Detectability of ultralight scalar field dark matter with gravitational-wave detectors

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An ultralight scalar field is one of the dark matter candidates. If it couples with Standard Model particles, it oscillates mirrors in gravitational-wave detectors and generates detectable signals. We focus on the case where the scalar field has nonminimal parity-even couplings with Standard Model particles and study the spectra of the signals, taking into account the motion of the detectors due to the Earth's rotation/the detectors' orbital motion around the Sun. Then, we formulate suitable data-analysis methods to detect it and estimate the future constraint with our methods. We find that gravitational-wave experiments can improve the existing constraints given by fifth-force experiments on one of the scalar field's coupling constants by factors of ~ 30 , ~ 100 , and ~ 350 for $m_\phi = 2 \times 10^{-17}$ eV, 10^{-14} eV, and 10^{-12} eV, respectively, where m_ϕ is the scalar field's mass. Our study demonstrates that experiments with gravitational-wave detectors are complementary to tests of the weak equivalence principle.

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I. INTRODUCTION

Although weakly interacting massive particles (WIMPs) are promising candidates of dark matter, null results from various experiments [1–4] cast doubt on WIMPs. Therefore, it is worth searching for other candidates. An ultralight scalar field is one of the other dark matter candidates. This type of dark matter can have mass down to $\sim 10^{-22}$ eV [5], and we especially consider a mass range of 10^{-19} eV $\lesssim m_\phi \lesssim 10^{-10}$ eV. The existence of such light scalar fields is motivated by string theory [6]. Ultralight scalar-field dark matter starts oscillating when $H \simeq m_\phi$, where H is the Hubble constant and m_ϕ is the scalar field's mass [7]. Since the occupation number is large for our mass range, it can be treated as a classical scalar field in the current Universe.

An oscillating classical scalar field causes various peculiar phenomena from which we can probe its existence. For instance, the energy-momentum tensor of the oscillating scalar field generates metric perturbations that oscillate with half the period of the oscillation of the scalar field. Reference [8] showed that such metric perturbations can be detected in pulsar timing array observations if the mass of

the scalar field is around 10^{-22} eV, and null results have already put constraints on its abundance [9,10]. If the scalar field couples with the Standard Model (SM) particles, it mediates additional force [11]. Such force is searched for in fifth-force experiments [12] and equivalence principle (EP) tests such as lunar laser ranging (LLR) experiments [13,14], experiments from the Eöt-Wash group [15,16], and the MICROSCOPE experiment [17]. The oscillation of the scalar field that couples with the SM particles causes time variations of physical constants such as the proton mass and the fine-structure constant. They produce the time variations of the atomic transitions as well as the oscillatory force on bodies, from which the constraint on the coupling to the SM particles can be obtained [18–27]. For example, hyperfine frequency comparison of ^{87}Rb and ^{133}Cs atoms has been used for this purpose [22]. Detecting variation of the fine-structure and particle masses with laser interferometer measurements is also discussed in Ref. [28].

In this paper, under the assumption that an ultralight scalar field comprises all dark matter and it weakly couples to the SM particles, we investigate the possibility of detecting such a field with gravitational-wave (GW) detectors. As far as we know, the idea of detecting such a field by GW detectors was first argued by Ref. [18]. The same idea was discussed in Ref. [29] to detect a light vector field, and a similar idea to detect particle dark matter was mentioned in Ref. [30]. The main idea is the following: An ultralight scalar field as dark matter varies spatially on the scale of

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the de Broglie wavelength $k^{-1} \sim 1/(m_\phi v_{\text{DM}})$ in the Galaxy, where $v_{\text{DM}} \sim 10^{-3}$ is the velocity dispersion of dark matter in the Galaxy.¹ Since the values of the physical constants depend on the value of the scalar field, the masses of optical equipments such as mirrors also depend on them. Then, the spatial variations of the scalar field exert position-dependent oscillatory force on every optical equipment. As a result, the optical equipments undergo position-dependent oscillatory motions, which ends up with non-vanishing signals in the GW detectors' outputs. Since the frequency of the motion is $f \simeq m_\phi/2\pi$ and the frequency dispersion is tiny, $\Delta f \sim m_\phi v_{\text{DM}}^2/2\pi$, detectors' signals are nearly monochromatic. It is worth mentioning that using the GW detectors to probe the ultralight scalar-field dark matter was also investigated in Ref. [32]. In this reference, not the effects caused by the scalar force but the effects of metric perturbations sourced by the scalar field were considered, and it was found that such effects are far from the detectable level even with future experiments.

To extract information on ultralight scalar-field dark matter from real data as much as possible, we need to understand its signal's characteristics and develop a suitable data-analysis method. In this paper, we formulate a data-analysis method toward a detection of ultralight scalar-field dark matter with GW detectors. First, we derive the expression of the signal in an output from a GW detector. Especially, we take into account the detectors' motion due to the Earth's rotation and the detectors' orbital motion around the Sun, which is not taken into account in the previous studies [18,29]. Next, we develop a data-analysis method suitable for the signal. Taking into account the possibility that we have only one detector, we develop a method which is applicable even with one detector. Finally, we update the previous estimate of the constraints.

The organization of the paper is as follows: In Sec. II, we explain the model we consider and how ultralight scalar-field dark matter behaves in our Galaxy. In Sec. III, we obtain a formula for the signal in an output from a gravitational-wave detector. In Sec. IV, we discuss the signal's characteristics and formulate a suitable data-analysis method to detect this signal. Then, we update the previous estimate of the constraints with our analysis method. Section V is devoted to the conclusion.

II. BASICS

We briefly review the model we consider and how the scalar-field dark matter behaves around the gravitational-wave detectors.

A. Model

We consider a scalar field, ϕ , which couples with particles in the Standard Model and accounts for the entire

amount of dark matter. We assume that ϕ linearly couples with the particles. Then the low-energy (~ 1 GeV) effective Lagrangian is given by [11]

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\phi\text{-SM}}, \quad (1)$$

where $\mathcal{L}_{\text{SM}}$ is the Lagrangian of the Standard Model. $\mathcal{L}_\phi$ and $\mathcal{L}_{\phi\text{-SM}}$ are given by

$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi), \quad (2)$$

$$\mathcal{L}_{\phi\text{-SM}} = \kappa\phi\left[\frac{d_e}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{d_g\beta_3}{2g_3}G_{\mu\nu}^AG^{A\mu\nu} - \sum_{i=e,u,d}(d_{m_i} + \gamma_{m_i}d_g)m_i\bar{\psi}_i\psi_i\right], \quad (3)$$

where $\kappa \equiv \sqrt{4\pi}/M_{\text{pl}}$; $F_{\mu\nu}$ and $G_{\mu\nu}^A$ are the electromagnetic and gluon field strength tensors, respectively; β_3 stands for the QCD beta functions; and γ_{m_i} are the anomalous dimensions of the electrons and the u and d quarks. With respect to the coupling between ϕ and quarks, it is more convenient to use

$$d_{\hat{m}} \equiv \frac{d_{m_d}m_d + d_{m_u}m_u}{m_d + m_u}, \quad d_{\delta m} \equiv \frac{d_{m_d}m_d - d_{m_u}m_u}{m_d - m_u} \quad (4)$$

instead of d_{m_d} and d_{m_u} in the context of equivalence principle tests [11]. Since the amplitude of ϕ attenuates as the Universe expands, we assume that the amplitude is so tiny in the current Universe that we can neglect higher-order terms in $V(\phi)$ and approximate it as

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2. \quad (5)$$

B. The scalar field around a detector

We let $(t, \vec{x})$ be the rest frame of the Solar System and discuss how ϕ behaves in this frame. Dark matter has a velocity dispersion of $v_{\text{DM}} \sim 10^{-3}$ in our Galaxy [31], and the Solar System is also moving with a velocity around v_{DM} . Therefore, the wave number of ϕ , $\vec{k}$ satisfies

$$|\vec{k}| \lesssim m_\phi v_{\text{DM}}. \quad (6)$$

Then ϕ can be written as

$$\phi = \int d^3k [\phi_{\vec{k}} e^{i(\omega_k t - \vec{k} \cdot \vec{x})} + \text{c.c.}], \quad (7)$$

where $|\phi_{\vec{k}}|$ is negligibly small for $|\vec{k}| \gg m_\phi v_{\text{DM}}$. The angular frequency $\omega_k \sim m_\phi + k^2/2m_\phi$ and its dispersion $\Delta\omega_k$ are

$$\omega_k \sim m_\phi, \quad \Delta\omega_k \sim m_\phi v_{\text{DM}}^2. \quad (8)$$

¹The local velocity dispersion of dark matter is discussed, e.g., in Ref. [31].

