Erratum: One-loop correction to heavy dark matter annihilation **[Phys. Rev. D *95*, 055001 (2017)]**

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In Sec. III, Eq. (13) should read

$$\begin{aligned}
 C_1(\mu) &= -\frac{\pi\alpha_2(\mu)}{m_\chi} + \frac{\alpha_2(\mu)^2}{4m_\chi} \left[2\ln^2 \frac{\mu^2}{4m_\chi^2} + 2\ln \frac{\mu^2}{4m_\chi^2} + 2i\pi \ln \frac{\mu^2}{4m_\chi^2} + 12 - \frac{11\pi^2}{6} \right], \\
 C_2(\mu) &= \frac{\pi\alpha_2(\mu)}{m_\chi} - \frac{\alpha_2(\mu)^2}{2m_\chi} \left[\ln^2 \frac{\mu^2}{4m_\chi^2} + 3\ln \frac{\mu^2}{4m_\chi^2} - i\pi \ln \frac{\mu^2}{4m_\chi^2} + 2 - \frac{5\pi^2}{12} \right].
 \end{aligned}
 \tag{13}$$

In detail the +12 and +2 in these equations were corrected from +8 and 0.

These changes result from corrections to the counter-terms and threshold corrections, which appeared in Appendix A, and affected results in our original Eqs. (A32), (A33), (A36), (A39), (A40), and (A41). For simplicity we repeat that entire part of the appendix below with the corrected equations, replacing the material between Eqs. (A31) and (A42) of the original paper.

Counter-terms

To begin with, as B_3 vanishes at tree level there are no counter-term corrections to its value at one loop. Instead we only need to consider graphs that would contribute to B_1 and B_2 , of which there are three:

The graph on the left corresponds to the internal wave-function and mass renormalization of the DM—renormalization factors denoted as Z_χ and Z_m —whilst the remaining two graphs account for the renormalization of the DM and electroweak gauge boson interaction vertex $g_2 \bar{\chi} \mathcal{W} \chi$ —here Z_1 (which includes coupling and external line wave function renormalization). Now if we calculate the above three graphs, we find a contribution proportional to the tree-level amplitude $\mathcal{M}_{\text{tree}}$, as well as a term that would contribute to B_3 . The contribution to B_3 is canceled by the additional s -channel type counter-term graphs not drawn, so the full counter-term contribution leaves only:

$$(2\delta_1 - \delta_\chi - \delta_m) \mathcal{M}_{\text{tree}},
 \tag{A31}$$

where we have used $Z_i = 1 + \delta_i$.

Next, when determining the δ_i we need to pick a scheme. As explained above, when calculating matching coefficients it is easiest to work in the on-shell scheme for wave function renormalization to ensure we do not have to worry about residues from the LSZ reduction. The meaning of the on-shell values of δ_χ and δ_m is clear, whereas for δ_1 we must write this out more explicitly. By definition we know $\delta_1 = \delta_{g_2} + \frac{1}{2}\delta_W + \delta_\chi$, where δ_{g_2} and δ_W are the counter-terms for the coupling and gauge boson wave functions, respectively. For the gauge boson wave function we use the on-shell scheme as usual. For the coupling counter-term, however, we define it in the $\overline{\text{MS}}$ scheme. Since our full theory is defined with the DM as a propagating degree of freedom, this coupling is defined above the m_χ . In the EFT the DM is integrated out, so the appropriate coupling for the matching is one defined below m_χ . We put this issue aside for now and return to it in the next section.

