Properties of excited charmed-bottom mesons

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We calculate the spectrum of B_c mesons using a nonrelativistic quark potential model. Using the calculated wave functions, we compute the radiative widths of B_c excited states. The strong decay widths are calculated in a modified 3P_0 model, assuming harmonic oscillator wave functions. The hadronic transition rates of B_c mesons are calculated using the Kuang-Yan approach. These results are used to determine branching ratios of possible decay channels of several B_c excited states. Calculated branching ratios are then combined with production cross section of B_c states at the LHC to suggest strategies to find missing excited states of B_c mesons.

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I. INTRODUCTION

The B_c^+ meson is the lowest-mass bound state of a charm quark and a bottom antiquark. This pseudoscalar mesonic ground state has no electromagnetic or strong decays as it cannot annihilate into gluons or photons. It was first observed by the CDF Collaboration at Fermilab through a semileptonic decay mode $B_c^\pm \rightarrow J/\psi l^\pm \nu$ in $p\bar{p}$ collisions. The measured mass and life time of B_c by CDF were $6.40 \pm 0.39 \pm 0.13$ GeV and $0.46_{-0.16}^{+0.18} \pm 0.03$ ps respectively [1]. It has also been observed by LHCb [2] and D0 [3] experiments through its different decay channels. In 2014 an excited state of B_c meson was observed by ATLAS experiment at LHC through the decay channel $B_c^\pm \rightarrow J/\psi \pi^\pm$ in pp collisions. The measured mass of the excited state was found to be $6842 \pm 4 \pm 5$ MeV [4], which is considered the second S-wave state of B_c . However, this excited state has not been confirmed by other experiments yet. Excited B_c states below BD threshold (≈ 7144 MeV) can only decay through radiative and hadronic transitions to B_c ground state, which decay through weak interaction. There are at least two S-wave, two P-wave, and one D-wave B_c multiplets lying below the threshold. Each of these states cascades into B_c ground state through emission of photons and/or pions only. This results into unique experimental signatures through which

we can identify them. This is particularly important when a large sample of B_c states is expected to be produced at the LHC. To predict event rates of various decay chains of excited B_c states lying below the BD threshold at the LHC, we require a knowledge of branching ratios of their electromagnetic and hadronic transitions along with their production cross sections.

There have been many calculations of the B_c spectrum using nonrelativistic and relativistic quark models [5–10]. The electromagnetic transitions of B_c are predicted in Refs. [6,8–11] and hadronic transitions are calculated in [8,12]. In Ref. [5], the open-flavor strong decay widths of B_c mesons are predicted in the 3P_0 model. Reference [5] studied only strong decays to pairs of S-wave mesons for many open flavor states, using the same value of the harmonic oscillator parameter β for all the flavor states. The present work provides a comprehensive theoretical study of B_c mesons properties: Here we calculate the spectrum, radiative transitions ($E1$ and $M1$), hadronic transitions, and strong decays of B_c mesons following a consistent approach. A nonrelativistic potential quark model is used to explain the mass spectrum of B_c mesons. The wave functions computed through this model are then used to find the decay widths of their $E1$ and $M1$ transitions. The hadronic transitions of B_c mesons are estimated by using the Kuang-Yan approach. We use the 3P_0 model to study the open-flavor strong decays of B_c states. In comparison to Ref. [5], we have calculated strong decay widths of all angularly excited B_c states while using different values of β for different flavor states. We combine radiative, hadronic, and strong widths to predict the branching ratios of all possible decay channels of several B_c excited states. The branching ratios of radiative and hadronic transitions of B_c excited states lying below the BD threshold are combined with their predicted production cross sections at the LHC energy to provide estimates of event rates of their possible

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decay chains through which these states can be identified in the experimental data.

The organization of the paper is as follows. First, we describe the potential model used to calculate the mass spectrum of charm-bottom mesons. In Sec. III, we review the 3P_0 decay model and evaluate the strong decay amplitudes. $E1$ and $M1$ radiative transitions are calculated in Sec. IV. This is followed by the estimates in Sec. V of hadronic transitions based on the Kuang-Yan approach. We discuss the best strategies for searching the excited B_c states lying below the BD threshold in Sec. VI, while our concluding remarks are given in Sec. VII.

II. MASS SPECTRUM

In this section we give the mass predictions of the nonrelativistic quark model for charm-bottom mesons. We use the standard “Coulomb + linear” potential, and spin-dependent corrections generated from vector gluon exchange and an effective scalar confinement interaction. The potential used in this paper is given by [13]

$$V_{q\bar{q}}(r) = -\frac{4\alpha_s}{3r} + br + \frac{32\pi\alpha_s}{9m_q m_{\bar{q}}} \left(\frac{\sigma}{\sqrt{\pi}} \right)^3 e^{-\sigma^2 r^2} \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \\ + \frac{4\alpha_s}{m_q m_{\bar{q}} r^3} T + \left(\frac{\mathbf{S}_q}{4m_q^2} + \frac{\mathbf{S}_{\bar{q}}}{4m_{\bar{q}}^2} \right) \cdot \mathbf{L} \left(\frac{4\alpha_s}{3r^3} - \frac{b}{r} \right) \\ + \frac{\mathbf{S}_q + \mathbf{S}_{\bar{q}}}{2m_q m_{\bar{q}}} \cdot \mathbf{L} \frac{4\alpha_s}{3r^3}. \quad (1)$$

Here α_s is the strong coupling constant, b is the string tension, and T is the tensor operator

$$T = \mathbf{S}_q \cdot \hat{r} \mathbf{S}_{\bar{q}} \cdot \hat{r} - \frac{1}{3} \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}, \quad (2)$$

with diagonal matrix elements given by

$$T = \begin{cases} -\frac{L}{6(2L+3)} & J = L + 1 \\ \frac{1}{6} & J = L \\ -\frac{(L+1)}{6(2L-1)} & J = L - 1. \end{cases} \quad (3)$$

The strong coupling constant α_s used in this potential for B_c mesons is taken to be 0.4. This value was obtained by our fit to the masses of two experimentally known states of B_c mesons (these are listed in the fourth column of Table I). The parameters $\sigma = 0.84$ GeV, $b = 0.0945$ GeV², $m_{u/d} = 0.325$ GeV, and $m_s = 0.422$ GeV were obtained from fits to light mesons [14], and $m_c = 1.4794$ GeV is from the charmonium sector. Finally $m_b = 4.825$ GeV is taken from Ref. [15].

The meson states with quark and antiquark of unequal mass are not charge conjugation eigenstates. Therefore, states with $J = L$ and $S = 0, 1$, i.e., $|n^1L_J\rangle$ and $|n^3L_J\rangle$ can

mix via spin-orbit interaction. For example 1P_1 and 3P_1 states can mix through the following linear combination:

$$|B(^1P_1)\rangle = +\cos(\phi_{1P})|1^1P_1\rangle + \sin(\phi_{1P})|1^3P_1\rangle, \\ |B(^1P'_1)\rangle = -\sin(\phi_{1P})|1^1P_1\rangle + \cos(\phi_{1P})|1^3P_1\rangle, \quad (4)$$

where ϕ_{1P} is the mixing angle. In the heavy quark limit $m_Q \rightarrow \infty$ the mixing angles become [17]

$$\phi_{m_Q \rightarrow \infty} = \arctan\left(\sqrt{\frac{L}{L+1}}\right). \quad (5)$$

This implies $\phi_{nP} = 35.3^\circ$, $\phi_{nD} = 39.2^\circ$, and $\phi_{nF} = 40.89^\circ$. The mixing angles in the heavy quark limit for $1D$ and $1F$ states are close to those produced by Ref. [8] whereas for the $1P$ state it is slightly different. The spectrum of B_c states obtained by solving the radial Schrödinger equation through the shooting method [18] is reported in Tables I and II.

III. OPEN FLAVOR STRONG DECAYS

In the 3P_0 model, the open-flavor strong decay of a meson takes place through production of a light $q\bar{q}$ pair ($q = u, d, s$) with vacuum quantum numbers ($J^{PC} = 0^{++}$). The interaction Hamiltonian for the 3P_0 model is obtained from the nonrelativistic limit of

$$H_I = 2m_q \gamma \int d^3\mathbf{x} \bar{\psi}_q(\mathbf{x}) \psi_q(\mathbf{x}), \quad (6)$$

where γ is a dimensionless pair production strength. The pair-production strength parameter γ is fitted to strong decay data. In the original 3P_0 model introduced by Micu [19], a new $q\bar{q}$ pair is produced by a constant pair production amplitude γ . Some variants of the 3P_0 model include an effective pair production strength (γ^{eff}) that suppresses heavy $q\bar{q}$ pair production [20,21].

