Production of P_c states from Λ_b decay

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In the present work, we investigate $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ production from Λ_b decay in a molecular scenario by using an effective Lagrangian approach. We predict the ratio of the branching fraction of $\Lambda_b \rightarrow P_c K$, which is weakly dependent on our model parameter. We also find the ratios of the productions of the branching fractions of $\Lambda_b \rightarrow P_c K$ and $P_c \rightarrow J/\psi p$ can be well interpreted in the molecular scenario. Moreover, the estimated branching fractions of $\Lambda_b \rightarrow P_c K$ are of order 10^{-6} , which could be tested by further measurements by the LHCb Collaboration.

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I. INTRODUCTION

Searching for hadrons beyond three-quark baryons and quark-antiquark mesons is one of the intriguing frontiers of hadron physics, even since the initial period of the quark model. Tremendous process has been made in recent decades. A growing number of tetraquark and pentaquark candidates have been observed experimentally (more details can be found in the recent review [1–3]). In 2015, the LHCb Collaboration reported two pentaquark candidates, $P_c(4380)$ and $P_c(4450)$, in the $J/\psi p$ invariant mass spectroscopy of the $\Lambda_b \rightarrow KJ/\psi p$ process [4]. The two-body mass spectroscopy and angular distributions of three-body final states has been analyzed, and the J^P quantum numbers of these two pentaquark candidates are preferred to be of opposite parity with $J = 3/2$ for one state and $J = 5/2$ for the other one.

It should be mentioned that before the LHCb observation there were theoretical predictions of molecular states composed by an anticharmed meson and charmed baryon [5–8], such as $\Sigma_c \bar{D}$ and $\Lambda_c \bar{D}$, which may have corresponded to the observed P_c states. Furthermore, in the vicinity of $P_c^+(4380)$ and $P_c^+(4450)$, there are abundant thresholds of a baryon and a meson, such as $\Sigma_c^{(*)} \bar{D}^{(*)}$, $\Lambda_c \bar{D}^*$, $\chi_{c1} p$, and $\psi(2S)p$. Along the lines of the molecular scenario, some interpretations related the above thresholds have been proposed [9–24]. It should be noted that the masses of $P_c^+(4380)$ and $P_c^+(4450)$ are above the threshold of $J/\psi p$ and were observed in the $J/\psi p$ mode; thus, these two P_c states more likely contain five constituent quarks,

that is, $c\bar{c}qqq$, where q is an up or down quark. In the pentaquark scenario, a series of interpretations with different quark configurations than $P_c^+(4380)$ and $P_c^+(4450)$ were proposed [25–32].

Very recently, the LHCb Collaboration updated its analysis of the $J/\psi p$ invariant mass spectroscopy of $\Lambda_b \rightarrow KJ/\psi p$ and found three pentaquark states, which are $P_c^+(4312)$, $P_c^+(4440)$, and $P_c^+(4457)$ [33]. After the new observation, some interpretation has been proposed immediately in the molecular [34–47] and pentaquark scenarios [48–54]. In addition, by fitting the LHCb data around 4312 MeV with the amplitudes satisfying the general principles of S -matrix theory, $P_c(4312)$ is assigned as a virtual state resulting from the weak interactions of $\bar{D}$ and Σ_c [55]. As listed in Table I, the mass of $P_c^+(4312)$ is very close to the threshold of $\Sigma_c \bar{D}$, while $P_c^+(4440)$ and $P_c^+(4457)$ are close to the $\Sigma_c \bar{D}^*$ threshold, and the small mass splitting of $P_c^+(4440)$ and $P_c^+(4457)$ may result from the spin-spin interactions of the components. Thus, one can assign $P_c^+(4312)$ as a $\Sigma_c \bar{D}$ molecular state with $J^P = \frac{1}{2}^-$, while $P_c^+(4440)$ and $P_c^+(4457)$ are $\Sigma_c \bar{D}^*$ molecular states with $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively. Such assignments are supported by the estimations in Refs. [36,37].

Besides the resonance parameters of the P_c states, the production ratio, $R \equiv \mathcal{B}(\Lambda_c \rightarrow P_c K) \times \mathcal{B}(P_c \rightarrow J/\psi p) / \mathcal{B}(\Lambda_c \rightarrow J/\psi p K)$, was also measured and are also listed in Table I. The new analysis indicates the production ratios are of order of 1%. The newly measured product ratios are much

TABLE I. The resonance parameters of the newly reported pentaquark states and the production ratio.