The above choices then define our scheme for δ_1 in a manner that ensures all residues are still 1. With this scheme, we can then calculate the relevant counter-terms for Majorana DM and find¹:

$$\begin{aligned}
\delta_\chi &= -\frac{\alpha_2}{4\pi} \left[\frac{2}{\epsilon_{\text{UV}}} + \frac{4}{\epsilon_{\text{IR}}} + 12L + 12\ln 2 + 8 \right], \\
\delta_m &= -\frac{\alpha_2}{4\pi} \left[\frac{6}{\epsilon_{\text{UV}}} + 12L + 12\ln 2 + 8 \right], \\
\delta_W &= -\frac{\alpha_2}{4\pi} \left[\frac{2n_D - 11}{6\epsilon_{\text{UV}}} + \frac{19 - 2n_D}{6\epsilon_{\text{IR}}} + \frac{8}{3}L + \frac{8}{3}\ln 2 \right], \\
\delta_{g_2} &= -\frac{\alpha_2}{4\pi} \left[\frac{35 - 2n_D}{12\epsilon_{\text{UV}}} \right], \\
\delta_1 &= -\frac{\alpha_2}{4\pi} \left[\frac{4}{\epsilon_{\text{UV}}} + \frac{67 - 2n_D}{12\epsilon_{\text{IR}}} + \frac{40}{3}L + \frac{40}{3}\ln 2 + 8 \right],
\end{aligned} \tag{A32}$$

where n_D is again the number of left-handed SM doublets. Recall that in determining the counter-terms we cannot neglect scaleless integrals as we did for the main calculation, so their contribution has been included here, and we explicitly distinguish ϵ_{UV} from ϵ_{IR} . Substituting these results into Eq. (A31), we find the crossed contribution is:

$$\begin{aligned}
B_1^{[\text{CT}]} &= \frac{\alpha_2}{4\pi} \left[\frac{2n_D - 43}{6\epsilon_{\text{IR}}} - \frac{8}{3}L - \frac{8}{3}\ln 2 \right], \\
B_2^{[\text{CT}]} &= -\frac{1}{2}B_1^{[\text{CT}]}, \\
B_3^{[\text{CT}]} &= 0.
\end{aligned} \tag{A33}$$

Interestingly the counter-term contribution is UV finite. This implies that the sum of all one-loop graphs before adding in counter-terms must be UV finite. Given that we used dimensional regularization to regulate both UV and IR divergences this cannot be immediately read off from our results, but going back to the integrals and keeping track of the UV divergences we confirmed that the sum is indeed UV finite.

Note if our DM field had instead been a Dirac fermion, the above result would vary, but in a way that is exactly canceled when we account for the threshold corrections in the coupling, as discussed in the next section. In detail, δ_χ and δ_m remain unchanged, whereas we now have

$$\begin{aligned}
\delta_W &= -\frac{\alpha_2}{4\pi} \left[\frac{2n_D - 3}{6\epsilon_{\text{UV}}} + \frac{19 - 2n_D}{6\epsilon_{\text{IR}}} + \frac{16}{3}L + \frac{16}{3}\ln 2 \right], \\
\delta_{g_2} &= -\frac{\alpha_2}{4\pi} \left[\frac{27 - 2n_D}{12\epsilon_{\text{UV}}} \right], \\
\delta_1 &= -\frac{\alpha_2}{4\pi} \left[\frac{4}{\epsilon_{\text{UV}}} + \frac{67 - 2n_D}{12\epsilon_{\text{IR}}} + \frac{44}{3}L + \frac{44}{3}\ln 2 + 8 \right],
\end{aligned} \tag{A34}$$

so that in the Dirac fermion case

$$\begin{aligned}
B_1^{[\text{CT}]} &= \frac{\alpha_2}{4\pi} \left[\frac{2n_D - 43}{6\epsilon_{\text{IR}}} - \frac{16}{3}L - \frac{16}{3}\ln 2 \right], \\
B_2^{[\text{CT}]} &= -\frac{1}{2}B_1^{[\text{CT}]}, \\
B_3^{[\text{CT}]} &= 0.
\end{aligned} \tag{A35}$$

¹As pointed out in [1], the original version of this paper had incorrect coefficients in some entries of these counter terms. This affected the constant terms appearing in C_1 and C_2 in Eq. (13), namely the +12 and +2 terms, respectively. The resulting changes to the figures are invisible to the eye. We thank the authors of that work for bringing this to our attention.