The 3P_0 model has been successfully applied to strong decays of light mesons [22], strange mesons [23,24], charmonium states [13], bottomonium states [25,26], open-charm [27,28], and open-bottom sectors [5,14,29,30]. In this study, we have computed strong decay widths of kinematically allowed open-flavor decay modes of all the B_c states mentioned in Tables I and II using the 3P_0 model. The interaction Hamiltonian for the 3P_0 model can be written in terms of the creation operators as

$$H_I = 2m_q \gamma \int d^3k [\bar{u}(\mathbf{k}, s) v(-\mathbf{k}, \bar{s})] b^\dagger(\mathbf{k}, s) d^\dagger(-\mathbf{k}, \bar{s}), \quad (7)$$

where $b^\dagger$ and $d^\dagger$ are the creation operators for quark and antiquark respectively. The pair production strength factor

TABLE I. Masses of ground and excited states of B_c mesons. The mixed states $P_1 - P'_1$, $D_2 - D'_2$, and $F_3 - F'_3$ and their mixing angles ϕ_{nP} , ϕ_{nD} , and ϕ_{nF} are defined according to Eq. (5). The SHO β values are listed in the last column which are obtained by fitting SHO wave functions to the quark model wave functions.

nL	Meson	Our calculated mass (GeV)	Expt. mass [16] (MeV)	β (GeV)
1S	$B_c(1^1S_0)$	6.318	6274.9 ± 0.8	0.653
	$B_c(1^3S_1)$	6.336	...	0.634
2S	$B_c(2^1S_0)$	6.741	6842 ± 4	0.515
	$B_c(2^3S_1)$	6.747	...	0.508
3S	$B_c(3^1S_0)$	7.014	...	0.442
	$B_c(3^3S_1)$	7.018	...	0.439
4S	$B_c(4^1S_0)$	7.239	...	0.402
	$B_c(4^3S_1)$	7.242	...	0.401
1P	$B_c(1^3P_0)$	6.631	...	0.468
	$B_c(1^3P_2)$	6.665	...	0.468
	$B_c(1P_1)$	6.650	...	0.471, 0.468
	$B_c(1P'_1)$	6.656	...	0.471, 0.468
	ϕ_{1P}	35.3°	...	
2P	$B_c(2^3P_0)$	6.915	...	0.428
	$B_c(2^3P_2)$	6.946	...	0.428
	$B_c(2P_1)$	6.930	...	0.430, 0.428
	$B_c(2P'_1)$	6.939	...	0.430, 0.428
	ϕ_{2P}	35.3°	...	
3P	$B_c(3^3P_0)$	7.147	...	0.395
	$B_c(3^3P_2)$	7.176	...	0.395
	$B_c(3P_1)$	7.162	...	0.397, 0.395
	$B_c(3P'_1)$	7.168	...	0.397, 0.395
	ϕ_{3P}	35.3°	...	
4P	$B_c(4^3P_0)$	7.350	...	0.373
	$B_c(4^3P_2)$	7.379	...	0.373
	$B_c(4P_1)$	7.364	...	0.374, 0.373
	$B_c(4P'_1)$	7.373	...	0.374, 0.373
	ϕ_{4P}	35.3°	...	
1D	$B_c(1^3D_1)$	6.841	...	0.417
	$B_c(1^3D_3)$	6.847	...	0.417
	$B_c(1D_2)$	6.845	...	0.417, 0.417
	$B_c(1D'_2)$	6.845	...	0.417, 0.417
	ϕ_{1D}	39.2°	...	
2D	$B_c(2^3D_1)$	7.080	...	0.395
	$B_c(2^3D_3)$	7.087	...	0.395
	$B_c(2D_2)$	7.084	...	0.395, 0.395
	$B_c(2D'_2)$	7.084	...	0.395, 0.395
	ϕ_{2D}	39.2°	...	
3D	$B_c(3^3D_1)$	7.289	...	0.374
	$B_c(3^3D_3)$	7.296	...	0.374
	$B_c(3D_2)$	7.293	...	0.374, 0.374
	$B_c(3D'_2)$	7.293	...	0.374, 0.374
	ϕ_{3D}	39.2°	...	
4D	$B_c(4^3D_1)$	7.478	...	0.357
	$B_c(4^3D_3)$	7.489	...	0.357
	$B_c(4D_2)$	7.482	...	0.358, 0.357
	$B_c(4D'_2)$	7.483	...	0.358, 0.357
	ϕ_{4D}	39.2°	...	

TABLE II. Masses of ground and excited states of B_c mesons (continued).

nL	Meson	Our calculated mass (GeV)	Expt. mass [16] (MeV)	β (GeV)
1F	$B_c(1^3F_2)$	6.9972	...	0.390
	$B_c(1^3F_4)$	6.9967	...	0.390
	$B_c(1F_3)$	6.994	...	0.390, 0.390
	$B_c(1F'_3)$	7.001	...	0.390, 0.390
	ϕ_{1F}	40.89°	...	
2F	$B_c(2^3F_2)$	7.2121	...	0.375
	$B_c(2^3F_4)$	7.2126	...	0.375
	$B_c(2F_3)$	7.211	...	0.375, 0.375
	$B_c(2F'_3)$	7.214	...	0.375, 0.375
	ϕ_{2F}	40.89°	...	

$\gamma = 0.35$ is obtained from a fit of known strong decay widths of the $c\bar{c}$ states above open-charm threshold [13]. In this work, we use a modified version of pair production strength that replaces γ with

$$\gamma^{\text{eff}} = \frac{m_{u/d}}{m} \gamma, \quad (8)$$

where m is the mass of the produced quark [20,21]. This mechanism suppresses those diagrams in which a heavy $q\bar{q}$ pair is created.

With the 3P_0 model, we use simple harmonic oscillator (SHO) wave functions and the SHO scale β is taken as the parameter of the model. In Refs. [27–30], the β parameter was obtained by equating the root-mean-squared (rms) radius of a harmonic oscillator wave function to the rms radius of the quark model wave function. In this work, we fit β of SHO wave functions to the wave functions obtained by numerically

FIG. 1. Plots of partial widths of some of the strong decays of B_c mesons in the 3P_0 model as a function of β for the initial meson. Solid curve: $B_c(4^1S_0) \rightarrow DB^*$; dotted curve: $B_c(3^3P_0) \rightarrow DB$; dashed curve: $B_c(4^3D_1) \rightarrow DB$; dot-dashed curve: $B_c(2^3F_2) \rightarrow DB$. Disk and rectangular marks on each curve correspond to the β values obtained by the fit to numerically calculated wave functions and to rms radii respectively.

solving the Schrödinger equation for the potential given in Eq. (1). The resulting β values, that are more accurate, are listed in Tables I–III for the initial B_c mesons, and the final D , D_s , B , and B_s mesons appearing in strong decays of B_c excited states. These two methods of finding β are compared in our earlier work [14]. In Fig. 1 we show the dependence of strong decay widths of few decay channels on the value of SHO parameter β . Disk and rectangular marks on each curve correspond to the β values obtained by fit to numerically calculated wave functions and to rms radii respectively. These plots show that fitted β values lie in the sensitive regions of the curves, which implies that an accurate method of determining the values of the β parameter can significantly improve the results.

To calculate the decay rate of a process $A \rightarrow B + C$, we evaluate the matrix element $\langle BC | H_I | A \rangle$ by using the interaction Hamiltonian of Eq. (7). In general, two different diagrams, shown in Fig. 2, contribute to the matrix element $\langle BC | H_I | A \rangle$. Using the relativistic phase space factor from Ref. [31] and performing the angular integration gives

$$\Gamma_{A \rightarrow BC} = 2\pi \frac{P E_B E_C}{M_A} \sum_{LS} |\mathcal{M}_{LS}|^2, \quad (9)$$

where $P = |\mathbf{P}_B| = |\mathbf{P}_C|$, M_A is the mass of the initial meson, and $E_B = \sqrt{M_B^2 + P^2}$ and $E_C = \sqrt{M_C^2 + P^2}$ are the energies of the final mesons B and C respectively. Where available, we use experimental masses [16] of B_c mesons; otherwise we use the theoretical masses given in Tables I and II. The masses of the final state mesons D ,

FIG. 2. Decay diagrams in the 3P_0 model.

TABLE III. Masses and SHO parameter (β) for open-charm and open-bottom mesons used in our strong decay width calculations. The experimental masses are taken from PDG [16]. The SHO β values are listed in the second column which we obtain by the fit of SHO wave function to the numerical solutions of Schrödinger equation.

Meson	β (GeV)	Expt. Mass [16] (MeV)	Mass used in calculations (GeV)
D	0.442	$D^\pm = 1869.58 \pm 0.09$ $D^0 = 1864.83 \pm 0.08$	1.867
D^*	0.338	$D^{*\pm} = 2010.26 \pm 0.05$ $D^{*0} = 2006.85 \pm 0.05$	2.007
D_s	0.463	$D_s^\pm = 1968.27 \pm 0.10$	1.968
D_s^*	0.369	$D_s^{*\pm} = 2112.1 \pm 0.4$	2.112
B	0.405	$B^\pm = 5279.31 \pm 0.15$ $B^0 = 5279.62 \pm 0.15$	5.279
B^*	0.372	$B^* = 5324.65 \pm 0.25$	5.325
B_s	0.429	$B_s^0 = 5366.82 \pm 0.22$	5.367
B_s^*	0.401	$B_s^{*0} = 5415.4^{+1.8}_{-1.5}$	5.415

D_s , B , and B_s are reported in Table III. The detailed formulism to calculate the matrix element $\langle BC|H_I|A \rangle$ and decay amplitude $\mathcal{M}_{LS}$ is described in our earlier work [14].

