Neutrino decoherence in a fermion and scalar background

José F. Nieves*

Laboratory of Theoretical Physics, Department of Physics, University of Puerto Rico, Río Piedras, Puerto Rico 00936

Sarira Sahu†

Instituto de Ciencias Nucleares, Universidad Nacional Autonoma de Mexico, Circuito Exterior, C. U., A. Postal 70-543, 04510 Mexico DF, Mexico

(Received 2 October 2019; published 30 December 2019)

We consider the decoherence effects in the propagation of neutrinos in a background composed of a scalar particle and a fermion due to the nonforward neutrino scattering processes. Using a simple model for the coupling of the form $\bar{f}_R \nu_L \phi$, we calculate the contribution to the imaginary part of the neutrino self-energy arising from the nonforward neutrino scattering processes in such backgrounds, from which the damping terms are determined. In the case we are considering, in which the initial neutrino state is depleted but does not actually disappear (the initial neutrino transitions into a neutrino of a different flavor but does not decay into a $f\phi$ pair, for example), we associate the damping terms with decoherence effects. For this purpose, we give a precise prescription to identify the decoherence terms, as used in the context of the master or Linblad equation, in terms of the damping terms we have obtained from the calculation of the imaginary part of the neutrino self-energy from the nonforward neutrino scattering processes. The results can be directly useful in the context of dark matter-neutrino interaction models in which the scalar and/or fermion constitute the dark matter and can also serve to guide the generalizations to other models and/or situations in which the decoherence effects in the propagation of neutrinos originate from the nonforward scattering processes may be important. As a guide to estimating such decoherence effects, the contributions to the absorptive part of the self-energy and the corresponding damping terms are computed explicitly in the context of the model we consider, for several limiting cases of the momentum distribution functions of the background particles.

DOI: [10.1103/PhysRevD.100.115049](https://doi.org/10.1103/PhysRevD.100.115049)

I. INTRODUCTION AND SUMMARY

Several extensions of the standard electroweak theory involve the coupling of neutrinos to scalar particles (ϕ) and fermions (f) of the generic form $\bar{f}_R \nu_L \phi$. Such couplings have been considered recently in the context of dark matter-neutrino interactions [1–9]. Those interactions can produce nonstandard contributions to the neutrino index of refraction and effective potential when the neutrino propagates in a background of those particles. In Ref. [10], we considered the real part of the self-energy of a neutrino that propagates in a medium consisting of fermions and scalars, with a coupling of that form. From the self-energy, the neutrino

and antineutrino effective potential and dispersion relations were then determined.

In the presence of these interactions, there can also be damping terms in the neutrino effective potential and index of refraction, as a consequence of processes such as $\nu + \phi \leftrightarrow f$ and $\nu + \bar{f} \leftrightarrow \bar{\phi}$, that may become possible depending on the kinematics conditions. In Ref. [11], we extended our previous work to calculate the imaginary part (or more precisely the absorptive part) of the neutrino self-energy, in a fermion and scalar background due to the $\bar{f}_R \nu_L \phi$ interaction. From the imaginary part of the self-energy, the damping terms in the effective potential and dispersion relation were obtained.

Here we note that, in addition to the effects we have mentioned, the presence of those couplings in general can induce decoherence effects in the propagation of neutrinos due to the neutrino nonforward scattering process [12–16]. To be more precise, here we consider various neutrino flavors (ν_{La}) interacting with a scalar and fermion with a coupling of the form

$$L_{\text{int}} = \sum_a \lambda_a \bar{f}_R \nu_{La} \phi + \text{H.c.} \quad (1.1)$$

*nieves@ltp.uprrp.edu
†sarira@nucleares.unam.mx

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

In this case, the scattering processes of the form $\nu_a + x \rightarrow \nu_b + x$, where $x = f, \phi$, can induce decoherence effects in the propagation of neutrinos, independently of the possible damping effects already mentioned. From the calculational point of view, the first step in our strategy is to determine the contribution of such processes to the absorptive part of the self-energy, from which we obtain the corresponding contribution to the damping matrix Γ by the usual method. However, in the present case, in which the initial neutrino state is depleted but does not actually disappear (the initial neutrino transitions into a neutrino of a different flavor but does not decay into a $f\phi$ pair, for example), the effects of the nonforward scattering processes are more properly interpreted in terms of decoherence phenomena rather than damping. The second step in our strategy is to give a precise prescription to identify the decoherence terms, as used in the context of the master or Linblad equation, in terms of the damping matrix Γ that we obtain from the calculation of the imaginary part of the neutrino self-energy due to the nonforward neutrino scattering processes. In writing Eq. (1.1), we assume the presence of only one scalar and one fermion field. Despite this simplification, our work illustrates some features that can serve as a guide when considering more general cases or situations not envisioned here. They can be applied, for example, in the context of models in which sterile (ν_{Ls}) neutrinos have *secret* gauge interactions of the form $\bar{\nu}_{Ls}\gamma^\mu\nu_{Ls}A'_\mu$ [17], when a sterile neutrino propagates in a background of sterile neutrinos and A' bosons. They can also be applied in models in which sterile neutrinos interact with the active neutrinos via coupling of the form $\lambda_a\bar{\nu}_{Rs}\nu_{La}\phi$ [3,6]. The formulas we obtain for the damping and decoherence terms can be applied in the context of such models with minor modifications. As usual, the formulas involve integrals over the momentum distribution functions of the background particles. As a guide to estimating such decoherence effects, the contributions to the absorptive part of the self-energy and the corresponding damping terms are computed explicitly in the context of the model we consider, for several limiting cases of the momentum distribution functions of the background particles.

In Sec. II, we review the method we used previously to determine the dispersion relation and damping term for a single neutrino generation propagating in an $f\phi$ background, and then extend it here to the case of several neutrino generations, in particular to determine the damping matrix from the calculation of the self-energy. In Sec. III, we carry out the calculation of the absorptive part of the self-energy that arises from the nonforward neutrino scattering processes. For definiteness, we consider the special situation in which there are no ϕ scalars in the background (the heavy ϕ limit), so the background consists of the f fermions and the antiparticles only. The final result in that section is the formula for the damping matrix, expressed in terms of integrals over the background fermion distribution functions and the coupling constants g_a defined in Eq. (1.1). In Sec. IV,

we formulate the interpretation of the damping matrix so determined as a decoherence effect and its relation to the Linblad equation and the stochastic evolution of the state vector [18–22]. The result is a well-defined formula for the *jump* operators in that context. In Sec. V, the integrals involved are evaluated explicitly for different conditions of the fermion background. Our conclusions and outlook are given in Sec. VI, and some details of the derivations are provided in the Appendix.

II. SELF-ENERGY AND THE DAMPING MATRIX

A. Dispersion relation for a single neutrino generation

In order to set down our notation and conventions, it is useful to first review briefly the case of only one neutrino generation coupled in Eq. (1.1), considered in Refs. [10,11]. We denote by u^μ the velocity four-vector of the background medium and by k^μ the momentum of the propagating neutrino. In the background medium's own rest frame,

$$u^\mu = (1, \vec{0}), \quad (2.1)$$

and in this frame, we also write

$$k^\mu = (\omega, \vec{k}). \quad (2.2)$$

In this work, we consider only one background medium, which can be taken to be at rest, and therefore we adopt Eqs. (2.1) and (2.2) throughout. The dispersion relation $\omega(\vec{k})$ and the spinor of the propagating mode are determined by solving the equation

$$(\not{k} - \Sigma_{\text{eff}})\psi_L(k) = 0, \quad (2.3)$$

where Σ_{eff} is the neutrino thermal self-energy. Σ_{eff} can be decomposed in the form

$$\Sigma_{\text{eff}} = \Sigma_r + i\Sigma_i, \quad (2.4)$$

where Σ_r is the dispersive part and Σ_i the absorptive part,

$$\begin{aligned} \Sigma_r &= \frac{1}{2}(\Sigma_{\text{eff}} + \bar{\Sigma}_{\text{eff}}), \\ \Sigma_i &= \frac{1}{2i}(\Sigma_{\text{eff}} - \bar{\Sigma}_{\text{eff}}), \end{aligned} \quad (2.5)$$

with

$$\bar{\Sigma}_{\text{eff}} = \gamma^0 \Sigma_{\text{eff}} \gamma^0. \quad (2.6)$$

In the context of thermal field theory,

$$\Sigma_r = \Sigma_{11r} \equiv \frac{1}{2}(\Sigma_{11} + \bar{\Sigma}_{11}), \quad (2.7)$$

where Σ_{11} is the 11 element of the thermal self-energy matrix. On the other hand, Σ_i is conveniently obtained from the formula

FIG. 1. One-loop diagram for the neutrino thermal self-energy matrix elements in an $f\phi$ background.

$$\Sigma_i = \frac{\Sigma_{12}}{2in_F(x_\nu)}, \quad (2.8)$$

where $\Sigma_{12}(k)$ is the 12 element of the neutrino thermal self-energy matrix, while

$$n_F(z) = \frac{1}{e^z + 1} \quad (2.9)$$

is the fermion distribution function, written in terms of a dummy variable z , and the variable x_ν is given by

$$x_\nu = \beta k \cdot u - \alpha_\nu. \quad (2.10)$$

To the lowest order, Σ_{11} and Σ_{12} are determined by evaluating the diagram shown in Fig. 1. The chirality of the neutrino interactions implies that [23]

$$\Sigma = V^\mu(\omega, \vec{\kappa}) \gamma_\mu L \quad (2.11)$$

and correspondingly

$$\Sigma_{r,i} = V_{r,i}^\mu(\omega, \vec{\kappa}) \gamma_\mu L, \quad (2.12)$$

with

$$V^\mu = V_r^\mu + iV_i^\mu. \quad (2.13)$$

We have indicated explicitly that, in general, both $V_{r,i}^\mu$ are functions of ω and $\vec{\kappa}$. Ordinarily, we will omit those arguments but we will restore them when needed.

