NLO QCD corrections to inclusive charmonium and B_c meson production in W^+ decays

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We calculate the next-to-leading order (NLO) quantum chromodynamics (QCD) corrections to inclusive processes $W^+ \rightarrow J/\psi(\eta_c) + c + \bar{s} + X$ and $W^+ \rightarrow B_c(B_c^*) + b + \bar{s} + X$ in the framework of nonrelativistic QCD (NRQCD) factorization formalism. The result indicates that the NLO corrections are significant, and the uncertainties in theoretical predictions with NLO corrections are greatly reduced. The charmonium and B_c meson yielding rates at the Large Hadron Collider (LHC) are given.

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I. INTRODUCTION

In the standard model (SM), the W boson mass is generated through the electroweak spontaneous breaking mechanism. Precise measurement of W boson mass and its decay width turns out to be a unique test of the SM and hence a probe for new physics. At the Large Hadron Collider (LHC), a huge number of W bosons are produced and recorded, which enables the research on W physics feasible and meaningful.

Heavy quarkonium and as well B_c meson production keeps on being an interesting and hot topic to study in high energy physics for decades, which may enrich our knowledge on the properties of quarkonium and the nature of perturbative QCD. Note, hereafter for simplicity the B_c represents for both scalar B_c and vector B_c^* unless specifically mentioned. Based on the nonrelativistic QCD (NRQCD) factorization formalism [1], direct hadroproduction of quarkonium and B_c meson was studied extensively [2–12]. In addition to the direct production, indirect production also stands as an independent and important source for those double-heavy mesons. The quarkonium and B_c meson production through top quark [13] and Z_0 decays [14–16] had been investigated at up to the next-to-leading order (NLO) accuracy. For indirect quarkonium and B_c in W decays, the leading order (LO) analyses were

performed in Refs. [17,18]. It turned out that the theoretical uncertainties at LO are very large, which suggests, and was partly confirmed, that the higher order QCD corrections in charmonium and B_c productions are usually very important, even crucial sometimes, for the sake of phenomenological use. To this end, we calculate in this work the NLO QCD corrections to the inclusive charmonium and B_c production in W^+ decays.

The rest of the paper is organized as follows. In Sec. II we present the LO calculation of W^+ decay to charmonium and B_c mesons. In Sec. III, some technical details in the calculation of NLO corrections are given. In Sec. IV, the numerical evaluation for concerned processes is performed at NLO QCD accuracy. The last section is remained for summary and conclusions.

II. THE LO DECAY WIDTH

At the LO in α_s , inclusive charmonium and B_c meson production through W^+ decays are described by the processes

$$\begin{aligned} W^+(p_W) &\rightarrow J/\psi(\eta_c)(p_H) + c(p_Q) + \bar{s}(p_s), \\ W^+(p_W) &\rightarrow B_c(B_c^*)(p_H) + b(p_Q) + \bar{s}(p_s) \end{aligned} \quad (1)$$

as shown in Fig. 1. The initial and final state particles are on the mass shell: $p_W^2 = m_W^2$, $p_H^2 = m_H^2$, $p_Q^2 = m_Q^2$ and $p_s^2 = 0$. We also introduce the Mandelstam variables: $s_1 = (p_H + p_Q)^2$, $s_2 = (p_H + p_s)^2$. The contributions from the color-octet (CO) heavy quark-antiquark intermediate state are not included in our calculation. According to the LO analysis [18], the contributions of CO channels are negligible in comparison to the color-singlet channels, although they may receive kinematic enhancement in the charmonium production case. For $B_c(B_c^*)$ production, the process in (1) indicate the production of $c\bar{s}$ pair with

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FIG. 1. The LO Feynman diagrams for charmonium and B_c meson production in W^+ decays.

subsequent $b\bar{b}$ pair creation. Note, the process W^+ to $c\bar{b}$ with subsequent $c\bar{c}$ creation, i.e., $W^+ \rightarrow B_c(B_c^*) + c + \bar{c}$, may have comparable contribution in rough estimation, due to the CKM suppression is partly balanced by the $c\bar{c}$ pair emission. However, practical numerical calculation [17,18] indicates that the decay width of this process is only about 1/10 of the $W^+ \rightarrow B_c(B_c^*) + b + \bar{s}$ process. Hence this process will not be considered in this work.