IV. RADIATIVE TRANSITIONS OF B_c MESONS

A. $E1$ radiative transitions

$E1$ radiative partial widths are computed with the following expression [32]:

$$\Gamma(n^{2S+1}L_J \rightarrow n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3} \langle e_Q \rangle^2 \alpha \omega^3 C_{fi} \delta_{SS'} |\langle n'^{2S'+1}L'_{J'} | r | n^{2S+1}L_J \rangle|^2 \frac{E_f}{M_i}, \quad (10)$$

where

$$\langle e_Q \rangle = \frac{m_b e_c - m_c e_{\bar{b}}}{m_b + m_c}. \quad (11)$$

Here e_c and $e_{\bar{b}}$ are the electric charges of the charm quark and bottom antiquark in units of $|e|$, m_b and m_c are the constituent masses of the charm and bottom quarks, α is the fine structure constant, ω is the final photon energy, M_i is mass of the initial meson, and E_f is the energy of the final state. Finally, the angular matrix element C_{fi} is given by

$$C_{fi} = \max(L, L')(2J' + 1) \begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix}^2.$$

Equation (10) is the result of Ref. [32] except for our inclusion of the relativistic phase space factor E_f/M_i from Refs. [13,27]. The matrix elements $\langle n'^{2S'+1}L'_{J'} | r | n^{2S+1}L_J \rangle$ are obtained using the quark model wave functions obtained in Sec. II. Wave function corrections due to perturbative spin-dependent interactions were neglected in this computation, as in Refs. [13,27]. Results for $E1$ radiative transitions for B_c mesons are given in Tables VII–XIV along with the matrix elements so that an interested reader can reproduce our results.

B. $M1$ radiative transitions

The $M1$ radiative partial widths are evaluated using the following expression [33]:

$$\Gamma(n^{2S+1}L_J \rightarrow n'^{2S'+1}L'_{J'} + \gamma) = \frac{\alpha}{3} \omega^3 (2J' + 1) \delta_{S,S'+1} \left| \frac{e_c}{m_c} \langle f | j_0 \left(\frac{m_b}{m_c + m_b} \omega r \right) | i \rangle - \frac{e_{\bar{b}}}{m_b} \langle f | j_0 \left(\frac{m_c}{m_c + m_b} \omega r \right) | i \rangle \right|^2, \quad (12)$$

where $j_0(x)$ is a spherical Bessel function. The definitions of the other parameters are the same as in the $E1$ radiative transitions. The results for $M1$ radiative transitions for B_c mesons are given in Tables VII–XIV.

V. $E1$ - $E1$ HADRONIC TRANSITIONS

Hadronic transitions are needed to estimate branching ratios and the event rates of decay chains of B_c states lying below the BD threshold. The differential rate for $E1$ - $E1$ hadronic transitions from an initial meson state Φ' to the final meson state Φ and a system of light hadrons h is given by [34,35]

$$\frac{d\Gamma}{d\mathcal{M}^2} [\Phi' \rightarrow \Phi + h] = (2J + 1) \sum_{k=0}^2 \left\{ \begin{matrix} k & l' & l \\ s & J & J' \end{matrix} \right\}^2 A_k(l', l), \quad (13)$$

where J' , J are the total angular momentum and l' , l are the orbital angular momentum of initial and final meson states respectively, $\left\{ \begin{matrix} \dots \end{matrix} \right\}$ is a 6-j symbol, $\mathcal{M}^2$ is the invariant mass squared of the light hadron system, and $A_k(l', l)$ are the reduced matrix elements [34]. Here we use scaling argument to predict the hadronic rates for $c\bar{b}$ mesons using measured rates of $c\bar{c}$ and/or $b\bar{b}$ as input. When measured rates are not available, we use predicted rates of hadronic transition of $b\bar{b}$ states. The scaling law for $E1$ - $E1$ hadronic transitions is given by [26,34]

$$\frac{\Gamma(c\bar{b})}{\Gamma(Q\bar{Q})} = \frac{\langle r^2(c\bar{b}) \rangle^2}{\langle r^2(Q\bar{Q}) \rangle^2} P, \quad (14)$$

TABLE IV. Estimates of reduced rates of hadronic transitions between $c\bar{b}$ mesons.

Transition	$(Q\bar{Q})$: rate (keV)	$\langle r^2(c\bar{b}) \rangle / \langle r^2(Q\bar{Q}) \rangle$	Our reduced $c\bar{b}$ rate (keV)	Reduced $c\bar{b}$ rate [8] (keV)
$2^3S_1 \rightarrow 1^3S_1 + \pi\pi$	$(c\bar{c})$: 155.84 ± 5.2^a $(b\bar{b})$: 8.46 ± 0.7^a average:	0.94 2.36	$A_0(0,0) = 61.59 \pm 2.1$ $A_0(0,0) = 26.06 \pm 2.2$ 43.83 ± 2.2	$A_0(0,0) = 82 \pm 8$ $A_0(0,0) = 33 \pm 5$ 57 ± 7
$3^3S_1 \rightarrow 1^3S_1 + \pi\pi$	$(b\bar{b})$: 1.34 ± 0.12^a	2.24	$A_0''(0,0) = 2.08 \pm 0.19$	$A_0''(0,0) = 4.2 \pm 0.6$
$2^3P_0 \rightarrow 1^3P_0 + \pi\pi$	$(b\bar{b})$: 0.44^b	2.23	$A_0(1,1) = 1.82$	$A_0(1,1) = 2.92$
$2^3P_2 \rightarrow 1^3P_1 + \pi\pi$	$(b\bar{b})$: 0.23^b	2.22	$A_2(1,1) = 0.328$	$A_2(1,1) = 0.164$
$1^3D_1 \rightarrow 1^3S_1 + \pi\pi$	$(b\bar{b})$: 0.14^b	2.23	$A_2(2,0) = 1.183$	$A_2(2,0) = 21$
$1^3F_2 \rightarrow 1^3P_0 + \pi\pi$	$(b\bar{b})$: 1.8×10^{-3b}	2.19	$A_2(3,1) = 0.114$	...

^aFrom PDG Ref. [16].^bFrom Ref. [26].

where $\langle r^2(Q\bar{Q}) \rangle$ is the expectation value of the square of the interquark separation and p is the phase space factor depending on the masses of initial and final states. The phase space factors for A_0 and A_2 reduced matrix elements are $\frac{G(\Phi'(c\bar{b}) \rightarrow \Phi(c\bar{b})\pi\pi)}{G(\Phi'(Q\bar{Q}) \rightarrow \Phi(Q\bar{Q})\pi\pi)}$ and $\frac{H(\Phi'(c\bar{b}) \rightarrow \Phi(c\bar{b})\pi\pi)}{H(\Phi'(Q\bar{Q}) \rightarrow \Phi(Q\bar{Q})\pi\pi)}$ respectively, where G and H are defined in Ref. [35].

For the $B_c(2S) \rightarrow B_c(1S) + \pi\pi$ reduced rates, we rescale the measured reduced rates of $\psi(2^3S_1) \rightarrow \psi(1^3S_1) + \pi\pi$ and $\Upsilon(2^3S_1) \rightarrow \Upsilon(1^3S_1) + \pi\pi$ [16] and take their average value. The reduced rates for $B_c(3S) \rightarrow B_c(1S) + \pi\pi$ are obtained by rescaling the corresponding measured reduced rates of $b\bar{b}$ [16]. There are 16 possible $B_c(2P) \rightarrow B_c(1P) + \pi\pi$ hadronic transitions, which can be expressed

in terms of three reduced rates $A_0(1,1)$, $A_1(1,1)$, and $A_2(1,1)$ using Eq. (13). In the soft-pion limit $A_1(l',l)$ contributions are suppressed, so we take $A_1(1,1) = 0$. $A_0(1,1)$ and $A_2(1,1)$ are obtained by rescaling the reduced rates of $\Upsilon(2^3P_0) \rightarrow \Upsilon(1^3P_0) + \pi\pi$ and $\Upsilon(2^3P_2) \rightarrow \Upsilon(1^3P_1) + \pi\pi$ predicted by Godfrey and Moats [26]. For calculating the hadronic transitions for $B_c(1D) \rightarrow B_c(1S) + \pi\pi$ and $B_c(1F) \rightarrow B_c(1P) + \pi\pi$, we rescale the reduced rates of $\Upsilon(1^3D_1) \rightarrow \Upsilon(1^3S_1) + \pi\pi$ and $\Upsilon(1^3F_2) \rightarrow \Upsilon(1^3P_0) + \pi\pi$ which are also taken from Ref. [26]. The scaling factors and reduced rates between $c\bar{b}$ mesons are given in Table IV. These reduced rates are used to determine the $c\bar{b}$ hadronic transitions which are

TABLE V. Rates for the $E1$ - $E1$ hadronic transitions of $2S$, $3S$, and $2P$ states of B_c mesons.

Transition	Expression for the rate	the $c\bar{b}$ rate (keV)
$2^1S_0 \rightarrow 1^1S_0 + \pi\pi$	$A_0(0,0)$	10.68
$2^3S_1 \rightarrow 1^3S_1 + \pi\pi$	$A_0(0,0)$	3.99
$3^1S_0 \rightarrow 1^1S_0 + \pi\pi$	$A_0''(0,0)$	0.97
$3^3S_1 \rightarrow 1^3S_1 + \pi\pi$	$A_0''(0,0)$	0.81
$2^3P_0 \rightarrow 1P_1 + \pi\pi$	$\frac{1}{3}A_1(1,1)$	0
$2^3P_0 \rightarrow 1P_1' + \pi\pi$	$\frac{1}{3}A_1(1,1)$	0
$2^3P_0 \rightarrow 1^3P_0 + \pi\pi$	$\frac{1}{3}A_0(1,1)$	0.002
$2^3P_2 \rightarrow 1^3P_2 + \pi\pi$	$\frac{1}{3}A_0(1,1) + \frac{1}{4}A_1(1,1) + \frac{7}{60}A_2(1,1)$	0.0007
$2^3P_2 \rightarrow 1P_1 + \pi\pi$	$\frac{1}{12}A_1(1,1) + \frac{3}{20}A_2(1,1)$	0.0002
$2^3P_2 \rightarrow 1P_1' + \pi\pi$	$\frac{1}{12}A_1(1,1) + \frac{3}{20}A_2(1,1)$	0.0002
$2^3P_2 \rightarrow 1^3P_0 + \pi\pi$	$\frac{1}{15}A_2(1,1)$	0.0005
$2P_1 \rightarrow 1P_1 + \pi\pi$	$\frac{1}{3}A_0(1,1) + \frac{1}{12}A_1(1,1) + \frac{1}{12}A_2(1,1)^a$	0.0003
$2P_1 \rightarrow 1^3P_0 + \pi\pi$	$\frac{1}{9}A_1(1,1)$	0
$2P_1' \rightarrow 1P_1 + \pi\pi$	$\frac{1}{3}A_0(1,1) + \frac{1}{3}A_1(1,1) + \frac{1}{3}A_2(1,1)^b$	3×10^{-6}
$2P_1' \rightarrow 1P_1' + \pi\pi$	$\frac{1}{3}A_0(1,1) + \frac{1}{3}A_1(1,1) + \frac{1}{3}A_2(1,1)^b$	0.0014
$2P_1' \rightarrow 1^3P_0 + \pi\pi$	$\frac{1}{9}A_1(1,1)$	0

^aThe expression is for $^3P_1 \rightarrow ^3P_1$ transition.^bThe expression is for $^1P_1 \rightarrow ^1P_1$ transition.