In addition, for ϕ to account for all of the dark matter, the following condition must be satisfied:

$$m_\phi^2 \langle \phi^2 \rangle = \rho_{\text{DM}} \simeq 0.3 \text{ GeV/cm}^3, \quad (9)$$

where $\langle \dots \rangle$ is an average over space with the volume of $V \gg (m_\phi v_{\text{DM}})^{-3}$ and time of $T \gg m_\phi^{-1}$, and we use a result from Ref. [33] as a value of ρ_{DM} .

III. RESPONSE OF GRAVITATIONAL-WAVE DETECTORS TO SCALAR WAVES

As we explained in the previous section, there is a scalar wave background in the Galaxy if an ultralight scalar field accounts for dark matter. In this section, we obtain the expression for the signal, $h(t)$, caused by the scalar waves in a GW detector's output. There are multiple sources of the signal in the output of the detector. One possible source is a time variation of the laser frequency due to the oscillation of the scalar field. This mimics the common motion of the arms of the detector. Since interferometric GW detectors are mainly sensitive for their differential motion, this effect is subdominant. The second possible source is modification of laser light's propagation, which is also negligible, as we prove in Appendix A. Then, we conclude that the signal is mainly sourced by the motion of optical equipments caused by the scalar waves. In what follows, we first calculate the response of the detectors to monochromatic waves,

$$\phi = \phi_{\vec{k}} \cos(\omega_k t - \vec{k} \cdot \vec{x} + \theta_{\vec{k}}). \quad (10)$$

Then we generalize the expression to that for the superposition of the waves.

In our work, we consider the Laser Interferometer Gravitational-Wave Observatory (LIGO) [34], Einstein Telescope (ET) [35], and Cosmic Explorer (CE) [36] as representative ground-based detectors and the DECi-hertz Interferometer Gravitational wave Observatory (DECIGO) [37] and Laser Interferometer Space Antenna (LISA) [38] as representative space-based gravitational-wave detectors. The sensitivity curves for ground-based detectors are taken from Ref. [36]. For the DECIGO's sensitivity curve, we use an analytical function in Ref. [39]. For LISA, we apply configuration parameter values summarized in Ref. [38] to calculate the sensitivity.

A. Motion of optical equipments

Due to the interaction with ϕ given by Eq. (3), atomic mass depends on the value of ϕ . Therefore, the action of the optical equipment in this theory is given by

$$S = - \int m(\phi) \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}, \quad (11)$$

where we neglect gravitational fields. Then the equation of motion of the optical equipment in the nonrelativistic limit becomes

$$\frac{d^2 x^i}{dt^2} \approx -\kappa\alpha(\phi) \left(\partial_i \phi + \dot{\phi} \frac{dx^i}{dt} \right), \quad (12)$$

$$\alpha(\phi) \equiv \frac{d \ln m(\phi)}{d(\kappa\phi)}. \quad (13)$$

Assuming that the equipment is at rest in the absence of ϕ , the second term on the right-hand side is second order in the amplitude of the scalar field and is subdominant compared to the first term. Thus, the equation of motion becomes

$$\frac{d^2 x^i}{dt^2} \simeq -\kappa\alpha(\phi) \partial_i \phi. \quad (14)$$

Since the masses of atoms are mostly determined by the QCD energy scale, $\alpha(\phi)$ is approximately given by [11]

$$\alpha(\phi) \simeq d_g^* \simeq d_g + 0.093(d_{\tilde{m}} - d_g). \quad (15)$$

Substituting Eqs. (10) and (15) into Eq. (14), we obtain

$$\frac{d^2 x^i}{dt^2} \simeq -d_g^* \kappa \phi_{\vec{k}} k^i \sin(\omega_k t - \vec{k} \cdot \vec{x}_0 + \theta_{\vec{k}}), \quad (16)$$

where $\vec{x}_0$ is the position at which the equipment is in the absence of ϕ . Solving this leads to

$$x^i \simeq d_g^* \kappa \phi_{\vec{k}} \frac{k^i}{m_\phi^2} \sin(\omega_k t - \vec{k} \cdot \vec{x}_0 + \theta_{\vec{k}}) + \text{const.} \quad (17)$$

B. Response of the interferometric gravitational-wave detectors

Let us next derive the expression of the signal caused by the motion of the optical equipments. First, we calculate the response of a Michelson interferometer with arms along the unit vectors, $\vec{n}$ and $\vec{m}$. Following the method used in Ref. [40], we calculate the time necessary for photon to make a round-trip through the arms. The oscillation of the mirror at $\vec{x}$ due to the scalar field is given by

$$\delta x^i(t, \vec{x}) \equiv d_g^* \kappa \phi_{\vec{k}} \frac{k^i}{m_\phi^2} \sin(\omega_k t - \vec{k} \cdot \vec{x} + \theta_{\vec{k}}). \quad (18)$$

Then the position of the front mirror, $\vec{x}_1$, and that of the end mirror, $\vec{x}_2$, in the arm along $\vec{n}$ are

$$x_1^i(t) = x^i + \delta x^i(t, \vec{x}), \quad (19)$$

$$x_2^i(t) = x^i + L n^i + \delta x^i(t, \vec{x} + L \vec{n}), \quad (20)$$

where L is the arm length in the absence of the scalar field. Therefore, the perturbation of the round-trip time due to the oscillation, $\delta t(t; L, \vec{n})$, is

$$\begin{aligned} \delta t(t; L, \vec{n}) \simeq & n_i(-\delta x^i(t, \vec{x}) \\ & + 2\delta x^i(t-L, \vec{x} + L\vec{n}) - \delta x^i(t-2L, \vec{x})), \end{aligned} \quad (21)$$

where t is the time at which the photon returns to the front mirror and higher-order terms in δx have been neglected. This can be separated into two parts:

$$\begin{aligned} \delta t(t; L, \vec{n}) = & n_i(-\delta x^i(t, \vec{x}) + 2\delta x^i(t-L, \vec{x}) - \delta x^i(t-2L, \vec{x})) \\ & + 2Ln_i n^j \partial_j \delta x^i(t-L, \vec{x}). \end{aligned} \quad (22)$$

The first part is

$$\begin{aligned} \delta t_1(t; L, \vec{n}) & \equiv n_i(-\delta x^i(t, \vec{x}) + 2\delta x^i(t-L, \vec{x}) - \delta x^i(t-2L, \vec{x})) \\ & \simeq 4d_g^* \kappa \phi_{\vec{k}} \frac{\vec{k} \cdot \vec{n}}{m_\phi^2} \sin^2\left(\frac{m_\phi L}{2}\right) \sin(\omega_k(t-L) - \vec{k} \cdot \vec{x} + \theta_{\vec{k}}), \end{aligned} \quad (23)$$

and the second part is

$$\begin{aligned} \delta t_2(t; L, \vec{n}) & \equiv 2Ln_i n^j \partial_j \delta x^i(t-L, \vec{x}) \\ & \simeq -2d_g^* \kappa \phi_{\vec{k}} L \frac{(\vec{k} \cdot \vec{n})^2}{m_\phi^2} \cos(\omega_k(t-L) - \vec{k} \cdot \vec{x} + \theta_{\vec{k}}). \end{aligned} \quad (24)$$

