Erratum: Relativistic corrections to exclusive $\chi_{cJ} + \gamma$ production from e^+e^- annihilation [Phys. Rev. D **97, 096001 (2018)]**

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In this erratum we correct for a missing factor $1/2$ in the Gremm-Kapustin relation, Eq. (49), and account for the fact that the chromoelectric production operators defined in Eqs. (12) are the opposite of the decay operators defined in [24], i. e., $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle = -\langle \chi_{cJ} | \mathcal{T}_{8\text{em}}(^3P_J) | \chi_{cJ} \rangle$. Furthermore, we correct for wrong signs in front of $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$ in Eqs. (24), a missing factor of 2π on the rhs of Eqs. (55)–(57) and an incorrect overall prefactor in Eq. (46k). In addition to that, we clarify the meaning of e_Q in our formulas. These changes lead to the correction of some formulas in Secs. IV A, V C, VI, VII, and VIII and to modifications of our final numerical predictions. All the corrections are detailed in the following.

In the abstract, the final cross section predictions at $\sqrt{s} = 10.6$ GeV should read $\sigma(e^+e^- \rightarrow \chi_{c0} + \gamma) = (1.6 \pm 0.6)$ fb, $\sigma(e^+e^- \rightarrow \chi_{c1} + \gamma) = (12.1 \pm 5.5)$ fb, and $\sigma(e^+e^- \rightarrow \chi_{c2} + \gamma) = (3.4 \pm 2.1)$ fb.

In the third lines of Eqs. (12a)–(12c) the i in front of the square brackets should be put inside the brackets.

In Eqs. (24) $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$ should be replaced by $-\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$. Accordingly, the rhs of Eqs. (25c), (25f), and (25i) should be multiplied by -1 .

In the first paragraph of Sec. V, the following sentence should be added after the second sentence: “In the following sections where we discuss the phenomenology of electromagnetic χ_{cJ} production, e_Q should be always understood as the charge of the charm quark, i. e., $e_Q \equiv e_c = +2/3$.”

In Eq. (46k) the overall prefactor on the rhs should read $i\lambda$ instead of $-\lambda$.

In Eq. (49) the prefactor in the second term on the rhs should be $m/2$ instead of m . Subsequently, the prefactor of $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle / \langle 0 | \mathcal{O}_1(^3P_J) | 0 \rangle$ in Eq. (50) should read $-1/2$ instead of -1 .

In the fourth sentence in Sec. V C the words “our LDMEs are identical to” should read “our LDMEs directly follow from.” In the same sentence the following footnote should be added after the words “we can use the Gremm-Kapustin relations”: “The factor $1/2$ in front of $m\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$ accounts for the fact that $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle = 2\langle \chi_{cJ} | \mathcal{F}_{\text{EM}}(^3P_J) | \chi_{cJ} \rangle$, with $\langle \chi_{cJ} | \mathcal{F}_{\text{EM}}(^3P_J) | \chi_{cJ} \rangle$ being the chromoelectric LDME from [23].”

In the second sentence of Sec. VI, the following footnote should be added after the words “in terms of production LDMEs”: “With the definitions of the decay and production LDMEs from [24] and Eqs. (12), respectively, we have $\langle 0 | \mathcal{O}_1(^3P_J) | 0 \rangle = \langle \chi_{cJ} | \mathcal{O}_{1\text{em}}(^3P_J) | \chi_{cJ} \rangle$, $\langle 0 | \mathcal{P}_1(^3P_J) | 0 \rangle = \langle \chi_{cJ} | \mathcal{P}_{1\text{em}}(^3P_J) | \chi_{cJ} \rangle$ and $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle = -\langle \chi_{cJ} | \mathcal{T}_{8\text{em}}(^3P_J) | \chi_{cJ} \rangle$.”