TABLE VI. Rates for the $E1$ - $E1$ hadronic transitions of $1D$ and $1F$ states of B_c mesons.

Transition	Expression for the rate	the $c\bar{b}$ rate (keV)
$1^3D_1 \rightarrow 1^3S_1 + \pi\pi$	$\frac{1}{5}A_2(2, 0)$	0.042
$1^3D_3 \rightarrow 1^3S_1 + \pi\pi$	$\frac{1}{5}A_2(2, 0)$	0.043
$1D_2 \rightarrow 1^1S_0 + \pi\pi$	$\frac{1}{5}A_2(2, 0)$	0.035
$1D_2 \rightarrow 1^3S_1 + \pi\pi$	$\frac{1}{5}A_2(2, 0)$	0.017
$1D'_2 \rightarrow 1^1S_0 + \pi\pi$	$\frac{1}{5}A_2(2, 0)$	0.23
$1D'_2 \rightarrow 1^3S_1 + \pi\pi$	$\frac{1}{5}A_2(2, 0)$	0.026
$1^3F_2 \rightarrow 1^3P_0 + \pi\pi$	$\frac{1}{15}A_2(3, 1)$	0.0004
$1^3F_2 \rightarrow 1^3P_2 + \pi\pi$	$\frac{1}{105}A_2(3, 1)$	0.00004
$1^3F_2 \rightarrow 1P_1 + \pi\pi$	$\frac{1}{15}A_2(3, 1)$	0.0001
$1^3F_2 \rightarrow 1P'_1 + \pi\pi$	$\frac{1}{15}A_2(3, 1)$	0.0002
$1F_3 \rightarrow 1^3P_2 + \pi\pi$	$\frac{1}{21}A_2(3, 1)$	0.0001
$1F_3 \rightarrow 1P_1 + \pi\pi$	$\frac{2}{21}A_2(3, 1)^a$	0.001
$1F_3 \rightarrow 1P'_1 + \pi\pi$	$\frac{2}{21}A_2(3, 1)^a$	~ 0
$1F'_3 \rightarrow 1^3P_2 + \pi\pi$	$\frac{1}{21}A_2(3, 1)$	0.0001
$1F'_3 \rightarrow 1P_1 + \pi\pi$	$\frac{1}{7}A_2(3, 1)^b$	0.00002
$1F'_3 \rightarrow 1P'_1 + \pi\pi$	$\frac{1}{7}A_2(3, 1)^b$	0.001
$1^3F_4 \rightarrow 1^3P_2 + \pi\pi$	$\frac{1}{7}A_2(3, 1)$	0.0005

^aThe expression is for $^3F_3 \rightarrow ^3P_1$ transition.^bThe expression is for $^1F_3 \rightarrow ^1P_1$ transition.

summarized in Tables V and VI. To calculate the transition rates for mixed states, mixing angles are incorporated, with the values taken from Tables I and II.

In Tables VII–XIV, we combine the widths of radiative decays, strong decays, and hadronic transitions to calculate the total widths and the branching ratios. These BR's are used in the next section to give estimates for the number of events expected at the LHC for different decay chains of B_c states below the threshold. In these tables $c_P = \cos \phi_{nP}$, $s_P = \sin \phi_{nP}$, $c_D = \cos \phi_{nD}$, $s_D = \sin \phi_{nD}$, $c_F = \cos \phi_{nF}$, and $s_F = \sin \phi_{nF}$, with n being the principal quantum number.

VI. EXPERIMENTAL SIGNATURES AND SEARCH STRATEGIES

Our calculated masses of B_c states show that there are three S -wave, two P -wave, two D -wave, and one F -wave B_c multiplets lying below the BD threshold (≈ 7144) MeV. These are the narrow states of the B_c spectrum because B_c cannot annihilate into gluons due to its nonzero flavor. All these excited states below the BD threshold will cascade decay into the ground state B_c through emission of photons and/or pions, which eventually decays through weak interaction. These photons and pions produced by electromagnetic and/or hadronic transitions carry unique signature

of initial B_c . Hence the production events of B_c excited states can be reconstructed by detecting and measuring the energies of produced photons and pions. On the other hand the excited states above the BD threshold will rapidly decay into a pair of $B(B_s)$ and $D(D_s)$ mesons through strong interaction processes. In order to predict observable event rates of B_c excited states (below BD threshold) in pp collision at the LHC, we require the knowledge of their production cross sections, branching ratios of their electromagnetic and hadronic transitions, and the branching ratios of weak decay channels of B_c ground states through which its production can be identified experimentally.

Inclusive production cross sections of B_c states in pp collision at LHC energy has been calculated in Refs. [36–39] using the fragmentation approach and in Refs. [40–45] using the pQCD approach. The results of the two approaches quantitatively agree for $p_T \geq 10$ GeV [44].

The fragmentation approach of Refs. [36,37] shows that production cross sections of $B_c(1^1S_0)$ and $B_c(1^3S_1)$ states are 0.72 and 1.21 nb respectively for transverse momentum $p_T(B_c) > 20$ GeV and rapidity $|y(B_c)| < 2.5$ at LHC. These values include the contribution of both $\bar{b}$ -quark and gluon fragmentation functions of the B_c states. When these values are extrapolated to kinematic cut $p_T(B_c) > 10$ GeV using the values reported in Table III of Ref. [38], we obtain the production cross sections 5.5 and 9.3 nb for the 1^1S_0 and 1^3S_1 states respectively. The production cross sections of the 2^1S_0 and 2^3S_1 states are obtained by multiplying the corresponding values of $1S$ states with the factor $|R_{2S}(0)/R_{1S}(0)|^2 \simeq 0.6$ [36]. In Ref. [38] production cross sections of $1P$ and $2P$ states are calculated using the fragmentation approach. However, for the LHC they report only the total cross section that includes the contribution from $1S$, $2S$, $1P$, and $2P$ states. The reported value 33.8 nb for kinematic cuts $p_T(B_c) > 10$ GeV and $|y(B_c)| < 2.5$ implies that total contribution of $1P$ and $2P$ states is 10.2 nb. We divide this value over eight $1P$ and $2P$ states using the distribution reported for Tevatron in Figs. 3 and 4 of Ref. [38]. It is pointed out in Ref. [37] that the distribution is not much changed at the LHC energy. In Ref. [39], it is shown that total fragmentation probability for a $\bar{b}$ -quark to produce the D -wave B_c mesons is about 2×10^{-5} , equivalent to 2% of the total inclusive cross section of all of B_c states lying below the BD threshold. These estimates of the cross sections are used to predict the number of events of various decay chains of excited B_c states.

The excited states below the BD threshold eventually decay into the B_c ground state; therefore, it is important to observe this state in order to reconstruct the events of originally produced states. Prominent weak decay modes of B_c ground states are summarized in Table 10 of Ref. [8]. We assume that ground state B_c is observed through two golden channels: (i) $B_c^\pm \rightarrow J/\psi + \pi^\pm \rightarrow l^- l^+ \pi^\pm$ having combined BR of 0.013% and detection efficiency

TABLE VII. Partial widths and branching ratios for strong, radiative, and hadronic transitions for the $1S$, $2S$, $3S$, and $4S$ states of B_c mesons. Column 4 labeled $\mathcal{M}$ gives the matrix element appropriate to the particular decay. For $E1$ and $M1$ transitions matrix elements are $\langle \psi_f | r | \psi_i \rangle$ and $\langle \psi_f | j_0(kr \frac{m_{b,c}}{m_c+m_b}) | \psi_i \rangle$ are in units of GeV^{-1} and the strong decay amplitudes are in units of $\text{GeV}^{-1/2}$. Details of the calculations are given in the text.