Since $|\vec{k} \cdot \vec{n}| \sim m_\phi v_{\text{DM}}$, the round-trip time can be approximated as follows:

$$\delta t(t; L, \vec{n}) \simeq \begin{cases} \delta t_1(t; L, \vec{n}), & \text{for } 2\frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi L} > v_{\text{DM}} \\ \delta t_2(t; L, \vec{n}), & \text{for } 2\frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi L} < v_{\text{DM}} \end{cases}. \quad (25)$$

Strictly speaking, both are invalid near $2\sin^2(m_\phi L/2)/(m_\phi L) = v_{\text{DM}}$, and we need to take into account both contributions there. Since its error on the final constraints shown in Fig. 2 is of a small factor and in a very narrow frequency range, we ignore it and use the above equations. From this, we can calculate the phase delay of the laser light after a round-trip as follows:

$$\varphi(t; L, \vec{n}) = 2\pi f_{\text{laser}}(2L + \delta t(t; L, \vec{n})), \quad (26)$$

where f_{laser} is the frequency of the laser light. The derivative expansion in Eq. (22) breaks down for $L > 1/(m_\phi v_{\text{DM}})$. This situation never realizes for any detectors considered in this paper. The phase delay for the other arm is obtained by replacing $\vec{n}$ with $\vec{m}$. Since the signal in the gravitational-wave channel is the difference of the phases of the laser light traveling in the two arms, it is

$$\begin{aligned} h(t) &= \frac{\varphi(t; L, \vec{m}) - \varphi(t; L, \vec{n})}{4\pi f_{\text{laser}} L} \\ &\simeq \begin{cases} h_1(t), & \text{for } 2\frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi L} > v_{\text{DM}} \\ h_2(t), & \text{for } 2\frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi L} < v_{\text{DM}} \end{cases}, \end{aligned} \quad (27)$$

where

$$\begin{aligned} h_1(t) &= 2d_g^* \kappa \phi_{\vec{k}} \frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi L} \frac{\vec{k} \cdot \vec{n} - \vec{k} \cdot \vec{m}}{m_\phi} \\ &\quad \times \sin(\omega_k(t-L) - \vec{k} \cdot \vec{x} + \theta_{\vec{k}}), \end{aligned} \quad (28)$$

$$\begin{aligned} h_2(t) &= -d_g^* \kappa \phi_{\vec{k}} \frac{(\vec{k} \cdot \vec{n})^2 - (\vec{k} \cdot \vec{m})^2}{m_\phi^2} \\ &\quad \times \cos(\omega_k(t-L) - \vec{k} \cdot \vec{x} + \theta_{\vec{k}}). \end{aligned} \quad (29)$$

Generalizing this expression into that for the superposition of the nonrelativistic waves, we obtain

$$h_1(t) = 2d_g^* \kappa \frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi^2 L} (n^i - m^i) \partial_i \phi(t, \vec{x}), \quad (30)$$

$$h_2(t) = d_g^* \kappa \frac{n^i n^j - m^i m^j}{m_\phi^2} \partial_i \partial_j \phi(t, \vec{x}). \quad (31)$$

Taking into account the time variations of the detectors' orientation, we must replace $\vec{n}$ and $\vec{m}$ with time-varying vectors, $\vec{n}(t)$ and $\vec{m}(t)$. For ground-based detectors, the time variation is due to the Earth's rotation. Then the expression for the signal is

$$\begin{aligned} h_1(t) &= 2d_g^* \kappa \frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi^2 L} \\ &\quad \times (g_{1,0}(t) + g_{1,1}(t)e^{i\Omega_d t} + g_{1,1}^*(t)e^{-i\Omega_d t}), \end{aligned} \quad (32)$$

$$\begin{aligned} h_2(t) &= d_g^* \frac{\kappa}{m_\phi^2} (g_{2,0}(t) + g_{2,1}(t)e^{i\Omega_d t} \\ &\quad + g_{2,1}^*(t)e^{-i\Omega_d t} + g_{2,2}(t)e^{2i\Omega_d t} + g_{2,2}^*(t)e^{-2i\Omega_d t}), \end{aligned} \quad (33)$$

where $\Omega_d = 2\pi/(\text{sidereal day})$. For the space-based detector, DECIGO, the expression is

$$\begin{aligned} h_1(t) &= 2d_g^* \kappa \frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi^2 L} (c_{1,0}(t) + c_{1,1}(t)e^{i\Omega_y t} \\ &\quad + c_{1,1}^*(t)e^{-i\Omega_y t} + c_{1,2}(t)e^{2i\Omega_y t} + c_{1,2}^*(t)e^{-2i\Omega_y t}), \end{aligned} \quad (34)$$

$$\begin{aligned}
h_2(t) = & d_g^* \frac{\kappa}{m_\phi^2} (c_{2,0}(t) + c_{2,1}(t)e^{i\Omega_y t} + c_{2,1}^*(t)e^{-i\Omega_y t} \\
& + c_{2,2}(t)e^{2i\Omega_y t} + c_{2,2}^*(t)e^{-2i\Omega_y t} + c_{2,3}(t)e^{3i\Omega_y t} \\
& + c_{2,3}^*(t)e^{-3i\Omega_y t} + c_{2,4}(t)e^{4i\Omega_y t} + c_{2,4}^*(t)e^{-4i\Omega_y t}),
\end{aligned} \quad (35)$$

where $\Omega_y = 2\pi/\text{year}$. The expressions of $g_{1,i}(t)$, $g_{2,i}(t)$, $c_{1,i}(t)$, and $c_{2,i}(t)$ are summarized in Appendix B.

Next, we calculate the response of LISA. There are several methods to cancel out laser-frequency noise in the LISA experiment, and a measured variable depends on which method we choose. In this paper, we apply the second-generation TDI variable $X_1(t)$ [41,42]. Note that the Doppler shift caused by scalar waves can be calculated by

$$y^{\text{SW}}(t; L, \vec{n}) \equiv -\frac{1}{2\pi f_{\text{laser}}} \frac{d\varphi(t; L, \vec{n})}{dt}, \quad (36)$$

and the contribution from scalar waves to $X_1(t)$ is approximately

$$\begin{aligned}
X_1^{\text{SW}}(t) \simeq & y(t; L, \vec{n}) - y(t; L, \vec{m}) - y(t - 2L; L, \vec{n}) \\
& + y(t - 2L; L, \vec{m}) - y(t - 4L; L, \vec{n}) \\
& + y(t - 4L; L, \vec{m}) + y(t - 6L; L, \vec{n}) \\
& - y(t - 6L; L, \vec{m}),
\end{aligned} \quad (37)$$

where $\vec{n}$ and $\vec{m}$ are unit vectors along two different arms of LISA and L is the arm length of LISA in this case. Then it is straightforward to calculate $X_1^{\text{SW}}(t)$, and the signal is

$$h(t) = X_1^{\text{SW}}(t) = \begin{cases} X_{1,1}(t), & \text{for } 2\frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi L} > v_{\text{DM}} \\ X_{1,2}(t), & \text{for } 2\frac{\sin^2(\frac{m_\phi L}{2})}{m_\phi L} < v_{\text{DM}} \end{cases}, \quad (38)$$

where

$$\begin{aligned}
X_{1,1}(t) \simeq & -\frac{16d_g^* \kappa}{m_\phi^2} \sin^2\left(\frac{m_\phi L}{2}\right) \sin(m_\phi L) \sin(2m_\phi L) \\
& \times (n^i - m^i) \partial_0 \partial_i \phi(t, \vec{x}),
\end{aligned} \quad (39)$$

$$\begin{aligned}
X_{1,2}(t) \simeq & -8d_g^* \kappa L \sin(m_\phi L) \sin(2m_\phi L) \\
& \times \frac{n^i n^j - m^i m^j}{m_\phi^2} \partial_0 \partial_i \partial_j \phi(t, \vec{x}).
\end{aligned} \quad (40)$$