At the leading order of the relative velocity expansion, it is legitimate to take $p_c = p_{\bar{c}}$, $m_H = 2m_c$ for charmonium production and $p_c = \frac{m_c}{m_b} p_{\bar{b}}$, $m_H = m_b + m_c$ for B_c production. The spin and color projection operator has the form

$$\Pi(n) = \frac{1}{2\sqrt{m_H}} \epsilon(n) (\not{p}_H + m_H) \otimes \left(\frac{1_c}{\sqrt{N_c}} \right), \quad (2)$$

where $\epsilon(^1S_0) = \gamma_5$, $\epsilon(^3S_1) = \not{\epsilon}$, 1_c stands for the unit color matrix, and $N_c = 3$ for the number of colors in QCD. The decay width at LO reads:

$$\Gamma_{\text{born}} = \frac{|\Psi_H(0)|^2}{2m_W} \frac{1}{3} \int \sum |\mathcal{M}_{\text{born}}|^2 d\text{PS}_3. \quad (3)$$

Here, $\sum$ sums over the polarizations and colors of the initial and final particles, $\frac{1}{3}$ comes from the spin average of the initial W^+ boson, $d\text{PS}_3$ stands for the three-body phase space. Taking the rest frame of W^+ and averaging over the spin of W^+ , the amplitude square is spherically symmetric, and the phase space can be expressed as

$$\int d\text{PS}_3 = \frac{1}{32\pi^3} \int_{m_H}^{E_H^+} dE_H \int_{E_s^-}^{E_s^+} dE_s. \quad (4)$$

Here, E_H and E_s represent the energy of final state hadron and s quark, respectively. The upper and lower bounds of the above integration are

$$E_H^+ = \frac{m_W^2 + m_H^2 - m_Q^2}{2m_W},$$

$$E_s^\pm = \frac{1}{2} \left(1 - \frac{m_Q^2}{M_{Qs}^2} \right) (m_W - E_H \pm \sqrt{E_H^2 - m_H^2}) \quad (5)$$

with

$$M_{Qs}^2 = m_W^2 + m_H^2 - 2m_W E_H. \quad (6)$$

III. THE NLO CORRECTIONS

At the NLO, the W^+ boson decay to charmonium and $B_c(B_c^*)$ meson include the virtual and real corrections of process (1), which are shown in Figs. 2 and 3, respectively. For η_c production, new subprocess $W^+ \rightarrow \eta_c + u + \bar{d} + g$, as shown in Fig. 4, should also be included. In the computation of NLO corrections, the conventional dimensional

FIG. 2. Typical Feynman diagrams in virtual corrections.

FIG. 3. Typical Feynman diagrams in real corrections.

FIG. 4. Feynman diagrams of $W^+ \rightarrow \eta_c + u + \bar{d} + g$ process.

regularization with $D = 4 - 2\epsilon$ is adopted to regularize the ultraviolet (UV) and infrared (IR) divergences. The method proposed in [19,20] is used to deal with the D dimensions γ_5 trace.

In the calculation, the package FeynArts [21] is used to generate Feynman diagrams; FeynCalc [22,23] and FORM [24,25] are used to perform algebraic calculation; FIRE [26,27] is employed to reduce the Feynman integrals into the master integrals (A_0 , B_0 , C_0 , D_0); With the help of Ref. [28] and Package-X [29], the master integrals are calculated analytically, and the results are checked by LoopTools [30]; The numerical phase space integration is performed by CUBA [31].

A. Virtual corrections

Typical Feynman diagrams in virtual corrections are shown in Fig. 2. The contribution from virtual corrections can be formulated as

$$\Gamma_{\text{virtual}} = \frac{|\Psi_H(0)|^2}{2m_W} \frac{1}{3} \int \sum 2\text{Re}(\mathcal{M}_{\text{virtual}} \mathcal{M}_{\text{born}}^*) d\text{PS}_3. \quad (7)$$

Here, $\text{Re}(\mathcal{M}_{\text{virtual}} \mathcal{M}_{\text{born}}^*)$ contains both UV and IR singularities. Since we set $p_c = p_{\bar{c}}$ and $p_c = \frac{m_c}{m_b} p_{\bar{b}}$ before the calculation of Feynman integrals, the Coulomb singularities, which are power IR divergences and may vanish in the dimensional regularization [32].