The results obtained in Ref. [11] are summarized as follows. Writing the neutrino and antineutrino dispersion relations in the form

$$\omega_r^{(\nu, \bar{\nu})} = \omega_r^{(\nu, \bar{\nu})} - \frac{i\gamma^{(\nu, \bar{\nu})}}{2}. \quad (2.14)$$

$\omega_r^{(\nu, \bar{\nu})}$ is given by

$$\omega_r^{(\nu, \bar{\nu})} = \kappa + V_{\text{eff}}^{(\nu, \bar{\nu})}, \quad (2.15)$$

where $V_{\text{eff}}^{(\nu, \bar{\nu})}$ are the effective potentials

$$\begin{aligned} V_{\text{eff}}^{(\nu)} &= n \cdot V_r(\kappa, \vec{\kappa}) = V_r^0(\kappa, \vec{\kappa}) - \hat{\kappa} \cdot \vec{V}_r(\kappa, \vec{\kappa}), \\ V_{\text{eff}}^{(\bar{\nu})} &= -n \cdot V_r(-\kappa, -\vec{\kappa}) = -V_r^0(-\kappa, -\vec{\kappa}) + \hat{\kappa} \cdot \vec{V}_r(-\kappa, -\vec{\kappa}), \end{aligned} \quad (2.16)$$

with

$$n^\mu = (1, \hat{\kappa}). \quad (2.17)$$

On the other hand, for the imaginary part,

$$\begin{aligned} -\frac{\gamma^{(\nu)}(\vec{\kappa})}{2} &= \frac{n \cdot V_i(\kappa, \vec{\kappa})}{1 - n \cdot \frac{\partial V_r(\omega, \vec{\kappa})}{\partial \omega} \big|_{\omega=\kappa}}, \\ -\frac{\gamma^{(\bar{\nu})}(\vec{\kappa})}{2} &= \frac{n \cdot V_i(-\kappa, -\vec{\kappa})}{1 - n \cdot \frac{\partial V_r(\omega, -\vec{\kappa})}{\partial \omega} \big|_{\omega=-\kappa}}, \end{aligned} \quad (2.18)$$

where n^μ is defined in Eq. (2.17). If the correction due to the $n \cdot \partial V_r(\omega, \vec{\kappa}) / \partial \omega$ in the denominator can be neglected, the formulas in Eq. (2.18) reduce to

$$\begin{aligned} -\frac{\gamma^{(\nu)}(\vec{\kappa})}{2} &= n \cdot V_i(\kappa, \vec{\kappa}), \\ -\frac{\gamma^{(\bar{\nu})}(\vec{\kappa})}{2} &= n \cdot V_i(-\kappa, -\vec{\kappa}). \end{aligned} \quad (2.19)$$

In any case, Eqs. (2.15) and (2.18) allow us to obtain the neutrino and antineutrino dispersion relation and damping from the self-energy.

B. Several generations—equation for the flavor spinors

Our aim here is to extend the above considerations to the case of several neutrino generations. In this case, $V_{r,i}^\mu$, as well as $\Sigma_{r,i}$, are matrices in flavor space. As already stated, in this work, we assume that the distribution functions of the background particles are isotropic. In this case, V^μ is a function only of k^μ and u^μ and no other vectors. One traditional way to take this into account is to parametrize V^μ in the form

$$V^\mu = ak^\mu + bu^\mu. \quad (2.20)$$

For our present purposes, we find more convenient to proceed as follows. The isotropy assumption is equivalent to assume that

$$V^\mu = (V^{(u)}, V^{(t)} \hat{\kappa}) \quad (2.21)$$

in that frame. For completeness, we note that this can be written in a general way by introducing

$$t^\mu = \frac{1}{\kappa} (k^\mu - \omega u^\mu), \quad (2.22)$$

where

$$\begin{aligned}\omega &= k \cdot u, \\ \kappa &= \sqrt{\omega^2 - k^2}\end{aligned}\quad (2.23)$$

and

$$V^\mu = V^{(u)}u^\mu + V^{(t)}t^\mu. \quad (2.24)$$

In what follows, we adopt the conventions defined by Eqs. (2.1), (2.2), and (2.21) throughout.

Our job here is to find out what is the Hamiltonian for the evolution equation of the flavor amplitudes. We do it in the following steps:

- (1) We write the ψ field in Eq. (2.3) schematically as the product $u\xi$, where u is a Dirac spinor and ξ is a flavor spinor.
- (2) From Eq. (2.3), we obtain an equation for ξ and the dispersion relation, leaving the Dirac matrix structure behind.
- (3) We then express the equation for ξ and the dispersion relation in the form

$$H\xi = \omega\xi, \quad (2.25)$$

which will identify the Hamiltonian.

The details follow.

As in the case of one generation, Eq. (2.3) has positive and negative frequency solutions. To distinguish them, we use the superscripts $\lambda = \pm$ on the relevant quantities. We introduce the positive ($u_L^{(+)}(\vec{k})$) and negative ($u_L^{(-)}(\vec{k})$) frequency left-handed chiral Dirac spinor satisfying

$$\not{k}u_L^{(\lambda)} = 0, \quad (2.26)$$

with

$$n^{(\lambda)\mu} = (1, \lambda\hat{k}). \quad (2.27)$$

To solve Eq. (2.3), we write the ansatz

$$\psi_L^{(\lambda)} = u_L^{(\lambda)}(\vec{k})\xi^{(\lambda)}(\vec{k}), \quad (2.28)$$

where $\xi^{(\lambda)}$ is a flavor spinor, representing the amplitude in flavor space. Equation (2.26) implies that

$$\begin{aligned}\not{k}u_L^{(\lambda)} &= (\omega - \lambda\kappa)\not{u}u_L^{(\lambda)}, \\ \not{V}Lu_L^{(\lambda)} &= (V^{(u)} - \lambda V^{(t)})\not{u}u_L^{(\lambda)}.\end{aligned}\quad (2.29)$$

Substituting Eq. (2.28) in Eq. (2.3) and using Eq. (2.29), we get the following equation for the flavor spinor:

$$[(\omega - \lambda\kappa) - n^{(\lambda)} \cdot V(\omega, \vec{k})]\xi^{(\lambda)} = 0, \quad (2.30)$$

where we have used the fact that

$$n^{(\lambda)} \cdot V = V^{(u)} - \lambda V^{(t)}, \quad (2.31)$$

and we have indicated explicitly that, in general, $V^{(u)}$, $V^{(t)}$ are functions of ω and $\vec{k}$ [24].

Equation (2.30) is an implicit equation that in principle determines the dispersion relations for $\omega^{(\lambda)}(\vec{k})$ for each mode. The next step is to linearize the equation, by substituting the zeroth order solution, $\omega = \lambda\kappa$ in $V^{(u)}$, $V^{(t)}$. Thus, Eq. (2.30) becomes

$$H^{(\lambda)}(\vec{k})\xi^{(\lambda)}(\vec{k}) = \omega\xi^{(\lambda)}(\vec{k}), \quad (2.32)$$

where

$$H^{(\lambda)}(\vec{k}) = \lambda\kappa + n^{(\lambda)} \cdot V(\lambda\kappa, \vec{k}). \quad (2.33)$$

Equation (2.32) determines the positive and negative frequency dispersion relations $\omega^{(\pm)}(\vec{k})$. According to the decomposition in Eq. (2.13), we define

$$H^{(\lambda)}(\vec{k}) = H_r^{(\lambda)}(\vec{k}) - \frac{i}{2}\Gamma^{(\lambda)}(\vec{k}), \quad (2.34)$$

where

$$\begin{aligned}H_r^{(\lambda)}(\vec{k}) &= \lambda\kappa + n^{(\lambda)} \cdot V_r(\lambda\kappa, \vec{k}), \\ -\frac{1}{2}\Gamma^{(\lambda)}(\vec{k}) &= n^{(\lambda)} \cdot V_i(\lambda\kappa, \vec{k}).\end{aligned}\quad (2.35)$$

The neutrino Hamiltonian is identified, as usual, by associating it with the positive frequency solution; that is, we set $\xi^{(\nu)}(\vec{k}) = \xi^{(+)}(\vec{k})$ and $\omega^{(\nu)}(\vec{k}) = \omega^{(+)}(\vec{k})$. For the antineutrino, we look at the equation for $\xi^{(\bar{\nu})}(\vec{k}) \equiv (\xi^{(-)}(-\vec{k}))^*$, with the identification $\omega^{(\bar{\nu})}(\vec{k}) = -(\omega^{(-)}(-\vec{k}))^*$. In this way, the equations are

$$H^{(\nu, \bar{\nu})}(\vec{k})\xi^{(\nu, \bar{\nu})}(\vec{k}) = \omega^{(\nu, \bar{\nu})}(\vec{k})\xi^{(\nu, \bar{\nu})}(\vec{k}), \quad (2.36)$$

with

$$\begin{aligned}H^{(\nu)}(\vec{k}) &= H_r^{(+)}(\vec{k}) - \frac{i}{2}\Gamma_i^{(+)}(\vec{k}), \\ H^{(\bar{\nu})}(\vec{k}) &= -(H_r^{(-)}(-\vec{k}))^* - \frac{i}{2}(\Gamma_i^{(-)}(-\vec{k}))^*,\end{aligned}\quad (2.37)$$

or explicitly

$$\begin{aligned}H^{(\nu)}(\vec{k}) &= \kappa + n \cdot V_r(\kappa, \vec{k}) + in \cdot V_i(\kappa, \vec{k}), \\ H^{(\bar{\nu})}(\vec{k}) &= \kappa - n \cdot V_r^*(-\kappa, -\vec{k}) + in \cdot V_i^*(-\kappa, -\vec{k}),\end{aligned}\quad (2.38)$$

where n^μ has been defined in Eq. (2.17).