Meson	Decay mode	Photon energy MeV	Amplitude ($\mathcal{M}$)	Γ_{thy} MeV	B.R (%)
B_c^*	$B_c \gamma$	60.81	$\langle 1^1S_0 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 1^3S_1 \rangle = 0.9983, 0.999$	0.15 keV	100
$B_c(2^1S_0)$	$B_c(1^1S_0) + \pi\pi$			10.68 keV	51.42
	$B_c^* \gamma$	392.83	$\langle 1^3S_1 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 2^1S_0 \rangle = 0.0316, -0.0182$	0.07 keV	0.34
	$B_c(1P_1) \gamma$	90.39	$\langle 1^1P_1 r 2^1S_0 \rangle = -2.8218$	7.11 keV	34.23
	$B_c(1P_1') \gamma$	84.46	$\langle 1^1P_1 r 2^1S_0 \rangle = -2.8218$	2.91 keV	14.01
	total			20.77 keV	100
$B_c(2^3S_1)$	$B_c(1^3S_1) + \pi\pi$			3.99 keV	26.76
	$B_c \gamma$	455.58	$\langle 1^1S_0 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 2^3S_1 \rangle = 0.0887, 0.0303$	0.41 keV	2.75
	$B_c(2^1S_0) \gamma$	6.0	$\langle 2^1S_0 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 2^3S_1 \rangle = 0.9994, 0.9995$	1.4×10^{-4} keV	0.001
	$B_c(1^3P_2) \gamma$	81.5	$\langle 1^3P_2 r 2^3S_1 \rangle = -2.8099$	4.31 keV	28.91
	$B_c(1P_1) \gamma$	96.3	$\langle 1^3P_1 r 2^3S_1 \rangle = -2.8099$	1.43 keV	9.59
	$B_c(1P_1') \gamma$	90.39	$\langle 1^3P_1 r 2^3S_1 \rangle = -2.8099$	2.35 keV	15.76
	$B_c(1^3P_0) \gamma$	115.0	$\langle 1^3P_0 r 2^3S_1 \rangle = -2.8099$	2.42 keV	16.23
	total			14.91 keV	100
$B_c(3^1S_0)$	$B_c(1^1S_0) + \pi\pi$			0.97 keV	4.71
	$B_c^* \gamma$	645.23	$\langle 1^3S_1 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 3^1S_0 \rangle = 0.0259, -0.0079$	0.24 keV	1.17
	$B_c(2^3S_1) \gamma$	261.92	$\langle 2^3S_1 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 3^1S_0 \rangle = 0.0546, -0.0101$	0.07 keV	0.34
	$B_c(1P_1) \gamma$	354.55	$\langle 1^3P_1 r 3^1S_0 \rangle = 0.1055$	0.6 keV	2.92
	$B_c(1P_1') \gamma$	348.86	$\langle 1^3P_1 r 3^1S_0 \rangle = 0.1055$	0.29 keV	1.41
	$B_c(2P_1) \gamma$	83.50	$\langle 2^3P_1 r 3^1S_0 \rangle = -4.3884$	13.56 keV	65.89
	$B_c(2P_1') \gamma$	74.60	$\langle 2^3P_1 r 3^1S_0 \rangle = -4.3884$	4.85 keV	23.57
	total			20.58 keV	100
$B_c(3^3S_1)$	$B_c(1^3S_1) + \pi\pi$			0.81 keV	4.12
	$B_c \gamma$	703.76	$\langle 1^1S_0 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 3^3S_1 \rangle = 0.0552, 0.0145$	0.57 keV	2.9
	$B_c(2^1S_0) \gamma$	271.53	$\langle 2^1S_0 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 3^3S_1 \rangle = 0.0889, 0.0246$	0.08 keV	0.41
	$B_c(3^1S_0) \gamma$	4.0	$\langle 3^1S_0 j_0(kr \frac{m_{b,c}}{m_c+m_b}) 3^3S_1 \rangle = 0.9996, 0.9996$	4.2×10^{-5} keV	0.0002
	$B_c(1^3P_2) \gamma$	344.12	$\langle 1^3P_2 r 3^3S_1 \rangle = 0.0926$	0.35 keV	1.78
	$B_c(1P_1) \gamma$	358.35	$\langle 1^3P_1 r 3^3S_1 \rangle = 0.0926$	0.08 keV	0.41
	$B_c(1P_1') \gamma$	352.66	$\langle 1^3P_1 r 3^3S_1 \rangle = 0.0926$	0.15 keV	0.76
	$B_c(1^3P_0) \gamma$	376.33	$\langle 1^3P_0 r 3^3S_1 \rangle = 0.0926$	0.09 keV	0.46
	$B_c(2^3P_2) \gamma$	71.63	$\langle 2^3P_2 r 3^3S_1 \rangle = -4.3738$	7.09 keV	36.03
	$B_c(2P_1) \gamma$	87.45	$\langle 2^3P_1 r 3^3S_1 \rangle = -4.3738$	2.59 keV	13.16
	$B_c(2P_1') \gamma$	78.56	$\langle 2^3P_1 r 3^3S_1 \rangle = -4.3738$	3.74 keV	19.0
	$B_c(2^3P_0) \gamma$	102.24	$\langle 2^3P_0 r 3^3S_1 \rangle = -4.3738$	4.13 keV	20.99
	total			19.68 keV	100
$B_c(4^1S_0)$	DB^*		$^3P_0 = +0.0305$	2.96	100
$B_c(4^3S_1)$	DB		$^1P_1 = -0.0067$	0.21	11.66
	DB^*		$^3P_1 = -0.022$	1.59	88.29
	total			1.8	100

TABLE VIII. Partial widths and branching ratios for strong, radiative, and hadronic transitions for the $1P$, $2P$, and $3P$ states of B_c mesons.

Meson	Decay mode	Photon energy MeV	Amplitude ($\mathcal{M}$)	Γ_{thy} MeV	B.R (%)
$B_c(1^3P_0)$	$B_c^*\gamma$	288.44	$\langle 1^3S_1 r 1^3P_0 \rangle = 1.896$	52.23 keV	100
$B_c(1P_1)$	$B_c\gamma$	323.71	$\langle 1^1S_0 r 1^1P_1 \rangle = 1.8362$	46.13 keV	68.78
	$B_c^*\gamma$	306.59	$\langle 1^3S_1 r 1^3P_1 \rangle = 1.896$	20.94 keV	31.22
	total			67.07 keV	100
$B_c(1P'_1)$	$B_c\gamma$	329.42	$\langle 1^1S_0 r 1^1P_1 \rangle = 1.8362$	24.37 keV	35.56
	$B_c^*\gamma$	312.31	$\langle 1^3S_1 r 1^3P_1 \rangle = 1.896$	44.16 keV	64.44
	total			68.53 keV	100
$B_c(1^3P_2)$	$B_c^*\gamma$	320.88	$\langle 1^3S_1 r 1^3P_2 \rangle = 1.896$	71.91 keV	100
$B_c(2^3P_0)$	$B_c(1^3P_0) + \pi\pi$			0.002 keV	0.004
	$B_c^*\gamma$	554.76	$\langle 1^3S_1 r 2^3P_0 \rangle = 0.4117$	17.52 keV	35.87
	$B_c(2^3S_1)\gamma$	165.96	$\langle 2^3S_1 r 2^3P_0 \rangle = 3.1266$	27.05 keV	55.38
	$B_c(1^3D_1)\gamma$	73.60	$\langle 1^3D_1 r 2^3P_0 \rangle = -2.9727$	4.27 keV	8.74
	total			48.84 keV	100
$B_c(2P_1)$	$B_c(1P_1) + \pi\pi$			0.0003 keV	0.0005
	$B_c\gamma$	584.98	$\langle 1^1S_0 r 2^1P_1 \rangle = 0.4258$	14.64 keV	23.16
	$B_c^*\gamma$	568.54	$\langle 1^3S_1 r 2^3P_1 \rangle = 0.4117$	6.3 keV	9.97
	$B_c(2^1S_0)\gamma$	186.42	$\langle 2^1S_0 r 2^1P_1 \rangle = 3.0728$	24.67 keV	39.02
	$B_c(2^3S_1)\gamma$	180.58	$\langle 2^3S_1 r 2^3P_1 \rangle = 3.1266$	11.64 keV	18.41
	$B_c(1D_2)\gamma$	84.48	$\langle 1^3D_2 r 2^3P_1 \rangle = -2.9727,$ $\langle 1^1D_2 r 2^1P_1 \rangle = -2.9929$	5.86 keV	9.27
	$B_c(1D'_2)\gamma$	84.48	$\langle 1^3D_2 r 2^3P_1 \rangle = -2.9727,$ $\langle 1^1D_2 r 2^1P_1 \rangle = -2.9929$	0.11 keV	0.17
	total			63.22 keV	100
	$B_c(1P_1) + \pi\pi$			3×10^{-6} keV	~ 0
	$B_c(1P'_1) + \pi\pi$			0.0014 keV	0.002
$B_c(2P'_1)$	$B_c\gamma$	593.21	$\langle 1^1S_0 r 2^1P_1 \rangle = 0.4258$	7.65 keV	10.83
	$B_c^*\gamma$	576.80	$\langle 1^3S_1 r 2^3P_1 \rangle = 0.4117$	13.12 keV	18.57
	$B_c(2^1S_0)\gamma$	195.18	$\langle 2^1S_0 r 2^1P_1 \rangle = 3.0728$	14.19 keV	20.08
	$B_c(2^3S_1)\gamma$	189.34	$\langle 2^3S_1 r 2^3P_1 \rangle = 3.1266$	26.76 keV	37.87
	$B_c(1^3D_1)\gamma$	97.31	$\langle 1^3D_1 r 2^3P_1 \rangle = -2.9727$	1.64 keV	2.32
	$B_c(1D_2)\gamma$	93.36	$\langle 1^3D_2 r 2^3P_1 \rangle = -2.9727,$ $\langle 1^1D_2 r 2^1P_1 \rangle = -2.9929$	1.5×10^{-4} keV	0.0002
	$B_c(1D'_2)\gamma$	93.36	$\langle 1^3D_2 r 2^3P_1 \rangle = -2.9727,$ $\langle 1^1D_2 r 2^1P_1 \rangle = -2.9929$	7.3 keV	10.33
	total			70.66 keV	100
	$B_c(1^3P_2) + \pi\pi$			0.0007 keV	0.0009
	$B_c(1P_1) + \pi\pi$			0.0002 keV	0.0003
$B_c(2^3P_2)$	$B_c(1P'_1) + \pi\pi$			0.0002 keV	0.0003
	$B_c(1^3P_0) + \pi\pi$			0.0005 keV	0.0007
	$B_c^*\gamma$	583.21	$\langle 1^3S_1 r 2^3P_2 \rangle = 0.4117$	20.36 keV	27.03
	$B_c(2^3S_1)\gamma$	196.15	$\langle 2^3S_1 r 2^3P_2 \rangle = 3.1266$	44.67 keV	59.31
	$B_c(1^3D_1)\gamma$	104.21	$\langle 1^3D_1 r 2^3P_2 \rangle = -2.9727$	0.12 keV	0.16
	$B_c(1^3D_3)\gamma$	98.29	$\langle 1^3D_3 r 2^3P_2 \rangle = -2.9727$	8.54 keV	11.34
	$B_c(1D_2)\gamma$	100.27	$\langle 1^3D_2 r 2^3P_2 \rangle = -2.9727$	0.65 keV	0.86
	$B_c(1D'_2)\gamma$	100.27	$\langle 1^3D_2 r 2^3P_2 \rangle = -2.9727$	0.97 keV	1.29
	total			75.31 keV	100
$B_c(3^3P_0)$	DB		$^1S_0 = +0.1157$	3.34	100
$B_c(3^3P_2)$	DB		$^1D_2 = -0.0215$	1.16	100

