Towards a minimal $SU(5)$ GUT

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We propose a simple $SU(5)$ model that connects the neutrino mass generation mechanism to the observed disparity between the masses of charged leptons and down-type quarks. The model is built out of 5-, 10-, 15-, 24-, and 35-dimensional representations of $SU(5)$ and comprises two (three) 3×3 (3×1) Yukawa coupling matrices to accommodate all experimentally measured fermion masses and mixing parameters. The gauge coupling unification considerations, coupled with phenomenological constraints inferred from experiments that probe neutrino masses and mixing parameters and/or look for proton decay, fix all relevant mass scales of the model. The proposed scenario places several multiplets at the scales potentially accessible at the LHC and future colliders and correlates this feature with the gauge-boson-mediated proton decay signatures. It also predicts that one neutrino is massless.

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I. INTRODUCTION

The promising proposal of the unification of the Standard Model (SM) matter fields and their interactions using the $SU(5)$ group as the supporting gauge structure has been around for more than four decades [1]. The initial efforts, naturally, were devoted to understanding what turned out to be a finite number of possible ways to generate masses for the SM charged fermions [2,3]. The subsequent need to accommodate the neutrino mass generation mechanism, on the other hand, has revealed that there are many potentially viable paths that could be taken, and the majority of studies within the $SU(5)$ theory framework, over the last two decades, have been focused on various ways of implementing it [4–21].

In this manuscript, we combine several known mechanisms of mass generation for both the charged and neutral fermions of the SM within the $SU(5)$ framework to produce an unexpectedly economical and predictive model that might take us a step closer to a minimal $SU(5)$. The scalar sector of our proposal contains only three

representations, whereas the fermion sector, besides the SM fields, contains only one vectorlike representation. In addition, the proposal allows for only two (three) 3×3 (1×3) Yukawa coupling matrices that need to accommodate all experimentally measured fermion masses and mixing parameters, where one of the 3×3 matrices can be taken (without loss of generality) to be diagonal and real, whereas the other 3×3 matrix is symmetric. Finally, the gauge coupling unification analysis, coupled with the need to accommodate experimentally observed parameters in the neutrino sector, constrains all relevant mass scales of the model to reside within very narrow ranges of viability.

The manuscript is organized as follows. We specify the particle content of our proposal in Sec. II. There we also discuss the particularities of the gauge coupling unification, spell out the connection between different mass generation mechanisms, and present a numerical study that showcases the phenomenological viability of the proposal. We conclude in Sec. III.

II. PROPOSAL

A. Particle content

We propose an extension of the original Georgi-Glashow (GG) model that connects the neutrino mass generation to the observed disparity between the masses of charged leptons and down-type quarks. The scalar sector of the GG model comprises one five-dimensional and one 24-dimensional representation. We add to it a single

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35-dimensional representation. The decompositions of these representations under the SM gauge group $SU(3) \times SU(2) \times U(1)$ are $5_H \equiv \Lambda = \Lambda_1(1, 2, 1/2) + \Lambda_3(3, 1, -1/3)$, $24_H \equiv \phi = \phi_0(1, 1, 0) + \phi_1(1, 3, 0) + \phi_3(3, 2, -5/6) + \phi_3(\bar{3}, 2, 5/6) + \phi_8(8, 1, 0)$, and $35_H \equiv \Phi = \Phi_1(1, 4, -3/2) + \Phi_3(\bar{3}, 3, -2/3) + \Phi_6(\bar{6}, 2, 1/6) + \Phi_{10}(\bar{10}, 1, 1)$, where the vacuum expectation value (VEV) of 24_H breaks $SU(5)$ down to the SM gauge group. Our normalization for the VEV of 24_H is such that $\langle 24_H \rangle \equiv \langle \phi \rangle = v_{24}/\sqrt{15} \text{diag}(-1, -1, -1, 3/2, 3/2)$. The electroweak VEV resides in 5_H and we denote it as v_H , where $v_H = 174$ GeV.