The UV singularities are removed by renormalization. For the renormalization of heavy quark field (Z_2), heavy quark mass (Z_m) and light quark field (Z_l), we take the on-shell (OS) scheme; for the renormalization of gluon field

(Z_3) and strong coupling constant (Z_g), the modified minimal-subtraction ($\overline{\text{MS}}$) schemes are used:

$$\begin{aligned} \delta Z_2^{\text{OS}} &= -C_F \frac{\alpha_s}{4\pi} \left[\frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} - 3\gamma_E + 3 \ln \frac{4\pi\mu^2}{m_Q^2} + 4 \right], \\ \delta Z_m^{\text{OS}} &= -3C_F \frac{\alpha_s}{4\pi} \left[\frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln \frac{4\pi\mu^2}{m_Q^2} + \frac{4}{3} \right], \\ \delta Z_l^{\text{OS}} &= -C_F \frac{\alpha_s}{4\pi} \left[\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right], \\ \delta Z_3^{\overline{\text{MS}}} &= \frac{\alpha_s}{4\pi} (\beta_0 - 2C_A) \left[\frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln(4\pi) \right], \\ \delta Z_g^{\overline{\text{MS}}} &= -\frac{\beta_0}{2} \frac{\alpha_s}{4\pi} \left[\frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln(4\pi) \right]. \end{aligned} \quad (8)$$

Here, μ is the renormalization scale, γ_E is the Euler's constant; $\beta_0 = (11/3)C_A - (4/3)T_f n_f$ is the one-loop coefficient of QCD beta function, n_f is the number of active quarks; $C_A = 3$, $C_F = 4/3$ and $T_F = 1/2$ are color factors. Note, final result is independent of δZ_3 , because terms proportional to δZ_3 from vertex correction cancel with that from self-energy correction.

In virtual corrections, the IR singularities arise when the gluon connecting two on shell partons is soft or collinear to final $\bar{s}$ quark. Due to $p_c = p_{\bar{c}}$ or $p_c = \frac{m_c}{m_b} p_{\bar{b}}$, parts of IR singularities cancel each other [1]. The remaining are canceled by the real corrections according to the Kinoshita-Lee-Nauenberg theorem [33,34].

B. Real corrections

Typical Feynman diagrams in real corrections are shown in Fig. 3. In the calculation of the real corrections, the phase space slicing method [35] is adopted to separate the IR singularities. By introducing soft cut δ_s and collinear cut δ_c , which satisfies $\delta_s \gg \delta_c$, the phase space can be separated into three regions:

- (i) Soft: $p_g^0 < \frac{m_W}{2} \delta_s$;
- (ii) Hard collinear: $p_g^0 > \frac{m_W}{2} \delta_s$, $M_{sg}^2 < m_W^2 \delta_c$;
- (iii) Hard noncollinear: $p_g^0 > \frac{m_W}{2} \delta_s$, $M_{sg}^2 > m_W^2 \delta_c$.

Here, $M_{sg} = (p_s + p_g)^2$ is the invariant mass of $\bar{s}$ and g system. Then the real corrections can be written as

$$\Gamma_{\text{real}} = \Gamma_{\text{real}}^{\text{S}} + \Gamma_{\text{real}}^{\text{HC}} + \Gamma_{\text{real}}^{\text{HNC}}, \quad (9)$$

where the superscripts “S,” “HC,” “HNC” represent the “soft,” “hard collinear,” “hard noncollinear” regions, respectively.

After neglecting terms of order δ_s and δ_c , the contributions from the soft part and hard collinear part read [35]

$$\begin{aligned}
 \Gamma_{\text{real}}^S &= \Gamma_{\text{born}} C_F \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{m_W^2} \right)^\epsilon \frac{1}{\delta_s^{2\epsilon}} \\
 &\times \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left(1 - \ln \frac{m_W^2}{m_Q^2} - 2 \ln \frac{m_H^2 + m_W^2 - s_1 - s_2}{m_W^2 - s_1} \right) \right. \\
 &\left. + \text{finite} \right], \\
 \Gamma_{\text{real}}^{\text{HC}} &= \Gamma_{\text{born}} C_F \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{m_W^2} \right) \\
 &\times \left[\frac{1}{\epsilon} \left(2 \ln \frac{m_W^2 \delta_s}{m_W^2 - s_1} + \frac{3}{2} \right) - \frac{3}{2} \ln \delta_c \right. \\
 &\left. - 2 \ln \delta_c \ln \frac{m_W^2 \delta_s}{m_W^2 - s_1} - \left(\ln \frac{m_W^2 \delta_s}{m_W^2 - s_1} \right)^2 - \frac{2\pi^2 - 21}{6} \right]. \quad (10)
 \end{aligned}$$

