Hierarchy independent sensitivity to leptonic δ_{CP} with atmospheric neutrinos

D. Indumathi,^{1,2,*} S. M. Lakshmi^{3,†} and M. V. N. Murthy^{1,‡}

¹*The Institute of Mathematical Sciences, Chennai 600 113, India*

²*Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400085, India*

³*Indian Institute of Technology Madras, Chennai 600 036, India*

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The Dirac leptonic CP phase δ_{CP} is one of the crucial unknown parameters in neutrino oscillation physics. In this paper we explore the possibility of using low energy atmospheric neutrino events to probe δ_{CP} . We show that at sub-GeV energies, when the events are binned as a function of the energy and direction of the final state leptons, a consistent distinction between various input δ_{CP} values is obtained. More importantly, we show that at these energies there is no sensitivity to the neutrino mass ordering/hierarchy, so that δ_{CP} can be measured without hierarchy ambiguity. A χ^2 analysis of the sensitivity to δ_{CP} using atmospheric neutrinos assuming an isoscalar detector that cannot separate ν_e and $\bar{\nu}_e$ events is done. While exceedingly high sensitivities are obtained in the ideal case, a modest sensitivity remains when the effects of energy resolution and systematic uncertainties are taken into account.

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I. INTRODUCTION

The Dirac leptonic phase δ_{CP} is one of the important unknown parameters in neutrino oscillation physics today. There is a hint that its value is approximately $\delta_{CP} \approx -145^\circ(-76^\circ)$ for normal (inverted) hierarchy [1]. Several experiments, mainly accelerator long baseline (LBL) experiments are running or are being designed to measure this parameter as precisely as possible [2–4]. These experiments have very good sensitivity to δ_{CP} but its extraction mostly depends on the hierarchy, whether normal or inverted, in the neutrino masses.

In this paper, we explore the possibility of using atmospheric neutrino events, especially in the sub-GeV region ($E_\nu \leq 1$ GeV) to probe the CP phase. Several atmospheric neutrino experiments are either currently running [5–7] or proposed [8–13]. Atmospheric neutrinos cover a wide range of baseline L_ν and energies E_ν and hence probe a wide variety of physics scenarios. It has been shown earlier [14–17] that atmospheric neutrino and antineutrino event rates at energies above a GeV or so are completely *insensitive* to the CP phase and hence can be used to measure the mass

ordering or hierarchy (MH) independent of the CP phase. This is in contrast to the LBL beam experiments where degeneracies give rise to ambiguities in the extraction of oscillation parameters. In particular, the MH and CP phase are entangled. Not so well known is the fact that the CP phase becomes independent of MH at sub-GeV energies.

We show for the first time that both the ν_e and ν_μ atmospheric neutrino events at low, i.e., sub-GeV, energies are sensitive to the CP phase. In addition, the dependence on the CP phase (of the angular distribution) of ν_e and ν_μ events is such that it systematically shifts the observed event rates in opposite directions in the two cases. Finally, this dependence on CP is independent of the mass hierarchy. Thus we propose that a measurement of sub-GeV atmospheric neutrino events, where event rates are large, will be an additional channel to determine δ_{CP} , and moreover, one that is independent of the neutrino MH. That is, atmospheric neutrino events are uniquely positioned so that low energy sub-GeV events are sensitive to δ_{CP} independent of MH, while higher energy (few GeV) events are sensitive to MH independent of δ_{CP} . We believe that this unique dependence has been discussed for the first time, in this paper.

In particular, we show in Sec. II that the events spectra for different values of the input δ_{CP} are different when the events are binned as a function of the energy and direction of the final state lepton in the charged current (CC) interaction, thus making it possible to have a good δ_{CP} sensitivity. In Sec. III we show analytically that at such low energies and larger baselines, which are relevant for atmospheric neutrinos, δ_{CP} can be determined irrespective of hierarchy. We quantify our results in Sec. IV with two χ^2

*indu@imsc.res.in

†slakshmi@physics.iitm.ac.in

‡murthy@imsc.res.in

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analyses, one for the idealized case and the other for the realistic case. For the idealized case we assume that CC ν_e , $\bar{\nu}_e$, ν_μ , and $\bar{\nu}_\mu$ events can be separated from one another and the detector has perfect resolutions and efficiencies. In the realistic case we consider that neutrino and anti-neutrino events cannot be separated from each other. Only CC $\nu_e + \bar{\nu}_e$ events are analyzed and the detector has a specific energy resolution. The effect of energy resolutions and systematic uncertainties on δ_{CP} sensitivity for the realistic case are also discussed; the effect of systematic uncertainties is to sharply reduce the sensitivity to δ_{CP} ; however, a modest sensitivity remains. We end with discussions and conclusions, which are presented in Sec. VI.