State	Mass (MeV)	Width (MeV)	R (%)
$P_c^+(4312)$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c^+(4440)$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c^+(4457)$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$0.53 \pm 0.16^{+0.15}_{-0.13}$

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smaller than those for $P_c^+(4380)$ and $P_c^+(4450)$ from their previous analysis, which are $(8.4 \pm 0.7 \pm 4.2)\%$ and $(4.1 \pm 0.5 \pm 1.1)\%$ for $P_c^+(4380)$ and $P_c^+(4450)$, respectively. With the Particle Data Group average of the branching ratio $\mathcal{B}(\Lambda_b \rightarrow J/\psi p K) = (3.2^{+0.6}_{-0.5}) \times 10^{-4}$, the production of the branching ratios for $\Lambda_b \rightarrow P_c K$ and $P_c \rightarrow J/\psi p$ are estimated to be

$$\begin{aligned} & \mathcal{B}(\Lambda_b \rightarrow P_c^+(4312)K) \times \mathcal{B}(P_c^+(4312) \rightarrow J/\psi p) \\ &= (0.96^{+1.12}_{-0.28}) \times 10^{-6}, \\ & \mathcal{B}(\Lambda_b \rightarrow P_c^+(4440)K) \times \mathcal{B}(P_c^+(4440) \rightarrow J/\psi p) \\ &= (3.55^{+1.43}_{-1.20}) \times 10^{-6}, \\ & \mathcal{B}(\Lambda_b \rightarrow P_c^+(4457)K) \times \mathcal{B}(P_c^+(4457) \rightarrow J/\psi p) \\ &= (1.70^{+0.77}_{-0.60}) \times 10^{-6}. \end{aligned} \quad (1)$$

Besides the mass spectra of the P_c states, how to understand the measured production ratios is an intriguing problem, which could help us reveal the inner structures of the pentaquark states. In Ref. [56], the partial widths of $P_c \rightarrow J/\psi p$ were estimated in a molecular scenario; thus, study of the production process $\Lambda_b \rightarrow P_c K$ in the same molecular scenario and compared with the measured production ratios listed in Eq. (1) can further test the molecular interpretations of P_c states, which is the main task of the present work.

The present work is organized as follows. After the Introduction, the formulas of the productions of $\Lambda_b \rightarrow P_c K$ are present, including the related effective Lagrangians and production amplitudes. In Sec. III, we present our numerical results and some discussions of the present results. A short summary is presented in Sec. IV.

II. PRODUCTIONS OF $\Lambda_b \rightarrow P_c K$

We can first analyze the production process of P_c states from the quark level. One should notice that the P_c production is accompanied with a K^- meson. In Fig. 1(a), the kaon is produced directly from the W^- meson. Since in the P_c states there are $\bar{c}$ and c quarks in $\bar{D}^{(*)}$ and Σ_c , respectively, the b quark should transition into a u quark via W^- emission, and the c and $\bar{c}$ quarks are created from the vacuum. This kind of diagram will be suppressed in the P_c production, since V_{ub} is about 1 order of magnitude smaller than V_{cb} and the charm quark pair creation from the vacuum is much more difficult than that of the light quark pairs. In the second kind of mechanism as shown in Fig. 1(b), the

FIG. 1. Possible diagrams contributing to $\Lambda_b \rightarrow P_c K$ at quark level in a molecular scenario.

subprocess of the weak decay is $b \rightarrow c \bar{c} s$. The $\bar{c}$ quark and the d quark in the initial Λ_b form an anticharmed meson, such as $\bar{D}^{(*)}$. Together with the $u\bar{u}$ quark pair created from the vacuum, the s and $\bar{u}$ quark can form a kaon, and the ucd quarks form a Σ_c baryon. Then, the Σ_c baryon and the anticharmed meson form a molecular state corresponding to P_c state. In Fig. 1(c), the subprocess are the same as the one in Fig. 1(b), but the $\bar{c}s$ quarks together with the $u\bar{u}$ quark pair form a kaon and an anticharmed meson. The anticharmed meson and Σ_c baryon formed by the cdu quark can make up a P_c state. Comparing to Fig. 1(c), the mechanism in Fig. 1(b) is suppressed due to color suppression in the hadronization process; thus, in the $\Lambda_b \rightarrow P_c K$ process, the mechanism in Fig. 1(c) is supposed to be dominant.

