1$, and the quartic coupling $\lambda_{H_1 H_1 H_1 H_1}$. For brevity, we include only those cubic couplings of H_1 with itself and with H_2 . The $H_1 AA$ and $H_1 H^+ H^-$ couplings are similar to $H_1 H_2 H_2$, as may be inferred from the tree-level cubics in Eq. (27) and Table I below. There are two types of one-loop corrections: (i) those to V_0 obtained by writing the zeroth-order CP -even fields in terms of H_1 and H_2 , Eqs. (13), and by using the one-loop extremal conditions, Eqs. (11); (ii) those obtained from V_1 in Eq. (9) by isolating the coefficients of H^3 , $H^2 H'$, etc.

³Of course, the electroweak Goldstone fields $w^\pm, z$ are absent in the unitary gauge but must be retained in renormalizable gauges.

$$\begin{aligned} V_0 = & -\lambda_{45}(\Phi^\dagger \Phi)(\Phi'^\dagger \Phi') - \lambda_{35}(\Phi^\dagger \Phi')(\Phi'^\dagger \Phi) \\ & -\frac{1}{2}\lambda_{34}[(\Phi^\dagger \Phi')^2 + (\Phi'^\dagger \Phi)^2] \\ & -2\lambda_{345} \cot 2\beta(\Phi'^\dagger \Phi')(\Phi^\dagger \Phi' + \Phi'^\dagger \Phi) \\ & -2\lambda_{345} \cot^2 2\beta(\Phi'^\dagger \Phi')^2. \end{aligned} \quad (26)$$

From this, the masses in Eq. (7) may be read off from the first three terms.

With foreknowledge, we now put $\phi = v = 246$ GeV. Then the nonzero cubic terms in the tree-level potential, written in terms of mass eigenstate scalars of V_0 , are³

(i) With ρ_i shifted by v_i , the cubic CP -even terms in V_0 are

$$\begin{aligned} V_0(\text{cubic}) = & \lambda_1 v_1 \rho_1^3 + \lambda_2 v_2 \rho_2^3 \\ & + \frac{1}{2}\lambda_{345}(v_1 \rho_1 \rho_2^2 + v_2 \rho_2 \rho_1^2) \\ = & -\lambda_{345}v(H(H')^2 + (H')^3 \cot 2\beta) \\ & -\frac{\Delta \hat{t}_1}{64\pi^2 c_\beta^2}[vc_\beta(Hc_\beta - H's_\beta)^3] \\ & -\frac{\Delta \hat{t}_2}{64\pi^2 s_\beta^2}[vs_\beta(Hs_\beta + H'c_\beta)^3]. \end{aligned} \quad (29)$$

Our convention for the triple and quartic couplings of H_1 , for example, is that they are the coefficients of H_1^3 and H_1^4 in these two types of corrections. Then,

TABLE I. Selected cubic and quartic couplings of the 125 GeV Higgs boson. Input masses are $M_H \cong M_{H_1}$ and $M_{H^\pm} = M_A$, with $M_{H'}$ taken from the sum rule Eq. (15) as explained in the text; M_{H_2} is the corresponding CP -even eigenvalue at one-loop order; $\tan\beta = 0.50$, and the misalignment angle $\delta = 0.0039, 0.0115, 0.0323$ for $M_{H^\pm} = 200, 400, 410$ GeV. Couplings $\lambda^{(0)}$ and $\lambda^{(1)}$ are contributions from the one-loop improved V_0 and full one-loop V_1 potentials. Comparisons are made to the Standard Model [$\kappa_\lambda = \lambda_{H_1 H_1 H_1}^{(0)+(1)}/(\lambda_{HHH})_{\text{SM}}$ and $\mu_\lambda = \lambda_{H_1 H_1 H_1 H_1}^{(0)+(1)}/(\lambda_{HHHH})_{\text{SM}}$] or to tree-level values ($\lambda_{H_1 H_2 H_2}^{(0)+(1)}/\lambda_{HH'H'}$). Masses and cubic couplings are in GeV units.