II. OSCILLATION PROBABILITIES AND ATMOSPHERIC NEUTRINO EVENTS AT SUB-GEV ENERGIES

In the case of atmospheric neutrinos, the muon neutrino (and antineutrino) fluxes $\Phi_\mu + \bar{\Phi}_\mu$ are larger than that of the

electron neutrino (and antineutrino) fluxes $\Phi_e + \bar{\Phi}_e$. The sensitivity to δ_{CP} in these events arises from the transition probabilities $P_{\mu e}$, $\bar{P}_{\mu e}$ and $P_{e\mu}$, $\bar{P}_{e\mu}$ [18–20]. Hence the sensitivity to δ_{CP} is larger for electronlike events as compared to the muonlike events. A comparison of the relevant oscillation probabilities in Earth matter for two different values of δ_{CP} as a function of the neutrino direction, $\cos\theta_\nu$, for a sample sub-GeV neutrino energy, $E_\nu = 0.65$ GeV, and normal mass hierarchy is shown in Fig. 1. The probabilities are generated using full three-flavor oscillations with the central values of the parameters given in Table I. The Preliminary reference Earth model (PREM) profile [21] is used to model the Earth matter density, and the neutrino time-dependent evolution equations are solved using a Runge-Kutta solver [22].

The probabilities are flipped for $P_{\mu e}$ ($\bar{P}_{\mu e}$) and $P_{e\mu}$ ($\bar{P}_{e\mu}$) when δ_{CP} changes from $90^\circ \leftrightarrow -90^\circ$. In each case, the probability with one value of δ_{CP} , say 90° , is not always smaller than that with -90° ; i.e., the trend is not

FIG. 1. Transition probabilities $P_{\mu e}$ (top left), $\bar{P}_{\mu e}$ (top right), $P_{e\mu}$ (bottom left) and $\bar{P}_{e\mu}$ (bottom right) as a function of $\cos\theta_\nu$ for a fixed neutrino energy, $E_\nu = 0.65$ GeV and input normal mass hierarchy.

TABLE I. Input values and 3σ ranges of parameters used to generate oscillated events. Values except that of δ_{CP} are taken as in Ref. [23]. The new value of θ_{13} is taken according to [24]. For the oscillation analysis, $\Delta m_{31}^2 = \Delta m_{\text{eff}}^2 + \Delta m_{21}^2 (\cos^2 \theta_{12} - \cos \delta_{CP} \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23})$; $\Delta m_{32}^2 = \Delta m_{31}^2 - \Delta m_{21}^2$, for normal hierarchy when $\Delta m_{\text{eff}}^2 > 0$. When $\Delta m_{\text{eff}}^2 < 0$, $\Delta m_{31}^2 \leftrightarrow -\Delta m_{32}^2$ for inverted hierarchy.

Parameter	Input value	Marginalization range
θ_{13}	$8.5 (8.63)^\circ$	Not marginalized
$\sin^2 \theta_{23}$	0.5	[0.39, 0.64]
Δm_{eff}^2	$2.4 \times 10^{-3} \text{ eV}^2$	$[2.3, 2.6] \times 10^{-3} \text{ eV}^2$
$\sin^2 \theta_{12}$	0.304	Not marginalized
Δm_{21}^2	$7.6 \times 10^{-5} \text{ eV}^2$	Not marginalized
δ_{CP}	$0, \pm 90^\circ, \pm 180^\circ$	Not marginalized

consistently larger or smaller in all bins, but shows an oscillatory pattern.

A similar behavior can be seen if we plot the events spectra as a function of $\cos \theta_\nu$, as in Fig. 2. The events are generated with parameters given in Table I. Here the oscillated events are plotted as a function of the neutrino direction $\cos \theta_\nu$ for bins of final state lepton energy and angle, $E_l = 0.5\text{--}0.8 \text{ GeV}$ and $\cos \theta_l = 0.6\text{--}0.7$, respectively, where $l = e, \mu$. The event spectrum follows the oscillatory behavior of the transition probabilities plotted in Fig. 1, where some bins have more events when $\delta_{CP} = +90^\circ$ than -90° , while it is reversed in other bins. But when we plot the events as a function of the lepton direction $\cos \theta_l$, which is the observable, the behavior changes as shown in Fig. 3.

FIG. 2. (Top) Oscillated electron type; (bottom) muon type events as a function of neutrino direction, $\cos \theta_\nu$, for events with final lepton energy, $E_l = 0.5\text{--}0.8 \text{ GeV}$, and direction $\cos \theta_l = 0.6\text{--}0.7$, with $\delta_{CP} = \pm 90^\circ$ and input NH. The left panels are for ν events and the right ones are for $\bar{\nu}$ events.

FIG. 3. (Top) Oscillated electron type; (bottom) muon type events as a function of the final state lepton angle, $\cos\theta_l$, for events with final lepton energy, $E_l = 0.5\text{--}0.8$ GeV, with $\delta_{CP} = \pm 90^\circ$ and input NH. The left panels are for ν events and the right ones are for $\bar{\nu}$ events. Note that all y-axes scales are different.

It can be seen from Fig. 3 that when the oscillated events are plotted as a function of $\cos\theta_l$, the spectrum with $\delta_{CP} = -90^\circ$ is always greater (less) than that with 90° for ν_e and $\bar{\nu}_e$ (ν_μ and $\bar{\nu}_\mu$), although the effect is smaller for muon neutrinos. That is, the oscillatory dependence on δ_{CP} seen in Figs. 1 and 2 has disappeared, giving rise to a systematic dependence on the CP phase. This effect is due to the kinematics of the interaction, which generates a final state lepton scattered at an angle that can be far different from that of the parent neutrino. This is especially so at low energies of interest here, where the dominant process is quasielastic (QE) neutrino-nucleus scattering.