With the above analysis, one can find that the Hamiltonian of the decay process (H) should include two parts, the first part being the weak decay process (H_W) and the second part being the transition process (H_T), in which the quarks involved in the weak decay, the remaining quarks in the initial Λ_b , and the quark pairs created from the vacuum transition into a kaon and a molecular state. It will be very difficult to estimate the diagrams in Fig. 1 at quark level; thus, in the present work, we can simplify the estimation at hadron level by inserting a complete basis formed by a baryon and a meson between H_T and H_W , and then the decay $\Lambda_b \rightarrow KP_c$ can occur via

$$\langle KP_c | H | \Lambda_b \rangle = \sum_{B,M} \langle KP_c | H_T | BM \rangle \langle BM | H_W | \Lambda_b \rangle. \quad (2)$$

In principle, one should consider all the possible loops which can connect the initial Λ_b and final $P_c K$. However, the dominant contributions are usually supposed to come from the ground states of the involved baryons and mesons. Considering the components of the P_c states, one can obtain the dominant diagrams contributing to $\Lambda_b \rightarrow P_c K$ as shown in Fig. 2 at the hadronic level.

A. Effective Lagrangians

We employ an effective Lagrangian approach to estimate the diagrams in Fig. 2. As for the $\Lambda_b \rightarrow \bar{D}_s^{(*)} \Sigma_c$, the interaction vertices are the same as the those of $\Lambda_b \rightarrow \bar{D}_s^{(*)} \Lambda_c$ and in the form [57,58]

FIG. 2. Diagrams contributing to a) $\Lambda_b \rightarrow P_c(4312)K$, b-c) $\Lambda_b \rightarrow P_c(4440)K$, and d-e) $\Lambda_b \rightarrow P_c(4457)K$.

$$\begin{aligned}\mathcal{L}_{\Lambda_b \Sigma_c D_s^*} &= D_s^{*\mu} \bar{\Sigma}_c \left(A_1 \gamma_\mu \gamma_5 + A_2 \frac{p_{2\mu}}{m} \gamma_5 + B_1 \gamma_\mu + B_2 \frac{p_{2\mu}}{m} \right) \Lambda_b, \\ \mathcal{L}_{\Lambda_b \Sigma_c D_s} &= i \bar{\Sigma}_c (A + B \gamma_5) \Lambda_b D_s,\end{aligned}\quad (3)$$

where A , B , A_1 , A_2 , B_1 , and B_2 are the recombinations of the form factors, which are

$$\begin{aligned}A &= -\lambda f_{D_s} \left[(m - m_2) f_1^V + \frac{m_1^2}{m} f_3^V \right], \\ B &= -\lambda f_{D_s} \left[(m + m_2) f_1^A - \frac{m_1^2}{m} f_3^A \right], \\ A_1 &= -\lambda f_{D_s^*} m_1 \left[f_1^A + f_2^A \frac{m - m_2}{m} \right], \\ A_2 &= -2\lambda f_{D_s^*} m_1 f_2^A, \\ B_1 &= \lambda f_{D_s^*} m_1 \left[f_1^V - f_2^V \frac{m + m_2}{m} \right], \\ B_2 &= 2\lambda f_{D_s^*} m_1 f_2^V,\end{aligned}\quad (4)$$

where $\lambda = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_1$ with $a_1 = 1.07$ [59] and $f_{D_s^{(*)}} = 0.247$ GeV is the decay constant of a charmed strange meson, which is estimated by twisted-mass Lattice QCD [60]. m , m_1 , and m_2 are the masses of Λ_b , $D_s^{(*)}$, and Σ_c , respectively. $f_i^{(A,V)}$ ($i = 1, 2, 3$) are the transition form factors of $\Lambda_b \rightarrow \Sigma_b$, which will be discussed in the next section.

The effective Lagrangians related to $D_s^{(*)} D^{(*)} K$ are [61]

$$\begin{aligned}\mathcal{L}_{KDD_s^*} &= i g_{KDD_s^*} D_s^{*\mu} [\bar{D} \partial_\mu \bar{K} - (\partial_\mu \bar{D}) \bar{K}] + \text{H.c.}, \\ \mathcal{L}_{KD_s D^*} &= i g_{KD_s D^*} D^{*\mu} [\bar{D}_s \partial_\mu K - (\partial_\mu \bar{D}_s) K] + \text{H.c.}, \\ \mathcal{L}_{KD_s^* D^*} &= -g_{KD_s^* D^*} \epsilon^{\mu\nu\alpha\beta} [\partial_\mu \bar{D}_s^* \partial_\alpha D_{s\beta}^* \bar{K} + \partial_\mu D_\nu^* \partial_\alpha \bar{D}_{s\beta}^* K],\end{aligned}\quad (5)$$

where the coupling constant are $g_{KDD_s^*} = g_{KD_s D^*} = 5.0$, $g_{KD_s^* D^*} = 7.0$ GeV⁻¹, which are estimated by SU(4) symmetry [61]. The effective Lagrangians of P_c and $\Sigma_c D^{(*)}$ are [62]