We also extend the fermion sector of the GG model with one 15-dimensional vectorlike set of fields, where these additional fermions are denoted as $15_F + \bar{15}_F \equiv \Sigma + \bar{\Sigma}$, with $15_F \equiv \Sigma = \Sigma_1(1, 3, 1) + \Sigma_3(3, 2, 1/6) + \Sigma_6(6, 1, -2/3)$. The SM fermions reside in $\bar{5}_{Fi}$ and 10_{Fi} , where $i (= 1, 2, 3)$ is the generation index. The scalar and fermion representations of our proposal thus comprise 5_H , 24_H , 35_H , $\bar{5}_{Fi}$, 10_{Fi} , and $15_F + \bar{15}_F$, where $i = 1, 2, 3$.

B. Gauge coupling unification

We want to demonstrate that the model generates gauge coupling unification at the one-loop level. The introduction of 35_H and $15_F + \bar{15}_F$ is crucial in that regard since the SM multiplets in 24_H and 5_H cannot produce viable unification on their own. There are, however, two mass relations between the multiplets in 35_H and $15_F + \bar{15}_F$ that one needs to take into account in order to perform a proper unification analysis, as we discuss next.

It is well known that the masses of $\phi_1 \in 24_H$ and $\phi_8 \in 24_H$ can be treated as arbitrary parameters, as far as the gauge coupling unification is concerned, since the potential contains enough nontrivial contractions of 24_H with itself to allow for that to happen. The model can also accommodate the splitting between the masses of $\Lambda_1 \in 5_H$ and $\Lambda_3 \in 5_H$, where we take that $M_{\Lambda_3} \geq 3 \times 10^{11}$ GeV in order to satisfy experimental bounds on the partial proton decay lifetimes [22], while the mass of Λ_1 is the mass of the SM Higgs field, i.e., $M_{\Lambda_1} \equiv M_H$.

The interaction of $15_F + \bar{15}_F$ with 24_H can induce mass splitting between fermions in the 15-dimensional representation, which is a rank-2 symmetric tensor. Namely, the two contractions

$$\mathcal{L} \supset M_{\Sigma} \Sigma^{\alpha\beta} \bar{\Sigma}_{\alpha\beta} + y \Sigma^{\alpha\beta} \phi_{\beta}^{\gamma} \bar{\Sigma}_{\alpha\gamma}, \quad (2.1)$$

where $\alpha, \beta, \gamma (= 1, 2, 3, 4, 5)$ are the $SU(5)$ indices, yield

$$M_{\Sigma_1} = M_{\Sigma} + \frac{y}{2} \sqrt{\frac{3}{5}} v_{24}, \quad (2.2)$$

$$M_{\Sigma_3} = M_{\Sigma} + \frac{y}{4\sqrt{15}} v_{24}, \quad (2.3)$$

$$M_{\Sigma_6} = 2M_{\Sigma_3} - M_{\Sigma_1}. \quad (2.4)$$

Even though Σ_3 and Σ_1 mix with the SM fermions (as we show later), Eq. (2.4) holds in the particular part of parameter space we are interested in and we accordingly use it in the one-loop gauge coupling unification analysis. In fact, the preferred unification scenario corresponds to the case when the masses of Σ_1 , Σ_3 , and Σ_6 are degenerate for all practical purposes. We note that y of Eq. (2.1) is, strictly speaking, a Yukawa coupling that, in the case of degenerate masses of Σ_1 , Σ_3 , and Σ_6 , does not affect the mechanisms that yield the correct values of the SM fermion masses and mixing parameters.

The masses of multiplets in 35_H , which is a rank-3 completely symmetric tensor, are determined by the interactions

$$\begin{aligned} \mathcal{L} \supset & \mu_{35}^2 \Phi \Phi^* + \lambda_0 (\Phi \Phi^*) \phi^2 + \lambda_1 \Phi^{\alpha\beta\gamma} \Phi^*_{\alpha\delta\epsilon} \phi_{\beta}^{\delta} \phi_{\gamma}^{\epsilon} \\ & + \lambda_2 \Phi^{\alpha\beta\epsilon} \Phi^*_{\alpha\beta\delta} \phi_{\epsilon}^{\gamma} \phi_{\gamma}^{\delta} + \mu'_{35} \Phi^{\alpha\beta\gamma} \Phi^*_{\delta\alpha\beta} \phi_{\gamma}^{\delta}, \end{aligned} \quad (2.5)$$