In summary, for either neutrinos or antineutrinos, we have the *eigenvalue* equation for ξ ,

$$\left(H_r - i\frac{\Gamma}{2}\right)\xi = \omega\xi, \quad (2.39)$$

with H_r and Γ being Hermitian matrices in flavor space, calculated in terms of the vector V_μ ,

$$H_r = \begin{cases} \kappa + n \cdot V_r(\kappa, \vec{\kappa}) & (\nu) \\ \kappa - n \cdot V_r^*(-\kappa, -\vec{\kappa}) & (\bar{\nu}) \end{cases} \\ -\frac{1}{2}\Gamma = \begin{cases} n \cdot V_i(\kappa, \vec{\kappa}) & (\nu) \\ n \cdot V_i^*(-\kappa, -\vec{\kappa}) & (\bar{\nu}) \end{cases}. \quad (2.40)$$

In coordinate space, this translates to the evolution equation

$$i\partial_t \xi(t) = \left(H_r - i\frac{\Gamma}{2}\right)\xi(t). \quad (2.41)$$

III. NONFORWARD SCATTERING TERMS

In the case that several neutrino flavors couple to $f\phi$, as indicated in Eq. (1.1), the damping matrix Γ receives another contribution, from the diagrams depicted in Fig. 2. From a physical point of view, these diagrams correspond to contributions to the damping matrix Γ due to the various neutrino nonforward scattering processes that can occur in the presence of the background particles $f\phi$, schematically of the form $\nu_a + x \leftrightarrow \nu_b + x$, where $x = f, \phi$ (and similar ones with f and/or ϕ crossed). This contrasts with the processes involved in the diagrams of Fig. 1, which are associated with decay-type process like $\nu_a + \phi \leftrightarrow f$ and related ones. To distinguish the two types of contributions

FIG. 2. Two-loop diagram for the neutrino thermal self-energy matrix element Σ_{12} in an $f\phi$ background. In principle, we have to consider the various thermal vertices $A = 1, 2$ and $B = 1, 2$. However, in the heavy ϕ limit, only the diagonal components of the ϕ thermal propagator are nonzero and therefore only one diagram, with $A = 1$ and $B = 2$, must be considered. In the labels referring to the various neutrino families, we use the indices a, b running over the neutrino flavors and i, j, k running over the neutrino modes with a definite dispersion relation in the medium. For simplicity of notation, we have labeled $k' = p - p' + k$.

to Γ , we denote by $\Gamma^{(1,2)}$ the contribution of the one-loop diagram (Fig. 1) and the two-loop diagram (Fig. 2), respectively. Our main observation is that $\Gamma^{(2)}$ has a structure that lends itself to a formulation as decoherence terms that in turn allows us to go beyond the evolution equation (2.41) to consider its effects. However, before going in that route, we calculate explicitly $\Gamma^{(2)}$ by direct evaluation of the diagrams in Fig. 2.

A. Calculation of Σ_{12} from Fig. 2

From Fig. 2, taking into account the sign difference between type 1 and type 2 vertices,

$$\begin{aligned} -i(\Sigma_{12}(k))_{ba} &= (-ig_a)(ig_b^*) \int \frac{d^4 p'}{(2\pi)^4} \sum_{A,B} i\Delta_{2B}^{(\phi)}(p' - k) \\ &\quad \times i\Delta_{A1}^{(\phi)}(p' - k) RiS_{12}^{(f)}(p')L \\ &\quad \times \sum_{cd} (ig_c)(-ig_d^*)(-1) \int \frac{d^4 p}{(2\pi)^4} \\ &\quad \times \text{Tr}(RiS_{BA}^{(f)}(p)L(iS_{AB}^{(\nu)}(k'))_{cd}), \end{aligned} \quad (3.1)$$

where

$$k' = p - p' + k. \quad (3.2)$$

Recall that $(\Sigma_i)_{ba}$ is given by Eq. (2.8) and then $(\Gamma^{(2)})_{ba}$ is obtained from Eq. (2.40) with $(V_i^\mu)_{ba}$ identified according to Eq. (2.12).

We will assume that m_ϕ is larger than both the background temperature and the incoming neutrino energy so that we can work in the heavy ϕ limit. In this case, only the diagonal elements of the thermal ϕ propagator are nonzero, and therefore in the vertices in Fig. 2 only the case $A = 1, B = 2$ has to be considered. Then using $\Delta_{22}^{(\phi)} = -\Delta_{11}^{(\phi)} = 1/m_\phi^2$,

$$\begin{aligned} -i(\Sigma_{12}(k))_{ba} &= -\frac{g_a g_b^*}{m_\phi^4} \sum_{cd} g_c g_d^* \int \frac{d^4 p'}{(2\pi)^4} RiS_{12}^{(f)}(p')L \\ &\quad \times \int \frac{d^4 p}{(2\pi)^4} \text{Tr}(RiS_{21}^{(f)}(p)L(iS_{12}^{(\nu)}(k'))_{cd}), \end{aligned} \quad (3.3)$$

which corresponds to the collapsed diagram shown in Fig. 3. The components of the propagator matrices are given by

$$\begin{aligned} S_{21}^{(f)}(p) &= 2\pi i \delta(p^2 - m_f^2) [\eta_F(p, \alpha_f) - \theta(p \cdot u)] \sigma_f(p), \\ S_{12}^{(f)}(p') &= 2\pi i \delta(p'^2 - m_f^2) [\eta_F(p', \alpha_f) - \theta(-p' \cdot u)] \sigma_f(p'), \end{aligned} \quad (3.4)$$

where

FIG. 3. Collapsed version of the two-loop diagram shown in Fig. 2 in the heavy ϕ limit. The various labels and indices are the same as those in Fig. 2.

$$\sigma_f(q) = \not{q} + m_f, \quad (3.5)$$

while

$$\begin{aligned} \eta_F(p, \alpha_f) &= \theta(p \cdot u) n_F(x_f) + \theta(-p \cdot u) n_F(-x_f), \\ \eta_F(p', \alpha_f) &= \theta(p' \cdot u) n_F(x'_f) + \theta(-p' \cdot u) n_F(-x'_f), \end{aligned} \quad (3.6)$$

with n_F defined in Eq. (2.9) and θ is the step function. We have defined

$$\begin{aligned} x_f &= \beta p \cdot u - \alpha_f, \\ x'_f &= \beta p' \cdot u - \alpha_f. \end{aligned} \quad (3.7)$$

For our purposes, it will be more convenient to use the identity

$$n_F(-x) = 1 - n_F(x) = e^x n_F(x) \quad (3.8)$$

and express $S_{21}^{(f)}(p)$ and $S_{12}^{(f)}(p')$ in the form

$$\begin{aligned} S_{21}^{(f)}(p) &= -2\pi i \delta(p^2 - m_f^2) \sigma_f(p) e^{x_f} n_F(x_f) \epsilon(p \cdot u), \\ S_{12}^{(f)}(p') &= 2\pi i \delta(p'^2 - m_f^2) \sigma_f(p') n_F(x'_f) \epsilon(p' \cdot u), \end{aligned} \quad (3.9)$$

where $\epsilon(z) = \theta(z) - \theta(-z)$.

We now consider the propagator to use for the internal neutrino line. In principle, we would use the formulas appropriate for the propagating neutrino including the background effects, taking into account the relationship between the neutrino flavor states and the mode states with definite dispersion relation. However, we adopt the perturbative approach and neglect the effect of the nonzero neutrino masses and/or dispersion relations in the calculation of Σ_{12} . In this case, the neutrino propagator $S_{AB}^{(\nu)}(k')$ is diagonal in flavor space, with all the elements actually being the same since all the neutrinos have the same mass (zero) and the same chemical potential. Specifically,

$$(S_{12}^{(\nu)}(k'))_{cd} = 2\pi i \delta(k'^2) \sigma_\nu(k') n_F(x'_\nu) \epsilon(k' \cdot u) \delta_{cd}, \quad (3.10)$$

where

$$x'_\nu = \beta k' \cdot u - \alpha_\nu \quad (3.11)$$

and

$$\sigma^{(\nu)}(k') = L \not{k}'. \quad (3.12)$$

We now work with the product of the fermion propagators in Eq. (3.3) as follows. Using the relation

$$x_\nu + x_f = x'_\nu + x'_f, \quad (3.13)$$

which follows from Eq. (3.2), the following identity is readily derived (see Appendix A):

$$\frac{1}{n_F(x_\nu)} e^{x_f} n_F(x_f) n_F(x'_f) n_F(x'_\nu) = E, \quad (3.14)$$

where

$$E \equiv n_F(x_f)(1 - n_F(x'_f)) - n_F(x'_\nu)(n_F(x_f) - n_F(x'_f)). \quad (3.15)$$

Substituting Eqs. (3.9) and (3.10) in Eq. (3.3), and using Eq. (3.14), from Eq. (2.8), we then obtain

$$\begin{aligned} (\Sigma_i(k))_{ba} &= -K_{ba} \int \frac{d^4 p'}{(2\pi)^3} \int \frac{d^4 p}{(2\pi)^3} (2\pi) \delta(p'^2 - m_f^2) \\ &\quad \times \delta(p^2 - m_f^2) \delta(k'^2) \epsilon(p \cdot u) \epsilon(p' \cdot u) \epsilon(k' \cdot u) \\ &\quad \times R \sigma_f(p') L \text{Tr}\{R \sigma_f(p) L \sigma_\nu(k')\} E, \end{aligned} \quad (3.16)$$

where

$$K_{ba} = \frac{g_a g_b^*}{2m_\phi^4} \left(\sum_c |g_c|^2 \right). \quad (3.17)$$

We now let k' be an arbitrary variable, but insert the factor $\delta^{(4)}(k' + p' - p - k)$ and integrate over k' . Thus,

$$\begin{aligned} (\Sigma_i(k))_{ba} &= -K_{ba} \int \frac{d^4 p'}{(2\pi)^3} \frac{d^4 p}{(2\pi)^3} \frac{d^4 k'}{(2\pi)^3} \delta(p'^2 - m_f^2) \\ &\quad \times \delta(p^2 - m_f^2) \delta(k'^2) \epsilon(p \cdot u) \epsilon(p' \cdot u) \epsilon(k' \cdot u) \\ &\quad \times (2\pi)^4 \delta^{(4)}(k' + p' - p - k) R \sigma_f(p') \\ &\quad \times L \text{Tr}\{R \sigma_f(p) L \sigma_\nu(k')\} E. \end{aligned} \quad (3.18)$$

The next step is to carry out the integrals over p^0, p'^0, k'^0 . We do the integral over k'^0 first. With the help of the delta function, we obtain

TABLE I. Correspondence between the $E_{\nu,\lambda\lambda'}$ and $E_{\bar{\nu},\lambda\lambda'}$ factors defined in Eq. (3.20), and the process that contributes to the $\nu(k)$ damping via Eq. (3.26). To simplify the notation, we are using the shorthands shown in Eq. (3.29) for the various distribution functions.

