$(g-2)_{\mu,e}$ and the ANITA anomalous events in a three-loop neutrino mass model

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The most recent measurement of the fine structure constant leads to a 2.4σ deviation in the electron anomalous magnetic moment $g_e - 2$, while the muon anomalous magnetic moment $g_\mu - 2$ has a long-standing 3.7σ deviation in the opposite direction. We show that these deviations can be explained in a three-loop neutrino mass model based on an E_6 Grand Unified Theory. We also study the impact such a model can have on the anomalous events observed by the ANITA experiment and find an insufficient enhancement of the event rate.

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I. INTRODUCTION

The Standard Model (SM) of particle physics, while a very successful and mathematically consistent theory, is challenged by a variety of experimental and theoretical puzzles. The neutrino oscillation data [1,2] conclusively require neutrinos to have small but nonzero masses, and although a plethora of models exists in the literature for generating such masses, keeping the new dynamics at an energy scale accessible by current and near-future experiments proves nontrivial.

There are, additionally, some less certain but equally intriguing puzzles. One of the long-standing deviations of the experimental data from the theoretical predictions of the SM is the anomalous magnetic moment of the muon, $a_\mu = (g_\mu - 2)$. There is a 3.7σ discrepancy between the experimental results [3,4] and theoretical predictions [5–8]. This has recently been compounded with a 2.4σ discrepancy between the experiment [9,10] and theory [11] values of a_e . Moreover, the deviations are in opposite directions, meaning that the most straightforward beyond the SM (BSM)

explanations are insufficient. Also, $\Delta a_e / \Delta a_\mu$ does not follow the lepton mass scaling $m_e^2 / m_\mu^2 \sim 2.25 \times 10^{-5}$, which means that a model with new flavor structure in the leptonic sector would be required to explain the discrepancy.

Another interesting experimental result is the observation of the two up-going ultra-high-energy cosmic ray air shower events by the ANITA Collaboration [12,13]. In principle, the events could be explained by a ν_τ that up-scatters inside the Earth into a τ and then decays hadronically upon emergence. However, the Earth is far too opaque for both the neutrino and the charged lepton at the measured energy (about 0.6 EeV) and angle (about 30° below the horizon) [14], requiring a neutrino flux that exceeds the current limits from the Pierre Auger Observatory [15] and IceCube [16].

From a big picture perspective, the unification of the electromagnetic and weak forces, the cancellation of gauge anomalies, and the near intersection of the gauge couplings at high energies in the SM all hint at a Grand Unified Theory (GUT). In a previous work, an E_6 GUT inspired model was shown to accommodate the neutrino masses and mixing parameters with TeV scale physics [17]. Such a low scale was made possible by forbidding mass diagrams below three-loop order. The flavor structure introduced in that work has the potential to explain the $g - 2$ anomalies. In this work, we show that the scalar and fermionic degrees of freedom in that model together with the correct flavor structure can provide the necessary corrections to $g - 2$ of both the muon and electron. We also study the implications on the ANITA observations and find that, while the expected event rate can be enhanced, explaining the two anomalous events remains challenging.

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The outline of the paper is as follows. In Sec. II, we present the model. We define all the necessary physical fermions and scalar particles in Sec. III. In Sec. IV, we briefly discuss the neutrino mass generation. We study the $g-2$ for both electron and muon in Sec. V. In Sec. VI, we discuss the ANITA anomalous events. Finally, we conclude in Sec. VII.

II. $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_Y$ MODEL FROM E_6 GUT

In this section, we briefly describe the gauge symmetry and the field content of the model introduced in Ref. [17]. A maximal subgroup of E_6 is $SU(6) \times SU(2)_N$, while a maximal subgroup of $SU(6)$ is $SU(5) \times U(1)'$. We assume the latter is broken down to generate the SM gauge groups while the $SU(2)_N$ survives down to low energy and is assumed not to affect the electric charge operator which retains the form $Q = T_{3L} + Y$. The particle content and the associate charges under $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_Y$ is as follows,

$$\begin{aligned} Q_i &\sim \begin{pmatrix} u_i \\ d_i \end{pmatrix} \sim \left(3, 2, 1, \frac{1}{6}\right), & U_i^c &\sim \left(\bar{3}, 1, 1, -\frac{2}{3}\right), \\ D_i^c &\sim (d_i^c \quad d_i^c) \sim \left(\bar{3}, 1, 2, \frac{1}{3}\right), \\ D_i &\sim \left(3, 1, 1, -\frac{1}{3}\right), & L_i &\sim \begin{pmatrix} E_i^0 & \nu_i \\ E_i^- & e_i^- \end{pmatrix} \sim \left(1, 2, 2, -\frac{1}{2}\right), \\ E_i^c &\sim (1, 1, 1, 1), \end{aligned}$$

$$L_i' \sim \begin{pmatrix} E_i^+ \\ \bar{E}_i^0 \end{pmatrix} \sim \left(1, 2, 1, \frac{1}{2}\right),$$

$$N_i^c \sim (n_{1i}^c \quad n_{2i}^c) \sim (1, 1, 2, 0),$$

$$F_i \sim \begin{pmatrix} F_{3i} & F_{1i} \\ F_{2i} & -F_{3i} \end{pmatrix} \sim (1, 1, 3, -1),$$

$$F_i^c \sim \begin{pmatrix} F_{3i}^c & F_{1i}^c \\ F_{2i}^c & -F_{3i}^c \end{pmatrix} \sim (1, 1, 3, 1),$$

where $i = 1, 2, 3$, are the generation index. The fundamental representation of E_6 , i.e., **27**, can accommodate Q_i , U_i^c , D_i^c , D_i , L_i , E_i^c , L_i' , and N_i^c , while the vectorlike fermions F_i and F_i^c come from the **351** and **$\bar{351}$** representations of E_6 , respectively. The embedding of all fields in full GUT multiplets ensures that the gauge anomalies are automatically canceled.

The scalar sector of the model has four scalar fields with the following charge assignment:

$$\begin{aligned} H_d &\sim \left(1, 2, 2, -\frac{1}{2}\right), & H_u &\sim \left(1, 2, 1, \frac{1}{2}\right), \\ S^0 &\sim (1, 1, 2, 0), & T &\sim \left(1, 2, 2, \frac{3}{2}\right). \end{aligned}$$

One **27** representation can give H_d , H_u , and S^0 , while one **650** gives the bidoublet scalar field T .