In the case of hard noncollinear part, the decay width reads

$$\Gamma_{\text{real}}^{\text{HNC}} = \frac{|\Psi_H(0)|^2}{2m_W} \frac{1}{3} \int^{\text{HNC}} \sum |\mathcal{M}_{\text{real}}|^2 d\text{PS}_4, \quad (11)$$

where the four-body phase space $d\text{PS}_4$ with cut can be written as

$$\begin{aligned}
 \int^{\text{HNC}} d\text{PS}_4 &= \frac{1}{512\pi^6} \int_{m_H}^{E_H^+} dE_H \int_{\sqrt{\delta_c} m_W}^{M_{sg}^+} dM_{sg} \\
 &\times \int_{E_{sg}^-}^{E_{sg}^+} dE_{sg} \frac{M_{sg}}{\sqrt{E_{sg}^2 - M_{sg}^2}} \\
 &\times \int_{\max(E_g^-, \delta_s m_W/2)}^{E_g^+} dE_g \Theta(E_g^+ - \delta_s m_W/2) \\
 &\times \int_0^{2\pi} d\eta_{sg}, \quad (12)
 \end{aligned}$$

with

$$\begin{aligned}
 E_H^+ &= \frac{m_W^2 + m_H^2 - (m_Q + \sqrt{\delta_c} m_W)^2}{2m_W}, \\
 M_{csg} &= \sqrt{m_W^2 + m_H^2 - 2m_W E_\psi}, \\
 M_{sg}^+ &= M_{csg} - m_Q, \\
 E_{sg}^\pm &= \frac{1}{4m_W M_{csg}^2} \left[(M_{csg}^2 - m_Q^2 + M_{sg}^2)(m_W^2 - m_H^2 + M_{csg}^2) \right. \\
 &\quad \left. \pm \sqrt{\lambda(M_{csg}^2, m_Q^2, M_{sg}^2)} \lambda(m_W^2, m_H^2, M_{csg}^2) \right], \\
 E_g^\pm &= \frac{1}{2} \left(E_{sg} \pm \sqrt{E_{sg}^2 - M_{sg}^2} \right), \quad (13)
 \end{aligned}$$

where $\lambda(s, m_a^2, m_b^2) = [s - (m_a + m_b)^2][s - (m_a - m_b)^2]$, and $\Theta(x)$ is the unit step function which return 1 when $x > 0$ and 0 for other case. After summing up these three

parts, the dependence on technical cuts will be eliminated as expected. Specifically, we vary δ_s from 10^{-3} to 10^{-6} with $\delta_c = \delta_s/1000$, and find the variations of final results are less than 0.5%.

For the subprocess $W^+ \rightarrow \eta_c + u + \bar{d} + g$, there are 4 diagrams, as shown in Fig. 4. The IR singularities are eliminated after summing all the amplitude square parts. The decay width can be calculated directly in 4 dimensions as $\Gamma_{\text{real}}^{\text{HNC}}$. Note, the real correction $W^+ \rightarrow \eta_c + c + \bar{s} + g$ with same topology but with u and $\bar{d}$ replaced by c and $\bar{s}$ is also considered.

IV. NUMERICAL RESULTS

For the numerical calculation, following input parameters are used [36]

$$\begin{aligned}
 \alpha &= 1/137.065, \quad \sin^2 \theta_W = 0.2312, \quad m_W = 80.399 \text{ GeV}, \\
 \bar{m}_c(\bar{m}_c) &= 1.275_{-0.035}^{+0.025} \text{ GeV}, \quad \bar{m}_b(\bar{m}_b) = 4.18_{-0.03}^{+0.04} \text{ GeV}. \quad (14)
 \end{aligned}$$

Here, θ_W is the Weinberg angle, $\bar{m}_Q(\mu)$ is the $\overline{\text{MS}}$ heavy quark mass. The relation between the $\overline{\text{MS}}$ quark mass and the pole mass m_Q up to $\mathcal{O}(\alpha_s)$ can be written as

$$m_Q = \bar{m}_Q(\mu) \left\{ 1 + \frac{\alpha_s(\mu)}{4\pi} C_F \left[3 \ln \frac{\mu^2}{\bar{m}_Q(\mu)^2} + 4 \right] \right\}. \quad (15)$$

Taking the $\bar{m}_Q(\bar{m}_Q)$ as input parameter, the pole mass m_Q can be determined order by order [37], which leads to the values collected in Table I.

