Nonleptonic beauty baryon decays and CP asymmetries based on an $SU(3)$ -flavor analysis

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We consider hadronic weak decays of beauty baryons into charmless baryons and pseudoscalar mesons in a general framework based on $SU(3)$ decomposition of the decay amplitudes. The advantage of this approach lies in the ability to perform an $SU(3)$ analysis of these decays without any particular set of dynamical assumptions while accounting for the effects of an arbitrarily broken $SU(3)$ flavor symmetry. Dictated by the symmetries of the effective Hamiltonian that allow us to relate or neglect reduced $SU(3)$ amplitudes, we derive several sum rule relations between amplitudes and relations between CP asymmetries in these decays and identify those that hold even if $SU(3)$ is broken.

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I. INTRODUCTION

LHCb is poised to collect a large data set of two-body weak decays of beauty baryons [1–3] into charmless baryons and pseudoscalar mesons, paving the way to a better understanding of heavy baryon decays. Significant progress has been made in the theoretical understanding of beauty meson decays [4–37] spurred by the experimental advances at the flavor factories Belle and BABAR [38,39] as well as at the LHCb [1,2,40–42]. The general framework of $SU(3)$ analysis in beauty mesons—as well as charm meson decays [43–53] into two pseudoscalars (PP), a pseudoscalar-vector boson (PV), and two vector mesons (VV)—has yielded several amplitude sum rules and relationships between CP asymmetries for various decay modes. While attempts have been made to analyze such decays for beauty baryons, a comprehensive analysis is so far missing in the literature. In this paper, we consider the hadronic beauty baryon decays into an octet or singlet of light baryons and a pseudoscalar meson based on the $SU(3)$ decomposition of the decay amplitudes approach pioneered for B -meson

decays by Grinstein and Lebed [12]. In contrast to the methodology employed in [54–75] for bottom and charmed hadron decays, our approach [12] facilitates an $SU(3)$ decomposition of the decays in terms of $SU(3)$ reduced amplitudes without any particular set of assumptions about the underlying dynamics.

The number of independent $SU(3)$ reduced amplitudes for any given initial and final state is exactly calculable, and relations between decay amplitudes emerge naturally once the set of independent $SU(3)$ reduced amplitudes is smaller than the total number of possible decays. The counting of independent $SU(3)$ reduced amplitudes draws on the choice of the effective Hamiltonian, which in the most general case, indicates 44 independent reduced $SU(3)$ amplitudes equaling the number of all possible $\Delta S = -1$ and $\Delta S = 0$ processes. In practice, the dimension-6 effective Hamiltonian that mediates such hadronic decays of bottom baryons predicts only 10 independent reduced $SU(3)$ amplitudes. One can therefore obtain amplitude relations between the decay modes that can be derived explicitly. Moreover, a systematic study of the $SU(3)$ -breaking effects at the level of decay amplitudes, expanded order by order in the $SU(3)$ -breaking parameter, is required to identify those amplitude relations that survive the $SU(3)$ -breaking effects. Starting with the symmetries of the effective Hamiltonian, we relate or neglect reduced $SU(3)$ amplitudes to derive several sum rule relations between amplitudes and relations between CP asymmetries while indicating more general relations that continue to hold when the $SU(3)$ symmetry is no longer exact.

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This study is crucial for a detailed analysis of the recent CP asymmetry measurements in bottom baryon decays at the CDF and LHCb [76–83].

The approach to decompose the decay amplitudes in terms of reduced $SU(3)$ amplitudes is presented in Sec. II. The relation between the $SU(3)$ Clebsch-Gordan (CG) coefficients in terms of the isoscalar factors and the $SU(2)$ CG coefficients is outlined in Appendix A. The results are summarized in Appendixes B and C. In Sec. III we perform the $SU(3)$ decomposition of unbroken effective hadronic weak decay Hamiltonians. The relations between the amplitudes for beauty baryon decays into octets of light baryons and pseudoscalar mesons are derived in Sec. IV. The effects of $SU(3)$ breaking from the s -quark mass are considered in Sec. IV A. The corresponding relations between CP asymmetries are derived in Sec. V. We finally conclude in Sec. VI.

II. APPLICATION OF $SU(3)$ TO DECAY AMPLITUDES

The $SU(3)$ decomposition of physical amplitudes describing a decay process involves writing it in terms of reduced matrix elements of explicit $SU(3)$ operators

$$\begin{aligned} \mathcal{A}(i \rightarrow f_b f_m) = & (-1)^{I_3 + \frac{Y}{2} + T} \sum_{\substack{\{f, R\} \\ Y^b + Y^m = Y^f, Y^f - Y^i = Y^H \\ I_3^b + I_3^m = I_3^f, I_3^f - I_3^i = I_3^H}} C_{I^b I^m I^f}^{I_3^b I_3^m I_3^f} \left(\begin{array}{ccc} \mathbf{f}_b & \mathbf{f}_m & \mathbf{f} \\ (Y^b, I^b, I_3^b) & (Y^m, I^m, I_3^m) & (Y^f, I^f, I_3^f) \end{array} \right) \\ & \times \left(\begin{array}{ccc} \mathbf{f} & \mathbf{i} & \mathbf{R} \\ (Y^f, I^f, I_3^f) & (-Y^i, I^i, -I_3^i) & (Y^H, I^H, I_3^H) \end{array} \right) C_{I^f I^i I^H}^{I_3^f - I_3^i I_3^H} \langle \mathbf{f} \| \mathbf{R}_I \| \mathbf{i} \rangle, \end{aligned} \quad (1)$$

where $C_{A,B,C}^{a,b,c}$ are the $SU(2)$ Clebsch-Gordan coefficients and

$$\left(\begin{array}{ccc} \mathbf{R}_a & \mathbf{R}_b & \mathbf{R}_c \\ (Y^a, I^a, I_3^a) & (Y^b, I^b, I_3^b) & (Y^c, I^c, I_3^c) \end{array} \right) \quad (2)$$

are the $SU(3)$ isoscalar coefficients obtained by coupling the representations $\mathbf{R}_a \otimes \mathbf{R}_b \rightarrow \mathbf{R}_c$. Here, T is the triality of an $SU(3)$ representation¹ that ensures the reality of the phase appearing in Eq. (1). The symmetry properties of the $SU(3)$ isoscalar factor and its role in obtaining the $SU(3)$ CG coefficients [84–89] are outlined in Appendix A. The amplitude is written with specific attention to the order in which the representations are coupled: The final state representations are coupled via $\mathbf{f}_b \otimes \mathbf{f}_m \rightarrow \mathbf{f}$, where the product (f) is then coupled through the conjugate of the initial representation or, equivalently, $\mathbf{f} \otimes \mathbf{i} \rightarrow \mathcal{H}$. This

with appropriate coefficients. The procedure is a straightforward application of the Wigner-Eckart theorem for the group $SU(3)$, where the reduced matrix elements are all possible $SU(3)$ invariants with CG coefficients connecting the basis involving physical states to the group theoretic basis.

The most general Hamiltonian $\mathcal{H}$ which connects [12] the initial and final states via the matrix elements $\langle f | \mathcal{H} | i \rangle$ consists of exactly those representations $\mathbf{R}$ appearing in $\mathbf{f} \otimes \mathbf{i}$, where the labels i and f denote both physical states and $SU(3)$ representations. It is important to note that in addition to the usual $SU(3)$ CG coefficients that arise from the coupling $\mathbf{f} \otimes \mathbf{i}$, the most general effective Hamiltonian ($\mathcal{H}$) itself involves unknown coefficients appearing in front of every $SU(3)$ representation. *A priori* these coefficients are all independent of each other and are determined once a particular form of the effective Hamiltonian is assumed. The states of $SU(3)$ representations are uniquely distinguished when, in addition to the I_3 and Y values, the isospin Casimir I^2 is specified. The full reduced $SU(3)$ amplitude is thus described by $\langle f | \mathcal{H} | i \rangle$. The expression of the amplitudes in terms of reduced $SU(3)$ amplitudes is concisely given as

ensures that all possible $SU(3)$ representations are indeed generated in the case of the most general effective Hamiltonian. Given a form of effective Hamiltonian ($\mathcal{H}_{\text{eff}}$), it can be $SU(3)$ decomposed,

$$\mathcal{H}_{\text{eff}} = \sum_{\substack{\{Y, I, I_3\} \\ \mathbf{R}}} \mathcal{F}_{\mathbf{R}}^{\{Y, I, I_3\}} \mathbf{R}_I, \quad (3)$$

where $\mathcal{F}_{\mathbf{R}}^{\{Y, I, I_3\}}$ depends on the $SU(3)$ CG coefficients appearing in front of the $SU(3)$ representations ($\mathbf{R}_I$). Moreover, $\mathcal{F}_{\mathbf{R}}^{\{Y, I, I_3\}}$ also contains additional factors entering Eq. (3) in the form of Wilson coefficients and CKM elements. It is also important to note that by knowing the dynamical coefficients for different isospin values in a given $SU(3)$ representation, one can drop the isospin Casimir label (I) and express the Wigner-Eckart reduced matrix element $\langle \mathbf{f} | \mathbf{R} | \mathbf{i} \rangle$ in its usual form, independent of the isospin I label. By using the completeness of $SU(3)$ CG coefficients up to a phase factor,

¹For an $SU(3)$ representation with m and n fundamental and antifundamental indices, the triality of the representation is given by $T((m, n)) = (m - n) \bmod 3$.

$$\langle \mathbf{f} \| \mathbf{R}_1 \| \mathbf{i} \rangle = \underbrace{\mathcal{F}_{\mathbf{R}}^{\{Y, I, I_3\}}}_{\text{dynamical coeff. of } \mathcal{H}} \sqrt{\frac{\dim \mathbf{f}}{\dim \mathbf{R}}} \langle \mathbf{f} \| \mathbf{R} \| \mathbf{i} \rangle. \quad (4)$$

Alternatively, one can start directly with the given form of effective Hamiltonian in Eq. (3) and perform an $SU(3)$ decomposition of the decay amplitude:

$$\begin{aligned} \mathcal{A}(i \rightarrow f_b f_m) = & \sum_{\substack{\{Y^H, I^H, I_3^H\} \\ \mathbf{R}}} \mathcal{F}_{\mathbf{R}}^{\{Y, I, I_3\}} \sum_{\substack{\{Y^f, I^f, I_3^f\} \\ \gamma^b + \gamma^m = \gamma^f, \gamma^f = \gamma^i + \gamma^H \\ I_3^b + I_3^m = I_3^f, I_3^f = I_3^i + I_3^H}} C_{I_3^b I_3^m I_3^f}^{I_3^b I_3^m I_3^f} \left(\begin{array}{ccc} \mathbf{f}_b & \mathbf{f}_m & \mathbf{f} \\ (Y^b, I^b, I_3^b) & (Y^m, I^m, I_3^m) & (Y^f, I^f, I_3^f) \end{array} \right) \\ & \times \left(\begin{array}{ccc} \mathbf{R} & \mathbf{i} & \mathbf{f} \\ (Y^H, I^H, I_3^H) & (Y^i, I^i, I_3^i) & (Y^f, I^f, I_3^f) \end{array} \right) C_{I_3^H I_3^i I_3^f}^{I_3^H I_3^i I_3^f} \langle \mathbf{f} \| \mathbf{R} \| \mathbf{i} \rangle. \end{aligned} \quad (5)$$