In the third line of Eq. (51a) the i in front of the square brackets should be put inside the brackets. In the third line of Eq. (51b) an i should be put inside the brackets. Furthermore, $\langle 0 | \mathcal{T}_8(^3P_0) | 0 \rangle$ and $\langle 0 | \mathcal{T}_8(^3P_2) | 0 \rangle$ that appear in the last lines of Eqs. (51a) and (51b) should be replaced with $-\langle 0 | \mathcal{T}_8(^3P_0) | 0 \rangle$ and $-\langle 0 | \mathcal{T}_8(^3P_2) | 0 \rangle$, respectively.

The rhs of Eqs. (55)–(57) should be multiplied with 2π .

The rhs of Eqs. (58c), (58f), (58i), (59c), (59f), and (59i) should be multiplied by -1 .

In Eqs. (60) and (61) $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$ should be replaced by $-\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$.

The coefficients multiplying $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$ in Eqs. (62) should be

$$\begin{aligned} \text{for } \langle 0 | \mathcal{T}_8(^3P_0) | 0 \rangle: & -\frac{(3 - 2r + 3r^2)}{4m(1 - r)(1 - 3r)}, \\ \text{for } \langle 0 | \mathcal{T}_8(^3P_1) | 0 \rangle: & +\frac{(3 - r)(1 + \cos^2\theta) + 2r(1 + r)(1 - \cos^2\theta)}{4m(1 - r)}, \\ \text{for } \langle 0 | \mathcal{T}_8(^3P_2) | 0 \rangle: & +\frac{(3 - r - 6r^2 + 18r^3)(1 + \cos^2\theta) + 6r(1 + r)(1 - \cos^2\theta)}{4m(1 - r)}, \end{aligned}$$

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and in Eqs. (63)

$$\begin{aligned} \text{for } \langle 0|\mathcal{T}_8(^3P_0)|0\rangle &: -\frac{(3-2r+3r^2)}{4m(1-4r+3r^2)}, \\ \text{for } \langle 0|\mathcal{T}_8(^3P_1)|0\rangle &: +\frac{(3+r^2)}{4m(1-r^2)}, \\ \text{for } \langle 0|\mathcal{T}_8(^3P_2)|0\rangle &: +\frac{(3+2r-3r^2+18r^3)}{4m(1-r)(1+3r+6r^2)}. \end{aligned}$$

In Eq. (69a), $\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$ should be replaced with $-\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$, while the prefactors of $\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$ and $\langle 0|\mathcal{T}_8(^3P_2)|0\rangle$ in Eqs. (69b) and (70b) should be $m/2$ instead of m . Equations (72) should read

$$\begin{aligned} \langle 0|\mathcal{P}_1(^3P_0)|0\rangle &= \frac{3}{16} M_{\chi_{c0}} \langle 0|\mathcal{O}_1(^3P_0)|0\rangle \left(M_{\chi_{c0}} + \frac{(3\pi^2-28)}{9\pi} M_{\chi_{c0}} \frac{\alpha_s}{\pi} + 2E_{\chi_{c0}} \right) \\ &\quad - \frac{M_{\chi_{c0}}^6}{512\pi\alpha^2 e_Q^4} \Gamma(\chi_{c0} \rightarrow \gamma\gamma), \\ \langle 0|\mathcal{T}_8(^3P_0)|0\rangle &= \frac{3}{4} \langle 0|\mathcal{O}_1(^3P_0)|0\rangle \left(M_{\chi_{c0}} + \frac{(3\pi^2-28)}{9\pi} M_{\chi_{c0}} \frac{\alpha_s}{\pi} - \frac{2}{3} E_{\chi_{c0}} \right) \\ &\quad - \frac{M_{\chi_{c0}}^5}{128\pi\alpha^2 e_Q^4} \Gamma(\chi_{c0} \rightarrow \gamma\gamma). \end{aligned}$$

In Eq. (73b), the correct prefactor of $M_{\chi_{c2}}$ should be $1/2$, not $1/4$, while the numerical factor in the denominator of the second term should be 256, not 512. In the caption of Fig. 2, “ $x_J = -1.1$ ” should be replaced by “ $x_2 = -1.0$,” while the words “for this value of x_J , the terms proportional to $\langle 0|\mathcal{O}_1(^3P_0)|0\rangle$ in Eqs. (72a) and (73a) become very small.” should read “for this value of x_2 , the terms proportional to $\langle 0|\mathcal{O}_1(^3P_0)|0\rangle$ in Eqs. (73a) become very small.”