$M_H \cong M_{H_1}$	$M_{H^\pm} = M_A$	$M_{H'} (M_{H_2})$	$\lambda_{H_1 H_1 H_1}^{(0)+(1)}$	κ_λ
125	200	532 (534)	51.9	1.64
125	400	301 (314)	66.6	2.10
125	410	115 (214)	115	3.63
$\lambda_{H_1 H_1 H_2}^{(0)+(1)}$	$\lambda_{HH'H'}$	$\lambda_{H_1 H_2 H_2}^{(0)+(1)}$	$\lambda_{H_1 H_2 H_2}^{(0)+(1)}/\lambda_{HH'H'}$	
3.84	1151	1252	1.09	
2.62	367	510	1.39	
7.92	54.1	349	6.46	
$\lambda_{H_1 H_1 H_1 H_1}^{(0)}$	$\lambda_{H_1 H_1 H_1 H_1}^{(1)}$	$\lambda_{H_1 H_1 H_1 H_1}^{(0)+(1)}$	μ_λ	
-0.942×10^{-3}	0.118	0.117	3.64	
0.0139	0.118	0.132	4.10	
0.0634	0.118	0.182	5.63	

the corrections to the triple-Higgs couplings from V_0 are⁴

$$\lambda_{H_1 H_1 H_1}^{(0)} = -\lambda_{345} v s_\delta^2 (c_\delta - s_\delta \cot 2\beta) - \frac{(\Delta \hat{t}_1 c_\beta^2 + \Delta \hat{t}_2 s_\beta^2) v}{64\pi^2}, \quad (30)$$

$$\lambda_{H_1 H_1 H_2}^{(0)} = +\lambda_{345} v s_\delta (2c_\delta^2 - 3s_\delta c_\delta \cot 2\beta - s_\delta^2) + \frac{3(\Delta \hat{t}_1 - \Delta \hat{t}_2) v s_\beta c_\beta}{64\pi^2}, \quad (31)$$

$$\lambda_{H_1 H_2 H_2}^{(0)} = -\lambda_{345} v c_\delta (c_\delta^2 - 3c_\delta s_\delta \cot 2\beta - 2s_\delta^2) - \frac{3(\Delta \hat{t}_1 s_\beta^2 + \Delta \hat{t}_2 c_\beta^2) v}{64\pi^2}. \quad (32)$$

- (ii) To calculate the contributions to the triple-Higgs couplings from V_1 , it is appropriate that we use the zeroth-order fields H and H' . Then,

⁴The corrections to the λ_{345} terms involve $\cos\delta$ and $\sin\delta$. We do not include δ dependence in the $\Delta \hat{t}_i$ terms because that would be a two-loop correction. Because δ and δ/β are at most a few percent [5], the effect of including them in these terms is negligibly small anyway.

$$\lambda_{H_1 H_1 H_1}^{(1)} = \frac{1}{6} \frac{\partial^3 V_1}{\partial H^3} \Big|_\langle, \quad (33)$$

$$\lambda_{H_1 H_1 H_2}^{(1)} = \frac{1}{2} \frac{\partial^3 V_1}{\partial H^2 \partial H'} \Big|_\langle, \quad (34)$$

$$\lambda_{H_1 H_2 H_2}^{(1)} = \frac{1}{2} \frac{\partial^3 V_1}{\partial H (\partial H')^2} \Big|_\langle, \quad (35)$$

where, again, $\langle \rangle$ means that the derivatives are evaluated at the vacuum expectation values of the fields. Write V_1 as

$$V_1 = \frac{1}{64\pi^2} \sum_X \alpha_X M_X^4 (\beta_X + \ln(M_X^2/\Lambda_{\text{GW}}^2)), \quad (36)$$

where $M_X^2 = g_X^2 (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)$, except that $m_t^2 = \Gamma_t^2 \Phi_1^\dagger \Phi_1$ with $\Gamma_t = \sqrt{2} m_t / v \cos\beta$. This m_t^2 affects $\lambda_{H_1 H_1 H_2}^{(1)}$ and $\lambda_{H_1 H_2 H_2}^{(1)}$. The constants α_X , β_X and g_X^2 can be read off from Eqs. (9) and (10). Then we obtain