and the squares of the masses read

$$M_{\Phi_1}^2 = \mu_{35}^2 + \frac{\lambda_0}{2} v_{24}^2 + \frac{3\lambda_1}{20} v_{24}^2 + \frac{3\lambda_2}{20} v_{24}^2 + \frac{1}{2} \sqrt{\frac{3}{5}} \mu'_{35} v_{24}, \quad (2.6)$$

$$M_{\Phi_3}^2 = \mu_{35}^2 + \frac{\lambda_0}{2} v_{24}^2 - \frac{\lambda_1}{60} v_{24}^2 + \frac{11\lambda_2}{90} v_{24}^2 + \frac{2}{3\sqrt{15}} \mu'_{35} v_{24}, \quad (2.7)$$

$$M_{\Phi_6}^2 = \mu_{35}^2 + \frac{\lambda_0}{2} v_{24}^2 - \frac{2\lambda_1}{45} v_{24}^2 + \frac{17\lambda_2}{180} v_{24}^2 - \frac{1}{6\sqrt{15}} \mu'_{35} v_{24}, \quad (2.8)$$

$$M_{\Phi_{10}}^2 = M_{\Phi_1}^2 - 3M_{\Phi_3}^2 + 3M_{\Phi_6}^2. \quad (2.9)$$

The last equality is the other mass relation that we need to use in our unification analysis.

To summarize, the relevant degrees of freedom for the gauge coupling unification considerations are the masses of $\Phi_1, \Phi_3, \Phi_6, \Phi_{10} \in 35_H$, $\Sigma_1, \Sigma_3, \Sigma_6 \in 15_F + \bar{15}_F$, $\phi_1, \phi_8 \in 24_H$, and $\Lambda_3 \in 5_H$ and the constraints that need to be satisfied are given by Eqs. (2.4) and (2.9) with $M_{\Lambda_3} \geq 3 \times 10^{11}$ GeV. The gauge coupling unification is successful if and when $M_{\text{GUT}} \geq 5 \times 10^{15}$ GeV, where M_{GUT} is the scale of unification, i.e., the GUT scale and, at the same time, the mass of the proton decay mediating $SU(5)$ gauge bosons. The aforementioned limit on M_{GUT} is due to the experimental constraint on the gauge-mediated $d = 6$ proton decay.

The gauge coupling unification analysis reveals that the field $\Phi_1(1, 4, -3/2)$ [$\Sigma_1(1, 3, 1)$] prefers to be very heavy (light) if M_{GUT} is to be sufficiently large. But, for reasons that will become clear when we discuss the neutrino mass generation mechanisms, the masses of Φ_1 and Σ_1 should be

FIG. 1. The gauge coupling unification, at the one-loop level, that corresponds to the case when $M_{\Sigma_1} = 10^{11}$ GeV and $M_{\Phi_1} = 10^{12}$ GeV, which yields the correct neutrino mass scale.

of the same order of magnitude if one is to have a potentially viable scenario. Also, Φ_1 cannot be too heavy since the neutrino mass scale would not come out right. Due to these conflicting needs we are left with a rather limited parameter space where one can simultaneously address experimental results on proton decay lifetimes and neutrino masses in a proper manner. Accordingly, in Fig. 1 we present the gauge coupling unification scenario that corresponds to the case when $M_{\Sigma_1} = 10^{11}$ GeV and $M_{\Phi_1} = 10^{12}$ GeV, which yields $M_{\text{GUT}} = 7.7 \times 10^{15}$ GeV under the assumption that the lightest new physics states cannot reside below 10 TeV. We use this particular unification scenario in our numerical study of the fermion masses and mixing parameters to showcase the viability of our proposal. The vertical lines in Fig. 1 correspond to the mass scales of the relevant multiplets that we explicitly specify for clarity of exposition.