$$\begin{aligned}\mathcal{L}_{P_{c1} \Sigma_c D} &= g_{P_{c1} \Sigma_c D} \bar{\Sigma}_c P_{1c} \bar{D}^0 + \text{H.c.}, \\ \mathcal{L}_{P_{c2} \Sigma_c D^*} &= g_{P_{c2} \Sigma_c D^*} \bar{\Sigma}_c \gamma_5 \left(g_{\mu\nu} - \frac{P_{4\mu} P_{4\nu}}{m_4^2} \right) \gamma^\nu P_{c2} \bar{D}^{*\mu}, \\ \mathcal{L}_{P_{c3} \Sigma_c D^*} &= g_{P_{c3} \Sigma_c D^*} \bar{\Sigma}_c P_{c3\mu} \bar{D}^{*\mu},\end{aligned}\quad (6)$$

where p_{c1} , p_{c2} , and p_{c3} denote $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ hereafter, respectively.

B. Decay amplitudes

With the effective Lagrangians listed above, we can obtain the amplitudes involved in the present work. The decay amplitude of $\Lambda_b(p) \rightarrow D_s^*(p_1) \Sigma_c(p_2) [D(q)] \rightarrow K(p_3) P_{c1}(p_4)$ corresponding to Fig. 2(a) is

$$\begin{aligned}\mathcal{M}_a &= i^3 \int \frac{d^4 q}{(2\pi)^4} [-i g_{P_{c1} \Sigma_c D} \bar{u}(p_4)] [\not{p}_2 + m_2] \left[\left(A_1 \gamma_\mu \gamma_5 + A_2 \frac{p_{2\mu}}{m} \gamma_5 + B_1 \gamma_\mu + B_2 \frac{p_{2\mu}}{m} \right) u(p) \right] [-g_{KDD_s^*} (p_3 - q)_\nu] \\ &\quad \times \left(-g^{\mu\nu} + \frac{p_1^\mu p_1^\nu}{m_1^2} \right) \frac{1}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_E^2} \mathcal{F}(q^2, m_E^2).\end{aligned}\quad (7)$$

The decay amplitude of $\Lambda_b(p) \rightarrow D_s^{(*)}(p_1) \Sigma_c(p_2) [D^*(q)] \rightarrow K(p_3) P_{c2}(p_4)$ corresponding to Figs. 2(b) and 2(c) are

$$\begin{aligned}\mathcal{M}_b &= i^3 \int \frac{d^4 q}{(2\pi)^4} \left[g_{P_{c2} \Sigma_c D^*} \bar{u}(p_4) \gamma^\nu \gamma_5 \left(g_{\mu\nu} - \frac{P_{4\mu} P_{4\nu}}{m_4^2} \right) \right] [\not{p}_2 + m_2] [i(A + B \gamma_5) u(p)] [-g_{KD_s^* D_s} (p_1 + p_3)_\alpha] \\ &\quad \times \left(-g^{\mu\alpha} + \frac{q^\mu q^\alpha}{m_E^2} \right) \frac{1}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_E^2} \mathcal{F}(q^2, m_E^2) \\ \mathcal{M}_c &= i^3 \int \frac{d^4 q}{(2\pi)^4} \left[g_{P_{c2} \Sigma_c D^*} \bar{u}(p_4) \gamma^\nu \gamma_5 \left(g_{\mu\nu} - \frac{P_{4\mu} P_{4\nu}}{m_4^2} \right) \right] [\not{p}_2 + m_2] \left[\left(A_1 \gamma_\alpha \gamma_5 + A_2 \frac{p_{2\alpha}}{m} \gamma_5 + B_1 \gamma_\alpha + B_2 \frac{p_{2\alpha}}{m} \right) \right. \\ &\quad \left. \times u(p) \right] [-g_{KD_s^* D_s^*} \epsilon_{\rho\lambda\eta\tau} q^\rho p_1^\eta] \left(-g^{\mu\lambda} + \frac{q^\mu q^\lambda}{m_E^2} \right) \left(-g^{\alpha\tau} + \frac{p_1^\alpha p_1^\tau}{m_1^2} \right) \frac{1}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_E^2} \mathcal{F}(q^2, m_E^2).\end{aligned}\quad (8)$$