The case we are interested in—namely, $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{8})P(\mathbf{8})$, where B_b , the initial antitriplet ($\bar{\mathbf{3}}$) beauty baryon, undergoes a charmless decay into an octet baryon (B) and an octet pseudoscalar meson (P)—is described by a Hamiltonian with $\Delta Q = 0$ and ΔS (equivalent to the ΔI_3 and ΔY representation). The possible decays can be divided into two subclasses, namely, the $\Delta S = 0$ and $\Delta S = -1$ transitions. The allowed final state $SU(3)$ representations ($\mathbf{f}$) are $\mathbf{1}, \mathbf{8}_1, \mathbf{8}_2, \mathbf{10}, \bar{\mathbf{10}}, \mathbf{27}$. There are 22 physical processes possible for $\Delta S = 0$ and another 22 for $\Delta S = -1$. In Appendixes B and C, respectively, each of these decay modes is decomposed in terms of the $SU(3)$ reduced amplitudes that add up to 44. Since the physical η and η' mesons are admixtures of octet η_8 and singlet η_1 mesons, a study of $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{8})P(\mathbf{1})$ is also necessary. Therefore, one has to take into account 8 (4 each for $\Delta S = -1$ and $\Delta S = 0$) additional independent $SU(3)$ amplitudes, which are also described in Appendixes B and C.

We emphasize that this method accounts for a complete set of reduced amplitudes [12], regardless of the specific form of interaction Hamiltonian. In particular, this decomposition holds even if the $SU(3)$ symmetry is arbitrarily broken and there is no physical reason to organize particles in $SU(3)$ multiplets. At this point, every process is independent, and finding relations among them requires that we assume a specific form of the interaction Hamiltonian.

III. $SU(3)$ DECOMPOSITION OF AN UNBROKEN EFFECTIVE HAMILTONIAN

The lowest order effective Hamiltonian [90–92] for charmless b -baryon decays consists of $\Delta S = -1$ and $\Delta S = 0$ parts. Each part is composed of the operators $Q_1, \dots, Q_{10}$. The complete Hamiltonian can be written as

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{4G_F}{\sqrt{2}} \left[\lambda_u^{(s)} (C_1(Q_1^{(u)} - Q_1^{(c)}) + C_2(Q_2^{(u)} - Q_2^{(c)})) - \lambda_t^{(s)} \sum_{i=1,2} C_i Q_i^{(c)} - \lambda_t^{(s)} \sum_{i=3}^{10} C_i Q_i^{(s)} \right. \\ & \left. + \lambda_u^{(d)} (C_1(Q_1^{(u)} - Q_1^{(c)}) + C_2(Q_2^{(u)} - Q_2^{(c)})) - \lambda_t^{(d)} \sum_{i=1,2} C_i Q_i^{(c)} - \lambda_t^{(d)} \sum_{i=3}^{10} C_i Q_i^{(d)} \right], \end{aligned} \quad (6)$$

where $V_{ub}V_{us}^* = \lambda_u^s$, $V_{ub}V_{ud}^* = \lambda_u^d$, $V_{tb}V_{ts}^* = \lambda_t^s$, $V_{tb}V_{td}^* = \lambda_t^d$ are the CKM elements, and C_i 's are the Wilson coefficients. Here, Q_1 and Q_2 are the “tree” operators:

$$\begin{aligned} Q_1^{(u)} &= (\bar{u}_L^i \gamma^\mu b_L^j) (\bar{s}_L^j \gamma_\mu u_L^i) \\ Q_1^{(c)} &= (\bar{c}_L^i \gamma^\mu b_L^j) (\bar{s}_L^j \gamma_\mu c_L^i) \\ Q_2^{(u)} &= (\bar{u}_L^i \gamma^\mu b_L^i) (\bar{s}_L^j \gamma_\mu u_L^j) \\ Q_2^{(c)} &= (\bar{c}_L^i \gamma^\mu b_L^i) (\bar{s}_L^j \gamma_\mu c_L^j). \end{aligned} \quad (7)$$

Note that $Q_3, \dots, Q_6$ are the “gluonic penguin” operators:

$$\begin{aligned} Q_3^{(s)} &= (\bar{s}_L^i \gamma^\mu b_L^i) \sum_{q=u,d,s} (\bar{q}_L^j \gamma_\mu q_L^j) \\ Q_4^{(s)} &= (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} (\bar{q}_L^j \gamma_\mu q_L^i) \\ Q_5^{(s)} &= (\bar{s}_L^i \gamma^\mu b_L^i) \sum_{q=u,d,s} (\bar{q}_R^j \gamma_\mu q_R^j) \\ Q_6^{(s)} &= (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} (\bar{q}_R^j \gamma_\mu q_R^i). \end{aligned} \quad (8)$$

Out of the four electroweak penguin (EWP) operators $Q_7, \dots, Q_{10}$, Q_7 and Q_8 ,

TABLE I. Operator decomposition.

	$15_{I=1}$	$15_{I=0}$	$\bar{6}_{I=1}$	$3_{I=0}^{(6)}$	$3_{I=0}^{(\bar{3})}$	$15_{I=3/2}$	$15_{I=1/2}$	$\bar{6}_{I=1/2}$	$3_{I=1/2}^{(6)}$	$3_{I=1/2}^{(\bar{3})}$
$us\bar{u}$	$-1/2$	$-1/\sqrt{8}$	$-1/2$	$-1/\sqrt{8}$	$-1/2$					
$su\bar{u}$	$-1/2$	$-1/\sqrt{8}$	$1/2$	$-1/\sqrt{8}$	$1/2$					
sdd	$1/2$	$-1/\sqrt{8}$	$-1/2$	$-1/\sqrt{8}$	$1/2$					
$ds\bar{d}$	$1/2$	$-1/\sqrt{8}$	$1/2$	$-1/\sqrt{8}$	$-1/2$					
$ss\bar{s}$		$1/\sqrt{2}$		$-1/\sqrt{2}$						
$ud\bar{u}$						$-1/\sqrt{3}$	$-1/\sqrt{24}$	$1/2$	$-1/\sqrt{8}$	$-1/2$
$du\bar{u}$						$-1/\sqrt{3}$	$-1/\sqrt{24}$	$-1/2$	$-1/\sqrt{8}$	$1/2$
$dd\bar{d}$						$1/\sqrt{3}$	$-1/\sqrt{6}$		$-1/\sqrt{2}$	
$ds\bar{s}$							$\sqrt{3}/\sqrt{8}$	$1/2$	$-1/\sqrt{8}$	$1/2$
$sd\bar{s}$							$\sqrt{3}/\sqrt{8}$	$-1/2$	$-1/\sqrt{8}$	$-1/2$

$$Q_7^{(s)} = \frac{3}{2} (\bar{s}_L^i \gamma^\mu b_L^i) \sum_{q=u,d,s} e_q (\bar{q}_R^j \gamma_\mu q_R^j),$$

$$Q_8^{(s)} = \frac{3}{2} (\bar{s}_L^i \gamma^\mu b_L^i) \sum_{q=u,d,s} e_q (\bar{q}_R^j \gamma_\mu q_R^i) \quad (9)$$

are typically ignored in hadronic decays because of the smallness of C_7 and C_8 with respect to the other Wilson coefficients.

The remaining EWP operators are

$$Q_9^{(s)} = \frac{3}{2} (\bar{s}_L^i \gamma^\mu b_L^i) \sum_{q=u,d,s} e_q (\bar{q}_L^j \gamma_\mu q_L^j),$$

$$Q_{10}^{(s)} = \frac{3}{2} (\bar{s}_L^i \gamma^\mu b_L^i) \sum_{q=u,d,s} e_q (\bar{q}_L^j \gamma_\mu q_L^i). \quad (10)$$

Note that $\mathcal{H}_{\text{eff}}$ is a linear combination of four-quark operators of the form $(\bar{q}_1 b)(\bar{q}_2 q_3)$. These operators transform as $3 \otimes 3 \otimes \bar{3}$ under $SU(3)$ flavor and can be decomposed into sums of irreducible operators corresponding to irreducible $SU(3)$ representations: $\mathbf{15}$, $\bar{\mathbf{6}}$, $\mathbf{3}^{(6)}$, $\mathbf{3}^{(\bar{3})}$, where the superscript index 6 ($\bar{3}$) indicates the origin of $\mathbf{3}$ out of the two possible representations arising from the tensor product of q_1 and q_2 . The $SU(3)$ triplet representation of quarks (q_i) and its conjugate denoting the antiquarks ($\bar{q}_i$) consists of the flavor states

$$q_i = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad \bar{q}_i = \begin{pmatrix} \bar{d} \\ -\bar{u} \\ \bar{s} \end{pmatrix}. \quad (11)$$

According to the sign convention chosen in Eq. (11), the meson wave functions are given as

$$K^+ = u\bar{s}, \quad K^- = -s\bar{u}, \quad K^0 = d\bar{s}, \quad \bar{K}^0 = s\bar{d},$$

$$\pi^+ = u\bar{d}, \quad \pi^- = -d\bar{u}, \quad \pi^0 = \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u}),$$

$$\eta_8 = -\frac{1}{2\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}),$$

$$\eta_1 = -\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}).$$

The physical mesons η, η' are related to the η_8 and η_1 through the $SO(2)$ rotation