Thus, when plotted as a function of $\cos\theta_l$, the features of the underlying oscillation probability are lost but the sensitivity to δ_{CP} is reinforced. This behavior, arising because of the different distributions of the final state

lepton due to the kinematics of the interaction, has not been discussed in the literature before. Again, the same systematic behavior is also seen in muon events. Although the effect is weaker, δ_{CP} dependence is opposite to that of electron events.

The cumulative sum of events as a function of $\cos\theta_l$ is shown in Fig. 4 to illustrate this. It can be seen that though the actual distribution of events in $\cos\theta_l$ follows the oscillation probabilities, scattering of neutrinos coming from several neutrino directions, $\cos\theta_\nu$, gives rise to leptons with the same scattering angle, $\cos\theta_l$. For instance, neutrinos with $\cos\theta_\nu$ practically from -1 to $+1$ contribute to the events in the bin with $0.6 \leq \cos\theta_l \leq 0.7$.

When we sum over these events, their contributions to each $\cos\theta_l$ bin do not resemble the probabilities, but follow a pattern in which the events spectrum with a particular

FIG. 4. Cumulative sum of oscillated (top) electron type (bottom) muon type events, which contribute to the $\cos \theta_l$ bin 0.6–0.7, as a function of $\cos \theta_\nu$ for the bin $E_l = 0.5\text{--}0.8$ GeV, $l = e, \mu$, with $\delta_{CP} = \pm 90^\circ$ and input NH. The last bin in each plot gives the total contribution to each $\cos \theta_l$ bin. The left panels are for ν events and the right ones are for $\bar{\nu}$ events.

value of δ_{CP} is always greater than the other because of the kinematics of the events. In addition, we have shown all plots so far assuming the normal hierarchy. We now show, both analytically and numerically, that these results hold, irrespective of the MH.

III. HIERARCHY (IN)DEPENDENCE AT LOW ENERGIES

In this section we show that at low energies there is no hierarchy ambiguity for atmospheric neutrinos and hence δ_{CP} can be measured irrespective of hierarchy. In fact, this can be established analytically as we show here.

A. Hierarchy independence: analytic approach

The three-flavor vacuum oscillation probability of a flavor $\nu_\alpha \rightarrow \nu_\beta$ is given by

$$P_{\alpha\beta}^{(-)\text{vac}} = \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin^2 \left(\frac{1.27 \Delta m_{ij}^2 L}{E} \right) \pm 2 \sum_{i>j} \text{Im}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin \left(\frac{2.53 \Delta m_{ij}^2 L}{E} \right), \quad (1)$$

where $\alpha, \beta = e, \mu, \tau$ are the flavor indices, and the $\pm$ sign corresponds to neutrinos and antineutrinos, respectively.

Here

$$U_{ai}^{\text{vac}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{CP}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}, \quad (2)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$; $i, j = 1, 2, 3$ are the mass eigenstates, $\Delta m_{ij}^2 = m_i^2 - m_j^2$ ($j < i$), θ_{ij} are the mixing angles, and δ_{CP} is the leptonic CP violation phase. Here L (in km) is the distance traveled by a neutrino of energy E (in GeV).

The survival probability $P_{\alpha\alpha}$ has no imaginary part, while for transition probabilities $\alpha \neq \beta$, the imaginary part changes sign with $P_{\alpha\beta} = \bar{P}_{\beta\alpha}$, the corresponding antineutrino probability. When E is small, of the order of a few hundred MeV, the corresponding oscillatory terms average out whenever L/E is large compared to Δm_{ij}^2 . Since $|\Delta m_{3j}^2| \sim 2.4 \times 10^{-3} \text{ eV}^2 \gg \Delta m_{21}^2 \sim 7.6 \times 10^{-5} \text{ eV}^2$, $j = 1, 2$; this applies to the atmospheric terms,

$$1.27 \Delta m_{3j}^2 \frac{L}{E} \approx \pi \frac{(L/100 \text{ km})}{(E/0.1 \text{ GeV})}, \quad (3)$$

rather than to “solar” terms,

$$1.27 \Delta m_{21}^2 \frac{L}{E} \approx \pi \frac{(L/3000 \text{ km})}{(E/0.1 \text{ GeV})}. \quad (4)$$

It immediately follows that the atmospheric event rates at these low energies and for $L \geq$ a few 100 km become independent of Δm_{32}^2 and Δm_{31}^2 and hence of their ordering. The solar mass-squared difference remains, but its magnitude and sign are well known. Hence the CP phase dependence can be studied with low energy atmospheric neutrinos, independent of the MH. In particular, as has been discussed in the literature before (see, for example, Refs. [18,25]), the probabilities involving e and μ are linear in $\sin \delta_{CP}$ and $\cos \delta_{CP}$. The survival probabilities are independent of $\sin \delta_{CP}$, which occurs in the imaginary part of the transition probabilities. In fact, P_{ee} is independent of both $\sin \delta_{CP}$ and $\cos \delta_{CP}$ while $P_{\mu\mu}$ depends on $\cos \delta_{CP}$ and $\cos 2\delta_{CP}$. The transition probabilities $P_{e\mu}$ and $P_{\mu e}$ are thus most sensitive to δ_{CP} , measurable in principle, via a CP asymmetry that can be expressed in vacuum as

$$A_{CP} = \frac{P_{e\mu} - P_{\mu e}}{P_{e\mu} + P_{\mu e}} = -\frac{C}{A + B \cos \delta} \sin \delta, \\ \bar{A}_{CP} = \frac{\bar{P}_{e\mu} - \bar{P}_{\mu e}}{\bar{P}_{e\mu} + \bar{P}_{\mu e}} = \frac{C}{A + B \cos \delta} \sin \delta, \quad (5)$$

for ν and $\bar{\nu}$, respectively. In matter, A, B, C are modified according to Earth matter effects on the oscillation parameters. The linear dependence on $\cos \delta_{CP}$ and $\sin \delta_{CP}$ remains unaltered. See Appendix for details. While such

an asymmetry serves to highlight the dependence of the transition probabilities on the CP phase, note, however, that this asymmetry is purely theoretical in origin since actual measurements would involve fluxes and cross sections that are different for terms involving $P_{e\mu}$ and P_{ee} . Hence we do not pursue this further.