The decay amplitudes of $\Lambda_b(p) \rightarrow D_s^{(*)}(p_1)\Sigma_c(p_2)[D^*(q)] \rightarrow K(p_3)P_{c3}(p_4)$ corresponding to Figs. 2(d) and 2(e) are

$$\begin{aligned}\mathcal{M}_d &= i^3 \int \frac{d^4 q}{(2\pi)^4} [-ig_{P_{c3}\Sigma_c D^*} \bar{u}_\mu(p_4)] [\not{p}_2 + m_2] [i(A + B\gamma_5)u(p)] [-g_{KD^* D_s}(p_1 + p_3)_\nu] \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_E^2}\right) \\ &\quad \times \frac{1}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_E^2} \mathcal{F}(q^2, m_E^2) \\ \mathcal{M}_e &= i^3 \int \frac{d^4 q}{(2\pi)^4} [-ig_{P_{c3}\Sigma_c D^*} \bar{u}_\sigma(p_4)] [\not{p}_2 + m_2] \left[\left(A_1 \gamma_\rho \gamma_5 + A_2 \frac{p_{2\rho}}{m} \gamma_5 + B_1 \gamma_\rho + B_2 \frac{p_{2\rho}}{m} \right) u(p) \right] \\ &\quad \times [-g_{KD^* D_s} \epsilon_{\mu\nu\alpha\beta} q^\mu p_1^\alpha] \left(-g^{\sigma\nu} + \frac{q^\sigma q^\nu}{m_E^2} \right) \left(-g^{\rho\beta} + \frac{p_1^\rho p_1^\beta}{m_1^2} \right) \\ &\quad \times \frac{1}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_E^2} \mathcal{F}(q^2, m_E^2).\end{aligned}\quad (9)$$

In the present work, a monopole form factor is introduced to depict the off-shell effect of the exchanged mesons, which is

$$\mathcal{F}(q^2, m_E^2) = \frac{m_E^2 - \Lambda_E^2}{q^2 - \Lambda_E^2}, \quad (10)$$

where $\Lambda_E = m_E + \alpha\Lambda_{\text{QCD}}$ with m_E being the mass of the exchanged meson, $\Lambda_{\text{QCD}} = 220$ MeV, and α is a model parameter, which is of order of unit [63–66].

With above amplitudes, one can estimate the partial width of $\Lambda_b \rightarrow P_c K$ by

$$\Gamma_{\Lambda_b} = \frac{1}{2} \frac{1}{8\pi} \frac{|\vec{p}|}{m^2} \overline{|\mathcal{M}|^2}, \quad (11)$$

where the factor $1/2$ results from the average of Λ_b spin and $\vec{p}$ is the momentum of P_c or K in the rest frame of Λ_b . The overline indicates the sum over the spins of final states.

III. NUMERICAL RESULTS AND DISCUSSIONS

Before we estimate the partial width of $\Lambda_b \rightarrow P_c K$ in the present scenario, we first discuss the transition form factors of $\Lambda_b \rightarrow \Sigma_c$. Unfortunately, there is no direct estimation of these transition form factors. One should notice the constituent quarks and the spatial part of the Σ_c and Λ_c are the same; thus, the transition form factors of $\Lambda_b \rightarrow \Sigma_c$ should be the same as those of $\Lambda_b \rightarrow \Lambda_c$, but smaller in magnitude due to light quark spin flipping in the transition $\Lambda_b \rightarrow \Sigma_c$. Here, we defined the suppression ratio R as

$$f_i^{A,V}(Q^2) = R F_i^{A,V}(Q^2), \quad \{i = 1, 2, 3\}, \quad (12)$$

where $f_i^{A,V}$ and $F_i^{A,V}$ with $i = (1, 2, 3)$ are the transition form factors of $\Lambda_b \rightarrow \Sigma_c$ and $\Lambda_b \rightarrow \Lambda_c$, respectively. The details of the transition form factors of $\Lambda_b \rightarrow \Lambda_c$ are presented in Appendix. Furthermore, the coupling constants related to P_c and $\Sigma_c \bar{D}^{(*)}$ will be discussed later.

In Fig 3, we plot the α dependence of $\mathcal{B}(\Lambda_b \rightarrow P_c K)/(R^2 g_{P_c}^2)$, which is of order 10^{-4} for $P_c^+(4440)$ and $P_c^+(4457)$ and 10^{-5} for $P_c^+(4312)$. As for the coupling constants g_{P_c} , they could be estimated by the compositeness condition with the assumption that all three observed P_c states are molecular states. In Ref. [56], the coupling constants are estimated depending on a model parameter Λ , which is of order 1 GeV. As in Ref. [56], the coupling constants g_{P_c} are estimated to be $2.25^{+0.01}_{-0.07}$, $1.72^{+0.15}_{-0.09}$, and $1.77^{+0.04}_{-0.06}$ for $P_c^+(4312)$, $P_c^+(4440)$, and $P_c^+(4457)$, respectively, where the center values are estimated with $\Lambda = 1.0$ GeV and the errors result from the variation of Λ from 0.8 to 1.2 GeV. The values of the coupling constants in Ref. [56] are consistent with those estimated in the framework of the effective range expansion and the resonance compositeness relations within error [44]. Moreover, the Λ dependences of g_{P_c} are similar; thus, their ratios are weakly dependent on the model parameter and are $g_{P_{c1}}/g_{P_{c2}} = 1.25^{+0.02}_{-0.03}$, $g_{P_{c1}}/g_{P_{c3}} = 1.22^{+0.02}_{-0.01}$, and

FIG. 3. The α dependence of the branching fractions of $\Lambda_b \rightarrow P_{c1}K$ (straight line), $\Lambda_b \rightarrow P_{c2}K$ (dashed line), and $\Lambda_b \rightarrow P_{c3}K$ (dotted line).