As a consequence of the above changes, Eqs. (81) become

$$\begin{aligned} \langle 0|\mathcal{P}_1(^3P_0)|0\rangle &= (0.103 \pm 0.092 \pm 0.070 \pm 0.026) \text{ GeV}^7, \\ \langle 0|\mathcal{T}_8(^3P_0)|0\rangle &= (0.0115 \pm 0.0750 \pm 0.0274 \pm 0.0305) \text{ GeV}^6, \end{aligned}$$

Eq. (82b) becomes

$$\langle 0|\mathcal{T}_8(^3P_0)|0\rangle = (-0.101 \pm 0.030 \pm 0.000 \pm 0.023) \text{ GeV}^6,$$

and Eqs. (83) become

$$\begin{aligned} \langle 0|\mathcal{P}_1(^3P_0)|0\rangle &= (0.058 \pm 0.074 \pm 0.086 \pm 0.033) \text{ GeV}^7, \\ \langle 0|\mathcal{T}_8(^3P_0)|0\rangle &= (-0.045 \pm 0.052 \pm 0.014 \pm 0.039) \text{ GeV}^6. \end{aligned}$$

The updated predictions for Table II are

	$\mathcal{O}(\alpha_s^0 v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$ and $\mathcal{O}(\alpha_s v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}^*(\alpha_s^0 v^2)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}(\alpha_s^0 v^2)$	$\frac{\sigma_8}{\sigma_1}$
$\sigma(\chi_{c0})$	1.09 ± 0.33	2.03 ± 0.61 $\pm 0.75 \pm 0.04$ ± 0.00	1.34 ± 0.02 $\pm 0.30 \pm 0.04$ ± 0.17	1.22 ± 0.49 $\pm 0.48 \pm 0.04$ ± 0.37	0.91 ± 0.38 $\pm 0.16 \pm 0.00$ ± 0.16
$\sigma(\chi_{c1})$	15.16 ± 4.53	17.47 ± 5.22 $\pm 5.87 \pm 0.69$ ± 0.00	12.78 ± 1.25 $\pm 2.83 \pm 0.71$ ± 1.10	13.87 ± 5.97 $\pm 1.10 \pm 0.68$ ± 0.77	1.09 ± 0.36 $\pm 0.15 \pm 0.01$ ± 0.15
$\sigma(\chi_{c2})$	6.30 ± 1.88	4.91 ± 1.47 $\pm 2.01 \pm 0.66$ ± 0.00	3.72 ± 0.46 $\pm 1.24 \pm 0.66$ ± 0.28	4.11 ± 2.13 $\pm 0.63 \pm 0.65$ ± 0.38	1.10 ± 0.44 $\pm 0.20 \pm 0.02$ ± 0.19