The most general renormalizable scalar potential is the following,¹

$$\begin{aligned} V = & m_1^2 \text{Tr}[H_d^\dagger \cdot H_d] + m_2^2 H_u^\dagger \cdot H_u + m_s^2 S^0 \cdot S^{0\dagger} + m_T^2 \text{Tr}[T^\dagger \cdot T] + \frac{\lambda_2}{2} (H_u^\dagger \cdot H_u)^2 + \frac{\lambda_1}{2} (\text{Tr}[H_d^\dagger \cdot H_d])^2 \\ & + \frac{\lambda_3}{2} \text{Tr}[H_d^\dagger \cdot H_d \cdot H_d^\dagger \cdot H_d] + \frac{\lambda_s}{2} (S^0 \cdot S^{0\dagger})^2 + \frac{\lambda_6}{2} (\text{Tr}[T^\dagger \cdot T])^2 + \frac{\lambda_7}{2} \text{Tr}[T^\dagger \cdot T \cdot T^\dagger \cdot T] \\ & + \lambda_4 (H_u^\dagger \cdot H_u) \text{Tr}[H_d^\dagger \cdot H_d] + \lambda_5 H_u^\dagger \cdot H_d \cdot H_d^\dagger \cdot H_u + \lambda_8 (H_u^\dagger \cdot H_u) (S^0 \cdot S^{0\dagger}) + \lambda_9 (S^0 \cdot S^{0\dagger}) \text{Tr}[H_d^\dagger \cdot H_d] \\ & + \lambda_{10} S^0 \cdot H_d^\dagger \cdot H_d \cdot S^{0\dagger} + \lambda_{11} (S^0 \cdot S^{0\dagger}) \text{Tr}[T^\dagger \cdot T] + \lambda_{12} S^0 \cdot T^\dagger \cdot T \cdot S^{0\dagger} + \lambda_{13} (H_u^\dagger \cdot H_u) \text{Tr}[T^\dagger \cdot T] + \lambda_{14} H_u^\dagger \\ & \cdot T \cdot T^\dagger \cdot H_u + \lambda_{15} \text{Tr}[H_d^\dagger \cdot H_d] \text{Tr}[T^\dagger \cdot T] + \lambda_{16} \text{Tr}[H_d^\dagger \cdot H_d \cdot T^\dagger \cdot T] + \lambda_{17} \text{Tr}[H_d^\dagger \cdot T] \text{Tr}[T^\dagger \cdot H_d] \\ & + \lambda' [\tilde{H}_u \cdot H_d \cdot \tilde{S}^0 + \text{H.c.}] + \lambda [\text{Tr}[\tilde{T}^T \cdot \tilde{H}_d] \text{Tr}[\tilde{H}_d^T \cdot \tilde{H}_d] + \tilde{T}^T \cdot \tilde{H}_d \cdot \tilde{H}_d^T \cdot \tilde{H}_d + \text{H.c.}], \end{aligned} \quad (1)$$

where all the parameters are real and $\tilde{X}$ denotes a single contraction with an antisymmetric tensor.

The Yukawa potential is given by

$$\begin{aligned} -\mathcal{L}_{\text{Yukawa}} = & y_{1ij} \text{Tr}[\tilde{L}_i \cdot \tilde{F}_j \cdot \tilde{T}^T] + y_{2ij} \text{Tr}[\tilde{L}_i \cdot \tilde{F}_j^\dagger \cdot \tilde{H}_d] + y_{3ij} \tilde{Q}_i \cdot H_d \cdot \tilde{D}_j^c + y_{4ij} \tilde{Q}_i \cdot H_u U_j^c \\ & + y_{5ij} \tilde{D}_i^c \cdot S^0 D_j + y_{6ij} \tilde{L}_j' \cdot L_i \cdot \tilde{S}^0 + y_{7ij} \tilde{L}_i' \cdot H_d \cdot \tilde{N}_j^c + y_{8ij} \text{Tr}[\tilde{L}_i^T \cdot \tilde{H}_d] E_j^c \\ & + y_{9ij} \text{Tr}[\tilde{H}_u \cdot L_i \cdot \tilde{N}_j^c] + \frac{1}{2} M_{ij} F_i F_j + m_{Nij} N_i^c N_j^c. \end{aligned} \quad (2)$$

For simplicity, we assume $M_{ij} = M_i \delta_{ij}$. We impose a discrete Z_2 symmetry such that only N_i^c is odd under the Z_2 symmetry while all the other particles are even. One implication of this Z_2 symmetry is that the y_{7ij} and y_{9ij}

¹We suppress the gauge indices and refer the reader to the previous work where this model was introduced [17].

terms in Eq. (2) are forbidden. The lightest component of N_i^c is therefore stable and may contribute to the dark matter density.

III. PHYSICAL SCALARS AND FERMIONS

We define the physical scalar particles and fermions in the mass basis necessary for the calculations in this section. The total number of scalar degrees of freedom is 24, out of which 6 are eaten by the massive gauge bosons. This leaves us with 18 physical scalars. After the spontaneous symmetry breaking, the Higgs scalars acquire vacuum expectation values (vevs), and we can write them as

$$\begin{aligned} H_d &\sim \begin{pmatrix} \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) & \frac{1}{\sqrt{2}}(\rho_3 + i\eta_3) \\ \phi_1^- & \phi_3^- \end{pmatrix}, & T &\sim \begin{pmatrix} T_1^{++} & T_2^{++} \\ T_1^+ & T_2^+ \end{pmatrix}, \\ H_u &\sim \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix}, & S^0 &\sim \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho_{1s} + i\eta_{1s}) & \frac{1}{\sqrt{2}}(v_s + \rho_{2s} + i\eta_{2s}) \end{pmatrix}, \end{aligned} \quad (3)$$

where the vev of S^0 (v_s) is $\mathcal{O}(10-100)$ TeV and breaks the $SU(2)_N$ symmetry, while the vevs of H_d and H_u (v_1 and v_2 , respectively) are $\mathcal{O}(10-100)$ GeV and break the electro-weak gauge symmetry.

The charged states $\phi_1^\pm$ and $\phi_2^\pm$ will mix with a mixing angle β and give the charged physical scalars $h_1^\pm$ and $h_2^\pm$, respectively, with masses m_{h_1} and m_{h_2} ,

$$\begin{aligned} h_1^\pm &= \cos\beta \phi_1^\pm + \sin\beta \phi_2^\pm, \\ h_2^\pm &= -\sin\beta \phi_1^\pm + \cos\beta \phi_2^\pm, \end{aligned} \quad (4)$$

where $\tan\beta = v_2/v_1$. Similarly, four more charged physical scalars $H_1^\pm$ and $H_2^\pm$ with masses m_{H_1} and

m_{H_2} , respectively, can arise from the mixing of $\phi_3^\pm$ and $T_2^\pm$:

$$\begin{aligned} H_1^\pm &= \cos\theta \phi_3^\pm + \sin\theta T_2^\pm, \\ H_2^\pm &= -\sin\theta \phi_3^\pm + \cos\theta T_2^\pm. \end{aligned} \quad (5)$$

There is a total of five neutral CP even states, out of which ρ_1 , ρ_2 , and ρ_{2s} will mix, resulting in three physical neutral scalars s_1 , s_2 , and s_{2s} with masses m_{s_1} , m_{s_2} , and $m_{s_{2s}}$, respectively. We identify the s_1 as the SM physical Higgs field, h , with mass m_h . The fields in the mass basis, s_i , are related to those in the interaction basis, ρ_i , by a rotation matrix R , which can be parametrized with three angles θ_1 , θ_2 , and θ_3 as follows,

$$R^{-1} = \begin{pmatrix} c_{\theta_1}c_{\theta_3} - c_{\theta_2}s_{\theta_1}s_{\theta_3} & -c_{\theta_1}s_{\theta_3} - c_{\theta_2}c_{\theta_3}s_{\theta_1} & s_{\theta_1}s_{\theta_2} \\ c_{\theta_3}s_{\theta_1} + c_{\theta_1}c_{\theta_2}s_{\theta_3} & c_{\theta_1}c_{\theta_2}c_{\theta_3} - s_{\theta_1}s_{\theta_3} & -c_{\theta_1}s_{\theta_2} \\ s_{\theta_2}s_{\theta_3} & c_{\theta_3}s_{\theta_2} & c_{\theta_2} \end{pmatrix} \equiv \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}, \quad (6)$$

where $\rho = R^{-1}s$, $c_{\theta_i} = \cos\theta_i$, and $s_{\theta_i} = \sin\theta_i$ ($i = 1, 2, 3$). We can parametrize the mixing of three neutral CP -odd states, η_1 , η_2 , and η_{2s} , in a similar way with $\eta = R'^{-1}s'$, and the three angles are denoted as θ'_1 , θ'_2 and θ'_3 . We only get one physical neutral pseudoscalar s'_1 with mass $m_{s'_1}$. The interaction states can be written as $\eta_1 = c'_{13}s'_1$, $\eta_2 = c'_{23}s'_1$, and $\eta_{2s} = c'_{33}s'_1$.