$g_{P_{c2}}/g_{P_{c3}} = 0.98^{+0.04}_{-0.03}$, respectively. With the above coupling constants, the ratios of the branching fractions $\Lambda_b \rightarrow P_c K$ are estimated to be

$$\begin{aligned}\mathcal{R}_{12}^P &\equiv \frac{\mathcal{B}(\Lambda_b \rightarrow P_{c1} K)}{\mathcal{B}(\Lambda_b \rightarrow P_{c2} K)} = 0.17^{+0.05}_{-0.05}, \\ \mathcal{R}_{13}^P &\equiv \frac{\mathcal{B}(\Lambda_b \rightarrow P_{c1} K)}{\mathcal{B}(\Lambda_b \rightarrow P_{c3} K)} = 0.17^{+0.05}_{-0.05}, \\ \mathcal{R}_{23}^P &\equiv \frac{\mathcal{B}(\Lambda_b \rightarrow P_{c2} K)}{\mathcal{B}(\Lambda_b \rightarrow P_{c3} K)} = 0.99^{+0.29}_{-0.29},\end{aligned}\quad (13)$$

which are independent on suppression ratio R . The center values correspond to $\alpha = 1.0$, $\Lambda = 1.0$ GeV, and the uncertainties result from the variation of the model parameter. Our estimation indicates the production ratios are very weakly dependent on the model parameter.

In Ref. [56], the partial widths of $P_c(4320) \rightarrow J/\psi p$, $P_c(4440) \rightarrow J/\psi p$, and $P_c(4450) \rightarrow J/\psi p$ are estimated depending on the model parameter Λ . The partial widths of $P_c \rightarrow J/\psi p$ are estimated to be $5.03^{+2.55}_{-2.05}$ MeV, $9.38^{+6.24}_{-4.11}$ MeV, and $2.89^{+1.91}_{-0.84}$ MeV for $P_c^+(4312)$, $P_c^+(4440)$, and $P_c^+(4457)$, respectively. The uncertainties of the partial widths are from the variation of model parameter Λ from 0.8 to 1.2 GeV, and the center values are estimated with $\Lambda = 1.0$ GeV. With these estimated partial widths and the measured total widths of P_c states, one can get the branching fractions of the $J/\psi p$ mode for P_c states, which are

$$\begin{aligned}\mathcal{B}(P_c(4312) \rightarrow J/\psi p) &= 0.51^{+0.35}_{-0.35}, \\ \mathcal{B}(P_c(4440) \rightarrow J/\psi p) &= 0.46^{+0.37}_{-0.32}, \\ \mathcal{B}(P_c(4457) \rightarrow J/\psi p) &= 0.45^{+0.52}_{-0.23}.\end{aligned}\quad (14)$$

With these branching fractions, we can estimate their ratios of the decay fraction of $P_c \rightarrow J/\psi p$ as

$$\begin{aligned}\mathcal{R}_{12}^D &= \frac{\mathcal{B}(P_{c1} \rightarrow J/\psi p)}{\mathcal{B}(P_{c2} \rightarrow J/\psi p)} = 1.13^{+0.76}_{-0.87}, \\ \mathcal{R}_{13}^D &= \frac{\mathcal{B}(P_{c1} \rightarrow J/\psi p)}{\mathcal{B}(P_{c3} \rightarrow J/\psi p)} = 1.14^{+1.20}_{-0.80}, \\ \mathcal{R}_{23}^D &= \frac{\mathcal{B}(P_{c2} \rightarrow J/\psi p)}{\mathcal{B}(P_{c3} \rightarrow J/\psi p)} = 1.01^{+1.07}_{-0.73},\end{aligned}\quad (15)$$

where the uncertainties of these ratios dominantly result from the measured error of P_c widths, while the uncertainties caused by the model-dependent estimations in Ref. [56] are much smaller since the Λ dependences of the partial widths are similar; thus, their ratios are very weakly dependent on the model parameter.