B. Hierarchy independence: Events spectra at low energies

It is known that Earth matter resonance occurs in atmospheric neutrinos at a few GeV energies, thus enabling the determination of the neutrino mass hierarchy. The advantage of using atmospheric neutrinos for hierarchy determination is that it can be determined unambiguously of δ_{CP} , especially using the $P_{\mu\mu}$ ($\bar{P}_{\mu\mu}$) survival channel. We have now shown that, at lower energies (sub-GeV range) this effect is reversed; i.e., δ_{CP} can be determined irrespective of the hierarchy. In addition, we examined the CP sensitivity of the events as a function of the final lepton scattering angle, $\cos \theta_l$. We now show that the systematic dependence on δ_{CP} remains when we integrate out the angular dependence and examine the events as a function of the final state lepton energy, E_l , alone.

This is illustrated with oscillated ν_e and $\bar{\nu}_e$ events binned in E_l in the ranges 0.1–2.0 and 2.0–11.0 GeV in Fig. 5. Here the events are averaged over all directions. Only ν_e and $\bar{\nu}_e$ events are shown in this figure. The effect is similar in ν_μ and $\bar{\nu}_\mu$ events, but the δ_{CP} sensitivity is smaller in ν_μ and $\bar{\nu}_\mu$ events. It can be seen from the figure that in the lower energy range 0.1–2.0 GeV, the NH and IH spectra with the same input δ_{CP} are the same, while the spectra for different input δ_{CP} values differ. In the higher energy range from 2.0–11.0 GeV, this effect is reversed. Thus in the lower energy range we can measure δ_{CP} irrespective of hierarchy and in the higher energy range hierarchy can be determined irrespective of δ_{CP} . This is a unique signature provided only by atmospheric neutrinos: that neutrinos of different energies from the same source can be used to probe different oscillation parameters unambiguously. The flux of atmospheric neutrinos is smaller than that of the accelerator neutrino experiments; however, the simultaneous availability of a wide range of energies (E) and baselines (L) and ν and $\bar{\nu}$ and of different neutrino flavors is a great advantage. In addition to the δ_{CP} sensitivity from the accelerator LBL experiments we can add the sensitivities from low atmospheric neutrino experiments, thus increasing the global sensitivity towards this parameter.

FIG. 5. Oscillated events with different δ_{CP} values in the E_e range (top) 0.1–2.0 GeV and (bottom) 2.0–11.0 GeV. The blue (pink) histograms are for normal (inverted) hierarchy. The left panels are for ν events and the right ones are for $\bar{\nu}$ events. Note that the y axes are kept different for visibility.

IV. SENSITIVITY TO δ_{CP} WITH LOW ENERGY ATMOSPHERIC NEUTRINOS

It has been shown in the previous sections that at low energies ($E_e \lesssim 1\text{--}2$ GeV) the events spectra from different values of δ_{CP} can be distinguished from one another independent of the neutrino mass hierarchy. This means that a good sensitivity to δ_{CP} can be obtained by analyzing low energy atmospheric neutrino events. We proceed to quantify this sensitivity through a χ^2 analysis. Two analyses are performed, an idealized case and a realistic case.

Events from charged current interactions of oscillated events—The events of interest here are those from the CC interactions of ν_e , $\bar{\nu}_e$, ν_μ , and $\bar{\nu}_\mu$. The number of CC ν_e events detected is given by

$$\mathcal{N}^e = t \times n_d \times \int d\sigma_{\nu_e} \times \left[P_{ee} \frac{d^2\Phi_e}{dE_\nu d\cos\theta_\nu} + P_{\mu e} \frac{d^2\Phi_\mu}{dE_\nu d\cos\theta_\nu} \right]; \quad (6)$$

that is, the ν_e events detected at the detector can be from the direct $\nu_e \rightarrow \nu_e$ survived events as well as from the $\nu_\mu \rightarrow \nu_e$ oscillated events. Here t is the exposure time, n_d is the number of targets in the detector, $d\sigma_{\nu_e}$ is the differential neutrino interaction cross section (typically differential in the final state charged lepton energy and/or angle, E_e and/or $\cos\theta_e$ respectively), and $d\Phi_{\nu_\mu}$ and $d\Phi_{\nu_e}$ are the ν_μ and ν_e fluxes. This is similar for $\bar{\nu}_e$, ν_μ , and $\bar{\nu}_\mu$ with relevant fluxes and probabilities.