The rest of the two neutral CP -even states, ρ_3 and ρ_{1s} , give one physical neutral scalar, s_3 , with mass m_{s_3} . The 2×2 mixing matrix can be parametrized by an angle ψ as

$$R_{2 \times 2}(\psi) = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix}. \quad (7)$$

The interaction states can be written as $\rho_3 = -\sin\psi s_3$ and $\rho_{1s} = \cos\psi s_3$. Similar mixing happens between the two neutral CP -odd states η_3 and η_{1s} . We parametrize the mixing matrix with an angle, ψ' . One physical pseudoscalar

s'_3 with mass $m_{s'_3}$ is generated. In terms of this physical pseudoscalar, we write $\eta_3 = -\sin\psi' s'_3$ and $\eta_{1s} = \cos\psi' s'_3$.

For the purpose of calculating the contributions to the anomalous magnetic moment and the ANITA observations, the relevant terms from Eq. (2) can be written as follows,

$$\begin{aligned} -\mathcal{L} &= y_{2ii}(-E_i^0\phi_1^- + \nu_i\phi_3^- + E_i^- \phi_1^0 - e_i^- \phi_3^0)F_{3i}^c \\ &+ y_{3ij}[(u_i\phi_1^- - d_i\phi_1^0)d_j^c - (u_i\phi_3^- - d_i\phi_3^0)d_j^c] \\ &+ y_{5ij}(d_i^c S_2^0 - d_i^c S_1^0)D_j + y_{6ii}[(E_i^0\bar{E}_i^0 - E_i^- E_i^+)S_2^0 \\ &- (\nu_i\bar{E}_i^0 - e_i^- E_i^+)S_1^0] + y_{8ii}(E_i^0\phi_3^- - \nu_i\phi_1^- - E_i^- \phi_3^0 \\ &+ e_i^- \phi_1^0)E_i^c + \frac{1}{2}M_i iF_i F_i^c + \text{H.c.}, \end{aligned} \quad (8)$$

where $\phi_{1,2,3}^0$ are neutral components of H_u and H_d [see Eq. (3)]. The vectorlike leptons E_i^- and F_{3i} mix and give rise to two charged physical vectorlike leptons given by

TABLE I. Brief descriptions of the physical scalars and fermions required for the calculations. Here, n_1 and n_2 are the dark matter candidates. The E^0 decays only to neutral fields, leading to a missing energy signal at the LHC.

Particle type	Particles	Mass parameters	Mass values	Possible final states at LHC
Charged scalars	$h_1^\pm, h_2^\pm$	m_{h_1}, m_{h_2}	$m_{h_{1,2}} \sim \mathcal{O}(500)$ GeV	$h_{1,2}^\pm \rightarrow u_i \bar{d}_j + E_i^0 + E_j^0, u_i \bar{d}_j$
	$H_1^\pm, H_2^\pm$	m_{H_1}, m_{H_2}	$m_{H_1} \sim \mathcal{O}(500)$ GeV $m_{H_2} \sim \mathcal{O}(5)$ TeV	$H_{1,2}^\pm \rightarrow u_i \bar{d}_j + \nu_i + E_j^0, d_i \bar{d}_j + e_j^- + E_i^0$
Neutral scalars	h, s_2	m_h, m_{s_2}	$m_h = 125$ GeV	$s_2, s_{2s} \rightarrow e_i^+ e_i^-, d_i \bar{d}_i$
	s_{2s}, s_3	$m_{s_{2s}}, m_{s_3}$	$m_s \sim \mathcal{O}(500)$ GeV	
Neutral pseudoscalar	s'_1, s'_3	$m_{s'_1}, m_{s'_3}$	$m_{s'} \sim \mathcal{O}(500)$ GeV	$s'_1, s'_3 \rightarrow e_i^+ e_i^-, d_i \bar{d}_i$
Charged vectorlike leptons	$f_1^\pm, f_2^\pm$	m_{f_1}, m_{f_2}	$m_f \sim \mathcal{O}(100)$ GeV	$f_{1,2i} \rightarrow u_i \bar{d}_j + E_i^0, d_i \bar{d}_j + e_i^+ + \bar{E}_i^0 + \bar{\nu}_i$
Neutral vectorlike leptons	$\bar{E}^0, E^0$	$m_{\bar{E}^0}, m_{E^0}$	$m_{\bar{E}^0} > m_{E^0} \sim \mathcal{O}(100)$ GeV	$\bar{E}_i^0 \rightarrow e_i^+ e_i^- + E_i^0, E_i^0 \rightarrow \nu_i + n_{1i} \bar{n}_{2j}$
New gauge bosons	$X_1^\mu, X_2^\mu, X_3^\mu$	$m_{X_1}, m_{X_2}, m_{X_3}$	$m_X \geq 3.6$ TeV	$X_3^\mu \rightarrow e_i^+ e_i^-, d_i \bar{d}_i$
Charged vectorlike quark	D	m_D	$m_D \sim \mathcal{O}(1)$ TeV	$D_i \rightarrow e_i^+ e_i^- + d_i + \nu_i + E_i, d_i \bar{d}_i + d_i + e_i^+ e_i^-$

$$\begin{aligned} f_{1i}^+ &= \cos \theta_f E_i^+ - \sin \theta_f F_{3i}^c \\ f_{2i}^+ &= \sin \theta_f E_i^+ + \cos \theta_f F_{3i}^c, \end{aligned} \quad (9)$$

respectively, with masses $m_{f_{1i}}$ and $m_{f_{2i}}$, where the mixing angle, θ_f , can be determined by diagonalizing the mass matrix. We summarize all the relevant fields in Table I.

IV. NEUTRINO MASSES

Here, we summarize the neutrino mass generation mechanism and the numerical results in our model and refer the reader to Ref. [17] for more details.

The tree-level Lagrangian in our model contains no mass terms for neutrinos, so one must rely on radiative mass generation (i.e., through loops). Normally, the particles running in such loops need to be made heavy to achieve the

vanishingly small neutrino masses, but due to the particle content and the associated Z_2 symmetry, the Majorana neutrino mass in our model cannot be generated below the three-loop level. The dimension-5 effective Majorana neutrino mass operator $L_i L_j H_d^* H_d^* / \mathcal{M}$, where $\mathcal{M}$ is some effective mass scale, can be realized at the three-loop level as shown in Fig. 1. The new heavy gauge bosons X_1 and X_2 play an important role in making the neutrino mass values small, and the mass matrix also gets suppressed from the loop factor $1/(16\pi)^3$. Due to all these suppressions, the new physics associated with the tiny neutrino masses can be kept at the TeV scale.