TABLE IX. Partial widths and branching ratios for strong, radiative, and hadronic transitions for the $4P$ states of B_c mesons.

Meson	Decay mode	Photon energy MeV	Amplitude ($\mathcal{M}$)	Γ_{thy} MeV	B.R
$B_c(4^3P_0)$	DB		$^1S_0 = +0.008$	0.45	9.03
	D^*B^*		$^1S_0 = -0.0196$	4.53	90.93
	$D_s B_s$		$^5D_0 = -0.0429$		
	total		$^1S_0 = +0.001$	0.002	0.04
$B_c(4P_1)$	DB^*		$^3S_1 = -0.0083c_P + 0.0117s_P$	1.03	14.21
			$^3D_1 = -0.0104c_P - 0.0072s_P$		
	D^*B		$^3S_1 = -0.0127c_P + 0.0176s_P$	0.94	12.97
			$^3D_1 = -0.0120c_P - 0.0082s_P$		
	D^*B^*		$^3S_1 = +0.0006c_P,$	5.28	72.83
			$^3D_1 = +0.0406c_P,$ $^5D_1 = -0.0491s_P$		
$B_c(4P'_1)$	DB^*		$^3S_1 = +0.0120c_P + 0.0086s_P$	1.44	13.81
			$^3D_1 = -0.0069c_P + 0.0100s_P$		
	D^*B		$^3S_1 = +0.0153c_P + 0.0111s_P$	1.67	16.01
			$^3D_1 = -0.0053c_P + 0.0079s_P$		
	D^*B^*		$^3S_1 = -0.0121s_P,$	7.32	70.18
			$^3D_1 = -0.0417s_P,$ $^5D_1 = -0.0503c_P$		
$B_c(4^3P_2)$	DB		$^1D_2 = -0.0013$	0.01	0.14
			$^3D_2 = +0.0088$	0.52	7.38
	D^*B		$^3D_2 = +0.0001$	0.1	1.42
			$^5S_2 = +0.0246$	6.41	91.01
	D^*B^*		$^1D_2 = +0.0126$		
			$^5D_2 = -0.0335$		
$B_c(4P_2)$	$D_s B_s$		$^1D_2 = -0.001$	0.003	0.004
	total			7.043	100

of $\sim 2\%$ and (ii) $B_c^\pm \rightarrow J/\psi l^\pm \nu_l \rightarrow l'^- l'^+ l^\pm \nu_l$ having combined BR of 0.21% and detection efficiency of $\sim 4\%$ (see Table XV for the branching ratios). We calculate the number of events of various decay chains of excited B_c states below the BD threshold at the LHC for integrated luminosity $L = 100 \text{ fb}^{-1}$. The values reported in Tables XVI–XXII include the events observed through both of the golden channels. The decay chains having yield less than 100 are not included in these tables.

We include a factor of 2 to incorporate both charge conjugate states of B_c . It is noted that our mass calculations show that $3S$, $2P$, $2D$, and $1F$ states are below but close to the BD threshold (energy gap $< 0.15 \text{ GeV}$). These results significantly differ from the mass predictions given in Refs. [5,6,8]. Reference [5] shows that $3S$, 2^3P_2 , $2D$, and $1F$ states are above the BD threshold, whereas Ref. [8] shows that three multiplets of $2P$ states are above the BD threshold along with $3S$, $2D$, and $1F$ states. Reference [6] also shows that $3S$ and three multiplets of $2P$ states are

above the BD threshold. The result is that these states, which are expected to be observed through radiative and hadronic transition according to our mass predictions, are unlikely to be observed in these references. This also gives significantly different branching ratios and predicted strong decay widths. Thus the experimenters should treat our predictions of branching ratios and strong decay widths of the states close to the BD threshold cautiously. Besides this caveat, there are no results available for the production cross sections of $3S$, $2D$, and $1F$ states in pp collision; therefore, we abstain from making any predictions of event rates for these states.

These results show that in the LHC it is possible to produce a sufficient number of events corresponding to different decay chains of the excited B_c states below the BD threshold. The task of event reconstruction becomes much easier when an excited B_c state directly decays to the ground state through $E1/M1$ or hadronic transitions. Tables VII and VIII show that 1^3S_1 , 2^3S_1 , $1P_1$, $1P'_1$, $2P_1$,

TABLE X. Partial widths and branching ratios for strong, radiative, and hadronic transitions for the $1D$ states of B_c mesons.

Meson	Decay Mode	Photon Energy MeV	Amplitude	Γ_{thy} MeV	B.R
$B_c(1^3D_1)$	$B_c(1^3S_1) + \pi\pi$			0.042 keV	0.07
	$B_c(1^3P_0)\gamma$	206.78	$\langle 1^3P_0 r 1^3D_1 \rangle = 3.3708$	40.55 keV	63.97
	$B_c(1P_1)\gamma$	188.33	$\langle 1^3P_1 r 1^3D_1 \rangle = 3.3708$	7.67 keV	12.1
	$B_c(1P'_1)\gamma$	182.50	$\langle 1^3P_1 r 1^3D_1 \rangle = 3.3708$	13.93 keV	21.98
	$B_c(1^3P_2)\gamma$	173.74	$\langle 1^3P_2 r 1^3D_1 \rangle = 3.3708$	1.2 keV	1.89
	total			63.39 keV	100
$B_c(1D_2)$	$B_c(1^1S_0) + \pi\pi$			0.035 keV	0.06
	$B_c(1^3S_1) + \pi\pi$			0.017 keV	0.03
	$B_c(1^3P_2)\gamma$	177.63	$\langle 1^3P_2 r 1^3D_2 \rangle = 3.3708$	4.62 keV	8.12
	$B_c(1P_1)\gamma$	192.22	$\langle 1^3P_1 r 1^3D_2 \rangle = 3.3708$, $\langle 1^1P_1 r 1^1D_2 \rangle = 3.3461$	52.26 keV	91.8
	$B_c(1P'_1)\gamma$	186.39	$\langle 1^3P_1 r 1^3D_2 \rangle = 3.3708$, $\langle 1^1P_1 r 1^1D_2 \rangle = 3.3461$	2.6×10^{-4} keV	0.0005
	total			56.93 keV	100
$B_c(1D'_2)$	$B_c(1^1S_0) + \pi\pi$			0.023 keV	0.04
	$B_c(1^3S_1) + \pi\pi$			0.026 keV	0.05
	$B_c(1^3P_2)\gamma$	177.63	$\langle 1^3P_2 r 1^3D_2 \rangle = 3.3708$	6.95 keV	13.32
	$B_c(1P_1)\gamma$	192.22	$\langle 1^3P_1 r 1^3D_2 \rangle = 3.3708$ $\langle 1^1P_1 r 1^1D_2 \rangle = 3.3461$	0.9 keV	1.72
	$B_c(1P'_1)\gamma$	186.39	$\langle 1^3P_1 r 1^3D_2 \rangle = 3.3708$ $\langle 1^1P_1 r 1^1D_2 \rangle = 3.3461$	44.29 keV	84.86
	total			52.19 keV	100
$B_c(1^3D_3)$	$B_c(1^3S_1) + \pi\pi$			0.043 keV	0.09
	$B_c(1^3P_2)\gamma$	179.58	$\langle 1^3P_2 r 1^3D_3 \rangle = 3.3708$	47.81 keV	99.91
	total			47.85 keV	100

TABLE XI. Partial widths and branching ratios for strong, radiative, and hadronic transitions for the $2D$ states of B_c mesons.