$E_{\nu,++} = f(1-f') - f'_\nu(f-f')$	$\nu_{a,b}(k) + f(p) \leftrightarrow \nu_i(k') + f(p')$
$E_{\nu,-+} = (1-\bar{f})(1-f') - f'_\nu(1-\bar{f}-f')$	$\nu_{a,b}(k) \leftrightarrow \nu_i(k') + \bar{f}(p) + f(p')$
$E_{\nu,+ -} = f\bar{f}' - f'_\nu(f + \bar{f}' - 1)$	$\nu_{a,b}(k) + f(p) + \bar{f}(p') \leftrightarrow \nu_i(k')$
$E_{\nu,--} = (1-\bar{f})\bar{f}' - f'_\nu(\bar{f}' - \bar{f})$	$\nu_{a,b}(k) + \bar{f}(p') \leftrightarrow \nu_i(k') + \bar{f}(p)$
$E_{\bar{\nu},++} = (1-f)f' + \bar{f}'_\nu(f-f')$	$\nu_{a,b}(k) + \bar{\nu}_i(\bar{k}') + f(p) \leftrightarrow f(p')$
$E_{\bar{\nu},-+} = \bar{f}f' + \bar{f}'_\nu(1-\bar{f}-f')$	$\nu_{a,b}(k) + \bar{\nu}_i(\bar{k}') \leftrightarrow \bar{f}(p) + f(p')$
$E_{\bar{\nu},+ -} = (1-f)(1-\bar{f}') + \bar{f}'_\nu(f + \bar{f}' - 1)$	$\nu_{a,b}(k) + \bar{\nu}_i(\bar{k}') + f(p) + \bar{f}(p') \leftrightarrow 0$
$E_{\bar{\nu},--} = \bar{f}(1-\bar{f}') + \bar{f}'_\nu(\bar{f}' - \bar{f})$	$\nu_{a,b}(k) + \bar{\nu}_i(\bar{k}') + \bar{f}(p') \leftrightarrow \bar{f}(p)$

$$\begin{aligned}
(\Sigma_i(k))_{ba} = & -K_{ba} \int \frac{d^4 p'}{(2\pi)^3} \frac{d^4 p}{(2\pi)^3} \frac{d^3 \kappa'}{(2\pi)^3 2\omega_{\kappa'}} \delta(p'^2 - m_f^2) \delta(p^2 - m_f^2) \epsilon(p \cdot u) \epsilon(p' \cdot u) \\
& \times (2\pi)^4 \{ \delta^{(4)}(k + p - k' - p') R\sigma_f(p') L\text{Tr}\{R\sigma_f(p) L\sigma_\nu(k')\} E_\nu \\
& - \delta^{(4)}(k + p + k' - p') R\sigma_f(p') L\text{Tr}\{R\sigma_f(p) L\sigma_\nu(-k')\} E_{\bar{\nu}} \},
\end{aligned} \tag{3.19}$$

where

$$\begin{aligned}
E_\nu &= n_F(x_f)(1 - n_F(x'_f)) - f_\nu(\omega_{\kappa'}) (n_F(x_f) - n_F(x'_f)), \\
E_{\bar{\nu}} &= n_F(x'_f)(1 - n_F(x_f)) + f_{\bar{\nu}}(\omega_{\kappa'}) (n_F(x_f) - n_F(x'_f)),
\end{aligned} \tag{3.20}$$

with

$$k'^\mu = (\omega_{\kappa'}, \vec{k}'), \quad \omega_{\kappa'} = |\vec{k}'|. \tag{3.21}$$

To arrive at Eq. (3.19), we have changed variables $\vec{k} \rightarrow -\vec{k}$ in the term with the $\sigma_\nu(-k')$ factor, and we have defined

$$E_\nu \equiv E|_{\omega'=\omega_{\kappa'}}, \quad E_{\bar{\nu}} \equiv E|_{\omega'=-\omega_{\kappa'}}, \tag{3.22}$$

with E defined in Eq. (3.15). Using the relations

$$\begin{aligned}
n_F(x'_f)|_{\omega'=\omega_{\kappa'}} &= f_\nu(\omega_{\kappa'}), \\
n_F(x'_f)|_{\omega'=-\omega_{\kappa'}} &= 1 - f_{\bar{\nu}}(\omega_{\kappa'}),
\end{aligned} \tag{3.23}$$

$E_{\nu,\bar{\nu}}$ reduce to the formulas given in Eq. (3.20).

Next we carry out the integration over p^0, p'^0 in a similar way. In analogy with Eq. (3.23), we will use

$$\begin{aligned}
n_F(x_f)|_{p^0=E_p} &= f_f(E_p), \\
n_F(x_f)|_{p^0=-E_p} &= 1 - f_{\bar{f}}(E_p),
\end{aligned} \tag{3.24}$$

with

$$E_p = \sqrt{\vec{p}^2 + m_f^2} \tag{3.25}$$

and the corresponding formulas for p' . Thus, from Eq. (3.19), we obtain

$$\begin{aligned}
(\Sigma_i(k))_{ba} = & -K_{ba} \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 \kappa'}{(2\pi)^3 2\omega_{\kappa'}} \\
& \times (2\pi)^4 \{ \delta^{(4)}(k + p - k' - p') R\sigma_f(p') L\text{Tr}\{R\sigma_f(p) L\sigma_\nu(k')\} E_{\nu,++} \\
& - \delta^{(4)}(k - p - k' - p') R\sigma_f(p') L\text{Tr}\{R\sigma_f(-p) L\sigma_\nu(k')\} E_{\nu,-+} \\
& - \delta^{(4)}(k + p + p' - k') R\sigma_f(-p') L\text{Tr}\{R\sigma_f(p) L\sigma_\nu(k')\} E_{\nu,+ -} \\
& + \delta^{(4)}(k + p' - k' - p) R\sigma_f(-p') L\text{Tr}\{R\sigma_f(-p) L\sigma_\nu(k')\} E_{\nu,--} \\
& - \delta^{(4)}(k + p + k' - p') R\sigma_f(p') L\text{Tr}\{R\sigma_f(p) L\sigma_\nu(-k')\} E_{\bar{\nu},++} \\
& + \delta^{(4)}(k + k' - p' - p) R\sigma_f(p') L\text{Tr}\{R\sigma_f(-p) L\sigma_\nu(-k')\} E_{\bar{\nu},-+} \\
& + \delta^{(4)}(k + p + p' + k') R\sigma_f(-p') L\text{Tr}\{R\sigma_f(p) L\sigma_\nu(-k')\} E_{\bar{\nu},+ -} \\
& - \delta^{(4)}(k + p' + k' - p) R\sigma_f(-p') L\text{Tr}\{R\sigma_f(-p) L\sigma_\nu(-k')\} E_{\bar{\nu},--} \},
\end{aligned} \tag{3.26}$$

with

$$p^\mu = (E_p, \vec{p}), \quad (3.27)$$

and similarly for p'^μ . In Eq. (3.26), we introduced the factors $E_{\nu,\lambda\lambda'}$ and $E_{\bar{\nu},\lambda\lambda'}$ (with λ, λ' being $\pm$), which are defined as follows:

$$E_{\nu,\lambda\lambda'} = E_\nu|_{p^0=\lambda E_p, p'^0=\lambda' E_{p'}} \quad (3.28)$$

and similarly for $E_{\bar{\nu},\lambda\lambda'}$. Using Eq. (3.24) and the corresponding formulas for $n_F(x'_f)$ in Eq. (3.20), the explicit formulas are given in Table I. To simplify the notation in the formulas summarized in Table I, we have introduced the shorthands

$$\begin{aligned} f &= f_f(E_p), & f' &= f_f(E_{p'}), & f'_\nu &= f_\nu(\omega_{k'}) \\ \bar{f} &= f_{\bar{f}}(E_p), & \bar{f}' &= f_{\bar{f}}(E_{p'}), & \bar{f}'_\nu &= f_{\bar{\nu}}(\omega_{k'}). \end{aligned} \quad (3.29)$$

The formulas for $E_{\bar{\nu},\lambda\lambda'}$ are obtained from those for $E_{\nu,\lambda\lambda'}$ by making the replacement $f'_\nu \rightarrow (1 - \bar{f}'_\nu)$.