$$\lambda_{H_1 H_1 H_1}^{(1)} = \frac{v}{16\pi^2} \sum_X \alpha_X \left[\frac{M_X^4}{v^4} \left(\beta_X + \frac{13}{6} + \ln \frac{M_X^2}{\Lambda_{\text{GW}}^2} \right) \right]_\langle, \quad (37)$$

$$\lambda_{H_1 H_1 H_2}^{(1)} = \frac{3v \tan\beta}{16\pi^2} \left[\frac{m_t^4}{v^4} \left(\frac{5}{2} + \ln \frac{m_t^2}{\Lambda_{\text{GW}}^2} \right) \right]_\langle, \quad (38)$$

$$\lambda_{H_1 H_2 H_2}^{(1)} = \frac{v}{16\pi^2} \sum_{X \neq t} \alpha_X \left[\frac{M_X^4}{v^4} \left(\beta_X + \frac{3}{2} + \ln \frac{M_X^2}{\Lambda_{\text{GW}}^2} \right) \right]_\langle - \frac{3v \tan^2\beta}{16\pi^2} \left[\frac{m_t^4}{v^4} \left(\frac{5}{2} + \ln \frac{m_t^2}{\Lambda_{\text{GW}}^2} \right) \right]_\langle. \quad (39)$$

The V_0 and V_1 contributions to the four-Higgs coupling $\lambda_{H_1 H_1 H_1 H_1}$ are

$$\lambda_{H_1 H_1 H_1 H_1}^{(0)} = -\frac{1}{2} \lambda_{345} s_\delta^2 (1 + s_\delta \cot 2\beta)^2 - \frac{(\Delta \hat{t}_1 c_\beta^2 + \Delta \hat{t}_2 s_\beta^2) v}{256\pi^2}, \quad (40)$$

$$\lambda_{H_1 H_1 H_1 H_1}^{(1)} = \frac{1}{64\pi^2} \sum_X \alpha_X \left[\frac{M_X^4}{v^4} \left(\beta_X + \frac{25}{6} + \ln \frac{M_X^2}{\Lambda_{\text{GW}}^2} \right) \right]_\langle. \quad (41)$$

In Fig. 4 we plot the allowed range of $\kappa_\lambda = \lambda_{H_1 H_1 H_1}^{(0)+(1)}/(\lambda_{HHH})_{\text{SM}}$ and $\mu_\lambda = \lambda_{H_1 H_1 H_1 H_1}^{(0)+(1)}/(\lambda_{HHHH})_{\text{SM}}$ for this GW-2HDM [where $(\lambda_{HHH})_{\text{SM}} = M_H^2/2v \cong 32$ GeV and $(\lambda_{HHHH})_{\text{SM}} = M_H^2/8v^2 \cong 0.0323$]. For this, we put

FIG. 4. The ratios $\kappa_\lambda = \lambda_{H_1 H_1 H_1}^{(0)+(1)} / (\lambda_{HHH})_{\text{SM}}$ [with $(\lambda_{HHH})_{\text{SM}} \cong 32 \text{ GeV}$] and $\mu_\lambda = \lambda_{H_1 H_1 H_1 H_1}^{(0)+(1)} / (\lambda_{HHHH})_{\text{SM}}$ [with $(\lambda_{HHHH})_{\text{SM}} \cong 0.0323$] as a function of $M_{H^\pm} = M_A$. The sharp rise starting near $M_{H^\pm} = 370 \text{ GeV}$ is an artifact of $M_{H'}$ starting its dive to zero.

$M_{H^\pm} = M_A$ to eliminate the scalars' contributions to the T parameter and then enforced the sum rule (15) so that $M_{H'} = [(540 \text{ GeV})^4 - 2M_{H^\pm}^4 - M_A^4]^{1/4}$. (We also set $\tan\beta = 0.5$, its current experimental upper limit [5]. There is no discernible effect on the cubic and quartic Higgs couplings for any plausible $\tan\beta > 0$.) From this plot, we see that $\kappa_\lambda \cong 1.6$ and $\mu_\lambda \cong 3.6$ below $M_{H^\pm} \cong 370 \text{ GeV}$. In this region, only 2%–10% of these cubic and quartic Higgs couplings comes from V_0 . Above it, these couplings approximately double as the sum rule forces $M_{H'}$ rapidly to zero at $M_{H^\pm} \cong 410 \text{ GeV}$; see Fig. 3. This is an artifact of the end point of the sum rule, with the sudden increase due entirely to the $\Delta\hat{t}_{1,2}$ terms in Eqs. (30) and (40).⁵