Based on the benchmark points determined in Ref. [17], the following parameter point can generate the correct neutrino observables:

$$\begin{aligned} m_{H_1} &= 5 \text{ TeV}, & m_{H_2} &= 500 \text{ GeV}, & m_X &= 5 \text{ TeV}, & m_{h_2} &= 268 \text{ GeV}, \\ m_{T_1} &= 500 \text{ GeV}, & \tan \beta &= 50, & v_1/\sqrt{2} &= 7 \text{ GeV}, & \theta &= 0.005^\circ, \\ m_f &= (110, 120, 130) \text{ GeV}, & m_{E^0} &= (105, 110, 115) \text{ GeV}. \end{aligned} \quad (10)$$

Since we assume all the gauge couplings to be unified at high energy, the value of g_2' is determined by renormalization group evolution which generically leads to values close to

FIG. 1. The three-loop diagram in the interaction basis responsible for the Majorana neutrino masses.

the SM gauge couplings. We therefore fix g_2' to be 0.35. Note that in Ref. [17] the value of $\tan \beta$ was set to be 2 in exchange for a smaller value of $y_1 \times y_2 \sim 10^{-3}-10^{-4}$ (the parametric dependence of the neutrino mass matrix is $y_1 \times y_2 \sin^2 \beta$). Since we will later need large Yukawa couplings to explain the magnetic moments, we have altered the original benchmark point while maintaining the neutrino masses. For the remainder of the paper, we assume parameters in the neighborhood of this benchmark point.

The particle spectrum is consistent with the LHC as shown in Ref. [17]. Of particular importance is that the new leptons can be abundantly produced at the LHC, but all of their decays include neutral states that are almost degenerate in mass, making them extremely difficult to find in missing energy searches. This holds true for all fields in our model

that are within the energy reach of the LHC and could, in principle, be subject to such constraints.

V. MUON AND ELECTRON ANOMALOUS MAGNETIC MOMENTS

A. Background

As mentioned in the Introduction, there is a 3.7σ discrepancy between the experimental results [3,4] and the theoretical predictions [5–8] of the anomalous magnetic moment of the muon, $a_\mu = (g_\mu - 2)$. The discrepancy was found to be

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (2.74 \pm .73) \times 10^{-9}. \quad (11)$$

The precision of the SM prediction will be improved in the future [18]. An updated measurement is expected soon from Fermilab [19,20] and J-PARC [21].

On the other hand, the direct experimental measurement of the electron anomalous magnetic moment, a_e^{exp} [9], had been in agreement with the SM prediction, a_e^{th} [11], at the level of 1.7σ until recently, when an updated value of the fine structure constant α was measured with high precision using cesium atoms [22],

$$\alpha^{-1}(Cs) = 137.035999046(27). \quad (12)$$

The result of this precise measurement of α leads to a 2.4σ discrepancy between the experiment [9,10] and theory [11] values of a_e ,

$$\Delta a_e = a_e^{\text{exp}} - a_e^{\text{th}} = (-8.7 \pm 3.6) \times 10^{-13}. \quad (13)$$

The simplest BSM explanations of these values, say a new mediator that couples to both electrons and muons, are expected to result in Δa_e and Δa_μ of the same sign, since the new physics couplings would appear twice in each diagram. Moreover, if coupling universality is assumed, then we additionally expect the corrections to scale with the lepton mass, that is $\Delta a_e/\Delta a_\mu \sim m_e^2/m_\mu^2 \sim 2.25 \times 10^{-5}$. Since neither of those is true, a more complex solution is needed. Several such solutions exist in the literature [23–29].

In this work, we rely on the diversity of Yukawa couplings in Eq. (8). These terms allow for a variety of new scalars and fermions to run in the loop, with couplings that are both chiral (different for left-handed and right-handed components) and flavor nonuniversal (different for each lepton). As we show in the next subsection, the chirality of these interactions leads certain couplings to appear only once in a given diagram, thereby allowing for corrections to a_e and a_μ in opposite directions, while the nonuniversality allows for modifying each independently of the other.

B. Calculations and results

We now proceed to present the calculations and results of the anomalous magnetic moments. We choose to work in the physical mass basis. Upon expanding Eq. (8) in terms of the physical fields, the Lagrangian that generates the necessary one-loop diagrams can be written as

$$\begin{aligned} -\mathcal{L} = & \bar{e}_i[C_{S1} + C_{P1}\gamma_5]f_{1i}s_3 + \bar{e}_i[C_{S2} + C_{P2}\gamma_5]f_{2i}s_3 + \bar{e}_i[C_{S3} + C_{P3}\gamma_5]f_{1i}s'_3 \\ & + \bar{e}_i[C_{S4} + C_{P4}\gamma_5]f_{2i}s'_3 + \bar{e}_i[C_{S5} + C_{P5}\gamma_5]e_i s_2 + \bar{e}_i[C_{S6} + C_{P6}\gamma_5]e_i s_{2s} \\ & + \bar{e}_i[C_{S7} + C_{P7}\gamma_5]e_i s'_1 + \bar{e}_i[C_{S8} + C_{P8}\gamma_5]E_i^0 H_1^- + \bar{e}_i[C_{S9} + C_{P9}\gamma_5]E_i^0 H_2^- \\ & + \bar{e}_i[C_{S10} + C_{P10}\gamma_5]\nu_i h_1^- + \bar{e}_i[C_{S11} + C_{P11}\gamma_5]\nu_i h_2^- + \text{H.c.}, \end{aligned} \quad (14)$$

where the coefficients are given by

$$\begin{aligned} C_{S1} &= \frac{1}{2\sqrt{2}}(y_{8i} \sin \psi \cos \theta_f - y_{2i} \sin \psi \sin \theta_f + y_{6i} \cos \psi \cos \theta_f), \\ C_{P1} &= \frac{1}{2\sqrt{2}}(y_{8i} \sin \psi \cos \theta_f - y_{2i} \sin \psi \sin \theta_f + y_{6i} \cos \psi \cos \theta_f), \\ C_{S2} &= \frac{1}{2\sqrt{2}}(y_{8i} \sin \psi \cos \theta_f - y_{2i} \sin \psi \sin \theta_f + y_{6i} \cos \psi \cos \theta_f), \\ C_{P2} &= \frac{1}{2\sqrt{2}}(y_{8i} \sin \psi \cos \theta_f - y_{2i} \sin \psi \sin \theta_f + y_{6i} \cos \psi \cos \theta_f), \\ C_{S3} &= \frac{1}{2\sqrt{2}}(y_{8i} \sin \psi \cos \theta_f - y_{2i} \sin \psi \sin \theta_f + y_{6i} \cos \psi \cos \theta_f), \\ C_{P3} &= \frac{1}{2\sqrt{2}}(y_{8i} \sin \psi \cos \theta_f - y_{2i} \sin \psi \sin \theta_f + y_{6i} \cos \psi \cos \theta_f), \\ C_{S4} &= \frac{1}{2\sqrt{2}}(y_{8i} \sin \psi \cos \theta_f - y_{2i} \sin \psi \sin \theta_f + y_{6i} \cos \psi \cos \theta_f), \end{aligned}$$

FIG. 2. One-loop diagrams in the mass basis which contributes to the anomalous magnetic moments of the muon and electron.

$$\begin{aligned}
 C_{P4} &= \frac{1}{2\sqrt{2}} (y_{8i} \sin \psi \cos \theta_f - y_{2i} \sin \psi \sin \theta_f + y_{6i} \cos \psi \cos \theta_f), \\
 C_{S5} &= \frac{1}{2\sqrt{2}} y_{8i} c_{12} = -C_{P5}; & C_{S6} &= \frac{1}{2\sqrt{2}} y_{8i} c_{13} = -C_{P6}, \\
 C_{S7} &= \frac{i}{2\sqrt{2}} y_{8i} c'_{13} = -C_{P7}; & C_{S8} &= \frac{1}{2} y_{8i} \cos \theta = -C_{P8}, \\
 C_{S9} &= -\frac{1}{2} y_{8i} \sin \theta = -C_{P9}; & C_{S10} &= -\frac{1}{2} y_{8i} \cos \beta = -C_{P10}, \\
 C_{S11} &= \frac{1}{2} y_{8i} \sin \beta = -C_{P11}.
 \end{aligned} \tag{15}$$

The 11 different terms in Eq. (14) can generate 11 different Feynman diagrams as shown in Fig. 2.² Note that the couplings $C_{S1}-C_{P11}$ are linear combinations of three Yukawa couplings y_2 , y_6 , and y_8 . This will lead to products of two different Yukawa couplings in various diagrams.