To generate Fig. 1 we use $\alpha_s(M_Z) = 0.1193$, $\alpha^{-1}(M_Z) = 127.906$, and $\sin^2 \theta_W = 0.23126$ [23] to maximize the GUT scale for the case when we set $M_{\Sigma_1} = 10^{11}$ GeV and $M_{\Phi_1} = 10^{12}$ GeV, while the masses of all other fields are allowed to freely move between 10 TeV and M_{GUT} except for the Λ_3 multiplet which needs to have $M_{\Lambda_3} \geq 3 \times 10^{11}$ GeV. Again, all of this is done after we implement the relations given in Eqs. (2.4) and (2.9).

It is clear from Fig. 1 that the proposal yields several multiplets with nontrivial assignments under $SU(3)$ and/or $SU(2)$ that could potentially reside at the TeV scale. These multiplets are Φ_6 , Φ_3 , ϕ_8 , and ϕ_1 . Note that in the regime of interest, i.e., when M_{Σ_1} is comparable to M_{Φ_1} , the masses of the fermions in $15_F + \overline{15}_F$ are degenerate for all practical purposes. The degeneracy of states in $15_F + \overline{15}_F$ that demands $y \approx 0$ is not guaranteed by any symmetry. It is simply an outcome of our proposal when the gauge coupling unification requirement and proton decay constraints are combined with neutrino oscillation data. Also,

FIG. 2. The gauge coupling unification, at the one-loop level, that corresponds to the case when $M_{\Sigma_1} = 10^{11}$ GeV, $M_{\Sigma_3} = 9.2 \times 10^{10}$ GeV, $M_{\Sigma_6} = 8.4 \times 10^{10}$ GeV, $M_{\Phi_1} = 10^{12}$ GeV, and $M_{\Lambda_3} = M_{\text{GUT}} = 7.7 \times 10^{15}$ GeV.

this unification scenario implies that the $d = 6$ proton decay contributions due to scalar mediation are completely negligible since Λ_3 is the only scalar leptoquark in the proposed model, where the maximization of the GUT scale yields $M_{\Lambda_3} \geq 3.1 \times 10^{14}$ GeV. If we set $M_{\Sigma_1} = 10^{11}$ GeV and $M_{\Phi_1} = 10^{12}$ GeV as in the unification scenario presented in Fig. 1 but, this time around, insist that Λ_3 is at the GUT scale, we obtain the unification of gauge couplings shown in Fig. 2. The impact of Λ_3 on the gauge coupling running is completely compensated by a tiny split between the masses of Σ_1 , Σ_3 , and Σ_6 with $y = 1.2 \times 10^{-6}$, whereas the GUT scale is marginally larger than in the scenario shown in Fig. 1 but rounds to the same numerical value.

Finally, we note that the multiplet $\Phi_1(1, 4, -3/2)$ cannot be below 10^{11} GeV (10^{10} GeV) under the assumption that the lightest new physics states introduced in this proposal do not reside below 10 TeV (1 TeV) if the GUT scale is to exceed 5×10^{15} GeV.

C. Neutrino mass generation

The $SU(5)$ contractions that generate contributions to the Majorana neutrino masses read

$$\mathcal{L} \supset \lambda' 5_H 5_H 35_H + Y_i^a 15_F \bar{5}_{Fi} 5_H^* + Y_i^b \overline{15}_F \bar{5}_{Fi} 35_H^*, \quad (2.10)$$

where Y^a and Y^b are arbitrary 1×3 Yukawa coupling matrices and λ' is a dimensionless parameter. The neutrino mass contributions are generated both at the tree level via a $d = 7$ operator and at the one-loop level via a $d = 5$ operator [24]. The Feynman diagram for the former (latter) contribution is shown in the left (right) panel of Fig. 3.