Taking into account the time variations of the detector's orientation, the expression of the signal becomes

$$\begin{aligned}
X_{1,1}(t) = & -16d_g^* \kappa \sin^2\left(\frac{m_\phi L}{2}\right) \sin(m_\phi L) \sin(2m_\phi L) \\
& \times (x_{1,0}(t) + x_{1,1}(t)e^{i\Omega_y t} + x_{1,1}^*(t)e^{-i\Omega_y t} \\
& + x_{1,2}(t)e^{2i\Omega_y t} + x_{1,2}^*(t)e^{-2i\Omega_y t}),
\end{aligned} \quad (41)$$

$$\begin{aligned}
X_{1,2}(t) = & -8d_g^* \kappa L \sin(m_\phi L) \sin(2m_\phi L) \\
& \times (x_{2,0}(t) + x_{2,1}(t)e^{i\Omega_y t} + x_{2,1}^*(t)e^{-i\Omega_y t} \\
& + x_{2,2}(t)e^{2i\Omega_y t} + x_{2,2}^*(t)e^{-2i\Omega_y t} + x_{2,3}(t)e^{3i\Omega_y t} \\
& + x_{2,3}^*(t)e^{-3i\Omega_y t} + x_{2,4}(t)e^{4i\Omega_y t} + x_{2,4}^*(t)e^{-4i\Omega_y t}).
\end{aligned} \quad (42)$$

The expressions of $x_{1,i}(t)$ and $x_{2,i}(t)$ are also summarized in Appendix B.

IV. DETECTION METHOD AND FUTURE CONSTRAINTS

In this section, we discuss an appropriate detection method for the signal derived in the previous section and the future constraints obtained by this method. In this section, we use $\hat{m}_\phi = m_\phi/2\pi$ instead of m_ϕ .

A. Characterization of the signal

Before moving to the detection method, we briefly summarize the characterization of the signal. For ground-based detectors, the signal expression is given by Eq. (32) for $\sin^2(\pi \hat{m}_\phi L)/(\pi \hat{m}_\phi L) > v_{\text{DM}}$. Since $g_{1,i}$ functions are proportional to ϕ , its spectrum is a sharp spectrum centered on $f \simeq \hat{m}_\phi$ with a width of $\sim \hat{m}_\phi v_{\text{DM}}^2$. Therefore, the resultant spectrum is the sum of three spectra centered on $f \simeq \hat{m}_\phi - f_{\text{det}}$, $f \simeq \hat{m}_\phi$, and $f \simeq \hat{m}_\phi + f_{\text{det}}$, with widths of $\sim \hat{m}_\phi v_{\text{DM}}^2$. Here f_{det} is the frequency of the detector's motion. On the other hand, if $\sin^2(\pi \hat{m}_\phi L)/(\pi \hat{m}_\phi L) < v_{\text{DM}}$, the signal expression, Eq. (33), has more terms, and the spectrum is a sum of 5 spectra centered on $f \simeq \hat{m}_\phi - 2f_{\text{det}}$, $f \simeq \hat{m}_\phi - f_{\text{det}}$, $f \simeq f_{\text{det}}$, $f \simeq \hat{m}_\phi + f_{\text{det}}$, and $f \simeq \hat{m}_\phi + 2f_{\text{det}}$, with widths of $\sim \hat{m}_\phi v_{\text{DM}}^2$.

Generalizing it to Eqs. (34), (35), (41), and (42), we find that the signal is the sum of $2N_{\text{det}}(\hat{m}_\phi) + 1$ spectra with a width of $\sim \hat{m}_\phi v_{\text{DM}}^2$, where

$$N_{\text{det}}(\hat{m}_\phi) = \begin{cases} 1, & \text{for } \frac{\sin^2(\pi \hat{m}_\phi L)}{\pi \hat{m}_\phi L} > v_{\text{DM}} \\ 2, & \text{for } \frac{\sin^2(\pi \hat{m}_\phi L)}{\pi \hat{m}_\phi L} < v_{\text{DM}} \end{cases} \quad (43)$$

for ground-based detectors and

$$N_{\text{det}}(\hat{m}_\phi) = \begin{cases} 2, & \text{for } \frac{\sin^2(\pi \hat{m}_\phi L)}{\pi \hat{m}_\phi L} > v_{\text{DM}} \\ 4, & \text{for } \frac{\sin^2(\pi \hat{m}_\phi L)}{\pi \hat{m}_\phi L} < v_{\text{DM}} \end{cases} \quad (44)$$

FIG. 1. The schematic picture of the signal's spectra. The top spectrum is for $\hat{m}_\phi v_{\text{DM}}^2 \gtrsim f_{\text{det}}$, and the bottom one is for $\hat{m}_\phi v_{\text{DM}}^2 \ll f_{\text{det}}$.

for space-based detectors. Each spectrum has a frequency band of $\hat{m}_\phi + j f_{\text{det}} < f < \hat{m}_\phi(1 + v_{\text{DM}}^2) + j f_{\text{det}}$ ($j = -N_{\text{det}}, -N_{\text{det}} + 1, \dots, N_{\text{det}}$), where

$$f_{\text{det}} \equiv \begin{cases} \Omega_{\text{d}}/2\pi, & \text{for ground-based detectors} \\ \Omega_{\text{v}}/2\pi, & \text{for space-based detectors} \end{cases}. \quad (45)$$

Therefore, if $\hat{m}_\phi v_{\text{DM}}^2 \gtrsim f_{\text{det}}$, the signal has one broadened spectrum whose frequency band is $\hat{m}_\phi - N_{\text{det}}(\hat{m}_\phi) f_{\text{det}} < f < \hat{m}_\phi(1 + v_{\text{DM}}^2) + N_{\text{det}}(\hat{m}_\phi) f_{\text{det}}$, and if $\hat{m}_\phi v_{\text{DM}}^2 < f_{\text{det}}$, the signal has $2N_{\text{det}}(\hat{m}_\phi) + 1$ separated spectra. The schematic spectra are drawn in Fig. 1.

B. Detection method

Next, we discuss an appropriate method to detect this signal. The data from a GW detector is the sum of the signal, $h(t)$, and the noise, $n(t)$:

$$s(t) = h(t) + n(t). \quad (46)$$

Here we assume that the noise is stationary Gaussian. The Fourier components of the data are

$$\tilde{s}(f_k) = \int_0^{T_{\text{obs}}} s(t) e^{-2\pi i f_k t} dt, \quad f_k = \frac{k}{T_{\text{obs}}}, \quad (47)$$

where T_{obs} is the observation time. From the discussions in Sec. IV A, the signal's power is in the following frequency bins:

$$B(\hat{m}_\phi) = \bigcup_{j=-N_{\text{det}}(\hat{m}_\phi)}^{N_{\text{det}}(\hat{m}_\phi)} B_j(\hat{m}_\phi), \quad (48)$$

$$B_j(\hat{m}_\phi) = \{f_k | R((\hat{m}_\phi + j f_{\text{det}}) T_{\text{obs}}) \leq k \leq R((\hat{m}_\phi(1 + v_{\text{DM}}^2) + j f_{\text{det}}) T_{\text{obs}}), k \in \mathbb{N}\}, \quad (49)$$

where $R(x)$ is the integer closest to x . In what follows, we present two efficient ways to extract this signal from the noise.

1. Single detector: Incoherent sum of spectra over $B(\hat{m}_\phi)$

One simple way is to take a sum of spectra over the signal's frequency range. The detection statistic in this method is

$$\rho(\hat{m}_\phi) \equiv \sum_{f_k \in B(\hat{m}_\phi)} \frac{2|\tilde{s}(f_k)|^2}{T_{\text{obs}} S(f_k)}, \quad (50)$$

where $S(f)$ is the one-sided power spectrum of the noise. Each term in the sum is normalized so that its expectation value is 1 if the data are pure noise. The benefit of this method is that it is applicable even with one detector.