²We have used the package `TikZ-Feynman` [30] to draw the diagrams.

The Feynman diagrams of Fig. 2 can be broadly categorized into two categories: one with a neutral scalar inside the loop and one with a charged scalar. Each term in the above Lagrangian takes the general form

$$-\mathcal{L} = \bar{e}_i [C_S + C_P \gamma_5] f_i s, \tag{16}$$

where f and S respectively denote the fermion and scalar fields that run in the loop.

Following Ref. [31], the contribution from the first type of diagrams with a neutral scalar can be written as

$$\Delta a_{ei}^1(C_S, C_P, m_{fi}, m_s) = \frac{-q_{fi} m_{ei}^2}{8\pi^2} \int_0^1 dx \frac{[C_S^2(x^2 - x^3 + \frac{m_{fi}}{m_{ei}} x^2) + C_P^2(m_{fi} \rightarrow -m_{fi})]}{m_{ei}^2 x^2 + (m_{fi}^2 - m_{ei}^2)x + m_s^2(1-x)}, \quad (17)$$

and the contribution of the second type of diagrams with a charged scalar can be written as

$$\Delta a_{ei}^2(C_S, C_P, m_{fi}, m_s) = \frac{-q_h m_{ei}^2}{8\pi^2} \int_0^1 dx \frac{[C_S^2(x^2 - x^3 + \frac{m_{fi}}{m_{ei}}(x^2 - x)) + C_P^2(m_{fi} \rightarrow -m_{fi})]}{m_{ei}^2 x^2 + (m_s^2 - m_{ei}^2)x + m_{fi}^2(1-x)}. \quad (18)$$

In the following, we discuss the various contributions to Δa_{ei} :

- (i) The first four terms of Eq. (14) give four diagrams where we have the vectorlike leptons, $f_{1,2}$, and the new neutral scalar particles, s_3 and s'_3 , inside the loop. Figure 2(a) shows these diagrams, and their contributions to Δa_{ei} is given by Eq. (17). From Eqs. (15) and (17), we get quadratic terms in the Yukawa couplings (y_{2i}^2 , y_{6i}^2 , and y_{8i}^2) as well as cross-terms ($y_{2i} \times y_{8i}$, $y_{2i} \times y_{6i}$, and $y_{6i} \times y_{8i}$). The quadratic terms are proportional to m_{ei}^2 , while the cross-terms are proportional to $m_{fi} \times m_{ei}$. With a fermion mass $m_{fi} \sim 100$ GeV, the cross-terms of Yukawa couplings can lead to contributions that are both large and of opposite signs for the muon and electron cases.
- (ii) The next three terms of Eq. (14) give three more diagrams in Fig. 2(b) with the new neutral scalars s_2 , s_{2s} , and s'_1 inside the loop along with the SM muons and electrons. Together with the Hermitian conjugates, these terms become simple, for example, $2C_{S5}\bar{e}_i e_i s_2$. Their contributions to Δa_{ei} are given by Eq. (17). All the diagrams are proportional to the SM Yukawa couplings, y_{8i}^2 , and hence suppressed by m_{ei}^2 . Therefore, their contribution to Δa_{ei} is small compared to the first four diagrams. Since we do not

rely on these contributions, the masses of the scalars involved are thus far not fixed.

- (iii) Figures 2(c) and 2(d) give four more diagrams arising from the last four terms in Eq. (14). All four diagrams have charged scalars ($H_{1,2}^\pm$ and $h_{1,2}^\pm$) and neutral fermions such as E_i^0 and ν_i inside the loop. Their contributions to Δa_{ei} are given in Eq. (18). These diagrams are also suppressed compared to the diagrams with cross-terms due to the SM Yukawa factors y_{8i}^2 . Additionally, since these particles also enter into the three-loop diagrams (Fig. 1) needed for neutrino mass generation, their masses are already fixed in our model, and so they do not play important roles in the Δa_{ei} calculations.
- (iv) In addition to these 11 scalar loop diagrams contributing to Δa_{ei} , we do get contributions from the gauge bosons associated with the new gauge group $SU(2)_N$. The lower limit on the new gauge boson masses is approximately 3.6 TeV [17,32,33], assuming the gauge coupling g'_2 is 0.35. These diagrams are also suppressed by the square of lepton masses. Therefore, these diagrams are small compared to the scalar diagrams of Fig. 2(a), and their contributions can be neglected.

For completeness, we consider the contributions to Δa_{ei} from all the scalar diagrams. The total contribution to Δa_{ei} can be expressed in a simple form:

$$\begin{aligned} \Delta a_{ei} = & \Delta a_{ei}^1(C_{S1}, C_{P1}, m_{f_{1i}}, m_{s_3}) + \Delta a_{ei}^1(C_{S2}, C_{P2}, m_{f_{2i}}, m_{s_3}) + \Delta a_{ei}^1(C_{S3}, C_{P3}, m_{f_{1i}}, m_{s'_3}) \\ & + \Delta a_{ei}^1(C_{S4}, C_{P4}, m_{f_{2i}}, m_{s'_3}) + \Delta a_{ei}^1(C_{S5}, m_{ei}, m_{s_2}) + \Delta a_{ei}^1(C_{S6}, m_{ei}, m_{s_{2s}}) \\ & + \Delta a_{ei}^1(C_{S7}, m_{ei}, m_{s'_1}) + \Delta a_{ei}^2(C_{S8}, C_{P8}, m_{E_i^0}, m_{H_1}) + \Delta a_{ei}^2(C_{S9}, C_{P9}, m_{E_i^0}, m_{H_2}) \\ & + \Delta a_{ei}^2(C_{S10}, C_{P10}, m_{\nu_i}, m_{h_1}) + \Delta a_{ei}^2(C_{S11}, C_{P11}, m_{\nu_i}, m_{h_2}). \end{aligned} \quad (19)$$

In order to find a working parameter point, we begin by fixing the dimensionless parameters that are not yet set by neutrino masses and then vary the masses of new fields. The y_8 's are fixed by the SM charged lepton masses by the relation $m_{ei} = y_{8i} v_1 / \sqrt{2}$, and we therefore set them to be $y_{8e} = -7 \times 10^{-5}$, $y_{8\mu} = 0.015$, and $y_{8\tau} = 0.25$. Other necessary coupling constants can be taken as follows: $y_{2e} = 0.80$, $y_{6e} = 2.50$, $y_{2\mu} = 0.50$, and $y_{6\mu} = 0.25$.