FIG. 6. Electron events for different input δ_{CP} values as a function of $\cos \theta_l^{\text{obs}}$ for $E_l^{\text{obs}} = 0.2\text{--}0.4$ and $0.5\text{--}0.8$ GeV; $l = e$ in a detector without any energy and direction smearing and has charge identification capabilities. The top row is for ν events and the bottom one for $\bar{\nu}$. The y axes are not the same. Here $\theta_{13} = 8.5^\circ$.

A. Ideal case

We assume the following for the ideal case. It is the best-case scenario when all four event types ν_e , $\bar{\nu}_e$, ν_μ , and $\bar{\nu}_\mu$ can be separated from one another. The detector has 100% reconstruction efficiency for all types of events. Unoscillated events are simulated in a 50 kton detector using the NUANCE [26] neutrino generator for an exposure of 100 years (500 kton years) using the Honda fluxes [27–29] and a generic isoscalar target. These 100 years of events are oscillated with the central values of the parameters given in Table I and then scaled down to 10 years to generate “data” events. The same 100 year sample is oscillated using the 3σ values of the oscillation parameters in Table I and scaled down to 10 years to generate “theory.” This is done to reduce fluctuations [no fluctuations (no fluct) case]. The parameters except Δm_{32}^2 and θ_{23} are kept fixed at their central values for both data and theory. In the ideal case, the value of θ_{13} is taken to be 8.5° .

Matter oscillations are applied event by event. Earth matter effects are calculated using the PREM profile of Earth [21]. For this initial analysis we consider the observed lepton energy range of $E_l^{\text{obs}} = 0.1\text{--}30.0$ GeV. The binning of events is done in energy and direction of the final state lepton, E_l^{obs} and $\cos \theta_l^{\text{obs}}$ [16,17].

The spectra of oscillated events as a function of $\cos \theta_l$ for different sample bins of E_l , $0.2\text{--}0.4$ GeV and $0.5\text{--}0.8$ GeV, respectively, are shown in Figs. 6 and 7 for charged current electronlike (CCE) and charged current muonlike (CCMU) events.

It is clearly evident that the effect of δ_{CP} is more in the electron-type events than in the muon-type events. Also, for a given type of event the separation between the spectra with different δ_{CP} is more at lower energies and decreases with the increase of energy. Also at very low energies the difference is consistent for both the up and down directions, whereas at higher energies, the effect is more in the up direction. Because

FIG. 7. As in Fig. 6, for muon events, $l = \mu$. Again, the y axes scales are not the same. Here also $\theta_{13} = 8.5^\circ$.

of these consistent differences we can distinguish different δ_{CP} values when the events are binned in the final state lepton direction. In the rest of the paper we discuss the sensitivity to δ_{CP} from CC ν_e and $\bar{\nu}_e$ events alone.

In the ideal analysis we assume perfect energy and direction resolutions for the detector and ignore systematic uncertainties. A Poissonian χ^2 [17] is used; i.e.,

$$\chi^2 = \sum_i \sum_j 2 \left[(T_{ij}^0 - D_{ij}) - D_{ij} \ln \left(\frac{T_{ij}^0}{D_{ij}} \right) \right], \quad (7)$$

where D_{ij} are the data events and T_{ij}^0 are the theory events with no systematic uncertainty. The indices i, j represent the final state lepton energy and direction, respectively.

1. δ_{CP} sensitivity in the ideal case

Both fixed parameter and marginalized analyses using CCE and CCMU type events were performed assuming

$\delta_{CP}^{\text{sel}} = -90^\circ$ as the central value. From Fig. 8 it is evident that the ability to distinguish between neutrino and anti-neutrino events improves the sensitivity to δ_{CP} . With $\nu_e - \bar{\nu}_e$ separation, the δ_{CP} region $\sim -30^\circ$ to $\sim 60^\circ$ can be ruled out at 5σ (2σ) with CCE (CCMU) events alone. When ν and $\bar{\nu}$ events cannot be separated, all regions of δ_{CP} are allowed at 5σ from CCE events alone. For CCMU events the ruled-out region shrinks to ~ 0 to 60° at 2σ in the absence of charge identification. This establishes the claimed δ_{CP} sensitivity in principle. In reality there is the effect of detector resolutions, systematic uncertainties, etc. The study of δ_{CP} sensitivity for a more realistic scenario is performed in Sec. V.

V. REALISTIC CASE

In this case we consider low energy CCE events only and include the effects of finite detector resolution and systematic errors. Also ν_e cannot be separated from $\bar{\nu}_e$.

FIG. 8. Sensitivity to δ_{CP} (left) with (wcid) and (right) without (no cid) charge identification in the ideal case. Here $\delta_{CP} = -90^\circ$ is selected as the input value. The left-hand figure shows the sensitivity when it is assumed that ν and $\bar{\nu}$ events can be separated using charge identification for both electron- (CCE) and muon- (CCMU) type neutrino events. The decrease in the sensitivity when there is no charge identification can be seen from the figure on the right. Here, the neutrino and antineutrino events are added together for the analysis since there is no charge identification.

The lepton direction can usually be well reconstructed; hence we take only energy smearing into account here. We assume a 50 kton detector and 10 year data accumulation for the analysis. Oscillated events are simulated taking fluctuations into account as follows. Of the 100 years of unoscillated CCE events generated using the NUANCE neutrino generator, a sample of 10 years of events is selected randomly. This sample is oscillated event by event using the central values of the parameters given in Table I to generate data. It was verified (for a small subset of the analysis) that similar results were obtained when a different 10 year sample was selected. The remaining 90 years of events are oscillated by varying the oscillation parameters in their 3σ ranges given in the same table and are scaled down to 10 years to generate theory events that are to be compared with data. Here also all parameters except δm^2_{32} and θ_{23} are kept fixed at their central values for both data and theory. Here we take θ_{13} as 8.63° , which is the current best fit value of this parameter [24].