This effect of the sum rule is seen numerically in Table I where we list triple and quartic couplings for three extreme values of $M_{H^\pm}$ in Fig. 4. The V_0 contribution to $\lambda_{H_1 H_1 H_2}$ is small and its V_1 contribution would vanish were it not for the fact that $m_t^2 = \Gamma_t \Phi_1^\dagger \Phi_1$ contains a linear term in H' . On the other hand, for almost the entire $M_{H^\pm}$ range, the contributions to $\lambda_{H_1 H_2 H_2}$ listed in the table are of normal size, $\mathcal{O}(\lambda_{HH'H'} = M_{H_2}^2/v)$. The interesting question of the effect this large coupling has on the production rate of $pp \rightarrow H_2 H_2$ is beyond the scope of this paper.

The value of the triple-Higgs coupling $\lambda_{H_1 H_1 H_1}^{(0)+(1)}$ in the GW-2HDM is close to its small SM value. As can be seen in Refs. [16–18], $\kappa_\lambda \cong 1.6$ –3.6 corresponds to $\sigma(pp \rightarrow HH) = 15$ –20 fb. This is the absolute minimum value of the di-Higgs production cross section for $\sqrt{s} = 13$ –14 TeV at the LHC. Because the sum rule (1) is

⁵Using M_{H_2} instead of $M_{H'}$ from the sum rule lessens somewhat this sharp rise in κ_λ and μ_λ , but that is not consistent loop-perturbation theory.

independent of the number or type of Higgs multiplets in the GW model, this result is true of *all* of them.

We are aware that there are many theoretical studies of the cubic and even quartic Higgs couplings—in the context of one-doublet models, multidoublet models, models with extra singlet “Higgses,” and so on—many more studies than we can note here. We apologize to their authors for not citing them. At perhaps the simplest level, this is the problem of the shape of the potential of the Higgs boson itself; specifically, what are λ_{HHH} and λ_{HHHH} ? One recent paper [6] studied a variety of new physics scenarios, their effect on these couplings, and the prospect of distinguishing them at the 14 TeV High Luminosity LHC (HL-LHC), the 27 TeV High Energy LHC (HE-LHC) and the 100 TeV Future Circular Hadron Collider (FCC-hh). These authors considered, *inter alia*, a Coleman-Weinberg-like potential. Compared to the SM values, they found $\kappa_\lambda = 5/3$ and $\mu_\lambda = 11/3$. These are close to our calculated values of κ_λ and μ_λ in Fig. 4 below $M_{H^\pm} = M_A \cong 370 \text{ GeV}$. According to the analysis in Ref. [6] of di-Higgs and tri-Higgs observability at the upgraded LHC and the FCC-hh, the HE-LHC is needed to detect and distinguish the triple Higgs coupling of the GW-2HDM and the FCC-hh is needed for the quartic coupling. This is a gloomy prospect.

IV. TESTING GILDENER-WEINBERG AT THE LHC

Much more immediately promising avenues of attack on GW models are searches for the new charged and neutral Higgs bosons that lie below 400–500 GeV. In the GW-2HDM, the new scalars are just $H^\pm$, A and H_2 . Assuming we have that $M_{H^\pm} = M_A$, the principal search modes are

$$H^\pm \rightarrow t\bar{b}(b\bar{t}) \quad \text{and} \quad W^\pm H_2; \quad (42)$$

$$A \rightarrow b\bar{b}, t\bar{t} \quad \text{and} \quad ZH_2; \quad (43)$$

$$H_2 \rightarrow b\bar{b}, t\bar{t} \quad \text{and} \quad ZA, W^\pm H^\mp. \quad (44)$$