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} \quad (12)$$

where the definition of θ is consistent with the overall notation for the meson wave functions and also agrees with the phenomenologically determined value of θ . In the following table the four-quark operators, which appear in $\mathcal{H}_{\text{eff}}$, are decomposed using $SU(3)$ Clebsch-Gordan tables. It is worthwhile to note that, in the Hamiltonian, operators appear as $\bar{q}_1 \bar{q}_2 q_3$, whereas in Table I, they are expressed conveniently as $q_1 q_2 \bar{q}_3$. With the help of Table I, the effective Hamiltonian can be expressed in terms of operators having definite $SU(3)$ transformation properties. In particular, the tree, gluonic, and electroweak penguin parts of the effective Hamiltonian consist of [21]

$$\begin{aligned} \frac{\sqrt{2}\mathcal{H}_T}{4G_F} = & \left\{ \lambda_u^s \left[\frac{(C_1 + C_2)}{2} \left(-\mathbf{15}_1 - \frac{1}{\sqrt{2}}\mathbf{15}_0 - \frac{1}{\sqrt{2}}\mathbf{3}_0^{(6)} \right) + \frac{(C_1 - C_2)}{2}(\mathbf{6}_1 + \mathbf{3}_0^{(\bar{3})}) \right] \right. \\ & \left. + \lambda_u^d \left[\frac{(C_1 + C_2)}{2} \left(-\frac{2}{\sqrt{3}}\mathbf{15}_{3/2} - \frac{1}{\sqrt{6}}\mathbf{15}_{1/2} - \frac{1}{\sqrt{2}}\mathbf{3}_{1/2}^{(6)} \right) + \frac{(C_1 - C_2)}{2}(-\mathbf{6}_{1/2} + \mathbf{3}_{1/2}^{(\bar{3})}) \right] \right\}, \quad (13) \end{aligned}$$

$$\frac{\sqrt{2}\mathcal{H}_g}{4G_F} = \{-\lambda_t^s[-\sqrt{2}(C_3 + C_4)\mathbf{3}_0^{(6)} + (C_3 - C_4)\mathbf{3}_0^{(\bar{3})}] - \lambda_t^d[-\sqrt{2}(C_3 + C_4)\mathbf{3}_{1/2}^{(6)} + (C_3 - C_4)\mathbf{3}_{1/2}^{(\bar{3})}] \\ - \lambda_t^s[-\sqrt{2}(C_5 + C_6)\mathbf{3}_0^{(6)} + (C_5 - C_6)\mathbf{3}_0^{(\bar{3})}] - \lambda_t^d[-\sqrt{2}(C_5 + C_6)\mathbf{3}_{1/2}^{(6)} + (C_5 - C_6)\mathbf{3}_{1/2}^{(\bar{3})}]\}, \quad (14)$$

$$\frac{\sqrt{2}\mathcal{H}_{\text{EWP}}}{4G_F} = \left\{ -\lambda_t^s \left[\frac{(C_9 + C_{10})}{2} \left(-\frac{3}{2}\mathbf{15}_1 - \frac{3}{2\sqrt{2}}\mathbf{15}_0 + \frac{1}{2\sqrt{2}}\mathbf{3}_0^{(6)} \right) + \frac{(C_9 - C_{10})}{2} \left(\frac{3}{2}\mathbf{6}_1 + \frac{1}{2}\mathbf{3}_0^{(\bar{3})} \right) \right] \right. \\ \left. - \lambda_t^d \left[\frac{(C_9 + C_{10})}{2} \left(-\sqrt{3}\mathbf{15}_{3/2} - \frac{1}{2}\sqrt{\frac{3}{2}}\mathbf{15}_{1/2} + \frac{1}{2\sqrt{2}}\mathbf{3}_{1/2}^{(6)} \right) + \frac{(C_9 - C_{10})}{2} \left(-\frac{3}{2}\mathbf{6}_{1/2} + \frac{1}{2}\mathbf{3}_{1/2}^{(\bar{3})} \right) \right] \right\}. \quad (15)$$

It is clear from Table I that higher $SU(3)$ representations like $\mathbf{24}$, $\mathbf{42}$, and $\mathbf{15}'$ are absent in the unbroken Hamiltonian.

IV. AMPLITUDE RELATIONS

From the tree and electroweak parts of the Hamiltonian, one can project out the coefficients corresponding to the $\mathbf{15}$ part of the Hamiltonian and write down the following relations between reduced matrix elements regardless of the initial and final states,

$$\frac{\langle \mathbf{f} \| \mathbf{15}_0 \| \mathbf{i} \rangle}{\langle \mathbf{f} \| \mathbf{15}_1 \| \mathbf{i} \rangle} = \frac{1}{\sqrt{2}}, \quad \frac{\langle \mathbf{f} \| \mathbf{15}_{\frac{1}{2}} \| \mathbf{i} \rangle}{\langle \mathbf{f} \| \mathbf{15}_{\frac{3}{2}} \| \mathbf{i} \rangle} = \frac{1}{2\sqrt{2}}, \\ \frac{\lambda_t^d \langle \mathbf{f} \| \mathbf{15}_0 \| \mathbf{i} \rangle_{\text{EWP}}}{\lambda_t^s \langle \mathbf{f} \| \mathbf{15}_{\frac{1}{2}} \| \mathbf{i} \rangle_{\text{EWP}}} = \sqrt{3}, \quad \frac{\lambda_u^d \langle \mathbf{f} \| \mathbf{15}_0 \| \mathbf{i} \rangle_{\text{T}}}{\lambda_u^s \langle \mathbf{f} \| \mathbf{15}_{\frac{1}{2}} \| \mathbf{i} \rangle_{\text{T}}} = \sqrt{3}. \quad (16)$$

In the case of several different operator structures contributing to the Hamiltonian, as is the case in Eq. (6), the relations between reduced matrix elements are expressed in the following way,

$$\frac{\langle \mathbf{f} \| \mathbf{R}_I \| \mathbf{i} \rangle}{\langle \mathbf{f} \| \mathbf{R}_{I'} \| \mathbf{i} \rangle} = \frac{\sum_l C_l C_l}{\sum_m C_m C'_m}, \quad (17)$$

where the $C_i^{(\prime)}$ are the coefficients of the different components of the Hamiltonian and the C_j 's are the CG coefficients; the sums extend over all the corresponding contributions to the Hamiltonian. In addition, the absence of some of the $SU(3)$ representations in the Hamiltonian is a consequence of the vanishing dynamical coefficients corresponding to the reduced matrix elements $\langle \mathbf{f} \| \mathbf{42} \| \mathbf{i} \rangle$, $\langle \mathbf{f} \| \mathbf{24} \| \mathbf{i} \rangle$, and $\langle \mathbf{f} \| \mathbf{15}' \| \mathbf{i} \rangle$, regardless of the I value and initial and final states. The $\Delta S = -1$ and $\Delta S = 0$ decay amplitudes and the reduced $SU(3)$ elements are expressed as column matrices $\mathcal{A}$ and $\mathcal{R}$, respectively, and are related by the matrix equation,

$$\mathcal{A}_{\text{tree}} = \mathbb{T}\mathcal{R}, \quad (18)$$

$$\mathcal{A}_{\text{penguin}} = \mathbb{P}\mathcal{R} \quad (19)$$

where $\mathbb{T}$ and $\mathbb{P}$ are the coefficient matrices related to the tree and penguin parts of the Hamiltonian described in Eqs. (13)–(15) and

$$\mathcal{A} = \mathcal{A}_{\text{tree}} + \mathcal{A}_{\text{penguin}}. \quad (20)$$

The rank of matrix $\mathbb{T}$ is lower than the total number of decay modes, suggesting that not all of the reduced $SU(3)$ matrix elements are independent. The number of actually independent reduced $SU(3)$ matrix elements is equal to the rank of matrix $\mathbb{T}$. Similarly, the coefficient matrix $\mathbb{P}$, which takes into account the contributions from the gluonic and EW penguin operators, also has a rank lower than the total number of decay modes. The number of amplitude relations can now be estimated unambiguously, which is the difference between the total number of decay modes and the rank of $\mathbb{T}(\mathbb{P})$. It is advantageous to factor out the CKM elements $\lambda_{u,t}^{s,d}$ and take into account the two possible, $l = 0$ and $l = 1$, relative angular momentum states of the final states while decomposing the decay amplitude in terms of tree and penguin reduced amplitudes,

$$\mathcal{A}^{\text{S}} = \lambda_u^q \mathcal{A}_{\text{tree}}^{\text{S}} + \lambda_t^q \mathcal{A}_{\text{penguin}}^{\text{S}}, \\ \mathcal{A}^{\text{P}} = \lambda_u^q \mathcal{A}_{\text{tree}}^{\text{P}} + \lambda_t^q \mathcal{A}_{\text{penguin}}^{\text{P}}, \quad (21)$$

where $q = s, d$ denote the $\Delta S = -1, 0$ process, and S and P denote the S -wave and P -wave amplitudes of the decay. The coefficient matrices are now redefined as $\mathcal{T}$ and $\mathcal{P}$, with entries that are nothing but products of Wilson coefficients (C_i) and Clebsch-Gordan coefficients. Of course, the number of independent rows remains unchanged, and the matrix equations take the form

$$\mathcal{A}_{\text{tree}}^i = \mathcal{T}\mathcal{R}, \quad \mathcal{A}_{\text{penguin}}^i = \mathcal{P}\mathcal{R} \quad (22)$$

for $i = \text{S-wave}$, $i = \text{P-wave}$ parts. At this point, it is important to recall the penguin part of the Hamiltonian described in Eqs. (14) and (15). In the case of gluonic penguins, $\bar{\mathbf{6}}$ and $\mathbf{15}$ of $SU(3)$ are absent, which results in a smaller set of independent reduced $SU(3)$ matrix elements. This implies additional amplitude relations between decay

modes, some of which are broken once the electroweak penguins are taken into account in the unbroken Hamiltonian. We include electroweak penguins that have parts transforming as **3**, $\bar{\mathbf{6}}$, and **15** of $SU(3)$ and retain all the reduced $SU(3)$ matrix elements. As a result, the amplitude relations derived hold for the gluonic penguin part as well as the electroweak penguin part of the unbroken Hamiltonian. We begin by identifying the identical rows of the $\mathcal{T}$ matrix, which readily gives the simplest amplitude relations for the tree part,

$$\mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^- K^+) = \mathcal{T}(\Xi_b^0 \rightarrow \Xi^- \pi^+), \quad (23)$$

$$\mathcal{T}(\Lambda_b^0 \rightarrow p^+ \pi^-) = \mathcal{T}(\Xi_b^0 \rightarrow \Sigma^+ K^-), \quad (24)$$

$$\mathcal{T}(\Xi_b^- \rightarrow n K^-) = \mathcal{T}(\Xi_b^- \rightarrow \Xi^0 \pi^-), \quad (25)$$

$$\mathcal{T}(\Xi_b^- \rightarrow \Xi^- K^0) = \mathcal{T}(\Xi_b^- \rightarrow \Sigma^- \bar{K}^0), \quad (26)$$

$$\mathcal{T}(\Xi_b^0 \rightarrow \Xi^- K^+) = \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^- \pi^+), \quad (27)$$