A. Choice of detector characteristics

For this analysis we have considered a detector with 100% reconstruction efficiency and no charge identification capability (i.e., the detector cannot distinguish between ν_e and $\bar{\nu}_e$ events). The final state electron energy is smeared according to the energy resolution of the detector to obtain the observed electron energy. For this analysis we have assumed three cases: (i) perfect electron energy resolution, (ii) an electron energy resolution given by

$$\frac{\sigma}{E} = \frac{15\%}{\sqrt{E}} \oplus 1\%, \quad (8)$$

as per the DUNE TDR [30,31], and (iii) and electron energy resolution according to the SuperKamiokande experiment [32],

$$\frac{\sigma}{E} = \frac{2.5\%}{\sqrt{E}} \oplus 0.5\%. \quad (9)$$

The detector is assumed to have perfect direction resolution since typically direction reconstruction in most detectors is excellent and the analysis is not sensitive to this parameter. Hence the oscillated events are binned in the energy and direction of the final state observed electron, as $(E_e^{\text{obs}}, \cos \theta_e^{\text{obs}})$. The bins of E_e^{obs} in the range 0.1–2.0 GeV are the same those given in Table II. The lepton direction is also binned from $\cos \theta_e^{\text{obs}} = [-1, 1]$ as in the same table. We have seen from the earlier analysis (Fig. 5) that at low energies, the oscillation probabilities are insensitive to the mass hierarchy. Hence energies only up to $E_e^{\text{obs}} \leq 2$ GeV are considered in the analysis.

A Poissonian χ^2 analysis assuming an isoscalar detector that cannot separate ν_e and $\bar{\nu}_e$ events from each other was performed. In addition to the effect of energy resolution incorporated by means of smearing, we also consider the effect of systematic uncertainties on δ_{CP} sensitivity. Three systematic uncertainties are taken into account, two on the fluxes and one on the cross section. These are incorporated by means of the pull method [33–35]. We have

$$\chi^2 = \sum_i \sum_j 2 \left[(T_{ij} - D_{ij}) - D_{ij} \ln \left(\frac{T_{ij}}{D_{ij}} \right) \right] + \sum_{l=1}^3 \xi_l^2, \quad (10)$$

TABLE II. The binning scheme used in the analysis in final state lepton energy and direction, E_l^{obs} , $\cos \theta_l^{\text{obs}}$.

Observable	Range	Bin width	No. of bins
E_l^{obs} (GeV) (20 bins)	[0.1, 0.2]	0.1	1
	[0.2, 0.4]	0.2	1
	[0.4, 0.5]	0.1	1
	[0.5, 0.8]	0.3	1
	[0.8, 1.0]	0.2	1
	[1, 2]	0.5	2
	[1, 4]	0.5	6
	[4, 7]	1	3
	[7, 11]	4	1
	[11, 12.5]	1.5	1
	[12.5, 15]	2.5	1
	[15, 30]	15	1
$\cos \theta_l^{\text{obs}}$ (20 bins)	[-1.0, 1.0]	0.10	20

where i, j are the indices corresponding to E_e and $\cos \theta_e$ bins, respectively. Here T_{ij} and D_{ij} are the (scaled) theory and data events, respectively, which includes both antineutrino events and neutrino events; $T_{ij} = T_{ij}^0(1 + \sum_{l=1}^3 \pi_{ij}^l \xi_l)$, where T_{ij}^0 is the corresponding number of events without systematic errors, π_{ij}^l is the l th systematic uncertainty in the ij th bin and ξ_l are the pulls corresponding to the three systematic uncertainties where $l = 1, 2, 3$ [15].

B. Results—effect of systematic uncertainties on δ_{CP} sensitivity

The sensitivity to the CP phase has been performed with the selected input value $\delta_{CP}^{\text{sel}} = -90^\circ$. The effect of systematic uncertainties alone was studied for the case without energy smearing. The three systematic uncertainties are the errors on flux normalization, cross section, and tilt, respectively. The tilt error reflects the uncertainty in the spectral shape of the flux and is implemented as follows. A shift in the neutrino spectrum due to an uncertainty in the energy dependence from the predicted value of atmospheric neutrino fluxes is given by the relation

$$\Phi_e(E) = \Phi_0(E) \left(\frac{E}{E_0} \right)^\epsilon \simeq \Phi_0(E) \left(1 + \epsilon \ln \frac{E}{E_0} \right), \quad (11)$$

where $E_0 = 2$ GeV and ϵ is the 1σ systematic tilt error taken as 5%. The difference $\Phi_e(E) - \Phi_0(E)$ gives the error in flux in each energy bin [33–35].

A comparison of the sensitivities to δ_{CP} with various combinations of systematic uncertainties is shown in Fig. 9. When there is no systematic uncertainty at all, regions between $\sim -40^\circ$ to 110° can be ruled out above 3σ . When tilt error due to flux shape uncertainty is taken into account, the region that can be ruled out reduces to roughly

FIG. 9. Sensitivity to δ_{CP} with different combinations of systematic uncertainties and marginalization for CCE events. Here the chosen value of δ_{CP} is -90° and lepton energy range is 0.1–2.0 GeV.