Using the relations

$$\begin{aligned} \sigma_f(p) &= \sum_s u_f(\vec{p}, s) \bar{u}_f(\vec{p}, s), \\ \sigma_f(-p) &= -\sum_s v_f(\vec{p}, s) \bar{v}_f(\vec{p}, s), \end{aligned} \quad (3.30)$$

and the analogous relations for $\sigma_\nu(k')$ makes it evident that the matrix element

$$\bar{u}_{\nu_b}(\vec{k})(\Sigma_i(k))_{ba} u_{\nu_a}(\vec{k}) \quad (3.31)$$

can be expressed as a sum of terms of the form

$$\begin{aligned} M(\nu_a(k) + f(p) \rightarrow \nu_i(k') + f(p')) M^*(\nu_b(k) \\ + f(p) \rightarrow \nu_i(k') + f(p')), \end{aligned} \quad (3.32)$$

involving the amplitudes for the processes

$$\nu_{a,b}(k) + f(p) \leftrightarrow \nu_i(k') + f(p'), \quad (3.33)$$

as well as the processes obtained by crossing $f(p), f(p'), \nu_i(k')$. Each term that appears within the brackets in Eq. (3.26) corresponds to one such process, and it is

inverse. The factors of $E_{\nu,\lambda\lambda'}, E_{\bar{\nu},\lambda\lambda'}$ incorporate the statistical effects of the background. As is well known [25], for Fermi systems, the inverse reactions are inhibited as a consequence of the Pauli blocking effect, and they contribute additively to the depletion of the state. The formulas given in Table I reflect the fact that the contributions from the direct and the inverse processes are given by the sum of their rates instead of their difference as in the bosonic case. For example, $E_{\nu,++}$ can be rewritten in the form

$$E_{\nu,++} = f(1 - f')(1 - f'_\nu) + f'_\nu f'(1 - f), \quad (3.34)$$

which is just the sum of the statistical factors for the direct and inverse processes indicated in Eq. (3.33). In a similar fashion, it can be verified that the terms in Eq. (3.26) and the associated $E_{\nu,\lambda\lambda'}$ and $E_{\bar{\nu},\lambda\lambda'}$ can be identified with the various processes as indicated in Table I. Similar identifications can be made for the antineutrino matrix element

$$\bar{v}_{\nu_b}(\vec{k})(\Sigma_i(-k))_{ba} v_{\nu_a}(\vec{k}). \quad (3.35)$$

For some conditions, some of these processes will be kinematically forbidden and will not contribute. We now assume that the conditions are such that, for $\omega > 0$, the only processes that are kinematically accessible are the one shown above, and the following one:

$$\nu_{a,b}(k) + \bar{f}(p') \rightarrow \nu_i(k') + \bar{f}(p). \quad (3.36)$$

These correspond to the first and the fourth terms, respectively, in the list of terms that appear within the bracket in Eq. (3.26). Alternatively, for $\omega < 0$, the only kinematically accessible processes are

$$\begin{aligned} \bar{\nu}_{a,b}(k) + f(p') &\rightarrow \bar{\nu}_i(k') + f(p), \\ \bar{\nu}_{a,b}(k) + \bar{f}(p) &\rightarrow \bar{\nu}_i(k') + \bar{f}(p'), \end{aligned} \quad (3.37)$$

which correspond to the fifth and eighth terms within the bracket in Eq. (3.26). In addition, we will assume that there no neutrinos or antineutrinos in the background; therefore, we set f_ν and $f_{\bar{\nu}}$ to zero. Then,

$$\begin{aligned} (\Sigma_i(k))_{ba} &= -K_{ba} \int \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 k'}{(2\pi)^3 2\omega_{k'}} \\ &\times (2\pi)^4 \{ \delta^{(4)}(k + p - k' - p') R\sigma_f(p') L\text{Tr}\{R\sigma_f(p) L\sigma_\nu(k')\} [f_f(E_p)(1 - f_f(E_{p'}))] \\ &+ \delta^{(4)}(k + p' - k' - p) R\sigma(-p') L\text{Tr}\{R\sigma(-p) L\sigma_\nu(k')\} [(1 - f_{\bar{f}}(E_p)) f_{\bar{f}}(E_{p'})] \} \\ &- \delta^{(4)}(k + p + k' - p') R\sigma_f(p') L\text{Tr}\{R\sigma_f(p) L\sigma_\nu(-k')\} [(1 - f_f(E_p)) f_f(E_{p'})] \\ &- \delta^{(4)}(k + p' + k' - p) R\sigma_f(-p') L\text{Tr}\{R\sigma_f(-p) L\sigma_\nu(-k')\} [f_{\bar{f}}(E_p)(1 - f_{\bar{f}}(E_{p'}))] \}, \end{aligned} \quad (3.38)$$

where, as we have mentioned, if $\omega > 0$ only the first two terms in the bracket contribute, while for $\omega < 0$ only the last two contribute.

Remembering Eq. (2.11), we identify $(V_i^\mu(\omega, \vec{\kappa}))_{ba}$ by writing

$$(\Sigma_i(k))_{ba} = (V_i^\mu(\omega, \vec{\kappa}))_{ba} \gamma_\mu L. \quad (3.39)$$

B. Formula for V_i^μ

We now express V_i^μ as follows. Using Eqs. (3.5) and (3.12),

$$\text{Tr}\{R\sigma_f(p)L\sigma_\nu(k')\} = -\text{Tr}\{R\sigma_f(p)L\sigma_\nu(-k')\} = 2p \cdot k', \quad (3.40)$$

and we have

$$(V_i^\mu)_{ba} = (V_i^{(+)\mu})_{ba} + (V_i^{(-)\mu})_{ba}, \quad (3.41)$$

where

$$\begin{aligned} (V_i^{(+)\mu})_{ba} &= -2K_{ba} \int \frac{d^3\kappa'}{(2\pi)^3 2\omega_{\kappa'}} I^{(+)\mu\nu}(k - k') k'_\nu \\ (V_i^{(-)\mu})_{ba} &= -2K_{ba} \int \frac{d^3\kappa'}{(2\pi)^3 2\omega_{\kappa'}} I^{(-)\mu\nu}(k + k') k'_\nu, \end{aligned} \quad (3.42)$$

with

$$\begin{aligned} I_{\mu\nu}^{(+)}(q) &= \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{d^3p}{(2\pi)^3 2E_p} \\ &\quad \times (2\pi)^4 \{ \delta^{(4)}(p + q - p') f_f(E_p) (1 - f_f(E_{p'})) \\ &\quad + \delta^{(4)}(p' + q - p) (1 - f_{\bar{f}}(E_p)) f_{\bar{f}}(E_{p'}) \} p'_\mu p_\nu, \\ I_{\mu\nu}^{(-)}(q) &= \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \frac{d^3p}{(2\pi)^3 2E_p} \\ &\quad \times (2\pi)^4 \{ \delta^{(4)}(p + q - p') (1 - f_f(E_p)) f_f(E_{p'}) \\ &\quad + \delta^{(4)}(p' + q - p) f_{\bar{f}}(E_p) (1 - f_{\bar{f}}(E_{p'})) \} p'_\mu p_\nu. \end{aligned} \quad (3.43)$$

It is useful to note that

$$I_{\mu\nu}^{(-)}(q) = I_{\mu\nu}^{(+)}(-q). \quad (3.44)$$

The remark below Eq. (3.38) is equivalent to say that $V_i^{(-)\mu}$ or $V_i^{(+)\mu}$ is zero if $\omega > 0$ or $\omega < 0$, respectively, that is

$$\begin{aligned} V_i^{(-)\mu} &= 0 \quad \text{if } \omega > 0, \\ V_i^{(+)\mu} &= 0 \quad \text{if } \omega < 0. \end{aligned} \quad (3.45)$$

Thus, finally, putting

$$k'^\mu = \omega_{\kappa'} n'^\mu, \quad (3.46)$$

with

$$n'^\mu = (1, \hat{\kappa}'), \quad (3.47)$$

we have

$$\begin{aligned} (V_i^{(+)\mu}(\omega, \vec{\kappa}))_{ba} &= -\frac{g_a g_b^*}{2m_\phi^4} \left(\sum_c |g_c|^2 \right) \\ &\quad \times \int \frac{d^3\kappa'}{(2\pi)^3} I^{(+)\mu\nu}(k - k') n'_\nu, \\ (V_i^{(-)\mu}(\omega, \vec{\kappa}))_{ba} &= -\frac{g_a g_b^*}{2m_\phi^4} \left(\sum_c |g_c|^2 \right) \\ &\quad \times \int \frac{d^3\kappa'}{(2\pi)^3} I^{(-)\mu\nu}(k + k') n'_\nu, \end{aligned} \quad (3.48)$$

where we have substituted the explicit expression for K_{ba} defined in Eq. (3.17).

C. Formula for $\Gamma^{(2)}$

From Eq. (2.40), remembering Eq. (3.45),

$$-\frac{1}{2}\Gamma^{(2)} = \begin{cases} n \cdot V_i^{(+)}(\kappa, \vec{\kappa}) & (\nu) \\ n \cdot V_i^{(-)*}(-\kappa, -\vec{\kappa}) & (\bar{\nu}) \end{cases}. \quad (3.49)$$

Denoting by $\Gamma^{(\nu)}$, $\Gamma^{(\bar{\nu})}$ the matrix for neutrinos or antineutrinos, respectively, explicitly using Eq. (3.48),

$$\begin{aligned} \Gamma^{(\nu)} &= g_a g_b^* \left(\sum_c |g_c|^2 \right) \gamma^{(\nu)}, \\ \Gamma^{(\bar{\nu})} &= g_a^* g_b \left(\sum_c |g_c|^2 \right) \gamma^{(\bar{\nu})}, \end{aligned} \quad (3.50)$$

with

$$\begin{aligned} \gamma^{(\nu)} &= \frac{1}{m_\phi^4} n_\mu \int \frac{d^3\kappa'}{(2\pi)^3} I^{(+)\mu\nu}(k - k') n'_\nu, \\ \gamma^{(\bar{\nu})} &= \frac{1}{m_\phi^4} n_\mu \int \frac{d^3\kappa'}{(2\pi)^3} I^{(+)\nu\mu}(k - k') n'_\nu, \end{aligned} \quad (3.51)$$

where we have used Eq. (3.44).