With the production and decay ratios estimated in the molecular scenario, we can get the product of these two ratios, i.e.,

FIG. 4. A Comparison of the ratio R_{ij} . The squares with error bars are measured data from the LHCb Collaboration [33], and the full circles with error bars are the estimations in the present work.

$$R_{ij} = \mathcal{R}_{ij}^P \times \mathcal{R}_{ij}^D, \quad (16)$$

and compare these ratios with the experimental measurement as listed in Eq. (1). We present a comparison of the measured R_{ij} from the LHCb Collaboration [33] and the estimation in the present work in Fig. 4. One can find our estimations are consistent with the experimental data from the LHCb Collaboration within error, which indicates all three P_c states could be interpreted as molecular states.

Moreover, taking the branching ratios estimated in the molecular scenario as listed in Eq. (14) back to Eq. (1), one can get the branching ratios of P_c production, which are

$$\begin{aligned}\mathcal{B}(\Lambda_b \rightarrow P_c^+(4312)^+ K) &= (1.87^{+2.54}_{-1.37}) \times 10^{-6}, \\ \mathcal{B}(\Lambda_b \rightarrow P_c^+(4440)^+ K) &= (7.80^{+7.15}_{-6.06}) \times 10^{-6}, \\ \mathcal{B}(\Lambda_b \rightarrow P_c^+(4457)^+ K) &= (3.77^{+4.66}_{-2.37}) \times 10^{-6}.\end{aligned}\quad (17)$$

In the present scenario, the ratio $\mathcal{B}(\Lambda_b \rightarrow P_c K)/(R^2 g_{P_c}^2)$ is estimated to be 10^{-5} for $P_c^+(4312)$ and 10^{-4} for $P_c^+(4440)$ and $P_c^+(4457)$. Considering the light quark spin-flip suppression in the $\Lambda_b \rightarrow \Sigma_c D^{(*)}$ process, one can suppose R much smaller than 1. In the present work, the estimated product ratio could be consistent with the experimental measurements within error when one takes $R = 0.09$, which are

$$\begin{aligned}\mathcal{B}(\Lambda_b \rightarrow P_c^+(4312)^+ K) &= (1.05-1.59) \times 10^{-6}, \\ \mathcal{B}(\Lambda_b \rightarrow P_c^+(4440)^+ K) &= (5.38-8.59) \times 10^{-6}, \\ \mathcal{B}(\Lambda_b \rightarrow P_c^+(4457)^+ K) &= (5.90-8.53) \times 10^{-6},\end{aligned}\quad (18)$$

respectively.

It is interesting to note that the $\Lambda_b \rightarrow \Sigma_c$ transition requires the spin flip of the involved light quark system. This has been widely expected to be power suppressed in $1/m_b$. A relevant counterpart is the transition of the bottom meson into a charmed scalar meson [67]. An analysis in the QCD sum rules indicates that the helicity-flipped transition form factor is smaller than the ordinary heavy-to-light

transition form factor by a factor 3 to 5. A similar power suppression might also happen in $\Lambda_b \rightarrow \Sigma_c$ compared to $\Lambda_b \rightarrow \Lambda_c$. If this were true, it indicates that the use of $R \sim 0.1$ is reasonable.

It should be mentioned that the GlueX Collaboration performed an analysis of the cross sections for $\gamma p \rightarrow J/\psi p$ near threshold [68]. Based on the present analysis, the upper limit of the branching ratios for the $J/\psi p$ channel are determined to be 4.6%, 2.3%, and 3.8% for $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, respectively [68]. With these upper limits, one can obtain the lower limits of the branching ratio $\Lambda_b \rightarrow P_c K$, which are $(2.08^{+2.43}_{-0.61}) \times 10^{-5}$, $(1.54^{+0.62}_{-0.52}) \times 10^{-4}$, and $(4.47^{+2.03}_{-1.58}) \times 10^{-5}$ for $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, respectively, which is about 1 order larger than the present estimation. However, on the experimental side, the measured upper limit is dependent on the interferences between the pentaquarks and the nonresonance continuum that is model dependent. On the theoretical side, there are no experimental measurement or theoretical estimations of the transition form factors of $\Lambda_b \rightarrow \Sigma_b$. In the present work, we borrow the theoretical estimation of the form factors of $\Lambda_b \rightarrow \Lambda_c$ and further argue that the transition form factors of $\Lambda_b \rightarrow \Sigma_b$ should be about 1 order smaller than the one of $\Lambda_b \rightarrow \Lambda_c$ due to the spin-flip suppression, which will also lead to some uncertainties of the present estimation of the partial width of $\Lambda_b \rightarrow J/\psi p$. Thus, we mainly focus on the ratios of the productions fractions, which are independent of the suppression factor R .