The wave functions at the origin are taken to be

$$\begin{aligned}
 |\Psi_{J/\psi}^{\text{LO}}(0)|^2 &= \frac{0.381}{4\pi} \text{ GeV}^3, \quad |\Psi_{J/\psi}^{\text{NLO}}(0)|^2 = \frac{0.882}{4\pi} \text{ GeV}^3, \\
 \Psi_{\eta_c}(0) &= \Psi_{J/\psi}(0), \\
 |\Psi_{B_c}(0)|^2 &= |\Psi_{B_c^*}(0)|^2 = \frac{1.642}{4\pi} \text{ GeV}^3. \quad (16)
 \end{aligned}$$

Here, the J/ψ wave function at the origin is extracted from its leptonic width:

$$\Gamma(J/\psi \rightarrow e^+ e^-) = \frac{16\pi\alpha^2}{9m_c^2} |\Psi_{J/\psi}(0)|^2 \left(1 - 4C_F \frac{\alpha_s}{\pi} \right), \quad (17)$$

TABLE I. Values of the pole masses m_c and m_b at LO and NLO, corresponding to $\bar{m}_c(\bar{m}_c) = 1.275_{-0.035}^{+0.025} \text{ GeV}$ and $\bar{m}_b(\bar{m}_b) = 4.18_{-0.03}^{+0.04} \text{ GeV}$.

	m_c (GeV)	m_b (GeV)
LO	$1.275_{-0.035}^{+0.025}$	$4.18_{-0.03}^{+0.04}$
NLO	$1.48_{-0.04}^{+0.02}$	$4.57_{-0.03}^{+0.04}$

TABLE II. The parameters A_H and B_H in Eq. (19). The units of heavy quark mass and A_H are suppressed for brevity, which are GeV and GeV^{-2} respectively.

	$m_c = 1.44, m_b = 4.54$		$m_c = 1.48, m_b = 4.57$		$m_c = 1.50, m_b = 4.61$	
	A_H	B_H	A_H	B_H	A_H	B_H
$W^+ \rightarrow Hq_i \bar{q}_j$						
$W^+ \rightarrow J/\psi c \bar{s}$	0.958	29.0	0.882	25.3	0.846	24.2
$W^+ \rightarrow \eta_c c \bar{s}$	0.927	31.8	0.853	29.0	0.818	27.7
$W^+ \rightarrow \eta_c u \bar{d} g$	0	3.59	0	3.21	0	3.03
$W^+ \rightarrow B_c b \bar{s}$	0.0257	0.738	0.0252	0.719	0.0245	0.697
$W^+ \rightarrow B_c^* b \bar{s}$	0.0222	0.532	0.0218	0.519	0.0212	0.503

TABLE III. The decay widths of W^+ inclusive decay to charmonium and $B_c(B_c^*)$ meson are presented in $\mu = m_H$ and $\mu = E_H^+$ with $E_H^+ = \frac{m_W^2 + m_H^2 - m_Q^2}{2m_W}$. The errors are caused by $m_c = 1.275_{-0.035}^{+0.025}$, $m_b = 4.18_{-0.03}^{+0.04}$ at LO and $m_c = 1.48_{-0.04}^{+0.02}$, $m_b = 4.57_{-0.03}^{+0.04}$ at NLO.

μ	$W^+ \rightarrow J/\psi c \bar{s}$		$W^+ \rightarrow \eta_c c \bar{s}$		$W^+ \rightarrow \eta_c u \bar{d} g$		$W^+ \rightarrow B_c b \bar{s}$		$W^+ \rightarrow B_c^* b \bar{s}$	
	m_H	E_H^+	m_H	E_H^+	m_H	E_H^+	m_H	E_H^+	m_H	E_H^+
$\Gamma_{\text{LO}} (\text{keV})$	$160.8_{-9.2}^{+14.2}$	$31.1_{-1.8}^{+2.8}$	$155.7_{-9.0}^{+13.8}$	$30.1_{-1.7}^{+2.7}$	...	...	$8.77_{-0.27}^{+0.20}$	$3.37_{-0.11}^{+0.07}$	$7.54_{-0.22}^{+0.16}$	$2.90_{-0.09}^{+0.06}$
$\Gamma_{\text{NLO}} (\text{keV})$	$129.4_{-6.2}^{+13.4}$	$60.5_{-2.6}^{+5.5}$	$171.7_{-8.3}^{+18.0}$	$64.9_{-2.8}^{+6.1}$	$33.6_{-1.8}^{+4.0}$	$4.61_{-0.25}^{+0.55}$	$6.34_{-0.20}^{+0.17}$	$3.64_{-0.11}^{+0.09}$	$4.43_{-0.15}^{+0.11}$	$2.83_{-0.08}^{+0.07}$