The integral expressions in Eq. (3.51) can be simplified as follows. The relevant integrals for neutrinos or antineutrinos are

$$\begin{aligned} n_\mu \int \frac{d^3\kappa'}{(2\pi)^3} I^{(+)\mu\nu}(k - k') n'_\nu &= I_1^{(f)} + I_2^{(\bar{f})}, \\ n_\mu \int \frac{d^3\kappa'}{(2\pi)^3} I^{(+)\nu\mu}(k - k') n'_\nu &= I_2^{(f)} + I_1^{(\bar{f})}, \end{aligned} \quad (3.52)$$

respectively, where, for $x = f, \bar{f}$, we define

$$\begin{aligned}
 I_1^{(x)} &= \int \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 \kappa'}{(2\pi)^3} (2\pi)^4 \delta^{(4)}(p + k - p' - k') f_x(E_p) (1 - f_x(E_{p'})) (p \cdot n') (p' \cdot n), \\
 &= \frac{2}{\omega_\kappa} \int \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 \kappa'}{(2\pi)^3 2\omega_{\kappa'}} (2\pi)^4 \delta^{(4)}(p + k - p' - k') f_x(E_p) (1 - f_x(E_{p'})) (p \cdot k') (p' \cdot k), \\
 I_2^{(x)} &= \int \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 \kappa'}{(2\pi)^3} (2\pi)^4 \delta^{(4)}(p' + k - p - k') (1 - f_x(E_p)) f_x(E_{p'}) (p \cdot n') (p' \cdot n) \\
 &= \frac{2}{\omega_\kappa} \int \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 \kappa'}{(2\pi)^3 2\omega_{\kappa'}} (2\pi)^4 \delta^{(4)}(p + k - p' - k') f_x(E_p) (1 - f_x(E_{p'})) (p' \cdot k') (p \cdot k). \quad (3.53)
 \end{aligned}$$

We have used Eq. (3.46) and the analogous relation between k^μ and n^μ , and in the expression for $I_2^{(x)}$ we have relabeled p and p' . For $I_1^{(x)}$, we use the fact that the delta function implies that $p' \cdot k = p \cdot k'$, while for $I_2^{(x)}$ we use $p \cdot k = p' \cdot k'$. Therefore,

$$\begin{aligned}
 I_1^{(x)} &= \frac{2}{\omega_\kappa} \int \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 \kappa'}{(2\pi)^3 2\omega_{\kappa'}} \\
 &\quad \times (2\pi)^4 \delta^{(4)}(p + k - p' - k') f_x(E_p) \\
 &\quad \times (1 - f_x(E_{p'})) (p \cdot k')^2, \\
 I_2^{(x)} &= \frac{2}{\omega_\kappa} \int \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 p'}{(2\pi)^3 2E_{p'}} \frac{d^3 \kappa'}{(2\pi)^3 2\omega_{\kappa'}} \\
 &\quad \times (2\pi)^4 \delta^{(4)}(p + k - p' - k') f_x(E_p) \\
 &\quad \times (1 - f_x(E_{p'})) (p \cdot k)^2. \quad (3.54)
 \end{aligned}$$

From Eqs. (3.51) and (3.52), we then have

$$\begin{aligned}
 \gamma^{(\nu)} &= \frac{1}{m_\phi^4} (I_1^{(f)} + I_2^{(\bar{f})}), \\
 \gamma^{(\bar{\nu})} &= \frac{1}{m_\phi^4} (I_2^{(f)} + I_1^{(\bar{f})}). \quad (3.55)
 \end{aligned}$$

IV. NONFORWARD SCATTERING AS A DECOHERENCE EFFECT

Our main observation here is that $\Gamma^{(2)}$, given in Eq. (3.50) by direct evaluation of the diagrams in Fig. 2, has a structure that lends itself to a formulation as decoherence terms in the context of the Linblad equation, and the notion of the stochastic evolution of the state vector [18–22]. Thus, we will assume that, for kinematic reasons, $\Gamma^{(1)}$ is zero and that $\Gamma^{(2)}$ is the only contribution to the damping matrix. The idea then is to assume that the evolution due to the damping effects described by $\Gamma^{(2)}$ is accompanied by a stochastic evolution that cannot be described by the coherent evolution of the state vector.

Let us then consider the evolution of the state vector (using a generic notation)

$$i\partial_t \phi(t) = H\phi(t), \quad (4.1)$$

with

$$H = H_r - \frac{i}{2}\Gamma. \quad (4.2)$$

In an interval dt , the state vector would have evolved coherently to

$$\phi_1(t + dt) \equiv (1 - iHdt)\phi. \quad (4.3)$$

The norm of this vector is

$$\phi_1^\dagger(t + dt)\phi_1(t + dt) = 1 - p, \quad (4.4)$$

where

$$p \equiv \phi^\dagger \Gamma \phi dt. \quad (4.5)$$

Thus, we interpret p as the probability that the system has decayed ($1 - p$ is the survival probability) due to the coherent (but non-Hermitian) evolution. We now assume that this coherent evolution is accompanied by stochastic processes that cause the system to jump from the initial state to a set of possible states, thus causing the damping.

To define the construction, suppose specifically that Γ has the form

$$\Gamma = \sum_i L_i^\dagger L_i. \quad (4.6)$$

In other words, suppose that we can find a set of matrices L_i such that Γ can be written in this form. As we will verify later on, this is indeed the case for $\Gamma^{(2)}$. Let us call

$$p_i \equiv \phi^\dagger L_i^\dagger L_i \phi. \quad (4.7)$$

Therefore, from Eq. (4.5),

$$p = dt \sum_i p_i. \quad (4.8)$$

Now we want to say that the stochastic processes cause the state vector to jump to any of the (normalized) state vectors

$$\phi^{(i)} \equiv \frac{L_i \phi}{\sqrt{\phi^\dagger L_i^\dagger L_i \phi}} = \frac{L_i \phi}{\sqrt{p_i}}, \quad (4.9)$$

with a probability π_i . Of course, the condition is that

$$\sum_i \pi_i = p, \quad (4.10)$$

which we satisfy by assuming that

$$\pi_i = p_i dt. \quad (4.11)$$

The main assumption is that the evolution of the system, taking into account both the coherent and stochastic evolution, is described by the density matrix (in the sense that we can use it to calculate averages of quantum expectation values)

$$\begin{aligned} \rho(t + dt) &= \phi_1 \phi_1^\dagger + \sum_i \pi_i \phi^{(i)} \phi^{(i)\dagger} \\ &= (1 - iHdt)\rho(1 + iH^\dagger dt) + dt \sum_i L_i \rho L_i^\dagger \\ &= \rho(t) - i(H\rho - \rho H^\dagger)dt + dt \sum_i L_i \rho L_i^\dagger \\ &= \rho(t) - i[H_r, \rho]dt - \frac{1}{2}\{\Gamma, \rho\}dt + dt \sum_i L_i \rho L_i^\dagger \end{aligned} \quad (4.12)$$

or

$$\partial_t \rho = -i[H_r, \rho] + \sum_i \left\{ L_i \rho L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho - \frac{1}{2} \rho L_i^\dagger L_i \right\}, \quad (4.13)$$

which is the Linblad equation [26].

From Eq. (3.50), it is immediately evident that $\Gamma^{(2)}$ has the form given in Eq. (4.6). Indeed, to be more precise, only one such matrix L is needed, for $\ell = \nu, \bar{\nu}$,

$$\begin{aligned} \Gamma^{(\ell)} &= L^{(\ell)\dagger} L^{(\ell)}, \\ \Gamma_{ba}^{(\ell)} &= \sum_c (L^{(\ell)\dagger})_{bc} L_{ca}^{(\ell)} = \sum_c L_{cb}^{(\ell)*} L_{ca}^{(\ell)}, \end{aligned} \quad (4.14)$$

with the identification

$$\begin{aligned} L_{ca}^{(\nu)} &\equiv \sqrt{\gamma^{(\nu)}} g_c g_a, \\ L_{ca}^{(\bar{\nu})} &\equiv \sqrt{\gamma^{(\bar{\nu})}} g_c^* g_a^*. \end{aligned} \quad (4.15)$$

In summary, we assert that, in the situation that $\Gamma^{(1)}$ is zero (or negligible) so that the damping matrix is given by $\Gamma^{(2)}$, determined from Fig. 2 and which has the form given in Eq. (3.50) (under the approximations and idealizations we have made), then its effects are more effectively taken into account in the context of the evolution equation for the flavor density matrix; in this case,

$$\begin{aligned} \partial_t \rho^{(\ell)} &= -i[H_r^{(\ell)}, \rho^{(\ell)}] + \left\{ L^{(\ell)} \rho^{(\ell)} L^{(\ell)\dagger} - \frac{1}{2} L^{(\ell)\dagger} L^{(\ell)} \rho^{(\ell)} \right. \\ &\quad \left. - \frac{1}{2} \rho^{(\ell)} L^{(\ell)\dagger} L^{(\ell)} \right\} \end{aligned} \quad (4.16)$$

for neutrinos or antineutrinos, with $L^{(\ell)}$ identified in Eq. (4.15). In what follows, we compute the integrals involved in the expressions for $\gamma^{(\nu, \bar{\nu})}$ explicitly for some idealized situations, which nevertheless should serve as starting point to consider more general and/or realistic cases.

V. DISCUSSION

A. Example of calculation of integrals

Equations (3.54) and (3.55) serve as the basis for the calculation of the matrix L using Eq. (4.15) in a number of useful cases. For illustrative purposes and a guide to applications to realistic and/or potentially important situations, here we evaluate explicitly the integrals involved for some specific simple cases of the background conditions.

We assume that $f_x \ll 1$, so that we can set $(1 - f_x(E_{p'})) \rightarrow 1$. Then,

$$\begin{aligned} I_1^{(x)} &= \frac{2}{\omega_\kappa} \left(\frac{1}{2\pi} \right)^5 \int \frac{d^3 p}{2E_p} f_x(E_p) J_1(p, k), \\ I_2^{(x)} &= \frac{2}{\omega_\kappa} \left(\frac{1}{2\pi} \right)^5 \int \frac{d^3 p}{2E_p} f_x(E_p) J_2(p, k), \end{aligned} \quad (5.1)$$

where

$$\begin{aligned} J_1 &= \int \frac{d^3 p'}{2E_{p'}} \frac{d^3 k'}{2\omega_{\kappa'}} \delta^{(4)}(p + k - p' - k') (p \cdot k')^2, \\ J_2 &= \int \frac{d^3 p'}{2E_{p'}} \frac{d^3 k'}{2\omega_{\kappa'}} \delta^{(4)}(p + k - p' - k') (p \cdot k)^2. \end{aligned} \quad (5.2)$$

The evaluation of the integrals $J_{1,2}$ is straightforward, as shown in Appendix B. Here we quote the results for the particular cases of an ultrarelativistic or a nonrelativistic fermion background. Although the idealizations and approximations we have made to arrive at these formulas may limit their applicability to realistic situations, the simplicity of these results can be used as a guide and benchmarks when considering specific applications of practical interest.