-30° to 100° at 3σ . When the other uncertainties are also nonzero, the sensitivity to δ_{CP} decreases further. For a given tilt error the sensitivity worsens with increase of the cross section and flux normalization error. The improvement in cross section or flux normalization error alone does not cause any improvement in δ_{CP} sensitivity. When all uncertainties are 5% each, the region roughly from -20° to 90° can be ruled out at 2σ . It is the tilt error that impacts δ_{CP} sensitivity most significantly. Hereafter we consider a conservative scenario where all three uncertainties are 5% each. The allowed δ_{CP} regions for different combinations of systematic uncertainties are detailed in Table III.

C. Effect of lepton energy resolution

In Sec. V B we saw how systematic uncertainties alone can impact the sensitivity to δ_{CP} while studying sub-GeV atmospheric neutrino events. In this section we study the effect of energy resolution of the final state lepton on the sensitivity to δ_{CP} . The comparison of the effect of energy resolution when there are no systematic uncertainties at all (indicated by *nop*) and when the three systematic uncertainties are 5% each, is shown in Fig. 10. The allowed regions of δ_{CP} for the different cases are shown in Table IV. In both cases the trend with the worsening energy resolution is similar; with perfect lepton energy resolution we get the maximum sensitivity to δ_{CP} . As the energy resolution worsens the sensitivity also reduces. For a given energy resolution, the systematic uncertainties severely affect the sensitivities. If the electron energy resolution

TABLE III. Range of allowed values of δ_{CP} at 2σ , for various combinations of systematic uncertainties when $\delta_{CP} = -90^\circ$ is selected as the input value.

Flux normalization error (%)	Cross section error (%)	Tilt error (%)	Lower allowed range of δ_{CP}	Upper allowed range of δ_{CP}
0	0	0	$[-180^\circ, -47.5^\circ]$	$[126^\circ, 180^\circ]$
0	0	5	$[-180^\circ, -50.3^\circ]$	$[131.6^\circ, 180^\circ]$
0.5	0.5	5	$[-180^\circ, -45^\circ]$	$[124.4^\circ, 180^\circ]$
2	1.5	5	$[-180^\circ, -25.4^\circ]$	$[99^\circ, 180^\circ]$
2	2	5	$[-180^\circ, -23.2^\circ]$	$[95.2^\circ, 180^\circ]$
5	2	5	$[-180^\circ, -15.2^\circ]$	$[84.7^\circ, 180^\circ]$
5	5	5	$[-180^\circ, -13.2^\circ]$	$[84^\circ, 180^\circ]$

TABLE IV. Allowed values of δ_{CP} at 2σ without and with pulls for different lepton energy resolutions, from CCE events in the energy range 0.1–2.0 GeV.

	No lepton energy smearing	$\frac{2.5\%}{\sqrt{E}} + 0.5\%$	$\frac{15\%}{\sqrt{E}} + 1\%$
No pulls	$[-180^\circ, -47^\circ], [126^\circ, 180^\circ]$	$[-180^\circ, -50.9^\circ], [127.7^\circ, 180^\circ]$	$[-180^\circ, -47.64^\circ], [122.42^\circ, 180^\circ]$
With pulls	$[-180^\circ, -13.2^\circ], [84^\circ, 180^\circ]$	$[-180^\circ, 12.6^\circ], [64.9^\circ, 180^\circ]$	$[-180^\circ, 0^\circ], [0^\circ, 180^\circ]$

is $15/\sqrt{E} + 1\%$ and the systematic uncertainties are 5% each all values of δ_{CP} are allowed at 2σ . Hence to have at least 2σ sensitivity the lepton energy resolution should be as small as $2.5\%/\sqrt{E} + 0.5\%$ or the systematic uncertainties should be constrained to less than 5% or both. This means that sensitive detectors with very good reconstruction efficiency and electron energy resolution

are needed to study δ_{CP} using atmospheric neutrinos. The systematic uncertainties should be as small as possible. It is possible to reduce the errors on cross sections from probes from dedicated cross section measurements on different targets. Unlike accelerator neutrino experiments, the uncertainties in fluxes are larger in atmospheric neutrinos and in turn affect the sensitivity to δ_{CP} from atmospheric neutrinos. Hence, in addition to improving the energy resolutions of the detectors it is necessary to reduce flux uncertainties to obtain a significant sensitivity to δ_{CP} from charged current ν_e and $\bar{\nu}_e$.

An important point to be noted here is that in the realistic case considered here, ν_e events cannot be separated from $\bar{\nu}_e$ events. When neutrino events can be separated from antineutrino events the sensitivity to δ_{CP} increases as illustrated in Fig. 8. One such technique for liquid argon (LAr) is discussed in [36]. Also, even though smaller compared to that from CCE events, addition of contributions from ν_μ and $\bar{\nu}_\mu$ events can improve the sensitivity to δ_{CP} . Thus it is important to have detectors with the ability to separate all four types of events very well and with less background.