with $\Gamma(J/\psi \rightarrow e^+ e^-) = 5.55 \text{ keV}$ [36]. The B_c wave function is estimated by using the Buchmueller-Tye potential [38].

The two-loop strong coupling of

$$\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0 L} - \frac{\beta_1 \ln L}{\beta_0^3 L^2} \quad (18)$$

is employed in the NLO calculation, in which, $L = \ln(\mu^2/\Lambda_{\text{QCD}}^2)$, $\beta_0 = (11/3)C_A - (4/3)T_F n_f$, $\beta_1 = (34/3)C_A^2 - 4C_F T_F n_f - (20/3)C_A T_F n_f$. We take $n_f = 4$, $\Lambda_{\text{QCD}} = 292 \text{ MeV}$ for J/ψ and η_c production; $n_f = 5$, $\Lambda_{\text{QCD}} = 210 \text{ MeV}$ for B_c and B_c^* production.

The NLO decay width can be expressed as

$$\Gamma_H^{\text{NLO}}(\mu) = \frac{\alpha_s(\mu)^2}{\sin^2 \theta_W} |\Psi_H(0)|^2 \times \left(A_H + \frac{\alpha_s(\mu)}{\pi} \left(B_H + A_H C_H \ln \frac{\mu^2}{m_W^2} \right) \right), \quad (19)$$

where $C_H = 25/6$ for $W^+ \rightarrow J/\psi(\eta_c) + c + \bar{s}$, $C_H = 0$ for $W^+ \rightarrow \eta_c + u + \bar{d} + g$, $C_H = 23/6$ for $W^+ \rightarrow B_c(B_c^*) + b + \bar{s}$. The parameters A_H and B_H are independent of μ , their values at different heavy quark mass are shown in Table II.

The decay widths at $\mu = m_H$ and $\mu = \frac{m_W^2 + m_H^2 - m_Q^2}{2m_W}$ are presented in Table III. The theoretical uncertainties caused by heavy quark mass are also exhibited. With NLO corrections, the renormalization scale dependence of the decay widths are greatly suppressed as expected. According to Ref. [36], total decay width for W^+ boson is about 2.195 GeV, the corresponding branching fractions are then shown in Table IV. The decay widths versus running renormalization scale are exhibited in Fig. 5.

To investigate the convergence behavior of perturbative expansion, we consider the measure $\mathcal{R} = |\frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}|$ introduced in Ref. [39] as well. For J/ψ or η_c production, the values of $\mathcal{R}$ may reach or even exceed 1 in the region $\mu \in [m_H, \frac{m_W^2 + m_H^2 - m_Q^2}{2m_W}]$. However, for $B_c(B_c^*)$ production, the

TABLE IV. Branching fractions of W^+ inclusive decay to charmonium and $B_c(B_c^*)$ meson. The upper and lower correspond to $\mu = m_H$ and $\mu = \frac{m_W^2 + m_H^2 - m_Q^2}{2m_W}$ respectively. The uncertainties caused by heavy quark mass are also included.

	$W^+ \rightarrow J/\psi c \bar{s}$	$W^+ \rightarrow \eta_c c \bar{s}$	$W^+ \rightarrow \eta_c u \bar{d} g$	$W^+ \rightarrow B_c b \bar{s}$	$W^+ \rightarrow B_c^* b \bar{s}$
$\text{Br}_{\text{LO}} \times 10^5$	1.33 ~ 7.97	1.29 ~ 7.72	...	0.149 ~ 0.409	0.128 ~ 0.351
$\text{Br}_{\text{NLO}} \times 10^5$	2.64 ~ 6.51	2.83 ~ 8.64	0.199 ~ 1.713	0.161 ~ 0.297	0.125 ~ 0.207

FIG. 5. The LO (dashed line) and NLO (solid line) decay widths versus running renormalization scale. The uncertainties caused by heavy quark mass are represented by color band.

convergence manifests well, where the values of $\mathcal{R}$ are less than 0.5, compatible with the results in Ref. [39].