Their main production cross sections at the 13 TeV LHC were discussed in Ref. [5] and they are displayed in Fig. 5 with the dependence on $\tan^2\beta$ scaled out. There seem to have been but a few searches for $pp \rightarrow H_2$ or $A \rightarrow b\bar{b}$, presumably because of the overwhelming continuum $b\bar{b}$ production. One recent search by CMS for a CP -even or -odd scalar with $M_{b\bar{b}} = 50$ –350 GeV and produced at high p_T is reported in Ref. [19]. No significant excess over SM backgrounds was found. For $M_{H_2, A} = 200$ –350 GeV, the 95% C.L. limits are $\sigma(pp \rightarrow H_2, A)B(H_2, A \rightarrow b\bar{b}) \cong 200$ –300 pb which translates into upper limits $\tan\beta \cong 3$ –6. It is important to note that the decays $H_2, A \rightarrow W^+W^-, ZZ$ and $H^\pm \rightarrow W^\pm Z$ are highly suppressed in GW models by the near alignment of the SM Higgs $H \cong H_1$. Likewise, alignment strongly suppresses

FIG. 5. The cross sections for $\sqrt{s} = 13$ TeV at the LHC for single Higgs production processes in the alignment limit ($\delta \rightarrow 0$) of the GW-2HDM with the dependence on $\tan\beta$ scaled out. Both charged Higgs states are included in $pp \rightarrow tH^-$. From Ref. [5].

$H_2, A \rightarrow ZH$ and $H^\pm \rightarrow W^\pm H$. Seeing these decay modes from a new, heavier *spinless* boson would be significant, if not fatal, blows to GW models.

The following is a summary of the current experimental situation for the new Higgs bosons' dominant decay modes:

- (i) The CMS search at 8 TeV for $H^\pm \rightarrow t\bar{b}$ [12] restricted $\tan\beta \lesssim 0.5$ for our type-I GW-2HDM with $180 \text{ GeV} < M_{H^\pm} < 550 \text{ GeV}$ [5]. Searches at 13 TeV for $H^\pm \rightarrow t\bar{b}$ by ATLAS [13] and CMS [20] extend down to $M_{H^\pm} = 200 \text{ GeV}$ but do not yet have the sensitivity to reach $\sigma(pp \rightarrow tH^\pm) = 0.50 \text{ pb}$ (0.033 pb) expected at 200 GeV (500 GeV) for $\tan\beta = 0.50$ and $B(H^\pm \rightarrow t\bar{b}) = 1$. At $M_{H^\pm} = 400 \text{ GeV}$ in the GW-2HDM, Fig. 3 gives $M_{H_2} = 314 \text{ GeV}$, while at $M_{H^\pm} = 408 \text{ GeV}$, it gives $M_{H_2} = 247 \text{ GeV}$. Between these two mass points, the $H^\pm \rightarrow W^\pm H_2$ decay rate increases by a factor of 70, overwhelming the $H^\pm \rightarrow t\bar{b}$ decay rate; see item (iii) below. The two processes $g\bar{b} \rightarrow H^+ \bar{t}, H^+ \rightarrow t\bar{b}$ and $g\bar{b} \rightarrow H^+ \bar{t}, H^+ \rightarrow W^+ H_2$, with $H_2 \rightarrow b\bar{b}$, have the same final state. Hence, $H^+ \rightarrow W^+ H_2$ may unintentionally be included in a search for $H^+ \rightarrow t\bar{b}$. Even if that happened, the model expectation $\sigma(g\bar{b} \rightarrow H^+ \bar{t}) = 0.075 \text{ pb}$ for $M_{H^\pm} \simeq 400 \text{ GeV}$ and $\tan\beta = 0.5$ is well below the 95% C.L. limits $\simeq 0.5$ (ATLAS) and 0.7 pb (CMS). There appear to be no dedicated searches released for $H^\pm \rightarrow W^\pm H_2 \rightarrow \ell^\pm b\bar{b}$ and for $H_2 \rightarrow W^\pm H^\mp \rightarrow \ell^\pm t\bar{b}$.
- (ii) CMS recently reported a search for a *CP*-even or -odd scalar ϕ with mass in the range 400–700 GeV and decaying to $t\bar{t}$ [21]. Results were presented in terms of allowed and excluded regions of the “coupling strength” $g_\phi = \lambda_{\phi t\bar{t}}/(m_t/v)$ and for fixed width-to-mass ratio $\Gamma_\phi/M_\phi = 0.5\%-25\%$. In the GW-2HDM, $g_\phi = \tan\beta$. For the *CP*-odd case, $\phi = A$, with