Meson	Decay mode	Photon energy MeV	Amplitude	Γ_{thy} MeV	B.R (%)
$B_c(2^3D_1)$	$B_c(2^3P_0)\gamma$	163.08	$\langle 2^3P_0 r 2^3D_1 \rangle = 4.4775$	35.09 keV	52.98
	$B_c(2P_1)\gamma$	148.41	$\langle 2^3P_1 r 2^3D_1 \rangle = 4.4775$	6.62 keV	10.0
	$B_c(2P'_1)\gamma$	139.60	$\langle 2^3P_1 r 2^3D_1 \rangle = 4.4775$	11.0 keV	16.61
	$B_c(2^3P_2)\gamma$	132.73	$\langle 2^3P_2 r 2^3D_1 \rangle = 4.4775$	0.95 keV	1.43
	$B_c(1^3P_0)\gamma$	434.76	$\langle 1^3P_0 r 2^3D_1 \rangle = 0.3469$	3.99 keV	6.02
	$B_c(1P_1)\gamma$	416.94	$\langle 1^3P_1 r 2^3D_1 \rangle = 0.3469$	0.88 keV	1.33
	$B_c(1P'_1)\gamma$	411.30	$\langle 1^3P_1 r 2^3D_1 \rangle = 0.3469$	1.69 keV	2.55
	$B_c(1^3P_2)\gamma$	402.84	$\langle 1^3P_2 r 2^3D_1 \rangle = 0.3469$	0.16 keV	0.24
	$B_c(1^3F_2)\gamma$	82.32	$\langle 1^3F_2 r 2^3D_1 \rangle = -3.1023$	5.85 keV	8.83
	total			66.23 keV	100

(Table continued)

TABLE XI. (Continued)

Meson	Decay mode	Photon energy MeV	Amplitude	Γ_{thy} MeV	B.R (%)
$B_c(2D_2)$	$B_c(2^3P_2)\gamma$	136.66	$\langle 2^3P_2 r 2^3D_2 \rangle = 4.4775$	3.71 keV	5.82
	$B_c(2P_1)\gamma$	152.33	$\langle 2^3P_1 r 2^3D_2 \rangle = 4.4775,$ $\langle 2^1P_1 r 2^1D_2 \rangle = 4.4392$	45.81 keV	71.84
	$B_c(2P_1')\gamma$	143.52	$\langle 2^3P_1 r 2^3D_2 \rangle = 4.4775,$ $\langle 2^1P_1 r 2^1D_2 \rangle = 4.4392$	3.2×10^{-4} keV	0.001
	$B_c(1^3P_2)\gamma$	406.61	$\langle 1^3P_2 r 2^3D_2 \rangle = 0.3469$	0.59 keV	0.93
	$B_c(1P_1)\gamma$	420.71	$\langle 1^3P_1 r 2^3D_2 \rangle = 0.3469,$ $\langle 1^1P_1 r 2^1D_2 \rangle = 0.3603$	6.17 keV	9.68
	$B_c(1P_1')\gamma$	415.07	$\langle 1^3P_1 r 2^3D_2 \rangle = 0.3469,$ $\langle 1^1P_1 r 2^1D_2 \rangle = 0.3603$	2.1×10^{-3} keV	0.003
	$B_c(1^3F_2)\gamma$	86.27	$\langle 1^3F_2 r 2^3D_2 \rangle = -3.1023$	0.3 keV	0.47
	$B_c(1F_3)\gamma$	89.43	$\langle 1^3F_3 r 2^3D_2 \rangle = -3.1023,$ $\langle 1^1F_3 r 2^1D_2 \rangle = -3.1085$	7.16 keV	11.23
	$B_c(1F_3')\gamma$	82.51	$\langle 1^3F_3 r 2^3D_2 \rangle = -3.1023,$ $\langle 1^1F_3 r 2^1D_2 \rangle = -3.1085$	0.02 keV	0.03
	total			63.76 keV	100
$B_c(2D_2')$	$B_c(2^3P_2)\gamma$	136.66	$\langle 2^3P_2 r 2^3D_2 \rangle = 4.4775$	5.58 keV	10.26
	$B_c(2P_1)\gamma$	152.33	$\langle 2^3P_1 r 2^3D_2 \rangle = 4.4775,$ $\langle 2^1P_1 r 2^1D_2 \rangle = 4.4392$	0.79 keV	1.45
	$B_c(2P_1')\gamma$	143.52	$\langle 2^3P_1 r 2^3D_2 \rangle = 4.4775,$ $\langle 2^1P_1 r 2^1D_2 \rangle = 4.4392$	35.64 keV	65.56
	$B_c(1^3P_2)\gamma$	406.61	$\langle 1^3P_2 r 2^3D_2 \rangle = 0.3469$	0.88 keV	1.62
	$B_c(1P_1)\gamma$	420.71	$\langle 1^3P_1 r 2^3D_2 \rangle = 0.3469,$ $\langle 1^1P_1 r 2^1D_2 \rangle = 0.3603$	0.14 keV	0.26
	$B_c(1P_1')\gamma$	415.07	$\langle 1^3P_1 r 2^3D_2 \rangle = 0.3469,$ $\langle 1^1P_1 r 2^1D_2 \rangle = 0.3603$	5.37 keV	9.88
	$B_c(1^3F_2)\gamma$	86.27	$\langle 1^3F_2 r 2^3D_2 \rangle = -3.1023$	0.45 keV	0.83
	$B_c(1F_3)\gamma$	89.43	$\langle 1^3F_2 r 2^3D_2 \rangle = -3.1023,$ $\langle 1^1F_2 r 2^1D_2 \rangle = -3.1085$	1.7×10^{-6} keV	~ 0
	$B_c(1F_3')\gamma$	82.51	$\langle 1^3F_2 r 2^3D_2 \rangle = -3.1023,$ $\langle 1^1F_2 r 2^1D_2 \rangle = -3.1085$	5.51 keV	10.14
	total			54.36 keV	100
$B_c(2^3D_3)$	$B_c(2^3P_2)\gamma$	139.60	$\langle 2^3P_2 r 2^3D_3 \rangle = 4.4775$	39.62 keV	74.5
	$B_c(1^3P_2)\gamma$	409.44	$\langle 1^3P_2 r 2^3D_3 \rangle = 0.3469$	6.0 keV	11.28
	$B_c(1^3F_4)\gamma$	89.72	$\langle 1^3F_4 r 2^3D_3 \rangle = -3.1023$	6.96 keV	13.09
	$B_c(1^3F_2)\gamma$	89.23	$\langle 1^3F_2 r 2^3D_3 \rangle = -3.1023$	0.02 keV	0.04
	$B_c(1F_3)\gamma$	92.39	$\langle 1^3F_3 r 2^3D_3 \rangle = -3.1023$	0.28 keV	0.53
	$B_c(1F_3')\gamma$	85.48	$\langle 1^3F_3 r 2^3D_3 \rangle = -3.1023$	0.3 keV	0.56
	total			53.18 keV	100

$2P_1'$ states can directly decay to B_c ground states through $E1/M1$ transitions. All these direct transitions also appear in Tables XVI, XVII, XIX, and XX of decay chains as their yield is much higher than 100. The case of $2^3S_1 \rightarrow 1^1S_0 + \gamma$ is particularly interesting. Only 2650 events are expected in this case owing to the small value of its BR ($\approx 2.75\%$)

despite having relatively large production cross section of 2^3S_1 state. Therefore, the best way to search this state is via $2^3S_1 \xrightarrow{\gamma} 1^3P_2 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} 1^1S_0$ or $2^3S_1 \xrightarrow{\pi\pi} 1^3S_1 \xrightarrow{\gamma} 1^1S_0$ for which the expected numbers of events are 2.78×10^4 and 2.58×10^4 respectively as shown in the Table XVI. Tables VII and X show that 2^1S_0 , $1D_2$, and $1D_2'$ states can directly decay to

TABLE XII. Partial widths and branching ratios for strong, radiative, and hadronic transitions for the $3D$ states of B_c mesons.

Meson	Decay mode	Photon energy MeV	Amplitude	Γ_{thy} MeV	B.R (%)
$B_c(3^3D_1)$	DB		$^1P_1 = -0.0126$	0.91	59.48
	DB^*		$^3P_1 = +0.0108$	0.55	35.95
	D^*B		$^3P_1 = +0.0122$	0.07	4.58
	total			1.53	100
$B_c(3D_2)$	DB^*		$^3P_2 = -0.0127c_D + 0.0156s_D$	1.81	99.81
			$^3F_2 = -0.0151c_D - 0.0123s_D$		
	D^*B		$^3P_2 = -0.0281c_D + 0.0344s_D$	3.5×10^{-3}	0.19
			$^3F_2 = -0.0014c_D - 0.0011s_D$		
	total			1.81	100
$B_c(3D'_2)$	DB^*		$^3D_2 = +0.0156c_D + 0.0127s_D$	1.93	47.3
			$^3F_2 = -0.0123c_D + 0.0151s_D$		
	D^*B		$^3D_2 = +0.0344c_D + 0.0281s_D$	2.15	52.7
			$^3F_2 = -0.0011c_D + 0.0014s_D$		
	total			4.08	100
$B_c(3^3D_3)$	DB		$^1F_3 = +0.0002$	0.0002	0.02
	DB^*		$^3F_3 = -0.0143$	0.99	98.98
	D^*B		$^3F_3 = -0.0027$	0.01	1.0
	total			1.0	100

TABLE XIII. Partial widths and branching ratios for strong, radiative, and hadronic transitions for the $4D$ states of B_c mesons.