$$\mathcal{T}(\Xi_b^0 \rightarrow \Sigma^- \pi^+) = \mathcal{T}(\Lambda_b^0 \rightarrow \Xi^- K^+), \quad (28)$$

$$\mathcal{T}(\Xi_b^0 \rightarrow \Sigma^+ \pi^-) = \mathcal{T}(\Lambda_b^0 \rightarrow p^+ K^-), \quad (29)$$

$$\mathcal{T}(\Xi_b^0 \rightarrow n \bar{K}^0) = \mathcal{T}(\Lambda_b^0 \rightarrow \Xi^0 K^0), \quad (30)$$

$$\mathcal{T}(\Xi_b^0 \rightarrow p^+ K^-) = \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-), \quad (31)$$

$$\mathcal{T}(\Xi_b^0 \rightarrow \Xi^0 K^0) = \mathcal{T}(\Lambda_b^0 \rightarrow n \bar{K}^0), \quad (32)$$

and the same set of relations derived from the matrix $\mathcal{P}$ corresponding to the penguin part of the effective Hamiltonian,

$$\mathcal{P}(\Lambda_b^0 \rightarrow \Sigma^- K^+) = \mathcal{P}(\Xi_b^0 \rightarrow \Xi^- \pi^+), \quad (33)$$

$$\mathcal{P}(\Lambda_b^0 \rightarrow p^+ \pi^-) = \mathcal{P}(\Xi_b^0 \rightarrow \Sigma^+ K^-), \quad (34)$$

$$\mathcal{P}(\Xi_b^- \rightarrow n K^-) = \mathcal{P}(\Xi_b^- \rightarrow \Xi^0 \pi^-), \quad (35)$$

$$\mathcal{P}(\Xi_b^- \rightarrow \Xi^- K^0) = \mathcal{P}(\Xi_b^- \rightarrow \Sigma^- \bar{K}^0), \quad (36)$$

$$\mathcal{P}(\Xi_b^0 \rightarrow \Xi^- K^+) = \mathcal{P}(\Lambda_b^0 \rightarrow \Sigma^- \pi^+), \quad (37)$$

$$\mathcal{P}(\Xi_b^0 \rightarrow \Sigma^- \pi^+) = \mathcal{P}(\Lambda_b^0 \rightarrow \Xi^- K^+), \quad (38)$$

$$\mathcal{P}(\Xi_b^0 \rightarrow \Sigma^+ \pi^-) = \mathcal{P}(\Lambda_b^0 \rightarrow p^+ K^-), \quad (39)$$

$$\mathcal{P}(\Xi_b^0 \rightarrow n \bar{K}^0) = \mathcal{P}(\Lambda_b^0 \rightarrow \Xi^0 K^0), \quad (40)$$

$$\mathcal{P}(\Xi_b^0 \rightarrow p^+ K^-) = \mathcal{P}(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-), \quad (41)$$

$$\mathcal{P}(\Xi_b^0 \rightarrow \Xi^0 K^0) = \mathcal{P}(\Lambda_b^0 \rightarrow n \bar{K}^0). \quad (42)$$

There are several triangle relations connecting the $\Delta S = -1$ decay modes:

$$\begin{aligned} \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-) + \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^- \pi^+) + 2\mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^0 \pi^0) &= 0, \\ \mathcal{T}(\Xi_b^- \rightarrow \Xi^- \pi^0) - \sqrt{3}\mathcal{T}(\Xi_b^- \rightarrow \Xi^- \eta_8) + \sqrt{2}\mathcal{T}(\Xi_b^- \rightarrow \Sigma^- \bar{K}^0) &= 0, \\ \mathcal{T}(\Xi_b^- \rightarrow \Sigma^0 K^-) - \sqrt{3}\mathcal{T}(\Xi_b^- \rightarrow \Lambda^0 K^-) + \sqrt{2}\mathcal{T}(\Xi_b^- \rightarrow \Xi^0 \pi^-) &= 0, \\ \mathcal{T}(\Xi_b^0 \rightarrow \Xi^- \pi^+) - \mathcal{T}(\Lambda_b^0 \rightarrow \Xi^- K^+) + \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^- \pi^+) &= 0, \\ \mathcal{T}(\Xi_b^0 \rightarrow \Sigma^+ K^-) - \mathcal{T}(\Lambda_b^0 \rightarrow p^+ K^-) + \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-) &= 0, \end{aligned} \quad (43)$$

as well as the $\Delta S = 0$ decay modes:

$$\begin{aligned} \mathcal{T}(\Xi_b^- \rightarrow \Sigma^0 \pi^-) - \sqrt{3}\mathcal{T}(\Xi_b^- \rightarrow \Lambda^0 \pi^-) - \sqrt{2}\mathcal{T}(\Xi_b^- \rightarrow n K^-) &= 0, \\ \mathcal{T}(\Xi_b^- \rightarrow \Sigma^- \pi^0) - \sqrt{2}\mathcal{T}(\Xi_b^- \rightarrow \Xi^- K^0) - \sqrt{3}\mathcal{T}(\Xi_b^- \rightarrow \Sigma^- \eta_8) &= 0, \\ \mathcal{T}(\Xi_b^0 \rightarrow \Sigma^- \pi^+) - \mathcal{T}(\Xi_b^0 \rightarrow \Xi^- K^+) - \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^- K^+) &= 0, \\ \mathcal{T}(\Xi_b^0 \rightarrow p^+ K^-) - \mathcal{T}(\Xi_b^0 \rightarrow \Sigma^+ \pi^-) + \mathcal{T}(\Lambda_b^0 \rightarrow p^+ \pi^-) &= 0. \end{aligned} \quad (44)$$

The simplest amplitude relations for the case of $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{8}_B \otimes \mathbf{1}_{\mathcal{M}}$ involving the $SU(3)$ singlet η_1 are indicated,

$$\begin{aligned} \mathcal{T}(\Xi_b^0 \rightarrow \Xi^0 \eta_1) &= \mathcal{T}(\Lambda_b^0 \rightarrow n \eta_1), \\ \mathcal{T}(\Xi_b^- \rightarrow \Xi^- \eta_1) &= \mathcal{T}(\Xi_b^- \rightarrow \Sigma^- \eta_1), \end{aligned} \quad (45)$$

along with the triangle relation for $\Delta S = -1$ processes,

$$\begin{aligned} \mathcal{T}(\Lambda_b^0 \rightarrow \Lambda \eta_1) - \frac{1}{\sqrt{3}} \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^0 \eta_1) \\ - \frac{\sqrt{2}}{\sqrt{3}} \mathcal{T}(\Xi_b^0 \rightarrow \Xi^0 \eta_1) = 0, \end{aligned}$$

and for $\Delta S = 0$ processes,

$$\begin{aligned} \mathcal{T}(\Lambda_b^0 \rightarrow n \eta_1) + \frac{\sqrt{3}}{\sqrt{2}} \mathcal{T}(\Xi_b^0 \rightarrow \Lambda^0 \eta_1) \\ - \frac{1}{\sqrt{2}} \mathcal{T}(\Xi_b^0 \rightarrow \Sigma^0 \eta_1) = 0. \end{aligned}$$

While there is no ground-state $SU(3)$ singlet Λ baryon, there can be $l = 1$ excited-state spin-3/2 Λ_s^{0*} baryons, for which one can derive amplitude relations in the case of $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{1}_B \otimes \mathbf{8}_M$:

$$\begin{aligned} \mathcal{T}(\Xi_b^0 \rightarrow \Lambda_s^{0*} \bar{K}^0) &= \mathcal{T}(\Lambda_b^0 \rightarrow \Lambda_s^{0*} K^0), \\ \mathcal{T}(\Xi_b^0 \rightarrow \Lambda_s^{0*} \eta_8) &= \mathcal{T}(\Xi_b^- \rightarrow \Lambda_s^{0*} K^-), \end{aligned} \quad (46)$$

triangle $\Delta S = -1$ relations,

$$\begin{aligned} \mathcal{T}(\Lambda_b^0 \rightarrow \Lambda_s^{0*} \pi_0) - \frac{1}{\sqrt{3}} \mathcal{T}(\Lambda_b^0 \rightarrow \Lambda_s^{0*} \eta_8) \\ + \frac{\sqrt{2}}{\sqrt{3}} \mathcal{T}(\Xi_b^0 \rightarrow \Lambda_s^{0*} \bar{K}^0) = 0, \end{aligned}$$

triangle $\Delta S = 0$ relations,

$$\begin{aligned} -\frac{1}{\sqrt{3}} \mathcal{T}(\Xi_b^0 \rightarrow \Lambda_s^{0*} \eta_8) + \mathcal{T}(\Xi_b^0 \rightarrow \Lambda_s^{0*} \pi_0) \\ - \frac{\sqrt{2}}{\sqrt{3}} \mathcal{T}(\Lambda_b^0 \rightarrow \Lambda_s^{0*} K^0) = 0. \end{aligned}$$

The S -wave and P -wave parts of the decay amplitudes behave identically under $SU(3)$ decomposition, and the relations hold individually for the tree and penguin parts of all the above-mentioned amplitude relations.

Finally, we consider the trivial case of $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{1}_B \otimes \mathbf{1}_M$ where the final state baryon and meson are both $SU(3)$ singlets. The only relevant decay, $\Lambda_b^0 \rightarrow \Lambda_s^{0*} \eta_1$, satisfying the $SU(3)$ quantum numbers involves a single reduced $SU(3)$ amplitude matching the counting of the number of possible independent $SU(3)$ reduced amplitudes. This concludes our discussion of all possible $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{8}_B \otimes \mathbf{8}_M$, $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{8}_B \otimes \mathbf{1}_M$, $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{1}_B \otimes \mathbf{8}_M$, $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{1}_B \otimes \mathbf{1}_M$ decays

of b -baryons. The most general $SU(3)$ relations can also be obtained in this approach by starting from the $\mathbb{T}(\mathbb{P})$ matrix and expressing the dependent rows as a linear combination of the independent ones. We do not list those relations here as they are not particularly illuminating. Nevertheless, in the next section, where $SU(3)$ -breaking effects are taken into account, we do consider a couple of interesting $SU(3)$ amplitude relations that should hold under some general dynamical assumptions.