IV. CONCLUSIONS

In the present work, we estimated the P_c production from Λ_b decay in the P_c molecular scenario, in which $P_c^+(4312)$ is considered as $\Sigma_c \bar{D}$ molecular with $J^P = \frac{1}{2}^-$, while $P_c^+(4440)$ and $P_c^+(4457)$ are interpreted as a $\Sigma_c \bar{D}^*$ molecule with $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively. By analyzing the P_c production at quark level, we find the dominant contributions to P_c productions come from the charmed hadron loops, where the initial Λ_b couples to $\Sigma_c \bar{D}^{(*)}$, then the $\bar{D}_s^{(*)}$ transits into $\bar{D}^{(*)}$ via kaon emission, and the recoil $\bar{D}^{(*)}$ and Σ_c couple to P_c states.

The P_c production processes are investigated at hadronic level with an effective Lagrangian approach. Unfortunately, the transition form factors related to $\Lambda_b \rightarrow \Sigma_c$ are unknown. In the present work, we borrow the form factors of $\Lambda_b \rightarrow \Lambda_c$ since the flavor and spatial parts of Λ_c and Σ_c are the same. However, the transition form factors of $\Lambda_b \rightarrow \Sigma_c$ should be smaller than those of $\Lambda_b \rightarrow \Lambda_c$ since the suppression is caused by the light quark spin flip. Here, we define a suppression factor R . Our estimation indicates that the R -independent ratios of $\mathcal{B}(\Lambda_b \rightarrow P_c K) \times \mathcal{B}(P_c \rightarrow J/\psi p)$ in the molecular scenario are consistent with the experimental measurements from the LHCb Collaboration [33]. Moreover, we find the magnitude of the production of the

branching ratios for $\Lambda_b \rightarrow P_c K$ and $P_c \rightarrow J/\psi p$ could be reproduced when one takes the suppression factor $R = 0.09$.

In the molecular scenario, the branching ratios of $\Lambda_b \rightarrow P_c K$ are estimated. Together with the branching fractions of $P_c \rightarrow J/\psi p$ estimated in Ref. [56], we find the ratios of the product of production and decay fractions of P_c states are consistent with the experimental measurements, which indicates that P_c states could be good candidates of $\Sigma_c \bar{D}^{(*)}$. However, the magnitudes of the production and decay fractions of P_c states in the present work and in Ref. [56] are model dependent, and on the experimental side, there are no direct measurements of these fractions. Direct comparison of theoretical estimations and experimental measurements of P_c production and decay fractions will be essential for understanding the nature of these states, which need further efforts from both the theoretical and experimental sides.

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APPENDIX: THE TRANSITION FORM FACTORS OF $\Lambda_b \rightarrow \Lambda_c$

The transition form factor pf of $\Lambda_b \rightarrow \Lambda_c$ could be parametrized in the form [69]

$$F(Q^2)_i^{A,V} = \frac{F(0)}{1 - a\zeta + b\zeta^2}, \quad (A1)$$

where $\zeta = Q^2/m^2$. In Table II, we collect the parameters related to the transition form factors of $\Lambda_b \rightarrow \Lambda_c$ [69], where F_i^V and F_i^A ($i = 1, 2, 3$) are the form factors of $\Lambda_b \rightarrow \Lambda_c$.

In the present estimation, we further parametrize the form factors in the form

$$F(Q^2) = F(0) \frac{\Lambda_1^2}{Q^2 - \Lambda_1^2} \frac{\Lambda_2^2}{Q^2 - \Lambda_2^2}, \quad (A2)$$

TABLE II. The values of the parameters $F(0)$, a , and b in the form factors of $\Lambda_b \rightarrow \Lambda_c$ transition [69].

	F_1^V	F_2^V	F_3^V	F_1^A	F_2^A	F_3^A
$F(0)$	0.549	-0.110	-0.023	0.542	-0.018	-0.123
a	1.459	1.680	1.181	1.443	0.921	1.714
b	0.571	0.794	0.276	0.559	0.255	0.828

TABLE III. Values of parameters Λ_1 and Λ_2 in the unit of GeV.

Parameter	$F_1^{V/A}$	$F_2^{V/A}$	$F_3^{V/A}$
Λ_1	6.613/6.649	6.218/8.320	7.268/6.160
Λ_2	6.598/6.635	6.146/8.246	7.223/6.085

which can avoid ultraviolet divergence in the loop integrals and evaluate the loop integrals with Feynman parametrization methods. The values of Λ_1 and Λ_2 in the above form factor are obtained by fitting Eq. (A1) with Eq. (A2), and

the fitted parameter values are list in Table III. From the fit, one can find the poles of $F(Q^2)$ are located above the mass of Λ_b , which is the unphysical region in the present work.

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