For simplicity, we choose the mixing angles ψ , ψ' , θ_f , $\theta_{1,2,3}$, and $\theta'_{1,2,3}$ to be 45° . In Table II, we give five different sets of values of the fermion and scalar masses that play an important role in the Δa_μ and Δa_e calculations. As we discussed earlier, the dominant contributions to Δa_{ei} are coming mainly from the diagrams of Fig. 2(a), in particular from the terms with the product of two different Yukawa couplings. This Yukawa structure allows for

TABLE II. Five benchmark points are shown and can account for the discrepancy in the anomalous magnetic moments of the muon and electron as well as be consistent with neutrino mass calculations.

Benchmark point	m_{f_1} (GeV)	m_{f_2} (GeV)	m_{s_3} (GeV)	$m_{s'_3}$ (GeV)
BP1	120	121	350	1985
BP2	120	135	350	1121
BP3	120	102	350	1578
BP4	120	118	350	570
BP5	120	145	350	2150

TABLE III. The scan range used to generate Fig. 3. The fixed parameters are $y_{2e} = 0.9$, $y_{2\mu} = 0.5$, $m_{f_1} = 120$ GeV, and $m_{s_3} = 350$ GeV.

Parameter	Range
y_{6e}	0.6–3.0
$y_{6\mu}$	0.001–2.0
m_{f_2}	100–150 GeV
$m_{s'_3}$	300–2500 GeV

the sought-after violation of the scaling dependence $\Delta a_e / \Delta a_\mu \propto m_e^2 / m_\mu^2$, leading instead to a ratio of m_e / m_μ and without a constraint on the sign. The particles necessary to produce these dominant contributions are $f_{1,2}$, s_3 , and s'_3 . This is true for any value of the parameters in Eq. (10).

In order to get of sense of how readily the model fits the observations, we perform a random scan over some of the parameters going into the $g - 2$ calculation. We limit the scan to a subset of four parameters for tractability. The diagrams in Fig. 2 suggest that the dependence on m_{f_1} and m_{s_3} is similar to that on m_{f_2} and $m_{s'_3}$, respectively.

We, therefore, fix the former and scan over the latter. We similarly choose to fix $y_{2e,\mu}$ and vary $y_{6e,\mu}$.

We sample 100,000 points at random from the range shown in Table III with $y_{2e} = 0.9$, $y_{2\mu} = 0.5$, $m_{f_1} = 120$ GeV, and $m_{s_3} = 350$ GeV. In Fig. 3, we show the results as a scatter plot in the $\Delta a_\mu - \Delta a_e$ plane along with the 1σ bands of Eqs. (11) and (13) ($\Delta a_\mu = (2.74 \pm .73) \times 10^{-9}$ and $\Delta a_e = (-8.7 \pm 3.6) \times 10^{-13}$). About 2800 points fall into the intersection of the two bands. We can see that, while a wide range of Δa_μ can be achieved, the values of Δa_e mostly lie on the upper end of the band.

In experimenting with other scanning schemes, we find that, indeed, allowing for the fixed parameters to vary leads to no significant expansion of the Δa_e reach. We also find that there is very strong dependence on the value of y_{6e} . Namely, expanding the lower scan limit of y_{6e} down to 0.01 leads to a dramatic drop in the density of viable points.

VI. ANITA

A. Background

ANITA is an Antarctic balloon experiment that looks for ultra-high-energy cosmic rays by detecting the associated geosynchrotron emissions using a series of radio antennas [34]. The flight altitude of over 30 km leads to a 10^6 km² coverage of the Antarctic ice, which compensates for the limited operation time compared to IceCube and the Pierre Auger Observatory and offers sensitivity to a complementary range of energies and phenomena. In particular, the polarity of the geosynchrotron radiation is correlated with the Earth's magnetic field, allowing ANITA to discriminate between showers emerging directly from the Earth and down-going showers that are reflected off the ice. During the first and third flights of ANITA, 2 out of the 36 detected events stood out.

FIG. 3. A scatter plot in the $\Delta a_\mu - \Delta a_e$ plane of about 16,800 points from a scan of 100,000 parameter points randomly selected from the range in Table III. The bands represent the 1σ deviation given by $\Delta a_\mu = (2.74 \pm .73) \times 10^{-9}$ and $\Delta a_e = (-8.7 \pm 3.6) \times 10^{-13}$. The number of points that fall into the intersection of the two bands is 2773.

During its first flight, ANITA-I [12], an event with an energy of 0.6 ± 0.4 EeV was detected at a zenith angle of $z = 117.4^\circ \pm 0.3^\circ$ with a noninverted polarity, suggesting that the event emerged from the Earth rather than reflected off the ice. More recently, a similar event, with noninverted polarity, has been found in the ANITA-III data [13], with an energy of $0.56^{+0.30}_{-0.20}$ EeV and a zenith angle of $z = 125^\circ \pm 0.3^\circ$. At such high energies, the survivability of neutrinos passing through such a long arc length within the Earth is very low, and the isotropic neutrino flux required to explain the events with a ν_τ that up-scatters to a τ has been found to be at least 2 [35] and possibly 6 [14] orders of magnitude larger than the limits set by the Pierre Auger Observatory and IceCube. The τ emergence probability has also been studied in Ref. [36].

One approach around this is to replace the τ with an intermediary field with a long lifetime and large survivability in matter which decays to a τ upon or right before emergence. In our case, we rely on the new neutral field $\bar{E}^0$, which is produced via the scattering of a ν_τ off of the Earth matter. The new heavy field can then propagate through the Earth, emerge at the South Pole, and generate a τ as part of its decay products, which can then decay hadronically, leading to an observable signal at ANITA. A similar approach has been considered in Refs. [37–42].

There are, however, several caveats to the story. It has been shown that solutions that rely on a neutrino flux still exceed the astrophysical bounds despite the higher rate of τ emergence compared to the SM-only scenario (see, for example, Ref. [43]). This is true for an isotropic neutrino flux or a long-lasting point source, so one needs to postulate a rather exotic astrophysical source of neutrinos to fully explain the ANITA observation. Explanations that rely on a source other than a neutrino flux, say dark matter, do not suffer from this drawback [43–50], although see the discussion in Ref. [51]. A summary of feasibility of various approaches can be found in the conference proceedings in Ref. [52].

A second caveat is that any BSM explanation of the ANITA events will have to confront the lack of similar events in IceCube [14]. Depending on the mechanism utilized, the dynamics could conspire to produce a signal in

one experiment but not the other. Intriguingly, building upon a proposal by Ref. [53], Ref. [14] shows that a small reported tension between the northern track and the full sky spectra measured by IceCube could be alleviated if some of the up-going muon events are interpreted as misidentified tau events, and they identify three such events by calculating the probability of emergence as a function of energy and angle. If true, these events would have higher energy than reported by IceCube, thus adding support to the ANITA observations. This interpretation, however, does not improve the viability of a neutrino flux explanation at either experiment.

A final caveat is that non-BSM explanations of the ANITA observations are still possible. Very recently, it was proposed that an additional electromagnetic component can be emitted during the shower's transit across the ice surface with a noninverted polarity [54]. Another recent paper proposed that some ice features can lead to reflection without an inversion of polarity [55]. Two earlier proposals that were not able to account for the anomaly were transition radiation through the air [56] and a more accurate treatment of the reflections of the electromagnetic waves [57].