We obtain the following contributions to the neutrino mass matrix elements:

FIG. 3. The Feynman diagrams of the tree-level (left panel) and one-loop-level (right panel) contributions to the Majorana neutrino masses.

$$(\mathcal{M}_\nu^{d=7})_{ij} = -\lambda' \frac{v_H^4}{M_{\Sigma_1} M_{\Phi_1}^2} (Y_i^a Y_j^b + Y_i^b Y_j^a), \quad (2.11)$$

$$(\mathcal{M}_\nu^{d=5})_{ij} = \frac{\lambda' v_H^2}{16\pi^2} \frac{(Y_i^a Y_j^b + Y_i^b Y_j^a) M_{\Sigma_1}}{M_{\Phi_1}^2 - M_H^2} \times \left(\frac{M_{\Phi_1}^2}{M_{\Sigma_1}^2 - M_{\Phi_1}^2} \log \left(\frac{M_{\Sigma_1}^2}{M_{\Phi_1}^2} \right) - \frac{M_H^2}{M_{\Sigma_1}^2 - M_H^2} \log \left(\frac{M_{\Sigma_1}^2}{M_H^2} \right) \right), \quad (2.12)$$

where both $\mathcal{M}_\nu^{d=7}$ and $\mathcal{M}_\nu^{d=5}$ exhibit exactly the same flavor dependence.

In the regime of interest, i.e., when $M_{\Sigma_1}, M_{\Phi_1} \gg M_H, v_H$, it is the $d=5$ contribution that dominates and we find that the entries of the neutrino mass matrix ($\mathcal{M}_\nu \equiv \mathcal{M}_\nu^{d=5}$) are

$$(\mathcal{M}_\nu)_{ij} \sim \frac{\lambda' v_H^2}{16\pi^2} (Y_i^a Y_j^b + Y_i^b Y_j^a) \frac{M_{\Sigma_1}}{M_{\Sigma_1}^2 - M_{\Phi_1}^2} \log \left(\frac{M_{\Sigma_1}^2}{M_{\Phi_1}^2} \right) = m_0 (Y_i^a Y_j^b + Y_i^b Y_j^a), \quad (2.13)$$

where, for $\lambda' = 1$, $M_{\Sigma_1} = 10^{11}$ GeV, and $M_{\Phi_1} = 10^{12}$ GeV, one finds that $m_0 \approx 4 \times 10^{-11}$ GeV. This, in turn, is the right neutrino mass scale if the entries in Y^a and Y^b are $\mathcal{O}(1)$ parameters. The tree-level contribution for the unification scenario under consideration is very suppressed compared to the one-loop-level one. Namely, we find that the prefactor of the term $(Y_i^a Y_j^b + Y_i^b Y_j^a)$ in Eq. (2.11) is $\lambda' (v_H^4)/(M_{\Sigma_1} M_{\Phi_1}^2) \approx 10^{-26}$ GeV. Note that one neutrino is always massless in our proposal since $\det(\mathcal{M}_\nu) = 0$.

D. Charged fermion masses

The contractions that are relevant for the charged fermion mass generation read

$$\mathcal{L} \supset Y_{ij}^d 10_{Fi} \bar{5}_{Fj} 5_H^* + Y_{ij}^u 10_{Fi} 10_{Fj} 5_H + Y_i^c 10_{Fi} \bar{15}_F 24_H + Y_i^a 15_F \bar{5}_{Fi} 5_H^* + M_{\Sigma} \bar{15}_F 15_F + y \bar{15}_F 15_F 24_H. \quad (2.14)$$

Here we use the freedom to freely rotate 10_{Fi} and $\bar{5}_{Fj}$ in an independent manner to go into the basis where Y^d is a real and diagonal matrix. Y^u is a symmetric 3×3 complex

matrix, whereas Y^c is a 3×1 complex matrix. The contraction proportional to the matrix Y^a also appears in Eq. (2.10) in regard to the discussion of the neutrino mass generation, while the last two terms in Eq. (2.14) appear in Eq. (2.1) and are relevant for the mass splitting of the multiplets within $15_F + \bar{15}_F$. These facts simply mean that there is a direct connection between the mass generation mechanisms for the charged and neutral fermions in our proposal. In fact, the role of $15_F + \bar{15}_F$ is to simultaneously generate neutrino masses and induce an experimentally observed mismatch between the masses of the charged leptons and the down-type quarks, as we discuss next. Note that the possibility to address the aforementioned mismatch with a vectorlike 15-dimensional representation within the context of supersymmetric $SU(5)$ has been studied in Ref. [25].