Likewise for Table III we obtain

	$\mathcal{O}(\alpha_s^0 v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$ and $\mathcal{O}(\alpha_s v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}^*(\alpha_s^0 v^2)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}(\alpha_s^0 v^2)$	$\frac{\sigma_8}{\sigma_1}$
$\sigma(\chi_{c_0})$	0.89 ± 0.27	1.71 ± 0.51 $\pm 0.64 \pm 0.03$ ± 0.00	1.50 ± 0.19 $\pm 0.13 \pm 0.09$ ± 0.10	1.89 ± 0.02 $\pm 0.13 \pm 0.15$ ± 0.22	1.26 ± 0.17 $\pm 0.02 \pm 0.03$ ± 0.07
$\sigma(\chi_{c_1})$	13.58 ± 4.06	15.46 ± 4.62 $\pm 5.18 \pm 0.61$ ± 0.00	14.05 ± 2.39 $\pm 1.63 \pm 0.24$ ± 0.69	10.31 ± 4.44 $\pm 1.63 \pm 0.89$ ± 0.52	0.73 ± 0.19 $\pm 0.03 \pm 0.05$ ± 0.07
$\sigma(\chi_{c_2})$	5.75 ± 1.72	4.46 ± 1.33 $\pm 1.77 \pm 0.57$ ± 0.00	4.10 ± 0.77 $\pm 0.87 \pm 0.47$ ± 0.18	2.77 ± 1.49 $\pm 0.87 \pm 0.70$ ± 0.25	0.68 ± 0.24 $\pm 0.07 \pm 0.09$ ± 0.09

In the fourth to last paragraph in Sec. VIII the text between the first and the ninth sentences should be replaced with “The first uncertainty originates from $\langle 0 | \mathcal{O}_1(^3P_0) | 0 \rangle$. It is clearly a large contribution to the total uncertainty of the production cross section for χ_{cJ} . The second uncertainty stems from the uncertainty in x_J and therefore in the binding energy. It is equally a large source of uncertainty for all produced states. The third uncertainty is obtained from varying the renormalization scale μ and is, with the exception of the χ_{c0} production in Table II, comparable with the uncertainty in the binding energy. Finally, the fourth source of uncertainty comes from the uncertainties in the experimental values of $M_{\chi_{cJ}}$ and $\Gamma(\chi_{cJ} \rightarrow \gamma\gamma)$. It leads to a large error in the case of χ_{c0} but is much less important for χ_{c1} and χ_{c2} . The surprisingly small errors for the production cross section of χ_{c0} in the determination from Eqs. (73) stem from a large numerical cancellation between the contributions from $\langle 0 | \mathcal{O}_1(^3P_0) | 0 \rangle$ and $\langle 0 | \mathcal{T}_8(^3P_0) | 0 \rangle$. This cancellation is accidental as it does not appear in the determination from Eqs. (72).”

The updated final predictions in Eqs. (90)–(92) and Eqs. (93)–(95) should read

$$\begin{aligned}\sigma(e^+e^- \rightarrow \chi_{c0} + \gamma) &= (1.55 \pm 0.26 \pm 0.30 \pm 0.09 \pm 0.43) \text{ fb}, \\ \sigma(e^+e^- \rightarrow \chi_{c1} + \gamma) &= (12.09 \pm 5.20 \pm 1.37 \pm 0.78 \pm 0.93) \text{ fb}, \\ \sigma(e^+e^- \rightarrow \chi_{c2} + \gamma) &= (3.44 \pm 1.81 \pm 0.75 \pm 0.68 \pm 0.46) \text{ fb},\end{aligned}$$

and

$$\begin{aligned}\sigma(e^+e^- \rightarrow \chi_{c0} + \gamma) &= (1.6 \pm 0.6) \text{ fb}, \\ \sigma(e^+e^- \rightarrow \chi_{c1} + \gamma) &= (12.1 \pm 5.5) \text{ fb}, \\ \sigma(e^+e^- \rightarrow \chi_{c2} + \gamma) &= (3.4 \pm 2.1) \text{ fb},\end{aligned}$$

respectively.

In the second to last paragraph of Sec. VIII the sentence “We see that the contribution of $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$ is small, but, at its central value, not negligible for χ_{c0} and potentially significant for χ_{c1} and χ_{c2} ” should read “We see that the contribution of $\langle 0 | \mathcal{T}_8(^3P_J) | 0 \rangle$ is small, but, at its central value, not negligible and thus potentially significant for all χ_{cJ} .”

The updated Fig. 2 is

The updated Fig. 3 is

The updated Fig. 4 is

The updated Fig. 5 is

The updated Fig. 6 is

Finally, the updated Fig. 7 is

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