Threshold corrections in the coupling

Throughout the above calculation we have treated the DM as a propagating degree of freedom and included its effects in loop diagrams. This implies that the coupling used so far above in this appendix implicitly depends on $n_D + 1$ flavors (n_D left-handed SM doublets and one Majorana DM fermion); i.e., we have used $\alpha_2 = \alpha_2^{(n_D+1)}(\mu)$. In the EFT however, the DM is no longer a propagating field and so the appropriate coupling is $\alpha_2^{(n_D)}(\mu)$. At order α_2^2 , which we are working to at one loop, the distinction will lead to a finite contribution because of the matching at the scale $\mu = m_\chi$, which we calculate in this section.

Let us start by reviewing the standard treatment of a running coupling in the $\overline{\text{MS}}$ scheme. This running is captured by the β -function, which is defined by $\beta(\alpha_2) = \mu d\alpha_2/d\mu$, where here α_2 is the renormalized coupling; the bare coupling is independent of μ . The β -function can be written as:

$$\beta(\alpha_2) = -2\epsilon\alpha_2 - \frac{b_0}{2\pi}\alpha_2^2 + \dots, \quad (\text{A36})$$

where we have expanded it to the order needed for this threshold matching analysis. At this order the LL solution for the running of the coupling is:

$$\alpha_2(\mu) = \frac{\alpha_2(\mu_0)}{1 + \alpha_2(\mu_0) \frac{b_0}{2\pi} \ln \frac{\mu}{\mu_0}}. \quad (\text{A37})$$

In order to determine the threshold matching correction at the one-loop order we are working it suffices to simply demand that the coupling is continuous at the scale m_χ , and this is captured by a difference in b_0 . For our problem we define $b_0^{(n_D+1)}$ to be the value above m_χ and $b_0^{(n_D)}$ the value below. Then using Eq. (A37) to define $\alpha_2^{(n_D+1)}(\mu)$ and $\alpha_2^{(n_D)}(\mu)$, it suffices to demand they match at a scale m_χ , which gives:

$$\alpha_2^{(n_D+1)}(\mu) = \alpha_2^{(n_D)}(\mu) \left[1 + \frac{\alpha_2^{(n_D)}(\mu)}{2\pi} (b_0^{(n_D)} - b_0^{(n_D+1)}) \ln \frac{\mu}{m_\chi} + \dots \right]. \quad (\text{A38})$$

So now we just need to determine $b_0^{(n_D)} - b_0^{(n_D+1)}$. In general for a theory containing just gauge bosons, Weyl fermions (WF), Dirac fermions (DF), Majorana fermions (MF) and charged scalars (CS), we can write:

$$b_0 = \frac{11}{3}C_A - \frac{2}{3} \sum_{i \in \text{WF}} C(R_i) - \frac{4}{3} \sum_{i \in \text{DF}} C(R_i) - \frac{2}{3} \sum_{i \in \text{MF}} C(R_i) - \frac{1}{3} \sum_{i \in \text{CS}} C(R_i). \quad (\text{A39})$$

Our calculation can involve all of these ingredients: electroweak gauge bosons, the left-handed SM fermions (which are Weyl because only one chirality couples to the gauge bosons), the Majorana or Dirac DM fermion and the Higgs. Then using $C_A = 2$, $C(R) = 1/2$ for the SM left-handed fermions and the Higgs, and $C(R) = 2$ for the adjoint Majorana Wino, we conclude:

$$\begin{aligned} b_0^{(n_D)} &= \frac{43 - 2n_D}{6}, \\ b_0^{(n_D+1)} &= \frac{35 - 2n_D}{6}. \end{aligned} \quad (\text{A40})$$