The decomposition of 10_{Fi} and $\bar{5}_{Fi}$ under the SM gauge group is $10_{Fi} = Q_{Li}(3, 2, 1/6) + u_i^c(\bar{3}, 1, -2/3) + e_i^c(1, 1, 1)$ and $\bar{5}_{Fi} = \ell_{Li}(1, 2, -1/2) + d_i^c(\bar{3}, 1, -1/3)$, respectively. After the electroweak symmetry breaking the isodoublet (isotriplet) fermions decompose under the $SU(3) \times U(1)_{\text{em}}$ group as $Q_{Li} = u_i(3, 2/3) + d_i(3, -1/3)$ and $\Sigma_3 = \Sigma^u(3, 2/3) + \Sigma^d(3, -1/3)$ [$\Sigma_1 = \Sigma^\nu(1, 0) + \Sigma^{e^c}(1, 1) + \Sigma^{e^c e^c}(1, 2)$], where the second number in the parentheses represents electric charge in units of absolute value of the electron charge. The mixing between the fermions in 10_{Fi} and $15_F + \bar{15}_F$ that appears as a result of the $SU(5)$ symmetry breaking reads

$$\mathcal{L} \supset -\frac{1}{4} \sqrt{\frac{5}{3}} v_{24} Y_i^c Q_{Li} \bar{\Sigma}_3, \quad (2.15)$$

whereas the electroweak symmetry breaking induces the following mixings among the fermions:

$$\begin{aligned} \mathcal{L} \supset & (u_i \Sigma^u) \begin{pmatrix} v_H (Y_{ij}^u)_{3 \times 3} & -\frac{1}{4} \sqrt{\frac{5}{3}} v_{24} (Y_i^c)_{3 \times 1} \\ (0)_{1 \times 3} & (M_{\Sigma_3})_{1 \times 1} \end{pmatrix} \begin{pmatrix} u_j^c \\ \bar{\Sigma}^u \end{pmatrix} \\ & + (d_i \Sigma^d) \begin{pmatrix} v_H^* (Y_{ij}^d)_{3 \times 3} & -\frac{1}{4} \sqrt{\frac{5}{3}} v_{24} (Y_i^c)_{3 \times 1} \\ v_H^* (Y_j^a)_{1 \times 3} & (M_{\Sigma_3})_{1 \times 1} \end{pmatrix} \begin{pmatrix} d_j^c \\ \bar{\Sigma}^d \end{pmatrix} \\ & + (e_i \bar{\Sigma}^{e^c}) \begin{pmatrix} v_H^* (Y_{ij}^{dT})_{3 \times 3} & v_H^* (Y_i^a)_{3 \times 1} \\ (0)_{1 \times 3} & (M_{\Sigma_1})_{1 \times 1} \end{pmatrix} \begin{pmatrix} e_j^c \\ \Sigma^{e^c} \end{pmatrix}. \quad (2.16) \end{aligned}$$

The states $\Sigma^{u,d,e^c} + \bar{\Sigma}^{u,d,e^c}$ need to be very heavy if one is to generate the correct neutrino mass scale, and can be safely integrated out. We accordingly find, in the limit when $v_{24}Y^c, M_{\Sigma_{1,3}} \gg v_H$, that the mass matrices for the up-type quarks (M_u), down-type quarks (M_d), and charged leptons (M_e) are

$$M_u = (\mathbb{I}_{3 \times 3} + \delta^2 Y^c Y^{c\dagger})^{-\frac{1}{2}} v_H Y^u, \quad (2.17)$$

$$M_d = (\mathbb{I}_{3 \times 3} + \delta^2 Y^c Y^{c\dagger})^{-\frac{1}{2}} v_H (Y^d + \delta Y^c Y^a), \quad (2.18)$$

$$M_e = v_H Y^{dT}, \quad (2.19)$$

where we define the dimensionless parameter $\delta \equiv \sqrt{5/3} v_{24}/(4M_{\Sigma_3})$ and, again, take v_H to be real. The gauge coupling unification scenario we present in Fig. 1 yields $\delta = 2.49 \times 10^4$. Note that without the mixing one retrieves the GG prediction $M_e = M_d^T$, at the unification scale, which is in conflict with experimental observations after the measured values of the down-type quarks and charged leptons are evolved to the GUT scale.