The energy distribution of charmonium and $B_c(B_c^*)$ meson are shown in Fig. 6. It can be seen from Fig. 6(b) that the η_c production rate are largely enhanced at small energy region. This enhancement occurs because the diagrams similar to Fig. 4, except u and $\bar{d}$ are replaced by c and $\bar{s}$, yield $g \rightarrow \eta_c + g$ fragmentation contributions. The contribution from the gluon propagator can be estimated as:

$$\int_{-1}^1 d\cos\theta_{\eta g} \frac{1}{(4m_c^2 + 2E_\eta E_g - 2|\vec{p}_\eta|E_g \cos\theta_{\eta g})^2} \sim \frac{1}{E_g E_\eta + E_g^2 + m_c^2}, \quad (20)$$

which indeed increases as E_η , the energy of η_c , decreases.

The instantaneous luminosity of LHC reach $2.06 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ in 2017 [40]. The production cross section of W^+ boson at the LHC can be estimated to be 100 nb

FIG. 6. The charmonium and $B_c(B_c^*)$ meson energy distribution in W^+ decay. The LO and NLO results are represented by double-dashed and double-solid lines, with the upper one referring to $\mu = m_H$, lower one referring to $\mu = \frac{m_W^2 + m_H^2 - m_Q^2}{2m_W}$. The uncertainties caused by heavy quark mass are represented by color band.

[41], then the number of W^+ events per year is about 6.5×10^{10} . Hence we can obtain about $(1.72 \sim 4.23) \times 10^6$ J/ψ events, $(1.97 \sim 6.73) \times 10^6$ η_c events and $(1.86 \sim 3.28) \times 10^5$ B_c events per year. Here, the B_c^* feed-down to B_c is taken into account. In experiment, the B_c meson can be fully reconstructed through $B_c \rightarrow J/\psi \pi^+$ decay, whose branching fraction is about 0.5% [42]. According to [36], the branching ratio $\text{Br}(J/\psi \rightarrow l^+ l^- (l = e, \mu)) = 12\%$,

$\text{Br}(\eta_c \rightarrow p \bar{p}) = 0.15\%$, then the numbers of J/ψ , η_c and B_c meson candidates per year are $(2.06 \sim 5.08) \times 10^5$, $(2.95 \sim 10.09) \times 10^3$ and $112 \sim 196$ respectively.

V. SUMMARY AND CONCLUSIONS

In this work we calculate the decay widths of W^+ to J/ψ , η_c and $B_c(B_c^*)$ mesons at the NLO QCD accuracy within

the NRQCD factorization framework. The theoretical uncertainties are estimated by varying the value of heavy quark mass and renormalization scale. Considering there are copious W data at the LHC, our results are hopefully to be tested in experiment.

Numerical calculation shows that the NLO corrections are significant, and the uncertainties in theoretical predictions with NLO corrections are greatly reduced. Since B_c^* almost all decays to B_c , assuming B_c is reconstructed through $B_c \rightarrow J/\psi \pi^+$, J/ψ is reconstructed through $J/\psi \rightarrow l^+ l^-$ ($l = e, \mu$), η_c is reconstructed through $\eta_c \rightarrow p \bar{p}$, the numbers of J/ψ , η_c and B_c meson candidates per year may reach $(2.06 \sim 5.08) \times 10^5$, $(2.95 \sim 10.09) \times 10^3$ and $112 \sim 196$ respectively at the LHC 2017 luminosity. However, it should be mentioned that despite of the inspiring events number, to observe these processes, especially the B_c production, would be a challenging task, due to the complexity of background.

In the end, it should be remarked that the large disparity of the scales m_W and m_{meson} suggests that a resummation of

logarithms of m_W/m_{meson} should be carried out. This asks for a further investigation.

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Note added.—When this work was finished and the manuscript was finalizing, there appeared a study on the web about the $B_c(B_c^*)$ meson production in W^+ decay with the NLO QCD corrections [43]. We numerically compared our results with that paper, and find that by taking the same inputs we can reproduce the Table I results there.¹

¹In Ref. [43], the two-loop α_s is used both in the LO and NLO calculation, while in our calculation, the one-loop and two-loop α_s are employed respectively.

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