A. $SU(3)$ -breaking effect

While isospin symmetry holds to a good approximation, the $SU(3)$ symmetry of the light quarks is broken by the mass of the s quark (m_s). To incorporate such $SU(3)$ -violating effects on decay amplitudes, one can parametrize the breaking of flavor $SU(3)$ by the following interaction [43,45,52,93–95]:

$$\delta \mathcal{H} = \epsilon \bar{q} \lambda_8 q \quad (47)$$

where λ_8 is the Gell-Mann matrix that contributes to the $SU(3)$ breaking and the breaking parameter ϵ depends on m_s . The $SU(3)$ structure of the unbroken Hamiltonian is modified by this term, and to the first order in strange quark mass, the broken Hamiltonian is made of the following $SU(3)$ representations [45],

$$\begin{aligned} (\mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}) \otimes (\mathbf{1} + \epsilon \mathbf{8}) &= (\mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}) \\ &+ \epsilon (\mathbf{3}_i \oplus \bar{\mathbf{6}}_i \oplus \mathbf{15}_1 \oplus \mathbf{15}_2 \oplus \mathbf{15}_3 \\ &\oplus \mathbf{15}_3^2 \oplus \mathbf{15}' \oplus \mathbf{24} \oplus \mathbf{42}), \end{aligned}$$

where the subscript $i = 1, 2, 3$ indicates the origin of that representation from $\mathbf{3}$, $\bar{\mathbf{6}}$, $\mathbf{15}$, respectively. The set of reduced $SU(3)$ amplitudes thus gets enlarged, and there are less relations as a result. The isospin relation

$$\begin{aligned} \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-) + \mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^- \pi^+) \\ + 2\mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^0 \pi^0) = 0 \end{aligned} \quad (48)$$

continues to hold even after including the $SU(3)$ -breaking effect to linear order.

There are other amplitude relations that can be derived on more general grounds. For instance, the isospin symmetry of the unbroken Hamiltonian forbids $\Delta I = 2$ and $\Delta I = 5/2$ transitions. As a consequence, the $SU(3)$ reduced matrix elements $\langle \mathbf{f} \| \mathbf{R}_{I=2} \| \mathbf{i} \rangle$ and $\langle \mathbf{f} \| \mathbf{R}_{I=5/2} \| \mathbf{i} \rangle$ must have a vanishing contribution to the decay amplitude for arbitrary initial and final states. Such $SU(3)$ breaking but isospin-conserving relations are given below,

$$\begin{aligned}
& \frac{\mathcal{T}(\Xi_b^0 \rightarrow \Sigma^0 \bar{K}^0)}{3} + \frac{\mathcal{T}(\Xi_b^0 \rightarrow \Sigma^+ K^-)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^0 \rightarrow \Xi^0 \pi^0)}{3} + \frac{\mathcal{T}(\Xi_b^0 \rightarrow \Xi^- \pi^+)}{3\sqrt{2}} \\
& + \frac{\mathcal{T}(\Xi_b^- \rightarrow \Sigma^0 K^-)}{3} + \frac{\mathcal{T}(\Xi_b^- \rightarrow \Sigma^- \bar{K}^0)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^- \rightarrow \Xi^0 \pi^-)}{3\sqrt{2}} + \frac{\mathcal{T}(\Xi_b^- \rightarrow \Xi^- \pi^0)}{3} \\
& + \frac{\sqrt{2}\mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^0 \pi^0)}{3} + \frac{\mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^- \pi^+)}{3\sqrt{2}} + \frac{\mathcal{T}(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-)}{3\sqrt{2}} = 0,
\end{aligned} \tag{49}$$

$$\begin{aligned}
& \frac{\mathcal{T}(\Xi_b^0 \rightarrow \Sigma^0 \bar{K}^0)}{\sqrt{6}} + \frac{\mathcal{T}(\Xi_b^0 \rightarrow \Sigma^+ K^-)}{2\sqrt{3}} - \frac{\mathcal{T}(\Xi_b^0 \rightarrow \Xi^0 \pi^0)}{\sqrt{6}} - \frac{\mathcal{T}(\Xi_b^0 \rightarrow \Xi^- \pi^+)}{2\sqrt{3}} \\
& + \frac{\mathcal{T}(\Xi_b^- \rightarrow \Sigma^0 K^-)}{\sqrt{6}} + \frac{\mathcal{T}(\Xi_b^- \rightarrow \Sigma^- \bar{K}^0)}{2\sqrt{3}} - \frac{\mathcal{T}(\Xi_b^- \rightarrow \Xi^0 \pi^-)}{2\sqrt{3}} - \frac{\mathcal{T}(\Xi_b^- \rightarrow \Xi^- \pi^0)}{\sqrt{6}} = 0.
\end{aligned} \tag{50}$$

The same set of relations holds for the penguin parts as well.

V. CP RELATIONS

The total decay rate for a two-body decay of a spin-1/2 antitriplet b -baryon ($\mathcal{B}_b$) to a spin-0 pseudoscalar ($\mathcal{M}$) and a spin-1/2 baryon ($\mathcal{B}$) has the following form [57,86,96–98]:

$$\Gamma(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M}) = \frac{|\mathbf{p}_B|}{8\pi m_{\mathcal{B}_b}^2} [|S|^2 + |P|^2]$$

where $|\mathbf{p}_B|$ is the momentum of the final state baryon. Since the decay products can be in one of two possible relative angular momentum states, $l = 0$ and $l = 1$, the amplitude can also be decomposed in terms of **S**-wave and **P**-wave parts. Including the phase space corrections, the **S**-wave and **P**-wave amplitudes are expressed with kinematic factors factored out [62,86,98] as

$$\begin{aligned}
S &= \sqrt{2m_{\mathcal{B}_b}(E_B + m_B)} \mathcal{A}^S, \\
P &= \sqrt{2m_{\mathcal{B}_b}(E_B - m_B)} \mathcal{A}^P
\end{aligned} \tag{51}$$

where $\mathcal{A}^S$ and $\mathcal{A}^P$ are the $SU(3)$ reduced amplitudes defined in Eq. (21). The decay rate is then expressed as

$$\Gamma = \frac{|\mathbf{p}_B|}{4\pi} \frac{(E_B + m_B)}{m_{\mathcal{B}_b}} \left[|\mathcal{A}^S|^2 + \left(\frac{|\mathbf{p}_B|}{E_B + m_B} \right)^2 |\mathcal{A}^P|^2 \right]. \tag{52}$$

Subsequently, A_{CP} is defined as [94]

$$\begin{aligned}
A_{CP} &= \frac{\Gamma(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M}) - \Gamma(\bar{\mathcal{B}}_b \rightarrow \bar{\mathcal{B}}\bar{\mathcal{M}})}{\Gamma(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M}) + \Gamma(\bar{\mathcal{B}}_b \rightarrow \bar{\mathcal{B}}\bar{\mathcal{M}})} \\
&= \frac{\Delta_{CP}(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M})}{2\tilde{\Gamma}(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M})},
\end{aligned} \tag{53}$$

where

$$\tilde{\Gamma}(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M}) = \frac{1}{2} (\Gamma(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M}) + \Gamma(\bar{\mathcal{B}}_b \rightarrow \bar{\mathcal{B}}\bar{\mathcal{M}})).$$

In order to express the CP relation among modes, we rely on the identity $\text{Im}(V_{ub}V_{ud}^*V_{tb}V_{td}) = -\text{Im}(V_{ub}V_{us}^*V_{tb}V_{ts}) = \mathbf{J}$, where $\mathbf{J}$ is the well-known Jarlskog invariant. Notice that Eqs. (51) and (53) imply that A_{CP} is the sum CP violation in the **S** and **P** waves. We define a quantity $\delta_{CP}^a = |\mathcal{A}^a|^2 - |\bar{\mathcal{A}}^a|^2$, for the partial wave a , where $a = \{\mathbf{S}, \mathbf{P}\}$ and $\mathcal{A}^a$ are defined in Eq. (51) with phase-space factors removed from the respective partial waves. By definition,

$$\begin{aligned}
& \delta_{CP}^a(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M}) \\
&= -4\mathbf{J}\text{Im}[\mathcal{A}_T^{a*}(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M})\mathcal{A}_P^a(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M})].
\end{aligned} \tag{54}$$

Based on amplitude relations for the tree and penguin parts obtained in Eqs. (23)–(32) and Eqs. (33)–(42), the following ten δ_{CP}^a relations are obtained,

$$\begin{aligned}
& \delta_{CP}^a(\Lambda_b^0 \rightarrow \Sigma^- K^+) = -\delta_{CP}^a(\Xi_b^0 \rightarrow \Xi^- \pi^+), \\
& \delta_{CP}^a(\Lambda_b^0 \rightarrow p^+ \pi^-) = -\delta_{CP}^a(\Xi_b^0 \rightarrow \Sigma^+ K^-), \\
& \delta_{CP}^a(\Xi_b^- \rightarrow n K^-) = -\delta_{CP}^a(\Xi_b^- \rightarrow \Xi^0 \pi^-), \\
& \delta_{CP}^a(\Xi_b^- \rightarrow \Xi^- K^0) = -\delta_{CP}^a(\Xi_b^- \rightarrow \Sigma^- \bar{K}^0), \\
& \delta_{CP}^a(\Xi_b^0 \rightarrow \Xi^- K^+) = -\delta_{CP}^a(\Lambda_b^0 \rightarrow \Sigma^- \pi^+), \\
& \delta_{CP}^a(\Xi_b^0 \rightarrow \Sigma^- \pi^+) = -\delta_{CP}^a(\Lambda_b^0 \rightarrow \Xi^- K^+), \\
& \delta_{CP}^a(\Xi_b^0 \rightarrow \Sigma^+ \pi^-) = -\delta_{CP}^a(\Lambda_b^0 \rightarrow p^+ K^-), \\
& \delta_{CP}^a(\Xi_b^0 \rightarrow n \bar{K}^0) = -\delta_{CP}^a(\Lambda_b^0 \rightarrow \Xi^0 K^0), \\
& \delta_{CP}^a(\Xi_b^0 \rightarrow p^+ K^-) = -\delta_{CP}^a(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-), \\
& \delta_{CP}^a(\Xi_b^0 \rightarrow \Xi^0 K^0) = -\delta_{CP}^a(\Lambda_b^0 \rightarrow n \bar{K}^0),
\end{aligned} \tag{55}$$

for both $a = S$ and $a = P$. Finally, we obtain A_{CP} relations using

$$A_{CP}(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M}) = \frac{\tau_{\mathcal{B}_b}}{\mathcal{BR}(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M})} \Delta_{CP}(\mathcal{B}_b \rightarrow \mathcal{B}\mathcal{M}), \quad (56)$$

where $\tau_{\mathcal{B}}$ is the lifetime of the beauty baryon. The relation between A_{CP} and δ_{CP} is

$$\Delta_{CP} = \frac{|\mathbf{p}_B|}{4\pi} \frac{(E_B + m_B)}{m_{\mathcal{B}_b}} \left[\delta_{CP}^S + \left(\frac{|\mathbf{p}_B|}{E_B + m_B} \right)^2 \delta_{CP}^P \right]. \quad (57)$$