1. Ultrarelativistic background

Specifically, we assume that

$$\alpha_f, \quad T, \quad \omega_\kappa \gg m_f. \quad (5.3)$$

As shown in Appendix B, in this case

$$\frac{1}{3} J_2 = J_1 = \frac{\pi}{6} \omega_\kappa^2 p^2 (1 - \cos \theta_p)^2, \quad (5.4)$$

where $p = |\vec{p}|$. Then, from Eq. (5.1),

$$\frac{1}{3}I_2^{(x)} = I_1^{(x)} = \frac{\kappa}{36\pi^3} \int_0^\infty dp p^3 f_x(p), \quad (5.5)$$

remembering that $\omega_\kappa = \kappa$. For a completely degenerate x background ($x = f$ or $\bar{f}$) putting $f_x = \theta(p_{F_x} - p)$, where p_{F_x} is the Fermi momentum,

$$\frac{1}{3}I_2^{(x)} = I_1^{(x)} = \frac{\kappa}{36\pi^3} \frac{p_{F_x}^4}{4} \quad (\text{Fermi gas}). \quad (5.6)$$

The Fermi momentum is given in terms of the number density f_x of the background fermions by $p_{F_x} = (3\pi^2 n_x)^{1/3}$. On the opposite side, for a classical background, putting $f_x = e^{-\beta p}$, where β is the inverse temperature (T), gives

$$\frac{1}{3}I_2^{(x)} = I_1^{(x)} = \frac{\kappa T^4}{6\pi^3} \quad (\text{classical gas}). \quad (5.7)$$

Using the above results in Eq. (3.55), we can consider some specific example situations. For example, for a completely degenerate f gas (and no $\bar{f}$ particles),

$$\frac{1}{3}\gamma^{(\bar{\nu})} = \gamma^{(\nu)} = \frac{\kappa p_{F_f}^4}{144\pi^3 m_\phi^4} \quad (\text{Fermi } f \text{ gas}), \quad (5.8)$$

or, for a completely degenerate $\bar{f}$ gas (and no f particles),

$$\frac{1}{3}\gamma^{(\nu)} = \gamma^{(\bar{\nu})} = \frac{\kappa p_{F_{\bar{f}}}^4}{144\pi^3 m_\phi^4} \quad (\text{Fermi } \bar{f} \text{ gas}), \quad (5.9)$$

while for a classical gas (equal number of f and $\bar{f}$)

$$\gamma^{(\nu)} = \gamma^{(\bar{\nu})} = \frac{2\kappa T^4}{3\pi^3 m_\phi^4}. \quad (5.10)$$

2. Nonrelativistic background

Here we assume that

$$\omega_\kappa \gg m_f \gg T. \quad (5.11)$$

As shown in Appendix B, in this case

$$I_1^{(x)} = \frac{1}{3}I_2^{(x)} = \frac{\kappa m_f n_x}{48\pi}, \quad (5.12)$$

where

$$n_x = 2 \int \frac{d^3 p}{(2\pi)^3} f_x(p) \quad (5.13)$$

is the total number density of f or $\bar{f}$. Thus, from Eq. (3.55),

$$\begin{aligned} \gamma^{(\nu)} &= \frac{1}{m_\phi^4} \left(\frac{\kappa m_f}{48\pi} \right) (n_f + 3n_{\bar{f}}), \\ \gamma^{(\bar{\nu})} &= \frac{1}{m_\phi^4} \left(\frac{\kappa m_f}{48\pi} \right) (3n_f + n_{\bar{f}}). \end{aligned} \quad (5.14)$$

B. Generalizations

As already mentioned in Sec. I, the method we have followed, and the formulas we have obtained for the jump operators, can be applied with minor modifications to other model interactions of potential interest. In particular, they can be applied to study the effects of the nonforward scattering of neutrinos when they propagate through a matter background due to the standard weak interactions of the neutrinos with the electrons and nucleons. For example, consider the contribution from the electron background. In the local limit of the weak interactions, the kinematics of the diagrams involved are similar to those of the heavy ϕ limit that we have assumed. The corresponding jump operators would be given by formulas of the same form as those in Eqs. (3.55) and (4.15), with obvious replacements. That is, $m_\phi \rightarrow m_W$, while the couplings g_a would have the standard weak coupling g as a common factor times another factor that is the same for $\nu_{\mu,\tau}$ but different for ν_e due to the charged-current interaction of the ν_e with the electrons. Similarly, the integrals would involve the background electron number density, but the specific kinematic factors involved must be determined by explicit calculation. The details of the calculation would be similar to those presented above.

In the phenomenological approaches to neutrino decoherence in the references we have cited, the decoherence terms are considered to be unknown phenomenological parameters, and the question of what particular model (e.g., neutrino interactions and mixing) can lead to one or another specific form of the decoherence matrices is not considered. In contrast, for example, in the context of the model we consider (one fermion f and one scalar ϕ), the matrices $L_{ab}^{(\nu,\bar{\nu})}$ would be diagonal of the form $\text{diag}(L_{11}^{(\nu,\bar{\nu})}, 0, 0)$ if only one neutrino couples to f and ϕ so that only one coupling g_a is nonzero (say g_1). In models with more than one f fermion, for example, it would be possible to have more general diagonal L matrices. In any case, in the approach that we have initiated, the decoherence matrices are determined in terms of the parameters of the theory (interactions, mixing, conditions of the environment), and in principle it allows to establish a link between the phenomenological form of the decoherence matrices and the fundamental parameters of the model.

VI. CONCLUSIONS AND OUTLOOK

In this work, we have considered the damping effects in the propagation of neutrinos in a background composed of

a scalar particle and a fermion with an interaction of the form given in Eq. (1.1), due to the nonforward neutrino scattering processes. Specifically, we calculated the contribution to the imaginary part of the neutrino thermal self-energy arising from the nonforward neutrino scattering processes in such backgrounds, from which the damping matrix is determined. Since in this case the initial neutrino state is depleted but does not actually disappear, we have argued that the damping matrix should be associated with decoherence effects. Following this suggestion, we have given a precise prescription to determine the decoherence terms, as used in the context of the master or Linblad equation, in terms of the damping terms we have obtained from the calculation of the nonforward neutrino scattering contribution to the imaginary part of the neutrino self-energy. The main result is a well-defined formula for the jump operators in that context, expressed in terms of integrals over the background fermion distribution functions and the couplings constants of the interaction of the neutrinos with the background particles in the model we consider. The results can be useful in the context of dark matter-neutrino interaction models in which the scalar and/or fermion constitute the dark matter and can also serve to guide the application to other models and/or situations that have been considered recently using the Linblad equation (e.g., [12]) in which the decoherence effects in the propagation of neutrinos may be important. For reference and guidance purposes, we have evaluated the integrals involved explicitly for some conditions of the background. Despite those simplifications, the results illustrate some features that can serve as a guide when considering more general cases or situations not envisioned here.

ACKNOWLEDGMENTS

The work of S.S. is partially supported by DGAPA-UNAM (Mexico) Project No. IN103019.

APPENDIX A: DERIVATION OF EQ. (3.14)

Here we show the details leading to Eq. (3.14). We will use the fact that the x' 's satisfy

$$x'_\nu + x'_f = x_\nu + x_f. \quad (\text{A1})$$

We then have

$$\begin{aligned} X &\equiv \frac{1}{n_F(x_\nu)} e^{x_f} n_F(x_f) n_F(x'_f) n_F(x'_\nu) \\ &= (e^{x_\nu} + 1) e^{x_f} n_F(x_f) n_F(x'_f) n_F(x'_\nu) \\ &= (e^{x_\nu + x_f} + e^{x_f}) n_F(x_f) n_F(x'_f) n_F(x'_\nu) \\ &= (e^{x'_\nu + x'_f} + e^{x_f}) n_F(x_f) n_F(x'_f) n_F(x'_\nu). \end{aligned} \quad (\text{A2})$$

Using

$$e^x = \frac{1}{n_F(x)} - 1, \quad (\text{A3})$$

we now work the first factor,

$$\begin{aligned} e^{x'_\nu + x'_f} + e^{x_f} &= \frac{1}{n_F(x'_\nu) n_F(x'_f)} + \frac{1}{n_F(x_f)} \\ &\quad - \frac{1}{n_F(x'_\nu)} - \frac{1}{n_F(x'_f)}, \end{aligned} \quad (\text{A4})$$

and using this in Eq. (A2), we get Eq. (3.14).