The threshold of $\rho(\hat{m}_\phi)$ to claim detection is determined by its statistical properties. If the data are pure noise, the cumulative distribution function for $\rho(\hat{m}_\phi)$ is

$$\text{CDF}[\rho(\hat{m}_\phi) < \rho_c] = \frac{\gamma(N(\hat{m}_\phi), \rho_c)}{(N(\hat{m}_\phi) - 1)!}, \quad (51)$$

where $N(\hat{m}_\phi)$ denotes the number of elements in $B(\hat{m}_\phi)$ and $\gamma(N, x)$ is an incomplete gamma function. Its derivation is explained in detail around Eq. (3.1) in Ref. [43]. Given a false alarm probability F , the threshold ρ_c is determined by

$$1 - \text{CDF}[\rho(\hat{m}_\phi) < \rho_c] = F. \quad (52)$$

Since the search is performed for various values of $\hat{m}_\phi$, the number of the trials, N_t , should be taken into account when F is determined. Assuming that the search is performed over the detector's frequency range at intervals of $\sim 1/T_{\text{obs}}$, the trial number is $(f_{\text{max}} - f_{\text{min}}) T_{\text{obs}} + 1$, where f_{min} and f_{max} are the minimum and maximum frequencies of the frequency range over which the search is performed. For example, to reject the existence of the signal with a 2σ level, F should be

$$F \simeq \frac{0.05}{N_t}. \quad (53)$$

The threshold of the signal's amplitude which satisfies $E[\rho(\hat{m}_\phi)] = \rho_c$ is

$$h_{\text{th}}(\hat{m}_\phi) = \sqrt{\frac{S(\hat{m}_\phi)}{2T_{\text{obs}}}} (\rho_c - N(\hat{m}_\phi)). \quad (54)$$

2. Two detectors: Narrow-band stochastic gravitational-wave background search

Another way is to take correlation between the outputs from two detectors, which is used to search for a stochastic GW background [44,45] and was considered in the previous study on the search for a light vector field [29]. The detection statistic is

$$\rho(\hat{m}_\phi) = \frac{1}{T_{\text{obs}}} \sum_{k=-\infty}^{\infty} \tilde{s}_1^*(f_k) \tilde{s}_2(f_k) \tilde{Q}(f_k; \hat{m}_\phi), \quad (55)$$

where $\tilde{s}_1(f_k)$ and $\tilde{s}_2(f_k)$ are Fourier components of the data from two detectors. $\tilde{Q}(f_k; \hat{m}_\phi)$ should be determined so that the search is optimal. Since the wavelength of the scalar field, $1/m_\phi v_{\text{DM}}$, is much longer than that of GWs with the same frequency, the overlap reduction function [44] is approximately constant. In addition, we only assume the signal's bandwidth and do not make any assumptions on its spectral shape. Under these considerations, we apply the following filter function:

$$\tilde{Q}(f_k; \hat{m}_\phi) = \frac{1}{\sqrt{S_1(f_k)S_2(f_k)}} \Theta(f_k; \hat{m}_\phi), \quad (56)$$

where

$$\Theta(f_k; \hat{m}_\phi) = \begin{cases} 1 & (|f_k| \in B(\hat{m}_\phi)) \\ 0 & (\text{otherwise}) \end{cases}. \quad (57)$$

Having explained the two detection methods, let us estimate the sensitivities of these methods. For the former method, the deviation of the expectation value of $\rho(\hat{m}_\phi)$ due to the presence of the signal is

$$E[\rho(\hat{m}_\phi)]_{s(t)=h(t)+n(t)} - E[\rho(\hat{m}_\phi)]_{s(t)=n(t)} \simeq \frac{T_{\text{obs}}}{S(\hat{m}_\phi)} \overline{h^2}, \quad (58)$$

where

$$\overline{h^2} = \frac{1}{T_{\text{obs}}} \int_0^{T_{\text{obs}}} h^2(t) dt. \quad (59)$$

On the other hand, its variance when the data are pure noise is

$$E[(\rho(\hat{m}_\phi) - E[\rho(\hat{m}_\phi)])^2] = 2N(\hat{m}_\phi). \quad (60)$$

Therefore, the threshold of $\sqrt{h^2}$ for detection is

$$h_{\text{th}}(\hat{m}_\phi) \sim \frac{(2N(\hat{m}_\phi))^{\frac{1}{4}}}{\sqrt{T_{\text{obs}}}} \sqrt{S(\hat{m}_\phi)}. \quad (61)$$

Next, we estimate the sensitivity of the latter method, assuming that the two detectors are parallel and their sensitivity curves are the same. The expectation value of the detection statistic when the signal is present is

$$E[\rho(\hat{m}_\phi)] \simeq \frac{T_{\text{obs}}}{S(\hat{m}_\phi)} \overline{h^2}, \quad (62)$$

while its variance when the data are pure noise is

$$E[(\rho(\hat{m}_\phi) - E[\rho(\hat{m}_\phi)])^2] = \frac{N(\hat{m}_\phi)}{2}. \quad (63)$$

Therefore, the threshold amplitude for detection is

$$h_{\text{th}}(\hat{m}_\phi) \sim \frac{N^{\frac{1}{4}}(\hat{m}_\phi)}{2^{\frac{1}{4}} \sqrt{T_{\text{obs}}}} \sqrt{S(\hat{m}_\phi)}, \quad (64)$$

which is different from Eq. (61) only by an $\mathcal{O}(1)$ factor.

Before moving on to the next subsection, we note that detection with multiple detectors is helpful to increase the significance of the signal even if detection with a single detector is possible with the former method in principle. In real experiments, there could be line noise, which has a sharp frequency spectrum and mimics the signal from the scalar field. It can significantly increase the background rate and degrade the significance in some frequency ranges. One of the ways to mitigate that effect is to utilize the coherence of the signal among multiple detectors.

We also note that these data-analysis methods might not be optimal, since we throw out the information on the relative phase between multiple spectral peaks generated by the detector's motion. In the all-sky continuous gravitational wave searches, the detector's motion is treated coherently for the significance of the signal [43,46]. Since the direction of the scalar waves is time-varying in our case, it is not obvious that it can be treated coherently. The development of the optimal method is the focus of future work.

C. Future constraints

Next, we estimate the future constraints on this dark matter model with the former detection method described in the previous subsection. Since the sensitivities of both data-analysis methods are of the same order, we focus on the former method utilizing the incoherent sum of the spectra. For simplicity, we consider a plane wave satisfying Eq. (9),

$$\phi = \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi} \cos(\omega_{\vec{k}} t - \vec{k} \cdot \vec{x} + \theta_{\vec{k}}), \quad |\vec{k}| = m_\phi v_{\text{DM}}. \quad (65)$$

Averaging the square of the signal over the directions of $\vec{k}$ and time, we obtain

$$\begin{aligned} \overline{h^2}(\hat{m}_\phi) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \int \frac{d\Omega_{\vec{k}}}{4\pi} h^2(t) \\ &= \begin{cases} \frac{8}{3} v_{\text{DM}}^2 d_g^2 \frac{\kappa^2 \rho_{\text{DM}}}{m_\phi^2 L^2} (1 - \vec{n} \cdot \vec{m}) \sin^4\left(\frac{m_\phi L}{2}\right), \\ \quad \text{for } 2\sin^2(m_\phi L/2)/(m_\phi L) > v_{\text{DM}} \\ \frac{4}{15} v_{\text{DM}}^4 d_g^2 \frac{\kappa^2 \rho_{\text{DM}}}{m_\phi^2} (1 - (\vec{n} \cdot \vec{m})^2), \\ \quad \text{for } 2\sin^2(m_\phi L/2)/(m_\phi L) < v_{\text{DM}} \end{cases} \end{aligned} \quad (66)$$

for detectors other than LISA and

$$\overline{h^2}(\hat{m}_\phi) = \begin{cases} \frac{256}{3} v_{\text{DM}}^2 d_g^2 \frac{\kappa^2 \rho_{\text{DM}}}{m_\phi^2} \sin^4\left(\frac{m_\phi L}{2}\right) \sin^2(m_\phi L) \sin^2(2m_\phi L), \\ \quad \text{for } 2\sin^2(m_\phi L/2)/(m_\phi L) > v_{\text{DM}} \\ \frac{64}{5} v_{\text{DM}}^4 d_g^2 \kappa^2 L^2 \rho_{\text{DM}} \sin^2(m_\phi L) \sin^2(2m_\phi L), \\ \quad \text{for } 2\sin^2(m_\phi L/2)/(m_\phi L) < v_{\text{DM}} \end{cases} \quad (67)$$

for LISA. Then, the criteria $\sqrt{\overline{h^2}(\hat{m}_\phi)} > h_{\text{th}}(\hat{m}_\phi)$ leads to the parameter region of d_g we will be able to probe with gravitational-wave detectors. Figure 2 compares this parameter region and the constraints given from the fifth-force experiments [12] and tests of the equivalence principle (EP tests) [16,17]. All the detectors will provide more stringent constraints than the fifth-force experiments do in a mass band. In particular, the constraints at $m_\phi = 2 \times 10^{-17}$ eV, 10^{-14} eV, and 10^{-12} eV are improved by factors of ~ 30 , ~ 100 , and ~ 350 , respectively.