Again, the neutrino mass matrix elements are

$$(\mathcal{M}_\nu)_{ij} = m_0 (Y_i^a Y_j^b + Y_i^b Y_j^a) = (U_{\text{PMNS}} \text{diag}(m_1, m_2, m_3) U_{\text{PMNS}}^T)_{ij}, \quad (2.20)$$

where m_i , $i = 1, 2, 3$, are neutrino mass eigenstates and U_{PMNS} represents the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) unitary mixing matrix. Since we work in the basis where Y^d is diagonal and real, the unitary matrix that diagonalizes the neutrino matrix must be identified with the PMNS matrix. Finally, the masses of the heavy states are

$$M_{\Sigma^u} = M_{\Sigma_3} (1 + \delta^2 Y^c Y^{c\dagger})^{\frac{1}{2}}, \quad M_{\Sigma^d} = M_{\Sigma^u}, \quad M_{\Sigma^{e^c}} = M_{\Sigma_1}. \quad (2.21)$$

To summarize, our proposal uses one vectorlike 15-dimensional representation that simultaneously generates neutrino masses with the aid of one 35-dimensional scalar representation and creates a viable mismatch between the masses of the down-type quarks and charged leptons. There are only two (three) 3×3 (3×1) Yukawa coupling matrices Y^u and Y^d (Y^a , Y^b , and Y^c) to accommodate the experimentally measured fermion masses and mixing parameters. Moreover, without the loss of generality, one can go into a basis where Y^d is a diagonal and real matrix while Y^u is a symmetric matrix. Note that the dimensionless parameter y is, strictly speaking, a Yukawa coupling, but the unification considerations imply that it can be neglected for all practical purposes.

TABLE I. The fit input for the GUT-scale values of the down-type quark and charged lepton masses [26] and the corresponding fit output.

Down-type quark & charged lepton masses		
Observable	Input (GeV)	Fit (GeV)
$m_d/10^{-3}$	1.14 ± 0.11	1.14
$m_s/10^{-2}$	2.15 ± 0.11	2.15
m_b	0.99 ± 0.009	0.99
$m_e/10^{-4}$	4.707 ± 0.0047	4.707
$m_\mu/10^{-2}$	9.936 ± 0.0099	9.936
m_τ	1.689 ± 0.0016	1.689

E. Numerical analysis

In this section, we perform a numerical fit of the SM fermion masses and mixing parameters that corresponds to the unification scenario of Fig. 1 to demonstrate the viability of our proposal. The fermion mass matrices are given in Eqs. (2.17), (2.18), (2.19), and (2.20). Since the charged lepton mass matrix is already diagonal in our basis, i.e., $M_e = v_H \text{diag}(y_e, y_\mu, y_\tau)$, we can trivially determine the entries of the $Y^d (= (y_e, y_\mu, y_\tau) = (2.703 \times 10^{-6}, 5.707 \times 10^{-4}, 9.70 \times 10^{-3}))$ matrix using the GUT-scale values of the observables that are listed in Table I.

Since the down-type quark and neutrino mass matrices share the same Yukawa couplings Y_i^a , $i = 1, 2, 3$, we perform a combined fit of these two sectors to reproduce the correct down-type quark masses and neutrino observables. We obtain the following fit parameters:

$$Y^a = (-0.0899, 0.551, 1), \quad (2.22)$$

$$Y^b = (0.975, 2.381, 1), \quad (2.23)$$

$$Y^c = -1.865 \times 10^{-7} (0.00137, 0.0942, 1), \quad (2.24)$$

where we normalize Y_3^a and Y_3^b to 1. To be consistent with our unification scenario and this normalization, the overall

TABLE II. The fit input for the neutrino observables [27] and the corresponding fit output.