APPENDIX B: CALCULATION OF INTEGRALS $J_{1,2}$ IN EQ. (5.2)

Since $J_{1,2}$ are a scalar integrals, we choose to do the integration in the frame in which $p^\mu = (m_f, \vec{0})$ (the *lab frame*). We label the quantities in that frame with an asterisk, $k^\mu = (\omega_k^*, \vec{k}^*)$ and similarly for k'^μ , and therefore

$$\begin{aligned} J_1 &= \int \frac{d^3 k'^*}{2\omega_{k'}^*} \delta[(p + k - k')^2 - m_f^2] \theta(m_f + \omega_k^* - \omega_{k'}^*) \\ &\quad \times (m_f \omega_{k'}^*)^2 \\ &= \int \frac{d^3 k'^*}{2\omega_{k'}^*} \delta[-2\omega_k^* \omega_{k'}^* (1 - \cos \theta_{k'}^*) + 2m_f(\omega_k^* - \omega_{k'}^*)] \\ &\quad \times \theta(m_f + \omega_k^* - \omega_{k'}^*) (m_f \omega_{k'}^*)^2, \end{aligned} \quad (\text{B1})$$

where $\theta_{k'}^*$ is the angle between $\vec{k}^*$ and $\vec{k}'^*$. Carrying out with the integration over $\cos \theta_{k'}^*$ first, with the help of the δ function, yields

$$\cos \theta_{k'}^* = 1 - \frac{m_f}{\omega_k^* \omega_{k'}^*} (\omega_k^* - \omega_{k'}^*) \quad (\text{B2})$$

and

$$J_1 = \frac{\pi m_f^2}{2\omega_k^*} \int_{\omega_{\min}^*}^{\omega_{\max}^*} d\omega_{k'}^* \omega_{k'}^{*2} = \frac{\pi m_f^2}{6\omega_k^*} (\omega_{\max}^{*3} - \omega_{\min}^{*3}), \quad (\text{B3})$$

where the requirement that $-1 \leq \cos \theta_{k'}^* \leq 1$ implies

$$\omega_{\min}^* = \frac{m_f \omega_k^*}{m_f + 2\omega_k^*}, \quad \omega_{\max}^* = \omega_k^*. \quad (\text{B4})$$

For J_2 , we proceed similarly, with the replacement $p \cdot k' \rightarrow p \cdot k = m_f \omega_k^*$ in the integrand, and thus,

$$J_2 = \frac{\pi m_f^2 \omega_k^*}{2} (\omega_{\max}^* - \omega_{\min}^*). \quad (\text{B5})$$

In order to use Eqs. (B3) and (B5) in Eq. (5.1), we express $\omega_{\min}^*$ and $\omega_{\max}^*$ in terms of E_p and $|\vec{p}|$ by means of the relation

$$\omega_k^* = \frac{1}{m_f} p \cdot k = \frac{\omega_k E_p}{m_f} (1 - v_p \cos \theta_p), \quad (\text{B6})$$

with $v_p = |\vec{p}|/E_p$. This allows the angular integration in Eq. (5.1) to be carried out in straightforward fashion, leaving only the integration over E_p , which depends on the distribution function, to be performed. As usual, we can consider special cases for illustrative purposes.

1. Ultrarelativistic background

Specifically, we assume that

$$\alpha_f, T, \omega_\kappa \gg m_f. \quad (\text{B7})$$

In this case,

$$\omega_{\min}^* = 0 \quad (\text{B8})$$

and therefore

$$J_1 = \frac{\pi}{6} (m_f \omega_\kappa^*)^2 \rightarrow \frac{\pi}{6} \omega_\kappa^2 |\vec{p}|^2 (1 - \cos \theta_p)^2. \quad (\text{B9})$$

Then, denoting $p = |\vec{p}|$, from Eq. (5.1),

$$\begin{aligned} I_1^{(x)} &= \frac{2}{\omega_\kappa} \left(\frac{1}{2\pi^2} \right)^5 \frac{\pi^2 \omega_\kappa^2}{6} \frac{8}{3} \int_0^\infty dp p^3 f_x(p) \\ &= \frac{\kappa}{36\pi^3} \int_0^\infty dp p^3 f_x(p), \end{aligned} \quad (\text{B10})$$

remembering that $\omega_\kappa = \kappa$. Similarly,

$$J_2 = \frac{\pi}{2} (m_f \omega_\kappa^*)^2 = 3J_1 \quad (\text{B11})$$

and therefore

$$I_2^{(x)} = \frac{\kappa}{12\pi^3} \int_0^\infty dp p^3 f_x(p), \quad (\text{B12})$$

a. Completely degenerate background

For a completely degenerate x background ($x = f$ or $\bar{f}$) putting $f_x = \theta(p_{F_x} - p)$, where p_{F_x} is the Fermi momentum,

$$\begin{aligned} I_1^{(x)} &= \frac{\kappa}{36\pi^3} \frac{p_{F_x}^4}{4}, \\ I_2^{(x)} &= \frac{\kappa}{12\pi^3} \frac{p_{F_x}^4}{4}. \end{aligned} \quad (\text{B13})$$

The Fermi momentum is given in terms of the number density f_x of the background fermions by $p_{F_x} = (3\pi^2 n_x)^{\frac{1}{3}}$.

b. Classical background

Putting $f_x = e^{-\beta p}$, where β is the inverse temperature (T), gives

$$\begin{aligned} I_1^{(x)} &= \frac{6\kappa}{36\pi^3 \beta^4} = \frac{\kappa T^4}{6\pi^3}, \\ I_2^{(x)} &= 3I_1^{(x)} = \frac{\kappa T^4}{2\pi^3}. \end{aligned} \quad (\text{B14})$$

2. Nonrelativistic background

Here we assume that

$$\omega_\kappa \gg m_f \gg T. \quad (\text{B15})$$

In this case, from Eq. (B6),

$$\omega_\kappa^* = \omega_\kappa, \quad (\text{B16})$$

and we have Eq. (B8) once again. Thus,

$$J_1 = \frac{\pi}{6} (m_f \omega_\kappa^*)^2 \rightarrow \frac{\pi}{6} m_f^2 \omega_\kappa^2 \quad (\text{B17})$$

and similarly we get

$$J_2 = 3J_1 \quad (\text{B18})$$

in this case as well. Thus, from Eq. (5.1), we arrive at the result quoted in Eq. (5.12).

-
- [1] G. Mangano, A. Melchiorri, P. Serra, A. Cooray, and M. Kamionkowski, Cosmological bounds on dark matter-neutrino interactions, *Phys. Rev. D* **74**, 043517 (2006).
 - [2] T. Binder, L. Covi, A. Kamada, H. Murayama, T. Takahashi, and N. Yoshida, Matter power spectrum in hidden neutrino interacting dark matter models: A closer look at the collision term, *J. Cosmol. Astropart. Phys.* **11** (2016) 043.
 - [3] R. Primulando and P. Uttayarat, Dark matter-neutrino interaction in light of collider and neutrino telescope data, *J. High Energy Phys.* **06** (2018) 026.

- [4] A. Olivares-Del Campo, C. Bhm, S. Palomares-Ruiz, and S. Pascoli, Dark matter-neutrino interactions through the lens of their cosmological implications, *Phys. Rev. D* **97**, 075039 (2018).
- [5] T. Brune and H. Päs, Massive Majorons and constraints on the Majoron-neutrino coupling, *Phys. Rev. D* **99**, 096005 (2019).
- [6] T. Franarin, M. Fairbairn, and J. H. Davis, JUNO sensitivity to resonant absorption of galactic supernova neutrinos by dark matter, [arXiv:1806.05015](https://arxiv.org/abs/1806.05015).

- [7] P. S. B. Dev *et al.*, Neutrino non-standard interactions: A status report, *SciPost Phys. Proc.* **2**, 001 (2019).
- [8] S. Pandey, S. Karmakar, and S. Rakshit, Interactions of astrophysical neutrinos with dark matter: A model building perspective, *J. High Energy Phys.* **01** (2019) 095.
- [9] S. Karmakar, S. Pandey, and S. Rakshit, Are we looking at neutrino absorption spectra at icecube? [arXiv:1810.04192](#).
- [10] J. F. Nieves and S. Sahu, Neutrino effective potential in a fermion and scalar background, *Phys. Rev. D* **98**, 063003 (2018).
- [11] J. F. Nieves and S. Sahu, Neutrino damping in a fermion and scalar background, *Phys. Rev. D* **99**, 095013 (2019).
- [12] P. Coloma, J. Lopez-Pavon, I. Martinez-Soler, and H. Nunokawa, Decoherence in neutrino propagation through matter, and bounds from IceCube/DeepCore, *Eur. Phys. J. C* **78**, 614 (2018).
- [13] J. A. Carpio, E. Massoni, and A. M. Gago, Revisiting quantum decoherence for neutrino oscillations in matter with constant density, *Phys. Rev. D* **97**, 115017 (2018).
- [14] M. M. Guzzo, P. C. de Holanda, and R. L. N. Oliveira, Quantum dissipation in a neutrino system propagating in vacuum and in matter, *Nucl. Phys. B* **908**, 408 (2016).
- [15] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, Probing non-standard decoherence effects with solar and KamLAND neutrinos, *Phys. Rev. D* **76**, 033006 (2007).
- [16] A. Capolupo, S. M. Giampaolo, and G. Lambiase, Decoherence in neutrino oscillations, neutrino nature and *CPT* violation, *Phys. Lett. B* **792**, 298 (2019).
- [17] X. Chu, B. Dasgupta, M. Dentler, J. Kopp, and N. Saviano, Sterile neutrinos with secret interactions cosmological discord?, *J. Cosmol. Astropart. Phys.* **11** (2018) 049.
- [18] A. J. Daley, Quantum trajectories and open many-body quantum systems, *Adv. Phys.* **63**, 77 (2014).
- [19] S. Weinberg, Collapse of the state vector, *Phys. Rev. A* **85**, 062116 (2012).
- [20] P. Pearle, Simple derivation of the Lindblad equation, *Eur. J. Phys.* **33**, 805 (2012).
- [21] M. B. Plenio and P. L. Knight, The quantum jump approach to dissipative dynamics in quantum optics, *Rev. Mod. Phys.* **70**, 101 (1998).
- [22] S. Lieu, Non-Hermitian Majorana modes protect degenerate steady states, *Phys. Rev. B* **100**, 085110 (2019).
- [23] Strictly speaking this is correct in the massless neutrino limit, which in practice is a valid approximation in the limit that the neutrino mass can be neglected in the calculation of the relevant diagrams.
- [24] Actually, under the isotropy assumption $V^{(u)}$, $V^{(t)}$ depends on ω and κ but not on the direction of $\vec{\kappa}$.
- [25] L. P. Kadanoff and G. Byam, *Quantum Statistical Mechanics* (Benjamin, New York, 1962), p. 37.
- [26] The author of Ref. [22] states it clearly like this: the Lindblad master equation (5) lends itself to a convenient physical interpretation known as the quantum stochastic wave function approach [46, 47]: in a time step dt , a system prepared in a pure state will either evolve coherently according to the non-Hermitian effective Hamiltonian H_{eff} or a “quantum jump event” will decohere the system by moving a pure state from $|\psi\rangle$ to $L_i|\psi\rangle$. Averaging over all such trajectories will produce the same expectation values as formally solving the Lindblad master equation for the evolution of the density matrix.