On the other hand, the future constraints are weaker than those from the EP tests in most of the band. However, this is the case only if the coupling constants other than d_g are zero. The EP tests constrain the following parameters:

$$D_{\hat{m}} = d_g^*(d_{\hat{m}} - d_g), \quad D_e = d_g^* d_e, \quad (68)$$

while we can constrain d_g^* with gravitational-wave detectors and fifth-force experiments. Especially, EP tests give no constraints if $d_g = d_{m_e} = d_{m_u} = d_{m_d}$ and $d_e = 0$. Figure 3 shows the parameter regions which pass the MICROSCOPE experiment and experiments with gravitational-wave detectors in the case where d_g and $d_{\hat{m}}$ are nonzero. We can see

FIG. 2. The future constraints we can obtain with various gravitational-wave detectors, the constraints given by the fifth-force experiments (Fifth-force), and the tests of the equivalence principle (EP test) are shown. The shaded region is or will be excluded. We consider the case where the coupling constants other than d_g are zero here. We assume that $T_{\text{obs}} = 1$ year and that all of them are 2σ limits. The constraints of EP tests shown here are given by an experiment from the Eöt-Wash group [16] and MICROSCOPE [17]. The constraints given by LLR experiments [13,14] and a hyperfine frequency comparison experiment [22] are less stringent in this frequency range. Note that the constraints from EP tests depend on the values of the other coupling constants, and they give no constraints if $d_g = d_{m_e} = d_{m_u} = d_{m_d}$ and $d_e = 0$.

that the latter experiments can exclude the parameter region along the line $d_g = d_{\hat{m}}$, which cannot be excluded by MICROSCOPE. In that sense, this search is complementary to the EP tests.

One of the example theories which maintain the equivalence principle is the scalar-tensor theory [47]. The action of this theory is given by

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \left(\frac{R}{4\kappa^2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \\ &\quad + \int d^4x \sqrt{-\tilde{g}} \mathcal{L}_m(\tilde{g}_{\mu\nu}, \Psi_m), \end{aligned} \quad (69)$$

where $\tilde{g}_{\mu\nu} \equiv A^2(\phi) g_{\mu\nu}$, and $A^2(\phi)$ is a free function. We again assume that ϕ is a massive scalar field linearly coupling with matter:

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2, \quad A^2(\phi) = 1 + 2d\kappa\phi. \quad (70)$$

The coupling constant, α , given in Eq. (13), is

$$\alpha(\phi) = \frac{d \ln(mA(\phi))}{d\kappa\phi} \simeq d. \quad (71)$$

FIG. 3. The parameter regions which pass experiments with gravitational-wave detectors and the MICROSCOPE experiment are shown. The top figure shows the regions at $m_\phi = 10^{-16}$ eV, and the bottom one shows those at $m_\phi = 10^{-13}$ eV. We assume that $T_{\text{obs}} = 1$ year and that all of them are 2σ limits.

Since this does not depend on the composition of the matter, this theory can be regarded as one of the dilaton theories, with²

$$d_g = d_{m_e} = d_{m_u} = d_{m_d} = d, \quad d_e = 0. \quad (72)$$

Therefore, the same constraints from the gravitational-wave detectors and the fifth-force experiments shown in Fig. 2 hold on d while there are no constraints from the EP tests.

²Conformal metric perturbation fluctuates the laser frequency and mimics the common motion of the mirrors. Since the interferometer is mainly sensitive to their differential motion, this effect is subdominant.

V. CONCLUSION

We investigated the possibility of utilizing gravitational-wave detectors to search for ultralight scalar-field dark matter. First, we derived an analytical expression of the signal and investigated its spectra. We carefully took into account the effect of the peculiar motion on the spectra for the first time. We also worked out the difference of the detector's responses between LISA and the other detectors and the difference of the peculiar motion between space-based and ground-based detectors. Then, we formulated suitable data-analysis methods to detect this signal, one of which is applicable even with one detector. Finally, we estimated the future constraints on the coupling constants between scalar-field dark matter and Standard Model particles with our method.

As a result, we found that all of the detectors we consider will provide more stringent constraints than the fifth-force experiments do in a mass band. In particular, the constraints on d_g at $m_\phi = 2 \times 10^{-17}$ eV, 10^{-14} eV, and 10^{-12} eV are improved by factors of ~ 30 , ~ 100 , and ~ 350 , respectively. Since experiments with gravitational-wave detectors constrain a different combination of the coupling constants from equivalence principle tests, they play complementary roles. We note that they are also different in phenomenological respects, since the equivalence tests measure the interaction between two macroscopic point particles in a vacuum, while the gravitational-wave experiments measure the interaction between a macroscopic point particle and the scalar field propagating in the Galaxy. The gravitational-wave experiment is phenomenologically similar to experiments proposed in Refs. [19], [24], and [29]. The experiments proposed by Refs. [19] and [24] measure temporal oscillations in the sizes of molecules and atomic transition energies, respectively, and constrain different combinations of the coupling constants. Reference [29] considers a different dark matter model from our model.

One of the next things to do is to apply our methods to real data. One of the issues to resolve for the application is line noise, which is sinusoidal noise and can mimic the signal we consider. Such noises also degrade the sensitivity of continuous-wave searches [48]. One characteristic of the signal to distinguish them can be an expected bandwidth of $\sim \hat{m}_\phi v_{\text{DM}}^2$. Another issue is nonstationarity of the detectors caused by environmental disturbances such as earthquakes [49]. We need to generalize our method so that it can be applied to data, a part of which is unavailable due to increased noise level. Improving our analysis method to resolve these issues and analyzing real data are left for future works.

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APPENDIX A: MODIFICATION OF LASER LIGHT PROPAGATION

In this section, we investigate how the scalar field affects the propagation of light through the coupling between the scalar field and photon. The Lagrangian for the photon is as follows:

$$\mathcal{L}_{\text{photon}} = -\frac{1+\psi}{4e^2} F_{\mu\nu} F^{\mu\nu}, \quad \psi \equiv -d_e \kappa \phi. \quad (\text{A1})$$

Then, the equation of motion for the vector potential is

$$\partial_\mu \{(1+\psi)\partial^\mu A^\nu\} - \partial_\mu \{(1+\psi)\partial^\nu A^\mu\} = 0. \quad (\text{A2})$$

In this paper, we consider the case where the scalar field behaves as the following nonrelativistic matter waves in the Galaxy:

$$\psi = \psi_0 \cos(-m_\phi u^\nu x_\nu + \theta_0), \quad u^\mu \simeq (1, \vec{v}), \quad (\text{A3})$$

where $|\vec{v}| \ll 1$. We obtain A_μ by solving Eq. (A2) under the following assumption:

$$|\psi| \ll 1, \quad \frac{m_\phi}{2\pi f_{\text{laser}}} \ll 1. \quad (\text{A4})$$

The first condition means that the amplitude of the scalar field is tiny enough, which is valid in our case. The second condition means the frequency of the laser light is much larger than the frequency of the oscillation we search for, which is valid in usual gravitational-wave experiments.