From this Eq. (A38) tells us that to the order we are working:

$$\alpha_2^{(n_D+1)}(\mu) = \alpha_2^{(n_D)}(\mu) \left[1 + \frac{\alpha_2^{(n_D)}(\mu)}{4\pi} \left(\frac{8}{3}L + \frac{8}{3}\ln 2 \right) \right]. \quad (\text{A41})$$

Now as there is only a difference between the couplings at next to leading order, this only corrects the tree level result stated in Eq. (A4). As such the impact of changing to the coupling defined below m_χ , which is relevant for the matching, is to add the following contribution:

$$\begin{aligned}
B_1^{[\text{Matching}]} &= \frac{\alpha_2}{4\pi} \left[\frac{8}{3}L + \frac{8}{3}\ln 2 \right], \\
B_2^{[\text{Matching}]} &= -\frac{1}{2}B_1^{[\text{Matching}]}, \\
B_3^{[\text{Matching}]} &= 0,
\end{aligned} \tag{A42}$$

where after adding this contribution now here and in all earlier one-loop results we can simply take $\alpha_2 = \alpha_2^{(n_D)}$.

In the Dirac case, the wino contributes differently to $b_0^{(n_D+1)}$, giving

$$b_0^{(n_D+1)} = \frac{27 - 2n_D}{6}, \tag{A43}$$

and accordingly we find

$$\begin{aligned}
B_1^{[\text{Matching}]} &= \frac{\alpha_2}{4\pi} \left[\frac{16}{3}L + \frac{16}{3}\ln 2 \right], \\
B_2^{[\text{Matching}]} &= -\frac{1}{2}B_1^{[\text{Matching}]}, \\
B_3^{[\text{Matching}]} &= 0.
\end{aligned} \tag{A44}$$

Note that these corrections from threshold matching for the coupling exactly cancel the L and $\ln 2$ terms that appear in $B_{1,2}^{[\text{CT}]}$. As alluded to above, although the counter-term and threshold matching results individually differ for the Dirac or Majorana cases, when combined these terms lead to the same result for the final $B_i^{(1)}$ coefficients given in Eq. (A45) below.

Combination

Combining the 25 graphs above with the counter-terms and the matching contributions, we arrive at the following result:

$$\begin{aligned}
B_1^{(1)} &= \frac{\alpha_2}{4\pi} \left[-\frac{4}{\epsilon^2} - \frac{48L + 12i\pi - 55 - 2n_D}{6\epsilon} - 8L^2 - 4L - 4i\pi L - 12 + \frac{11\pi^2}{6} \right], \\
B_2^{(1)} &= \frac{\alpha_2}{4\pi} \left[\frac{2}{\epsilon^2} + \frac{48L - 12i\pi + 79 - 2n_D}{12\epsilon} + 4L^2 + 6L - 2i\pi L + 2 - \frac{5\pi^2}{12} \right], \\
B_3^{(1)} &= \frac{\alpha_2}{4\pi} \left[\frac{n_D - 72\ln 2 - 71 + 3\pi^2}{12} \right],
\end{aligned} \tag{A45}$$

where recall $L = \ln \mu/2m_\chi$, n_D is the number of SM left-handed doublets and now all $\epsilon = \epsilon_{\text{IR}}$.

As explained in detail at the outset of the calculation, the one-loop contribution to the matching coefficient is just the finite part of this result. Combining this with the tree-level term in Eq. (A4) and mapping back to C_r using Eq. (A3) then gives us the Wilson coefficients in Eq. (13), which we set out to justify.

If instead we had a Dirac DM triplet rather than a Majorana, then the only impact on the above would be for $B_3^{(1)}$, and we would instead have

$$B_3^{(1)} = \frac{\alpha_2}{4\pi} \left[\frac{n_D - 72\ln 2 - 43}{12} \right]. \tag{A46}$$

[1] M. Beneke, A. Broggio, C. Hasner, and M. Vollmann, *Phys. Lett. B* **786**, 347 (2018).