With all of that mind, we content ourselves with an estimate of the emergence fraction of taus in our model, using the SM fraction from Ref. [14] as a reference point.

B. Calculations and results

To enhance the rate of τ emergence, we identify a particle that can be produced from neutrino scattering off of nuclei, survive passage through Earth along a chord length of about $l \sim 5740\text{--}7210$ km corresponding to the angles of two ANITA events, and decay to a τ lepton. The neutral vectorlike lepton $\bar{E}^0$ from the doublet L' can serve such a purpose. It can be produced in the ν_τ -nucleon scattering mediated by the neutral scalars s_3 and s'_3 along with a heavy quark D_i as shown in Fig. 4(a). It can then propagate through the Earth without significant attenuation and finally decay into a τ lepton pair and another heavy neutral particle as shown in Fig. 4(b).

We start our analysis by writing the necessary terms of the Lagrangian in the mass basis

$$\begin{aligned}
 -\mathcal{L} = & -\frac{y_{5ij} \cos \psi}{2\sqrt{2}} \bar{d}_i^c (1 - \gamma_5) D_j s_3 - \frac{iy_{5ij} \cos \psi'}{2\sqrt{2}} \bar{d}_i^c (1 - \gamma_5) D_j s'_3 - \frac{y_{6ij} \cos \psi}{2\sqrt{2}} \bar{E}_i^0 (1 - \gamma_5) \nu_j s_3 - \frac{iy_{6ij} \cos \psi'}{2\sqrt{2}} \bar{E}_i^0 (1 - \gamma_5) \nu_j s'_3 \\
 & + \frac{y_{6ij} c_{31}}{2\sqrt{2}} \bar{E}_i^0 (1 - \gamma_5) E_j^0 h + \frac{y_{6ij} c_{32}}{2\sqrt{2}} \bar{E}_i^0 (1 - \gamma_5) E_j^0 s_2 + \frac{y_{6ij} c_{33}}{2\sqrt{2}} \bar{E}_i^0 (1 - \gamma_5) E_j^0 s_{2s} + \frac{iy_{6ij} c'_{33}}{2\sqrt{2}} \bar{E}_i^0 (1 - \gamma_5) E_j^0 s'_1 \\
 & - \frac{y_{3ij} c_{11}}{2\sqrt{2}} \bar{d}_i (1 - \gamma_5) d_j h - \frac{y_{3ij} c_{12}}{2\sqrt{2}} \bar{d}_i (1 - \gamma_5) d_j s_2 - \frac{y_{3ij} c_{13}}{2\sqrt{2}} \bar{d}_i (1 - \gamma_5) d_j s_{2s} - \frac{iy_{3ij} c'_{13}}{2\sqrt{2}} \bar{d}_i (1 - \gamma_5) d_j s'_1 \\
 & + \frac{y_{8ij} c_{11}}{2\sqrt{2}} \bar{e}_i (1 - \gamma_5) e_j h + \frac{y_{8ij} c_{12}}{2\sqrt{2}} \bar{e}_i (1 - \gamma_5) e_j s_2 + \frac{y_{8ij} c_{13}}{2\sqrt{2}} \bar{e}_i (1 - \gamma_5) e_j s_{2s} + \frac{iy_{8ij} c'_{13}}{2\sqrt{2}} \bar{e}_i (1 - \gamma_5) e_j s'_1 + \text{H.c.} \quad (20)
 \end{aligned}$$

FIG. 4. Feynman diagrams in mass basis for (a) neutrino-nucleon interaction and (b) the decay of the neutral vectorlike lepton to tau final state.

The first four terms of Eq. (20) are needed for the neutrino-nucleon cross sections, and the rest of the terms are responsible for the decay of $\bar{E}^0$. The Feynman diagrams of the scattering process and decay channel in the mass basis are shown in Figs. 4(a) and 4(b), respectively.

We consider cosmic neutrinos of energies about $\mathcal{O}$ (EeV) which scatter off of matter producing $\bar{E}^0$ along with hadrons. In the limit of large momentum transfer, the differential cross section is given by

$$\frac{d^2\sigma}{dx dy} = \frac{3y_5^2 y_{6\tau}^2}{256\pi M E_\nu x^2} (\cos^2\psi - \cos^2\psi')^2 \times \sum_i [x f_i(x, \sqrt{2ME_\nu xy}) + x \bar{f}_i(x, \sqrt{2ME_\nu xy})], \quad (21)$$

where $Q^2 = 2ME_\nu xy$ is the momentum transfer and $f_i(x, Q^2)$ and $\bar{f}_i(x, Q^2)$ are the parton distribution functions (pdfs) of the quarks and antiquarks, respectively. M is the nucleon mass, E_ν is the incoming neutrino energy, while x and y are dimensionless variables defined as

$$x = \frac{Q^2}{2M\nu} \quad \text{and} \quad y = \frac{\nu}{E_\nu}, \quad (22)$$

where $\nu = E_\nu - E_{\bar{E}^0}$ is the energy loss in the laboratory frame. The momentum transfer is always greater than the nucleon mass. y is the fractions of the initial neutrino energy transferred to the hadrons. The total cross section can be obtained by integrating Eq. (21) over x and y ,

$$\sigma = \int_{y_{\min}}^1 dy \int_{x_{\min}}^1 dx \frac{d^2\sigma}{dx dy}, \quad (23)$$

where $x_{\min} = Q_{\min}^2/2ME_\nu$ and $y_{\min}$ is determined by the relation $Q^2 = 2ME_\nu xy$ as $y_{\min} = x_{\min}/x$. The total cross section in Eq. (23) can be calculated numerically with the given input parameters of the model and the pdfs of the quarks and antiquarks. We use the CTEQ5 parton distribution [58] in our numerical calculations. We also cross-checked our calculations against the analytical form of the pdfs [59] and found agreement. The cross sections corresponding to three different benchmark points are shown in Table IV. The coupling constants are taken to be $y_5 = 0.23$ and $y_{6\tau} = 0.17$. The cross section in Eq. (21) is sensitive to the mixing angles ψ and ψ' . These angles also appear in the $g-2$ calculations, but Eq. (19) is not sensitive enough to these angles to change the numerical results. Therefore, we can vary them to get different cross sections from Eq. (23). We choose three different values of $\psi = 55^\circ, 60^\circ$, and 65° , and $\psi' = 5^\circ$ corresponding to the three benchmark points in Table IV.