We separate the vector potential into two parts,

$$A^\mu = \bar{A}^\mu + \delta A^\mu, \quad (\text{A5})$$

where the second part is the correction due to ψ . To simplify the calculation, we apply a gauge transformation

$$A_{\text{NEW}}^\mu = A_{\text{OLD}}^\mu - \partial^\mu \Lambda \quad (\text{A6})$$

to move to a suitable gauge. To obtain $\bar{A}^\mu$, we apply the Lorentz gauge,

$$\partial_\mu \bar{A}^\mu = 0. \quad (\text{A7})$$

Then the equation of motion for $\bar{A}^\mu$ is

$$\partial_\nu \partial^\nu \bar{A}^\mu = 0, \quad (\text{A8})$$

and its solution is

$$\bar{A}^\mu = a^\mu e^{ik^\nu x_\nu}, \quad k^\mu k_\mu = 0. \quad (\text{A9})$$

To obtain δA^μ , we apply the following gauge:

$$\partial_\mu \delta A^\mu + \partial_\mu \psi \bar{A}^\mu = 0. \quad (\text{A10})$$

To move to this gauge, we choose Λ such that it satisfies

$$\partial_\mu \partial^\mu \Lambda = \partial_\mu \delta A_{\text{OLD}}^\mu + \partial_\mu \psi \bar{A}^\mu. \quad (\text{A11})$$

Since the right-hand side is the sum of two modes with four-momenta of $k^\mu \pm m_\phi u^\mu$ and

$$(k^\mu \pm m_\phi u^\mu)(k_\mu \pm m_\phi u_\mu) \simeq \pm 4\pi m_\phi f_{\text{laser}} \neq 0, \quad (\text{A12})$$

it is easy to show that Eq. (A11) is solvable and that

$$\Lambda = (\partial_\nu \partial^\nu)^{-1} [\partial_\mu \delta A_{\text{OLD}}^\mu + \partial_\mu \psi \bar{A}^\mu]. \quad (\text{A13})$$

In this gauge, the equation of motion for δA^μ is

$$\partial_\nu \partial^\nu \delta A^\mu + \partial_\nu \psi \partial^\nu \bar{A}^\mu + \partial_\nu \partial^\mu \psi \bar{A}^\nu = 0. \quad (\text{A14})$$

Since the third term is negligible due to the second condition in Eq. (A4), this equation can be approximated as

$$\partial_\nu \partial^\nu \delta A^\mu + \partial_\nu \psi \partial^\nu A^\mu \simeq 0. \quad (\text{A15})$$

The solution of this equation is

$$\begin{aligned} \delta A^\mu &\simeq \frac{1}{2} \psi_0 a^\mu \cos[-m_\phi u^\mu x_\mu + \theta_0] e^{ik^\nu x_\nu} \\ &= -\frac{1}{2} d_e \kappa \phi a^\mu e^{ik^\nu x_\nu}. \end{aligned} \quad (\text{A16})$$

Therefore, the solution for Eq. (A2) is

$$A^\mu \simeq \left(1 - \frac{1}{2} d_e \kappa \phi(t, \vec{x})\right) a^\mu e^{ik^\nu x_\nu}, \quad (\text{A17})$$

which means the amplitude of the vector potential modulates.

Next, we estimate the effect of this modulation on the output of the gravitational-wave detectors. Due to the second condition in Eq. (A11), the electromagnetic tensor can be approximated as

$$F_{\mu\nu} \simeq \left(1 - \frac{1}{2} d_e \kappa \phi(t, \vec{x})\right) \bar{F}_{\mu\nu}, \quad \bar{F}_{\mu\nu} \equiv \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu. \quad (\text{A18})$$

Therefore, laser power, I , modulates, and the amplitude of this modulation is

$$\frac{\delta I}{I} \sim d_e \kappa \phi_{\text{amp}}, \quad (\text{A19})$$

where ϕ_{amp} is the amplitude of the scalar field's oscillation. Assuming the scalar field accounts for the entire amount of dark matter and $\phi_{\text{amp}} \sim \sqrt{2\rho_{\text{DM}}}/m_\phi$, it can be estimated as

$$\frac{\delta I}{I} \sim 1 \times 10^{-16} d_e \left(\frac{1 \text{ Hz}}{\hat{m}_\phi} \right), \quad (\text{A20})$$

where $\hat{m}_\phi = m_\phi/2\pi$. On the other hand, the power spectrum density of shot noise in I is $S_I(f) = 2\hbar\omega I$, where ω is the angular frequency of laser light. Then

$$\frac{\sqrt{S_I(f)}}{I} \simeq 2 \times 10^{-3} \left(\frac{1 \text{ W}}{I} \right)^{\frac{1}{2}} \left(\frac{100 \text{ nm}}{\lambda_{\text{laser}}} \right)^{\frac{1}{2}} / \sqrt{\text{Hz}}, \quad (\text{A21})$$

where λ_{laser} is the wavelength of laser light. Since the time for which this signal keeps its coherence is $t_{\text{coh}} \sim 1/\hat{m}_\phi v_{\text{DM}}^2$, the detectable amplitude of this modulation is improved by a factor of $1/\sqrt{t_{\text{coh}}} \sim (\hat{m}_\phi v_{\text{DM}}^2)^{\frac{1}{2}}$,

$$\begin{aligned} \frac{\delta I_{\text{th}}}{I} &\sim \frac{1}{\sqrt{t_{\text{coh}}}} \frac{\sqrt{S_I(f)}}{I} \\ &\sim 2 \times 10^{-6} \left(\frac{\hat{m}_\phi}{1 \text{ Hz}} \right)^{\frac{1}{2}} \left(\frac{1 \text{ W}}{I} \right)^{\frac{1}{2}} \left(\frac{100 \text{ nm}}{\lambda_{\text{laser}}} \right)^{\frac{1}{2}}. \end{aligned} \quad (\text{A22})$$

Therefore, for the modulation due to the scalar field's oscillation to be detected, the laser power must satisfy

$$I \gtrsim \frac{4 \times 10^{19}}{d_e^2} \left(\frac{1000 \text{ nm}}{\lambda_{\text{laser}}} \right) \left(\frac{\hat{m}_\phi}{1 \text{ Hz}} \right)^3 \text{ W}. \quad (\text{A23})$$

In the experiments we consider, $\lambda_{\text{laser}} \lesssim \mathcal{O}(1000) \text{ nm}$. On the other hand, the equivalence principle tests [15–17] have already provided the constraints, $d_e \lesssim 10^{-2}$ for the ground-based detectors' band and $d_e \lesssim 10^{-4}$ for the space-based detectors' band, if it is assumed that miraculous cancellation between the coupling constants does not occur. Therefore, the required laser power [Eq. (A23)] is much higher than laser power in the gravitational-wave experiments we consider. In reality, we cannot integrate the signal for a longer time than the observational time. It makes the requirement for laser power more severe. On the other hand, if $1/\hat{m}_\phi v_{\text{DM}}^2$ is shorter than the observational time, we can improve the signal-to-noise ratio by incoherently summing the spectra as we proposed in Sec. IV B 1. However, the difference is $\mathcal{O}(1)$, and it does not change the conclusion. Therefore, the modification of laser light propagation does not cause detectable signals.

APPENDIX B: TIME VARIATION OF THE DETECTORS' ORIENTATIONS

In this appendix, we obtain the expressions of the functions on the right-hand sides of Eqs. (32), (33), (34), (35), (41), and (42). Here we refer to Appendix C of Ref. [50]. The detectors' configurations are shown in Fig. 4.