Since Δ_{CP} depends on the masses of the initial and final baryons as well as the final state meson [61,98], some approximation is needed to obtain A_{CP} relations between various modes. Ignoring $\mathbf{p}_B$ and m_B differences between the various modes, CP violation relations between various modes can be experimentally verified using the relation

$$\frac{A_{CP}(\mathcal{B}_{bi} \rightarrow \mathcal{B}_j \mathcal{M}_k)}{A_{CP}(\mathcal{B}_{bl} \rightarrow \mathcal{B}_m \mathcal{M}_n)} \simeq - \frac{\tau_{\mathcal{B}_{bi}} \mathcal{BR}(\mathcal{B}_{bl} \rightarrow \mathcal{B}_m \mathcal{M}_n)}{\tau_{\mathcal{B}_{bl}} \mathcal{BR}(\mathcal{B}_{bi} \rightarrow \mathcal{B}_j \mathcal{M}_k)}, \quad (58)$$

where i, j, k and l, m, n are indices corresponding to the various baryons belonging to the above-mentioned δ_{CP} relations. There is a further simplification in the case $i = l$, resulting in

$$\frac{A_{CP}(\mathcal{B}_{bi} \rightarrow \mathcal{B}_j \mathcal{M}_k)}{A_{CP}(\mathcal{B}_{bl} \rightarrow \mathcal{B}_m \mathcal{M}_n)} \simeq - \frac{\mathcal{BR}(\mathcal{B}_{bl} \rightarrow \mathcal{B}_m \mathcal{M}_n)}{\mathcal{BR}(\mathcal{B}_{bi} \rightarrow \mathcal{B}_j \mathcal{M}_k)}, \quad (59)$$

where the uncertainties due to the lifetime measurement also cancel out [57]. Alternatively, if the longitudinal polarization of the daughter baryon can be measured from an angular distribution study of the final states, one can estimate the relative strength of the P -wave contribution [62,98] in the total decay width. The longitudinal polarization of the daughter baryon is given by

$$\alpha = \frac{2\text{Re}(\mathcal{A}^{S*} \mathcal{A}^P) |\mathbf{p}_B| / (E_B + m_B)}{|\mathcal{A}^S|^2 + |\mathcal{A}^P|^2 (|\mathbf{p}_B| / (E_B + m_B))^2}. \quad (60)$$

The P -wave contribution can now be systematically taken into account, resulting in a more reliable prediction for A_{CP} relations. These relations serve as an important test of flavor $SU(3)$ symmetry in beauty baryon nonleptonic decays, and one can compare these findings with the analogous decays of bottom mesons to have a better understanding of the $SU(3)$ flavor symmetry-breaking pattern.

VI. CONCLUSIONS

We consider a general framework for hadronic beauty baryon decays into the octet or singlet of a light baryon and a pseudoscalar meson, based on $SU(3)$ decomposition of

the decay amplitudes. We show that in the most general case, the 44 distinct decay modes require 44 independent reduced $SU(3)$ amplitudes to describe all possible $\Delta S = -1$ and $\Delta S = 0$ processes. In practice, the dimension-6 effective Hamiltonian that mediates such nonleptonic decays of bottom baryons predicts only 10 independent reduced $SU(3)$ amplitudes. As a consequence, there must exist relations between the decay amplitudes. We explicitly derive several sum rule relations between decay amplitudes as well as relations between CP asymmetries. Moreover, we systematically study the $SU(3)$ -breaking effects in the decay amplitudes at leading order in the $SU(3)$ -breaking parameter. We further identify an amplitude relation that survives even when the $SU(3)$ flavor symmetry is no longer exact.

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APPENDIX A: $SU(3)$ ISOSCALAR FACTORS

The isoscalar factors depend of the identity of the representations, and on the hypercharges and isospins of the isomultiplets that are coupled. Let us denote them by the following notation:

$$\left(\begin{array}{ccc} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (y, i, i_3) & (y', i', i'_3) & (Y, I, I_3) \end{array} \right). \quad (A1)$$

The $SU(3)$ Clebsch-Gordan coefficients are found as products of isoscalar factors and $SU(2)$ Clebsch-Gordan coefficients:

$$\begin{aligned} & \langle \mathbf{R}, Y, I, I_3 | \mathbf{r}, y, i, i_3, \mathbf{r}', y', i', i'_3 \rangle \\ &= \left(\begin{array}{ccc} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (y, i, i_3) & (y', i', i'_3) & (Y, I, I_3) \end{array} \right) \\ & \times \langle I, I_3 | i, i_3, i', i'_3 \rangle \end{aligned}$$

where $\langle I, I_3 | i, i_3, i', i'_3 \rangle$ are the $SU(2)$ CG coefficients. The order in which the $SU(3)$ representations are coupled is $\mathbf{r} \otimes \mathbf{r}' \rightarrow \mathbf{R}$. The two symmetry relations involving the $SU(3)$ isoscalar factors are as follows:

- (1) If the order in which the representations are coupled is reversed (i.e., $\mathbf{r}' \otimes \mathbf{r} \rightarrow \mathbf{R}$), then the isoscalar

factors pick up a phase factor:

$$\begin{pmatrix} \mathbf{r}' & \mathbf{r} & \mathbf{R} \\ (y', i', i'_3) & (y, i, i_3) & (Y, I, I_3) \end{pmatrix} = (-1)^{I-i-i'} \xi(\mathbf{R}; \mathbf{r}, \mathbf{r}') \times \begin{pmatrix} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (y, i, i_3) & (y', i', i'_3) & (Y, I, I_3) \end{pmatrix}. \quad (\text{A2})$$

Here $\xi(\mathbf{R}; \mathbf{r}, \mathbf{r}')$ is the phase factor [88] that depends only on the identity element of $\mathbf{r}$, $\mathbf{r}'$, and $\mathbf{R}$, and the phase convention chosen for the highest weight state.

- (2) Conjugation operation on all three representations also gives rise to a phase factor:

$$\begin{pmatrix} \bar{\mathbf{r}} & \bar{\mathbf{r}}' & \bar{\mathbf{R}} \\ (y, i, i_3) & (y', i', i'_3) & (Y, I, I_3) \end{pmatrix} = (-1)^{I-i-i'} \xi(\mathbf{R}; \mathbf{r}, \mathbf{r}') \times \begin{pmatrix} \mathbf{r} & \mathbf{r}' & \mathbf{R} \\ (-y, i, -i_3) & (-y', i', -i'_3) & (-Y, I, -I_3) \end{pmatrix}. \quad (\text{A3})$$

Similar to the previous case, $\xi(\mathbf{R}; \mathbf{r}, \mathbf{r}')$ is the phase factor [88] that depends only on the identity element

of $\mathbf{r}$, $\mathbf{r}'$, and $\mathbf{R}$, and the phase convention chosen for the highest weight state. As a corollary of Eqs. (A2) and (A3),

$$\xi(\bar{\mathbf{R}}; \bar{\mathbf{r}}, \bar{\mathbf{r}}') = \xi(\mathbf{R}; \mathbf{r}, \mathbf{r}'), \quad (\text{A4})$$

$$\zeta(\mathbf{R}; \mathbf{r}', \mathbf{r}) = \zeta(\mathbf{R}; \mathbf{r}, \mathbf{r}'). \quad (\text{A5})$$

Following [88], we have adopted the Condon-Shortley and de Swart phase convention [84,85], which requires eigenvalues of the isospin (I) as well as V spin raising and lowering operators to be real and positive. An additional requirement on the highest weight state is that the Clebsch-Gordan coefficient between these three states be real and positive, i.e.,

$$\langle \mathbf{R}, Y_h, I_h, I_{h3} | \mathbf{r}, y_h, i_h, i_{h3}, \mathbf{r}', y'_h, i'_h, i'_{h3} \rangle > 0.$$

These conditions ensure that $SU(3)$ CG coefficients and isoscalar factors are all real.

APPENDIX B: $SU(3)$ DECOMPOSITION OF $\Delta S=0$ PROCESSES

The $\Delta S=0$ decay amplitudes for $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{8}_B \mathbf{8}_{\mathcal{M}}$ are $SU(3)$ decomposed in the following way:

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow n\eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^- K^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow p^+ \pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow n\pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Lambda^0 K^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^0 K^0) \\ \mathcal{A}(\Xi_b^- \rightarrow nK^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma^- \eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Lambda^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \Xi^- K^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma^- \pi^0) \\ \mathcal{A}(\Xi_b^0 \Xi^- K^+) \\ \mathcal{A}(\Xi_b^0 \Xi^- \pi^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^+ \pi^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow n\bar{K}^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 \pi^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow p^+ K^-) \\ \mathcal{A}(\Xi_b^0 \Xi^0 K^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Lambda^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Lambda^0 \pi^0) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{2} & \frac{1}{\sqrt{10}} & -\frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & -\frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{3}}{3} & \frac{1}{3\sqrt{3}} & \frac{\sqrt{3}}{3} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{13}} & \frac{3}{4\sqrt{3}} & \frac{3}{4\sqrt{3}} & \frac{\sqrt{2}}{4\sqrt{5}} & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{6}} & \frac{\sqrt{2}}{6} & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{6} & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{3}}{3} & \frac{1}{3\sqrt{3}} & \frac{\sqrt{3}}{3} & -\frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{13}} & \frac{3}{4\sqrt{3}} & \frac{3}{4\sqrt{3}} & \frac{\sqrt{2}}{4\sqrt{5}} & \frac{1}{4} & \frac{1}{4} & \frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{6}} & \frac{\sqrt{2}}{6} & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{6} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3\sqrt{15}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt{15}} & \frac{1}{2\sqrt{5}} & \frac{1}{\sqrt{30}} & -\frac{3}{4\sqrt{10}} & -\frac{3}{4\sqrt{10}} & -\frac{\sqrt{2}}{4\sqrt{5}} & -\frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{5}} & -\frac{1}{2} & -\frac{1}{2\sqrt{5}} & \frac{\sqrt{2}}{3} & \frac{2}{3\sqrt{3}} & \frac{\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{\sqrt{5}} & -\frac{\sqrt{2}}{2} & -\frac{1}{\sqrt{10}} & -\frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & -\frac{1}{4\sqrt{5}} & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{3\sqrt{15}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt{15}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{\sqrt{30}} & -\frac{3}{4\sqrt{10}} & -\frac{3}{4\sqrt{10}} & -\frac{\sqrt{2}}{4\sqrt{5}} & \frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{5}} & \frac{1}{2} & \frac{1}{2\sqrt{5}} & \frac{\sqrt{2}}{3} & \frac{2}{3\sqrt{3}} & \frac{\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 \\ \frac{2\sqrt{3}}{3} & -\frac{2\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{2}}{3} & \frac{1}{\sqrt{15}} & -\frac{3}{4\sqrt{5}} & \frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & \frac{1}{4} & -\frac{1}{12} & -\frac{1}{2\sqrt{6}} & -\frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{6}} & \frac{1}{6\sqrt{15}} & \frac{1}{3\sqrt{6}} & -\frac{\sqrt{2}}{6} & \frac{1}{3} & -\frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{10}} & -\frac{1}{3\sqrt{2}} & 0 \\ \frac{2}{3\sqrt{5}} & -\frac{2}{3\sqrt{5}} & -\frac{1}{3\sqrt{5}} & -\frac{1}{\sqrt{15}} & \frac{1}{\sqrt{10}} & \frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{30}} & -\frac{1}{2\sqrt{5}} & 0 & 0 & 0 & \frac{1}{4\sqrt{5}} & -\frac{1}{4} & \frac{1}{6\sqrt{10}} & \frac{1}{6} & -\frac{\sqrt{2}}{6} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{15}} & 0 & 0 \\ \frac{2}{3\sqrt{5}} & -\frac{2}{3\sqrt{5}} & -\frac{1}{3\sqrt{5}} & \frac{1}{\sqrt{15}} & -\frac{1}{\sqrt{10}} & \frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{30}} & -\frac{1}{2\sqrt{5}} & 0 & 0 & 0 & -\frac{1}{4\sqrt{5}} & \frac{1}{4} & \frac{1}{6\sqrt{10}} & \frac{1}{6} & -\frac{\sqrt{2}}{6} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{15}} & 0 & 0 \\ \frac{2\sqrt{6}}{3} & -\frac{2\sqrt{6}}{3} & -\frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & -\frac{1}{\sqrt{15}} & -\frac{3}{4\sqrt{5}} & \frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & -\frac{1}{4} & \frac{1}{12} & \frac{1}{2\sqrt{6}} & \frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{6}} & \frac{1}{6\sqrt{15}} & \frac{1}{3\sqrt{6}} & -\frac{\sqrt{2}}{6} & -\frac{1}{3} & \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{10}} & \frac{1}{3\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{3\sqrt{5}} & -\frac{1}{3\sqrt{10}} & 0 & 0 & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{5}} & \frac{1}{12} & -\frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{30}} & -\frac{1}{3\sqrt{2}} & \frac{1}{6} & 0 & -\frac{1}{3} & -\frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 0 & -\frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{10}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{5}} & -\frac{1}{12} & \frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{30}} & \frac{1}{3\sqrt{2}} & -\frac{1}{6} & 0 & \frac{1}{3} & -\frac{1}{\sqrt{5}} & 0 \\ \frac{1}{6\sqrt{30}} & -\frac{\sqrt{2}}{3} & \frac{\sqrt{3}}{3} & \frac{1}{3\sqrt{10}} & -\frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & 0 & 0 & -\frac{1}{3} & \frac{1}{\sqrt{6}} & -\frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt{6}} & \frac{\sqrt{2}}{6} & -\frac{1}{6} & -\frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{10}} & -\frac{1}{3\sqrt{2}} & 0 \\ \frac{1}{6\sqrt{30}} & \frac{1}{3\sqrt{30}} & -\frac{1}{3\sqrt{30}} & -\frac{1}{3\sqrt{10}} & \frac{1}{2\sqrt{15}} & \frac{1}{4\sqrt{5}} & -\frac{3}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & -\frac{1}{4} & \frac{1}{12} & \frac{1}{2\sqrt{6}} & \frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{6}} & \frac{\sqrt{2}}{6} & -\frac{1}{3\sqrt{6}} & -\frac{1}{6\sqrt{15}} & \frac{1}{6} & \frac{1}{3\sqrt{2}} & 0 & -\frac{\sqrt{2}}{3} & \frac{1}{\sqrt{10}} & \frac{1}{2\sqrt{2}} \\ \frac{1}{6\sqrt{30}} & \frac{1}{3\sqrt{30}} & -\frac{1}{3\sqrt{30}} & \frac{1}{3\sqrt{10}} & -\frac{1}{2\sqrt{15}} & \frac{1}{4\sqrt{5}} & -\frac{3}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & \frac{1}{4} & -\frac{1}{12} & -\frac{1}{2\sqrt{6}} & -\frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{6}} & \frac{\sqrt{2}}{6} & -\frac{1}{3\sqrt{6}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{6} & -\frac{1}{3\sqrt{2}} & 0 & \frac{\sqrt{2}}{3} & \frac{1}{\sqrt{10}} & \frac{1}{2\sqrt{2}} \\ \frac{7}{6\sqrt{30}} & \frac{1}{3\sqrt{30}} & -\frac{2\sqrt{3}}{3} & -\frac{1}{3\sqrt{10}} & \frac{1}{2\sqrt{15}} & -\frac{1}{4\sqrt{5}} & -\frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & \frac{1}{4} & -\frac{5}{12} & \frac{1}{2\sqrt{6}} & \frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt{6}} & \frac{\sqrt{2}}{6} & \frac{1}{6} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{10}} & \frac{1}{3\sqrt{2}} & 0 & -\frac{1}{2\sqrt{2}} \\ \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{3} & -\frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{30}} & \frac{1}{2\sqrt{3}} & \frac{\sqrt{2}}{4} & -\frac{1}{4\sqrt{15}} & -\frac{1}{2\sqrt{10}} & 0 & 0 & 0 & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{3}} & -\frac{1}{3\sqrt{2}} & \frac{\sqrt{2}}{6} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{15} & 0 & 0 & 0 \\ -\frac{1}{6\sqrt{30}} & -\frac{1}{3\sqrt{30}} & \frac{1}{3\sqrt{30}} & 0 & 0 & -\frac{1}{4\sqrt{5}} & \frac{3}{4\sqrt{5}} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{3} & -\frac{1}{3\sqrt{15}} & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{3} & -\frac{1}{2\sqrt{2}} \\ \frac{1}{6\sqrt{30}} & -\frac{\sqrt{2}}{3} & \frac{\sqrt{3}}{3} & -\frac{1}{3\sqrt{10}} & \frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & 0 & 0 & \frac{1}{3} & -\frac{1}{\sqrt{6}} & \frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt{6}} & \frac{\sqrt{2}}{6} & \frac{1}{6} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{10}} & \frac{1}{3\sqrt{2}} & 0 & \frac{1}{2\sqrt{2}} \\ \frac{7}{6\sqrt{30}} & \frac{1}{3\sqrt{30}} & -\frac{2\sqrt{3}}{3} & \frac{1}{3\sqrt{10}} & -\frac{1}{2\sqrt{15}} & -\frac{1}{4\sqrt{5}} & -\frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & -\frac{1}{4} & \frac{5}{12} & -\frac{1}{2\sqrt{6}} & -\frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{6}} & -\frac{1}{6\sqrt{15}} & -\frac{1}{3\sqrt{6}} & \frac{\sqrt{2}}{6} & -\frac{1}{6} & -\frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{10}} & -\frac{1}{3\sqrt{2}} & 0 & -\frac{1}{2\sqrt{2}} \\ -\frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{3} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{30}} & 0 & 0 & \frac{1}{4\sqrt{5}} & -\frac{3}{4\sqrt{5}} & \frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} \\ \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{3} & -\frac{1}{3\sqrt{10}} & \frac{1}{3\sqrt{30}} & -\frac{1}{2\sqrt{5}} & \frac{\sqrt{2}}{4} & -\frac{1}{4\sqrt{15}} & -\frac{1}{2\sqrt{10}} & 0 & 0 & 0 & \frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{3}} & -\frac{1}{3\sqrt{2}} & \frac{\sqrt{2}}{6} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{15} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \langle 27 || \mathbf{15}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{24}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \bar{\mathbf{10}} || \mathbf{24}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \bar{\mathbf{10}} || \mathbf{6}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8}_1 || \mathbf{3}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8}_1 || \mathbf{15}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8}_1 || \mathbf{6}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8}_2 || \mathbf{3}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8}_2 || \mathbf{15}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8}_2 || \mathbf{6}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{10} || \mathbf{15}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{10} || \mathbf{15}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{15}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{24}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle \bar{\mathbf{10}} || \mathbf{15}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle \bar{\mathbf{10}} || \mathbf{24}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8}_1 || \mathbf{15}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8}_2 || \mathbf{15}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle \mathbf{1} || \mathbf{3}_{1/2} || \bar{\mathbf{3}} \rangle \end{pmatrix}$$

The $\Delta S = 0$ decay amplitudes for $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{8}_B \mathbf{1}_{\mathcal{M}}$ are $SU(3)$ decomposed in the following way:

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow n\eta_1) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Lambda^0\eta_1) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0\eta_1) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma^-\eta_1) \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{4} & -\frac{\sqrt{3}}{2} & \frac{3}{4} & 0 \\ \frac{\sqrt{3}}{4} & -\frac{1}{2\sqrt{2}} & -\frac{1}{4\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \langle \mathbf{8} \| \mathbf{3}_{1/2} \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \bar{\mathbf{6}}_{1/2} \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \mathbf{15}_{1/2} \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \mathbf{15}_{3/2} \| \bar{\mathbf{3}} \rangle \end{pmatrix}. \quad (\text{B1})$$

The $\Delta S = 0$ decay amplitudes for $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{1}_B \mathbf{8}_{\mathcal{M}}$ are $SU(3)$ decomposed in the following way:

$$\begin{pmatrix} \mathcal{A}(\Xi_b^0 \rightarrow \Lambda_s^{0*}\eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Lambda_s^{0*}\pi_0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Lambda_s^{0*}\pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Lambda_s^{0*}K^0) \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & -\frac{\sqrt{3}}{2} & \frac{3}{4} & 0 \\ \frac{\sqrt{3}}{4} & -\frac{1}{2\sqrt{2}} & -\frac{1}{4\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} \begin{pmatrix} \langle \mathbf{8} \| \mathbf{3}_{1/2} \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \bar{\mathbf{6}}_{1/2} \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \mathbf{15}_{1/2} \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \mathbf{15}_{3/2} \| \bar{\mathbf{3}} \rangle \end{pmatrix}. \quad (\text{B2})$$

APPENDIX C: $SU(3)$ DECOMPOSITION OF $\Delta S = -1$ PROCESSES

The $\Delta S = -1$ decay amplitudes for $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{8}_B \mathbf{1}_{\mathcal{M}}$ are $SU(3)$ decomposed in the following way:

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Lambda^0\eta_1) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^0\eta_1) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi^-\eta_1) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi^0\eta_1) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{1}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \langle \mathbf{8} \| \mathbf{3}_0 \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \bar{\mathbf{6}}_0 \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \mathbf{15}_0 \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \mathbf{15}_1 \| \bar{\mathbf{3}} \rangle \end{pmatrix}. \quad (\text{C1})$$

The $\Delta S = -1$ decay amplitudes for $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{1}_B \mathbf{8}_{\mathcal{M}}$ are $SU(3)$ decomposed in the following way:

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Lambda_s^{0*}\pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Lambda_s^{0*}\eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Lambda_s^{0*}K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Lambda_s^{0*}\bar{K}^0) \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{1}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \langle \mathbf{8} \| \mathbf{3}_0 \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \bar{\mathbf{6}}_0 \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \mathbf{15}_0 \| \bar{\mathbf{3}} \rangle \\ \langle \mathbf{8} \| \mathbf{15}_1 \| \bar{\mathbf{3}} \rangle \end{pmatrix}. \quad (\text{C2})$$

The $\Delta S = -1$ decay amplitudes for $\bar{\mathbf{3}}_{B_b} \rightarrow \mathbf{8}_B \mathbf{8}_M$ are $SU(3)$ decomposed in the following way:

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^0 \eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Lambda^0 \eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^- \pi^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Lambda^0 \pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^0 \pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow p^+ K^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow n \bar{K}^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Xi^- K^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Xi^0 K^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma^- \bar{K}^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi^- \eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Lambda^0 K^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi^- \pi^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma^0 K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi^- \pi^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 \bar{K}^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^+ K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Lambda^0 \bar{K}^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 \pi^0) \end{pmatrix} = \begin{pmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{3}{2\sqrt{10}} & \frac{3}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{2}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & 0 & 0 & 0 & -\frac{1}{\sqrt{15}} & -\frac{1}{2\sqrt{15}} & 0 & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{10}} & -\frac{1}{6\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{2}}{2} & \frac{1}{3} & \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & 0 & 0 & 0 & \frac{1}{\sqrt{15}} & \frac{1}{2\sqrt{15}} & 0 & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{10}} & -\frac{1}{6\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{2}}{2} & \frac{1}{3} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{6\sqrt{10}} & \frac{1}{6\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{\sqrt{2}}{2} & \frac{2}{3} & \frac{\sqrt{2}}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{15}} & \frac{1}{2\sqrt{15}} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{5}} & \frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{4} & 0 & 0 & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 \\ -\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{2}} & \frac{1}{\sqrt{15}} & \frac{1}{2\sqrt{15}} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{10}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{4\sqrt{5}} & -\frac{\sqrt{2}}{4} & 0 & 0 & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & 0 & 0 \\ \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{15}} & -\frac{1}{2\sqrt{15}} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{5}} & \frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{4} & 0 & 0 & -\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & 0 & 0 \\ \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{5}} & \frac{1}{3\sqrt{2}} & -\frac{1}{\sqrt{15}} & -\frac{1}{2\sqrt{15}} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{10}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{4\sqrt{5}} & -\frac{\sqrt{2}}{4} & 0 & 0 & -\frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{4} & \frac{\sqrt{3}}{4} & 0 & 0 \\ \frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & 0 & \frac{1}{2\sqrt{15}} & -\frac{1}{\sqrt{15}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -\frac{1}{3\sqrt{10}} & \frac{1}{6\sqrt{5}} & -\frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{4} & \frac{1}{6} & -\frac{1}{3\sqrt{2}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{4} & -\frac{1}{4\sqrt{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & 0 & -\frac{1}{2\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & -\frac{1}{3\sqrt{10}} & \frac{1}{6\sqrt{5}} & -\frac{1}{4\sqrt{5}} & \frac{\sqrt{2}}{4} & \frac{1}{6} & -\frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{4} & \frac{1}{4\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2\sqrt{30}} & \sqrt{\frac{2}{15}} & -\frac{1}{2\sqrt{3}} & 0 & 0 & \frac{1}{4\sqrt{5}} & -\frac{1}{4\sqrt{5}} & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{4} & \frac{1}{4\sqrt{10}} & 0 & 0 & \frac{1}{4} & -\frac{1}{4} & -\frac{\sqrt{2}}{4} & \frac{1}{4\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\ -\frac{1}{4} & \frac{1}{4} & \frac{1}{2\sqrt{30}} & \sqrt{\frac{2}{15}} & -\frac{1}{2\sqrt{3}} & 0 & 0 & \frac{1}{4\sqrt{5}} & -\frac{1}{4\sqrt{5}} & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{4} & \frac{1}{4\sqrt{10}} & 0 & 0 & -\frac{1}{4} & \frac{1}{4} & \frac{\sqrt{2}}{4} & -\frac{1}{4\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 \\ -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{7}{6\sqrt{10}} & -\frac{1}{3\sqrt{10}} & -\frac{1}{6} & \frac{1}{\sqrt{30}} & -\sqrt{\frac{2}{15}} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & \frac{1}{6\sqrt{5}} & -\frac{1}{6\sqrt{10}} & \frac{3}{4\sqrt{10}} & -\frac{\sqrt{2}}{4} & \frac{1}{3\sqrt{2}} & -\frac{1}{3} & \frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{3}} & \frac{7}{6\sqrt{10}} & -\frac{1}{3\sqrt{10}} & -\frac{1}{6} & -\frac{1}{\sqrt{30}} & \sqrt{\frac{2}{15}} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & \frac{1}{6\sqrt{5}} & -\frac{1}{6\sqrt{10}} & \frac{3}{4\sqrt{10}} & -\frac{\sqrt{2}}{4} & \frac{1}{3\sqrt{2}} & -\frac{1}{3} & -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{6}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & 0 & \frac{1}{2\sqrt{15}} & -\frac{1}{\sqrt{15}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & \frac{1}{3\sqrt{10}} & -\frac{1}{6\sqrt{5}} & \frac{3}{4\sqrt{5}} & -\frac{\sqrt{2}}{4} & -\frac{1}{6} & \frac{1}{3\sqrt{2}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & -\frac{1}{4} & \frac{1}{4\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} \\ \frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{3}} & \frac{7}{6\sqrt{10}} & -\frac{1}{3\sqrt{10}} & -\frac{1}{6} & -\frac{1}{\sqrt{30}} & \sqrt{\frac{2}{15}} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & -\frac{1}{6\sqrt{5}} & \frac{1}{6\sqrt{10}} & -\frac{3}{4\sqrt{10}} & \frac{\sqrt{2}}{4} & -\frac{1}{3\sqrt{2}} & \frac{1}{3} & -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & 0 & -\frac{1}{2\sqrt{15}} & \frac{1}{\sqrt{15}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & \frac{1}{3\sqrt{10}} & -\frac{1}{6\sqrt{5}} & \frac{3}{4\sqrt{5}} & -\frac{\sqrt{2}}{4} & -\frac{1}{6} & \frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{6}} & -\frac{1}{2\sqrt{6}} & \frac{1}{4} & -\frac{1}{4\sqrt{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} \\ -\frac{1}{4} & \frac{1}{4} & -\frac{1}{2\sqrt{30}} & -\sqrt{\frac{2}{15}} & \frac{1}{2\sqrt{3}} & 0 & 0 & -\frac{1}{4\sqrt{5}} & \frac{1}{4\sqrt{5}} & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{4} & \frac{1}{4\sqrt{10}} & 0 & 0 & -\frac{1}{4} & \frac{1}{4} & -\frac{\sqrt{2}}{4} & \frac{1}{4\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{2\sqrt{30}} & -\sqrt{\frac{2}{15}} & \frac{1}{2\sqrt{3}} & 0 & 0 & -\frac{1}{4\sqrt{5}} & \frac{1}{4\sqrt{5}} & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{4} & \frac{1}{4\sqrt{10}} & 0 & 0 & \frac{1}{4} & -\frac{1}{4} & \frac{\sqrt{2}}{4} & -\frac{1}{4\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 \\ -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{7}{6\sqrt{10}} & -\frac{1}{3\sqrt{10}} & -\frac{1}{6} & \frac{1}{\sqrt{30}} & -\sqrt{\frac{2}{15}} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & -\frac{1}{6\sqrt{5}} & \frac{1}{6\sqrt{10}} & -\frac{3}{4\sqrt{10}} & \frac{\sqrt{2}}{4} & -\frac{1}{3\sqrt{2}} & \frac{1}{3} & \frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{6}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} \langle 10 | \mathbf{15}_1 | \bar{\mathbf{3}} \rangle \\ \langle 10 | \mathbf{15}_1' | \bar{\mathbf{3}} \rangle \\ \langle 27 | \mathbf{15}_1 | \bar{\mathbf{3}} \rangle \\ \langle 27 | \mathbf{24}_1 | \bar{\mathbf{3}} \rangle \\ \langle 27 | \mathbf{42}_1 | \bar{\mathbf{3}} \rangle \\ \langle \bar{10} | \mathbf{24}_1 | \bar{\mathbf{3}} \rangle \\ \langle \bar{10} | \mathbf{6}_1 | \bar{\mathbf{3}} \rangle \\ \langle 8_1 | \mathbf{15}_1 | \bar{\mathbf{3}} \rangle \\ \langle 8_1 | \mathbf{6}_1 | \bar{\mathbf{3}} \rangle \\ \langle 1 | \mathbf{3}_0 | \bar{\mathbf{3}} \rangle \\ \langle 27 | \mathbf{15}_0 | \bar{\mathbf{3}} \rangle \\ \langle 27 | \mathbf{42}_0 | \bar{\mathbf{3}} \rangle \\ \langle 8_1 | \mathbf{3}_0 | \bar{\mathbf{3}} \rangle \\ \langle 8_1 | \mathbf{15}_0 | \bar{\mathbf{3}} \rangle \\ \langle 27 | \mathbf{24}_2 | \bar{\mathbf{3}} \rangle \\ \langle 27 | \mathbf{42}_2 | \bar{\mathbf{3}} \rangle \\ \langle 8_2 | \mathbf{15}_1 | \bar{\mathbf{3}} \rangle \\ \langle 8_2 | \mathbf{6}_1 | \bar{\mathbf{3}} \rangle \\ \langle 8_2 | \mathbf{3}_0 | \bar{\mathbf{3}} \rangle \\ \langle 8_2 | \mathbf{15}_0 | \bar{\mathbf{3}} \rangle \\ \langle 10 | \mathbf{15}_0 | \bar{\mathbf{3}} \rangle \\ \langle \bar{10} | \mathbf{24}_2 | \bar{\mathbf{3}} \rangle \end{pmatrix}$$

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