ν parameters	Input		
$\Delta m_{21}^2/10^{-5}$	7.57 ± 0.18 (eV ²)		
$\Delta m_{31}^2/10^{-3}$	2.50 ± 0.03 (eV ²)		
$\sin^2 \theta_{12}^{\text{PMNS}}$	0.322 ± 0.018		
$\sin^2 \theta_{23}^{\text{PMNS}}$	0.542 ± 0.025		
$\sin^2 \theta_{13}^{\text{PMNS}}/10^{-2}$	2.219 ± 0.075		
ν masses	Fit (eV)	ν angles	Fit (°)
m_1	0	$\theta_{12}^{\text{PMNS}}$	34.57
$m_2/10^{-3}$	8.70	$\theta_{23}^{\text{PMNS}}$	47.41
$m_3/10^{-2}$	4.99	$\theta_{13}^{\text{PMNS}}$	8.56

scale for the neutrino mass matrix is fixed to be $m_0 = 9.28 \times 10^{-12}$ GeV which, in turn, requires $\lambda' = 0.239$. We summarize the best-fit values in Tables I and II. Clearly, all of the observables can be fitted to their experimentally measured central values given in Tables I and II, while one neutrino is predicted to be massless.

In general the Yukawa couplings in $Y^{a,b,c}$ are complex numbers, but for simplicity we have taken them to be real. Note that the up-type quark mass matrix is proportional to the complex symmetric matrix Y^u . This provides enough freedom for one to simultaneously reproduce the up-type quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) parameters. It should be pointed out that to fit the CKM parameters one needs to keep track of the unitary transformations that take the down-type quark mass matrix given in Eq. (2.18) into a diagonal form. Note that the fit implies that $(\delta Y_i^c) \ll 1$, where, again, $\delta \equiv \sqrt{5/3} v_{24}/(4M_{\Sigma_3})$. This means that the mass relation of Eq. (2.4) is not affected by the fermion mixing mechanism, as can be seen from Eq. (2.21). More importantly, it means that the up-type quark mass matrix [given in Eq. (2.17)] is a symmetric one to great accuracy. Our proposal thus predicts that there are two proton decay channels that depend only on the CKM parameters and the unification input [28] of Fig. 1. These channels are $p \rightarrow K^+ \bar{\nu}$ and $p \rightarrow \pi^+ \bar{\nu}$.

III. CONCLUSION

We proposed a simple $SU(5)$ model that relates the neutrino mass generation mechanism to the observed disparity between the masses of charged leptons and down-type quarks. The entire structure of the model is

based on the first five nontrivial $SU(5)$ representations of the lowest possible dimension. The scalar sector is made up of three representations of dimensions 5, 24, and 35, while the fermion sector—besides the usual matter content of the minimal $SU(5)$ —contains only one set of 15-dimensional vectorlike fields. The role of the vectorlike fields is twofold. They generate neutrino masses and, at the same time, create the required mismatch between the down-type quarks and charged leptons. The proposal successfully accommodates all experimentally measured fermion masses and mixing parameters with two (three) 3×3 (3×1) Yukawa coupling matrices, where without loss of generality one of the 3×3 matrices can be taken to be diagonal and real, whereas the other 3×3 matrix is symmetric.

The gauge coupling unification considerations, coupled with phenomenological constraints inferred from experiments that probe neutrino masses and mixing parameters and/or look for proton decay, fix all relevant mass scales of the model. The proposed scenario places several multiplets with nontrivial assignments under $SU(3)$ and/or $SU(2)$ at the scales potentially accessible at the LHC and future colliders and correlates this feature with the gauge-boson-mediated proton decay signatures. Two particular decay channels, i.e., $p \rightarrow K^+ \bar{\nu}$ and $p \rightarrow \pi^+ \bar{\nu}$, depend only on the scale of unification and the SM parameters. The model contains only one scalar leptoquark whose contribution towards proton decay is negligible. It also predicts the existence of one massless neutrino.

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