The $\bar{E}^0$ produced in the collision propagates through the Earth and eventually decays dominantly into a τ lepton pair and E^0 . The three-body decay channel is mediated by the neutral scalars h, s_2, s_{2s} , and s'_1 . The decay width is given by

$$\Gamma_{\bar{E}^0 \rightarrow E^0 \tau \tau} = \frac{y_{6\tau}^2 y_{8\tau}^2}{128\pi^3} \left(\frac{c_{31}c_{11}}{m_h^2} + \frac{c_{32}c_{12}}{m_{s_2}^2} + \frac{c_{33}c_{13}}{m_{s_{2s}}^2} - \frac{c'_{33}c'_{13}}{m_{s'_1}^2} \right)^2 \times \int_{E_{\tau}^{\min}}^{E_{\tau}^{\max}} dE_{\tau} \int_{E_{E^0}^{\min}}^{E_{E^0}^{\max}} dE_{E^0} f(E_{E^0}), \quad (24)$$

where

$$f(E_{E^0}) = \frac{E_{E^0}}{2} [m_{\bar{E}^0}^2 - 2E_{E^0}m_{\bar{E}^0} + m_{E^0}^2 - 2m_\tau^2], \quad (25)$$

$$E_{\tau}^{\max, \min} = \frac{1}{2}(m_{\bar{E}^0} - E_{E^0}) \pm \frac{1}{2} \sqrt{(E_{E^0}^2 - m_{E^0}^2) \left(1 - \frac{4m_\tau^2}{m_{\bar{E}^0}^2 - 2E_{E^0}m_{\bar{E}^0} + m_{E^0}^2} \right)},$$

$$m_{E^0} \leq E_{E^0} \leq \frac{m_{\bar{E}^0}^2 + m_{E^0}^2 - 4m_\tau^2}{2m_{\bar{E}^0}}. \quad (26)$$

Equation (24) gives the sum over all four contributions for h, s_2, s_{2s} and s'_1 . The same decay channel is open for all

TABLE IV. Three benchmark points of parameter space are shown. The parameter space can explain the ANITA events and is consistent with neutrino mass and $g-2$ calculations. The masses are in the GeV unit.

Point	m_{s_3}	$m_{s'_3}$	m_h	m_{s_2}	$m_{s_{2s}}$	$m_{s'_1}$	m_D	m_{E^0}	$m_{\bar{E}^0}$	σ (pb)	$\tau_{\bar{E}^0}$ (ns)	l_{BSM} (km)	l_{decay} (km)	ϵ_{BSM}
BP1	500	1200	125	500	500	500	1000	105	110	708	2	4705	6387	2×10^{-5}
BP2	500	600	125	400	400	500	1000	110	115	887	2	3757	6368	3×10^{-5}
BP3	600	800	125	500	600	800	1000	115	120	1066	2	3126	6350	4×10^{-5}

charged leptons and down type quarks in the SM. We choose $m_{\bar{E}^0}$ and m_{E^0} such that the bottom quark final state is kinematically forbidden. The decays to muons and strange quarks are suppressed by a factor 100 due to the SM Yukawas. The lifetime of $\bar{E}^0$ in the rest frame is $\tau_{\bar{E}^0} = 1/\Gamma$, where Γ is the total decay width of $\bar{E}^0$. Note that the masses of the new scalar fields are set to be $\mathcal{O}(500)$ GeV, making their contributions to the decay width subleading compared to that of the SM Higgs. We show the lifetime for a few benchmark points in Table IV, taking the coupling constants to be $y_{8\tau} = 0.25$ and $y_{6\tau} = 0.17$.

Let us now define the survival probability of emergence, ϵ_{BSM} . We follow closely the treatment in Ref. [40]. We assume that a fraction of neutrinos has survived the SM interactions after propagating a distance of l_1 km. We take the SM interaction length to be $l_{\text{SM}} = 275$ km [14]. These surviving neutrinos can produce $\bar{E}^0$ as a result of the collision with nucleons inside the Earth. The interaction length of the ν_τ -nucleon scattering process can be defined as $l_{\text{BSM}} = 1/(\sigma N_A \rho)$, where σ is the scattering cross section, N_A is the Avogadro number, 6.022×10^{23} , and ρ is the density of the target material. For simplicity, we assume the Earth to have uniform density and take the value to be $\rho = 5$ gm/cm³. The $\bar{E}^0$ then travels a distance of $(l - l_1)$ km, where $l = 6475$ km is the average chord length along the Earth corresponding to the two ANITA events, and finally decays below an altitude of about 10 km; otherwise, the air shower will not get a chance to fully develop [40]. The decay length in the Earth frame is $l_{\text{decay}} = \gamma c \tau_{\bar{E}^0}$. We can now define the survival probability as

$$\begin{aligned} \epsilon_{\text{BSM}} = & \int_0^l dl_1 \int_{l-l_1}^{l-l_1+d} dl_2 \int_0^{l-l_1-l_2+d} dl_3 \\ & \times \left[\left(\frac{e^{-\frac{l_3}{l_{\text{decay},\tau}}}}{l_{\text{decay},\tau}} \text{Br}(\tau \rightarrow \text{hadrons}) \right) \right. \\ & \times \left. \frac{e^{-\frac{l_2}{l_{\text{decay}}}}}{l_{\text{decay}}} \frac{e^{-\frac{l_1}{l_{\text{BSM}}}}}{l_{\text{BSM}}} \left(1 - \int_0^{l_1} dl_4 \frac{e^{-\frac{l_4}{l_{\text{SM}}}}}{l_{\text{SM}}} \right) \right], \quad (27) \end{aligned}$$

where from right to left the first expression is the fraction of ν_τ s that survive passage through matter until the interaction point, l_1 , where $\bar{E}^0$ is produced. The second factor is the fraction of neutrinos converted to $\bar{E}^0$ anywhere inside the Earth between the emergence point $l_1 = 0$ and the entire chord length $l_1 = l$. The third factor is the fraction of those $\bar{E}^0$ that decay after a sufficient distance l_2 to emerge from the Earth, $l_2 = l - l_1$, and a further distance, d , beyond which it would exceed the 10 km altitude. We take $d = 10/\sin 30^\circ = 20$ km. The fourth expression is the fraction of τ that decays into hadrons within the distance $d = 20$ km. The decay length of τ is $l_{\text{decay},\tau} = \gamma c \tau_\tau$, where

the lifetime of τ is $\tau_\tau = 0.3 \times 10^{-12}$ s and the branching ratio of τ to hadrons is $\text{Br}(\tau \rightarrow \text{hadrons}) = 0.65$ [4].

We dial our parameters such that both l_{BSM} and l_{decay} are comparable to the chord length. The corresponding scattering cross section, lifetime, and estimated survival probability are shown in Table IV. The expression above could yield a higher value if we choose a much shorter l_{BSM} and longer l_{decay} , but then one must account for the probability of ν regeneration. The interaction length of regeneration is similar to l_{BSM} at such energies, and once accounted for, the gains from reducing l_{BSM} are countered, and one gets similar values of ϵ_{BSM} . For comparison, the SM survivability found by Ref. [14] is 4.4×10^{-7} and 3.2×10^{-8} for the ANITA-I and ANITA-III events, respectively, at $E_\nu = 1$ EeV. Note that the results in Table IV are far less sensitive to the chord length compared to the survivability in the SM.

VII. CONCLUSION

We were able to explain the observed values of the anomalous magnetic moments of the muon and electron within the framework of an E_6 inspired GUT model. In addition to the theoretical appeal of fitting into a unification picture, the model has been shown in an earlier work to generate neutrino masses radiatively at the three-loop level, thereby allowing for the new physics mass scale to be within the reach of the current and proposed collider experiments. The flavor structure of the model, which allowed the neutrino fit, played a crucial role in explaining the anomalous magnetic moments of the muon and electron. In this model, the ratio of Δa_μ and Δa_e is proportional to m_μ/m_e with a relative sign difference between them as required by the experiments. We have also studied the model's contribution to the anomalous events observed by the ANITA experiments using the mass scale of new physics set by the neutrino fitting and found a modest improvement over the SM predictions.

We stress that in explaining all the aforementioned observations we have introduced various new, SM charged particles in the 100 GeV to 1 TeV range. The primary reason these have not been ruled out is that they are too mass degenerate with one of their decay products to be covered by existing LHC searches. This adds to the list of motivations of why the efforts given in closing this mass gap in collider searches are worth pursuing.

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