For ground-based detectors, we consider the coordinate $(t, \vec{x})$ as a celestial coordinate whose origin is the center of the Earth, whose z axis coincides with the Earth's rotation axis and points toward the North Pole, whose x and y axes lie in the Earth's equatorial plane, and whose x axis points toward the vernal point. Then, we have

$$\vec{n}(t) = O_2^T(t) O_3^T \hat{n}, \quad \vec{m}(t) = O_2^T(t) O_3^T \hat{m}. \quad (\text{B1})$$

The matrices and vectors on the right-hand sides are

$$O_2(t) = \begin{pmatrix} \sin\delta\cos(\delta_r + \Omega_d t) & \sin\delta\sin(\delta_r + \Omega_d t) & -\cos\delta \\ -\sin(\delta_r + \Omega_d t) & \cos(\delta_r + \Omega_d t) & 0 \\ \cos\delta\cos(\delta_r + \Omega_d t) & \cos\delta\sin(\delta_r + \Omega_d t) & \sin\delta \end{pmatrix}, \quad (\text{B2})$$

FIG. 4. The configurations of a ground-based detector (top) and a space-based detector (bottom) are shown.

$$O_3 = \begin{pmatrix} -\sin(\gamma - \frac{\zeta}{2}) & \cos(\gamma - \frac{\zeta}{2}) & 0 \\ -\cos(\gamma - \frac{\zeta}{2}) & -\sin(\gamma - \frac{\zeta}{2}) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (\text{B3})$$

and

$$\hat{n} = (1, 0, 0)^T, \quad \hat{m} = (\cos \zeta, \sin \zeta, 0)^T, \quad (\text{B4})$$

where δ is the geodetic latitude of the detector's site, δ_r is the phase defining the position of the Earth in its diurnal motion at $t = 0$, γ is the angle measured counterclockwise from east to the bisector of the interferometer arms, and ζ is the angle between the interferometer arms. Then the functions on the right-hand sides of Eqs. (32) and (33) are

$$g_{1,0}(t) = -2 \cos \gamma \cos \delta \sin\left(\frac{\zeta}{2}\right) \partial_z \phi, \quad (\text{B5})$$

$$g_{1,1}(t) = e^{i\delta_r} (\sin \gamma - i \cos \gamma \sin \delta) \sin\left(\frac{\zeta}{2}\right) (i\partial_x + \partial_y) \phi, \quad (\text{B6})$$

$$g_{2,0}(t) = \cos \gamma \sin \gamma \cos^2 \delta \sin \zeta (\partial_{xx} + \partial_{yy} - 2\partial_{zz}) \phi, \quad (\text{B7})$$

$$g_{2,1}(t) = -e^{i\delta_r} \cos \delta (\cos(2\gamma) + i \sin(2\gamma) \sin \delta) \times \sin \zeta (i\partial_{xz} + \partial_{yz}) \phi, \quad (\text{B8})$$

$$g_{2,2}(t) = \frac{1}{2} e^{2i\delta_r} (i \sin \gamma + \cos \gamma \sin \delta) (\cos \gamma + i \sin \gamma \sin \delta) \times \sin \zeta (i\partial_{xx} + 2\partial_{xy} - i\partial_{yy}) \phi, \quad (\text{B9})$$

where $\partial_{i_1 i_2 \dots i_n} \equiv \partial_{i_1} \partial_{i_2} \dots \partial_{i_n}$.

For space-based detectors, we consider the coordinate $(t, \vec{x})$ as an ecliptic coordinate whose origin is the center of the Sun. Then we have

$$\vec{n}(t) = O_2(t) \hat{n}, \quad \vec{m}(t) = O_2(t) \hat{m}. \quad (\text{B10})$$

The matrix and vectors on the right-hand sides are

$$O_2(t) = \begin{pmatrix} \sin \eta(t) \cos \xi(t) - \sin \iota \cos \eta(t) \sin \xi(t) & -\sin \eta(t) \sin \xi(t) - \sin \iota \cos \eta(t) \cos \xi(t) & -\cos \iota \cos \eta(t) \\ -\cos \eta(t) \cos \xi(t) - \sin \iota \sin \eta(t) \sin \xi(t) & \cos \eta(t) \sin \xi(t) - \sin \iota \sin \eta(t) \cos \xi(t) & -\cos \iota \sin \eta(t) \\ \cos \iota \sin \xi(t) & \cos \iota \cos \xi(t) & -\sin \iota \end{pmatrix} \quad (\text{B11})$$

and

$$\hat{n} = -(\cos \sigma_3, \sin \sigma_3, 0), \quad \hat{m} = (\cos \sigma_2, \sin \sigma_2, 0), \quad (\text{B12})$$

where

$$\iota = -\frac{\pi}{6}, \quad \eta(t) = \Omega_y t + \eta_0, \quad \xi(t) = -\Omega_y t + \xi_0, \quad (\text{B13})$$

and

$$\sigma_a = -\frac{3}{2}\pi + \frac{2}{3}(a-1)\pi. \quad (\text{B14})$$

Then the functions on the right-hand sides of Eqs. (34), (35), (41), and (42) are

$$c_{1,0}(t) = \frac{3}{4} (\cos(\eta_0 + \xi_0) \partial_x \phi + \sin(\eta_0 + \xi_0) \partial_y \phi), \quad (\text{B15})$$

$$c_{1,1}(t) = \frac{\sqrt{3}}{4} e^{-i\xi_0} \partial_z \phi, \quad (\text{B16})$$

$$c_{1,2}(t) = -\frac{1}{8} e^{i(\eta_0 - \xi_0)} (\partial_x - i\partial_y) \phi, \quad (\text{B17})$$

$$c_{2,0}(t) = \frac{9\sqrt{3}}{32} (2 \cos(2(\eta_0 + \xi_0)) \partial_{xy} \phi + \sin(2(\eta_0 + \xi_0)) (-\partial_{xx} + \partial_{yy}) \phi), \quad (\text{B18})$$

$$c_{2,1}(t) = \frac{9}{16} e^{-i(\eta_0 + 2\xi_0)} (-i\partial_{xz} + \partial_{yz}) \phi, \quad (\text{B19})$$

$$c_{2,2}(t) = \frac{3\sqrt{3}}{32} i e^{-2i\xi_0} (\partial_{xx} + \partial_{yy} - 2\partial_{zz}) \phi, \quad (\text{B20})$$

$$c_{2,3}(t) = \frac{3}{16} e^{i(\eta_0 - 2\xi_0)} (i\partial_{xz} + \partial_{yz}) \phi, \quad (\text{B21})$$

$$c_{2,4}(t) = -\frac{\sqrt{3}}{64} i e^{2i(\eta_0 - \xi_0)} (\partial_{xx} - 2i\partial_{xy} - \partial_{yy}) \phi, \quad (\text{B22})$$

$$x_{1,0}(t) = \frac{3}{4} (\cos(\eta_0 + \xi_0) \partial_{tx} \phi + \sin(\eta_0 + \xi_0) \partial_{ty} \phi), \quad (\text{B23})$$

$$x_{1,1}(t) = \frac{\sqrt{3}}{4} e^{-i\xi_0} \partial_{tz} \phi, \quad (\text{B24})$$

$$x_{1,2}(t) = -\frac{1}{8}e^{i(\eta_0 - \xi_0)}(\partial_{tx} - i\partial_{ty})\phi, \quad (\text{B25})$$

$$x_{2,0}(t) = \frac{9\sqrt{3}}{32}(2\cos(2(\eta_0 + \xi_0))\partial_{txy}\phi + \sin(2(\eta_0 + \xi_0))(-\partial_{txx} + \partial_{tyy})\phi), \quad (\text{B26})$$

$$x_{2,1}(t) = \frac{9}{16}e^{-i(\eta_0 + 2\xi_0)}(-i\partial_{txz} + \partial_{tyz})\phi, \quad (\text{B27})$$

$$x_{2,2}(t) = \frac{3\sqrt{3}}{32}ie^{-2i\xi_0}(\partial_{txx} + \partial_{tyy} - 2\partial_{txz})\phi, \quad (\text{B28})$$

$$x_{2,3}(t) = \frac{3}{16}e^{i(\eta_0 - 2\xi_0)}(i\partial_{txz} + \partial_{tyz})\phi, \quad (\text{B29})$$

$$x_{2,4}(t) = -\frac{\sqrt{3}}{64}ie^{2i(\eta_0 - \xi_0)}(\partial_{txx} - 2i\partial_{txy} - \partial_{tyy})\phi. \quad (\text{B30})$$

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