TABLE II. 95% C.L. upper limits on $\sigma(pp \rightarrow A(H_2))B(A(H_2) \rightarrow ZH_2(A))B(H_2(A) \rightarrow b\bar{b})$ via gluon fusion from ATLAS [22], CMS [23] and GW-2HDM calculations for two cases of large M_A and M_{H_2} . The CMS limits include $B(Z \rightarrow e^+e^-, \mu^+\mu^-)$; the ATLAS limits and GW-2HDM predictions do not. Masses are in GeV and σB in femtobarns. $M_A = M_{H^\pm}$ is assumed and M_{H_2} is taken from Fig. 3 as explained in the text. Model cross sections are taken from Fig. 5 multiplied by $\tan^2\beta = 0.25$.

$M_A = M_{H^\pm}$	M_{H_2}	ATLAS	CMS	GW-2HDM
400	300	255	75	65
300	500	105	50	100

$400 \text{ GeV} < M_A < 500 \text{ GeV}$ and all Γ_A/M_A considered, the region $\tan\beta < 0.5$ is not excluded.⁶ This is possibly due to an excess at 400 GeV that corresponds to a global (local) significance of $1.9(3.5 \pm 0.3)\sigma$ for $\Gamma_A/M_A \simeq 4\%$. Reference [21] also notes that $t\bar{t}$ threshold effects may account for the excess.

- (iii) Searches for $pp \rightarrow A(H_2) \rightarrow ZH_2(A) \rightarrow \ell^+\ell^- b\bar{b}$ via gluon fusion have been reported by ATLAS [22] and CMS [23]. Two examples of observed 95% upper limits on cross sections and the corresponding GW-2HDM predictions are given in Table II. A word of caution is in order here: These decay rates are dominated by the emission of longitudinally polarized weak bosons and are proportional to $p^3/M_{W,Z}^2$, hence sensitive to the available phase space.

At the LHC there are now 140 fb^{-1} of pp collision data at 13 TeV from run 2 and another 200 fb^{-1} at 14 TeV are expected from run 3 by the time it concludes at the end of 2024. With masses in the range 200–500 GeV, GW Higgs production rates are $\sigma(pp \rightarrow H^+ + H^-) = (0.1\text{--}1.0) \text{ pb} \times \tan^2\beta$, $\sigma(pp \rightarrow A) = (4.0\text{--}20) \text{ pb} \times \tan^2\beta$ and $\sigma(pp \rightarrow H_2) = (2.0\text{--}7.0) \text{ pb} \times \tan^2\beta$. Thus, unless $\tan\beta \lesssim 0.2$, there will be anywhere from 10^3 to several 10^6 of these GW Higgs bosons produced by the end of run 3. Given the large SM production of $b\bar{b}$, direct detection of $H_2 \rightarrow b\bar{b}$ via gluon fusion is the most difficult. There is no doubt that improved sensitivity in the low-mass region of $H^\pm \rightarrow t\bar{b}$ is needed to access the expected cross sections. The decays $A(\text{or } H_2) \rightarrow ZH_2(\text{or } A) \rightarrow \ell^+\ell^- b\bar{b}$, $H^\pm \rightarrow W^\pm H_2 \rightarrow \ell^\pm b\bar{b} + E_T$ are helped by the narrow $b\bar{b}$ resonance and lepton kinematics. They may be easier than $H^\pm \rightarrow t\bar{b}$, but they cover a slimmer portion of $(M_{H^\pm}, M_A, M_{H_2})$ space, the upper and lower ends of the allowed $M_{H^\pm} = M_A$ region.

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⁶The same appears to be true for $\phi = H_2$ with $\Gamma_{H_2}/M_{H_2} \gtrsim 1\%$.

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