VI. CONCLUSIONS

A study of charged current events from low energy atmospheric neutrino interactions in a detector sensitive to sub-GeV energies shows that the CP violating phase δ_{CP} in the leptonic sector can be determined from such a study. Detailed simulations were performed for the case of CC atmospheric electron neutrino events since these were maximally sensitive to this parameter. It is seen that the events spectra binned according to the final state electron direction show a consistent distinctive pattern for different

FIG. 10. Sensitivity to δ_{CP} with different combinations of systematic uncertainties and energy resolutions for CCE events. Here the chosen value of δ_{CP} is -90° and lepton energy range is 0.1–2.0 GeV.

values of δ_{CP} . This allows, in principle, a precise determination of δ_{CP} . Also at such low energies, the events are completely insensitive to the major issue of the neutrino mass hierarchy, thus enabling an unambiguous measurement of δ_{CP} irrespective of this unknown quantity; we believe this has been pointed out here for the first time.

The sensitivity dependence was carried out for three choices of detector—a perfect detector and a more realistic one with energy resolution similar to that of SuperK and DUNE detectors, respectively. A very good χ^2 is obtained for a selected input $\delta_{CP}^{\text{sel}} = -90^\circ$. While uncertainties in the current measurements of the remaining neutrino oscillation parameters as well as the detector energy resolution do not spoil this result, inclusion of systematics (from atmospheric neutrino fluxes and cross sections, especially the former) considerably worsens the sensitivity.

Muonlike events also contribute, though in a less sensitive way. But it is very important to analyze all possible events since neutrino experiments are low counting ones and every event adds to the statistics of the experiment. Hence addition of these events (not done in the present analysis) can lead to further improvements and may especially help to constrain the neutrino fluxes and hence decrease the corresponding systematic uncertainties.

It is also seen that when ν_e , $\bar{\nu}_e$, ν_μ , and $\bar{\nu}_\mu$ can be separated from one another, the δ_{CP} sensitivity is higher than the case when ν and $\bar{\nu}$ cannot be separated from each other. This necessitates having special detectors that can distinguish between ν_e and $\bar{\nu}_e$; an analysis is beyond the scope of this work.

Accelerator long baseline experiments like DUNE themselves can determine δ_{CP} with very high precision because of the large statistics and the excellent resolutions of the detector. But the sub-GeV energy atmospheric neutrinos should not be abandoned since they provide an alternate method of determining δ_{CP} , independent of the neutrino mass hierarchy, and the results add up to the global sensitivity to δ_{CP} thus increasing the overall sensitivity to the parameter. It will be interesting to study how well different atmospheric neutrino detectors can probe δ_{CP} especially in the sub-GeV range and whether any modification to detector configurations will improve the sensitivity to this parameter. A detailed realistic analysis with detectors that can separate ν_e , $\bar{\nu}_e$, ν_μ , and $\bar{\nu}_\mu$ is beyond the scope of this paper and is a work in progress.

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Note added.—Recently, Ref. [36] has discussed a similar more detailed analysis of atmospheric neutrino events in DUNE. The hierarchy dependence has been noticed but its origin not commented upon, in contrast to this paper.

APPENDIX: DETAILS OF HIERARCHY INDEPENDENCE AT LOW ENERGIES

The transition probability in vacuum can be expressed as

$$P_{\alpha\beta}^{\text{vac}} = -4\text{Re}[U_{\alpha 2}U_{\beta 2}^*U_{\alpha 1}^*U_{\beta 1}]\sin^2(1.27\Delta m_{21}^2 L/E) \\ - 2\text{Re}[U_{\alpha 3}U_{\beta 3}^*(\delta_{\alpha\beta} - U_{\alpha 3}^*U_{\beta 3})] \\ + 2\text{Im}[U_{\alpha 2}U_{\beta 2}^*U_{\alpha 1}^*U_{\beta 1}]\sin(2.53\Delta m_{21}^2 L/E). \quad (\text{A1})$$

Since the probability is averaged over L/E , it is independent of Δm_{32}^2 , so that there is no hierarchy ambiguity. In particular, the transition probabilities are given by

$$P_{e\mu} = A + B \cos \delta - C \sin \delta = \bar{P}_{\mu e}; \\ P_{\mu e} = A + B \cos \delta + C \sin \delta = \bar{P}_{e\mu}, \quad (\text{A2})$$

where the coefficients are given by

$$A = c_{13}^2 \sin^2(2\theta_{12})(c_{23}^2 - (s_{23}s_{13})^2) \sin^2(\delta_{21}/2) \\ + \frac{1}{2}s_{23}^2 \sin^2(2\theta_{13}), \\ B = \frac{1}{4}c_{13} \sin(4\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \sin^2(\delta_{21}/2), \\ C = \frac{1}{4}c_{13} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \sin(\delta_{21}), \quad (\text{A3})$$

where $\delta_{21} = 2.53\Delta m_{21}^2 L/E$. Here A , B , C are limited only by precision measurements of oscillation parameters. The CP asymmetry can be expressed as

$$A_{CP} = \frac{P_{e\mu} - P_{\mu e}}{P_{e\mu} + P_{\mu e}} = -\frac{C}{A + B \cos \delta} \sin \delta, \\ \bar{A}_{CP} = \frac{\bar{P}_{e\mu} - \bar{P}_{\mu e}}{\bar{P}_{e\mu} + \bar{P}_{\mu e}} = \frac{C}{A + B \cos \delta} \sin \delta, \quad (\text{A4})$$

for ν and $\bar{\nu}$, respectively.

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