Meson	Decay mode	Photon energy MeV	Amplitude ($\mathcal{M}$)	Γ_{thy} MeV	B.R (%)
$B_c(4^3D_1)$	DB		$^1P_1 = -0.0178$	2.95	72.25
	DB^*		$^3P_1 = +0.0023$	0.05	1.22
	D^*B		$^3P_1 = +0.0061$	0.27	6.61
	D^*B^*		$^1P_1 = +0.0009$	0.81	19.84
			$^5P_1 = -0.0004$		
			$^5F_1 = -0.0114$		
	$D_s B_s$		$^1P_1 = -0.0007$	0.003	0.07
	$D_s B_s^*$		$^3P_1 = +0.0002$	1.5×10^{-4}	~ 0
	total			4.08	100
$B_c(4D_2)$	DB^*		$^3D_2 = -0.0028c_D + 0.0040s_D$	0.61	14.94
			$^3F_2 = 0.0063c_D + 0.0055s_D$		
	D^*B		$^3D_2 = -0.0072c_D + 0.0089s_D$	1.03	25.23
			$^3F_2 = 0.0092c_D + 0.0074s_D$		
	D^*B^*		$^3P_2 = -0.0005c_D, ^3F_2 = -0.0093c_D$	0.65	15.92
			$^5P_2 = +0.0001s_D, ^5F_2 = +0.0109s_D$		
	$D_s B_s^*$		$^3D_2 = -0.0001c_D + 0.0001s_D$	7.0×10^{-4}	0.02
			$^3F_2 = +0.0003c_D + 0.0003s_D$		
$D_s^* B_s$			$^3D_2 = -0.0021c_D + 0.0025s_D$	3.7×10^{-5}	~ 0
			$^3F_2 = +0.0001c_D + 0.0001s_D$		
	total			2.29	100

(Table continued)

TABLE XIII. (Continued)

Meson	Decay mode	Photon energy MeV	Amplitude ($\mathcal{M}$)	Γ_{thy} MeV	B.R. (%)
$B_c(4D'_2)$	DB^*		${}^3D_2 = 0.0040c_D + 0.0028s_D$ ${}^3F_2 = 0.0055c_D - 0.0063s_D$	0.21	11.28
	D^*B		${}^3D_2 = 0.0089c_D + 0.0072s_D$ ${}^3F_2 = 0.0074c_D - 0.0092s_D$	0.96	51.58
	D^*B^*		${}^3P_2 = +0.0005s_D, {}^3F_2 = +0.0093s_D$ ${}^5P_2 = +0.0001c_D, {}^5F_2 = +0.0109c_D$	0.68	36.54
	$D_s B_s^*$		${}^3D_2 = +0.0001c_D + 0.0001s_D$ ${}^3F_2 = +0.0003c_D - 0.0003s_D$	1.5×10^{-4}	0.01
	$D_s^* B_s$		${}^3D_2 = +0.0025c_D + 0.0021s_D$ ${}^3F_2 = +0.0001c_D - 0.0001s_D$	0.011	0.59
	total			1.86	100
	DB		${}^1F_3 = +0.0117$	1.3	44.19
	DB^*		${}^3F_3 = -0.0074$	0.48	16.32
	D^*B		${}^3F_3 = -0.0088$	0.58	19.72
	D^*B^*		${}^5P_3 = -0.0024$ ${}^1F_3 = +0.0038$ ${}^5F_3 = -0.0083$	0.58	19.72
$B_c(4^3D_3)$	$D_s B_s$		${}^1F_3 = +0.0003$	4.4×10^{-4}	0.01
	$D_s B_s^*$		${}^3F_3 = -0.0004$	8.9×10^{-4}	0.03
	$D_s^* B_s$		${}^3F_3 = -0.0005$	3.5×10^{-4}	0.01
	total			2.94	100

TABLE XIV. Partial widths and branching ratios for strong, radiative, and hadronic transitions for the $1F$ and $2F$ states of B_c mesons.

Meson	Decay mode	Photon energy MeV	Amplitude ($\mathcal{M}$)	Γ_{thy} MeV	B.R. (%)
$B_c(1^3F_2)$	$B_c(1^3P_0) + \pi\pi$			0.0004 keV	0.0007
	$B_c(1^3P_2) + \pi\pi$			0.00004 keV	0.0001
	$B_c(1P_1) + \pi\pi$			0.0001 keV	0.0002
	$B_c(1P'_1) + \pi\pi$			0.0002 keV	0.0004
	$B_c(1^3D_1)\gamma$	154.46	$\langle 1^3D_1 r 1^3F_2 \rangle = 4.4823$	48.41 keV	85.02
	$B_c(1D_2)\gamma$	150.54	$\langle 1^3D_2 r 1^3F_2 \rangle = 4.4823$	3.32 keV	5.83
	$B_c(1D'_2)\gamma$	150.54	$\langle 1^3D_2 r 1^3F_2 \rangle = 4.4823$	4.98 keV	8.75
	$B_c(1^3D_3)\gamma$	148.59	$\langle 1^3D_3 r 1^3F_2 \rangle = 4.4823$	0.23 keV	0.4
	total			56.94 keV	100
$B_c(1F_3)$	$B_c(1^3P_2) + \pi\pi$			0.0001 keV	0.0002
	$B_c(1P_1) + \pi\pi$			0.001 keV	0.002
	$B_c(1^3D_3)\gamma$	145.46	$\langle 1^3D_3 r 1^3F_3 \rangle = 4.4823$	2.29 keV	4.59
	$B_c(1D_2)\gamma$	147.41	$\langle 1^3D_2 r 1^3F_3 \rangle = 4.4823,$ $\langle 1^1D_2 r 1^1F_3 \rangle = 4.4769$	47.64 keV	95.41
	$B_c(1D'_2)\gamma$	147.41	$\langle 1^3D_2 r 1^3F_3 \rangle = 4.4823,$ $\langle 1^1D_2 r 1^1F_3 \rangle = 4.4769$	0.0001 keV	0.0002
	total			49.93 keV	100

(Table continued)

TABLE XIV. (Continued)

Meson	Decay mode	Photon energy MeV	Amplitude ($\mathcal{M}$)	Γ_{thy} MeV	B.R. (%)
$B_c(1F'_3)$	$B_c(1^3P_2) + \pi\pi$			0.0001 keV	0.0002
	$B_c(1P_1) + \pi\pi$			2×10^{-5} keV	~ 0
	$B_c(1P'_1) + \pi\pi$			0.001 keV	0.002
	$B_c(1^3D_3)\gamma$	152.31	$\langle 1^3D_3 r 1^3F_3 \rangle = 4.4823$	3.51 keV	6.13
	$B_c(1D_2)\gamma$	154.26	$\langle 1^3D_2 r 1^3F_3 \rangle = 4.4823$, $\langle 1^1D_2 r 1^1F_3 \rangle = 4.4769$	0.18 keV	0.31
	$B_c(1D'_2)\gamma$	154.26	$\langle 1^3D_2 r 1^3F_3 \rangle = 4.4823$, $\langle 1^1D_2 r 1^1F_3 \rangle = 4.4769$	53.53 keV	93.55
	total			57.22 keV	100
$B_c(1^3F_4)$	$B_c(1^3P_2) + \pi\pi$			0.0005 keV	0.001
	$B_c(1^3D_3)\gamma$	148.10	$\langle 1^3D_3 r 1^3F_4 \rangle = 4.4823$	50.8 keV	~ 100
	total			50.8 keV	100
$B_c(2^3F_2)$	DB		$^1D_2 = +0.0277$	2.89	76.66
	DB^*		$^3D_2 = +0.0206$	0.88	23.34
	total			3.77	100
$B_c(2F_3)$	DB^*		$^3D_3 = -0.0205c_F + 0.0237s_F$ $^3G_3 = -0.0017c_F - 0.0012s_F$	0.01	100
$B_c(2F'_3)$	DB^*		$^3D_3 = +0.0262c_F + 0.0227s_F$ $^3G_3 = -0.0019c_F + 0.0022s_F$	2.61	100
$B_c(2^3F_4)$	DB		$^1G_4 = +0.0099$	0.37	97.37
	DB^*		$^3G_4 = -0.002$	0.01	2.63
	total			0.38	100

TABLE XV. Branching ratios for the two golden channels of B_c along the branching ratio of $J/\psi \rightarrow l^+l^-$.

Decay process	Branching ratio (%)
$B_c^\pm \rightarrow J/\psi \pi^\pm$	$0.111^{+0.009}_{-0.010}$ [46]
$B_c^\pm \rightarrow J/\psi l^\pm \nu_l$	1.73 ± 0.05 [46]
$J/\psi \rightarrow l^+l^-$	11.9 ± 0.06 [16]

B_c ground state through hadronic transitions. However, due to small BRs of hardonic transitions of $1D_2$ and $1D'_2$ (less than 1%), the resultant number of events are less than 100 and are not included in Table XXII. The best way to detect D states is via double or triple photon emission as given in Table XXII. Our results given in Tables XVI–XXII of decay chains can help experimentalists in adopting the best strategies to discover and study properties of the excited B_c states below the BD threshold.

VII. CONCLUDING REMARKS

In this paper we studied the properties of charmed-bottom mesons including masses, radiative transitions, hadronic transitions, and the Okubo-Zweig-Iizuka (OZI) allowed strong decays. We have computed the spectrum of B_c mesons up to $2F$ states with a nonrelativistic quark

TABLE XVI. Decay chains of 1S and 2S states and expected number of events at the LHC.

Initial state	Decay chain	Number of events
1^3S_1	$\xrightarrow{100\%} B_c$	1.6×10^5
2^1S_0	$\xrightarrow{51.42\%} B_c$	2.96×10^4
	$\xrightarrow{34.23\%} 1P_1 \xrightarrow{68.78\%} B_c$	1.35×10^4
	$\xrightarrow{34.23\%} 1P_1 \xrightarrow{31.22\%} 1^3S_1 \xrightarrow{100\%} B_c$	6.15×10^3
	$\xrightarrow{14.01\%} 1P'_1 \xrightarrow{35.56\%} B_c$	2.86×10^3
	$\xrightarrow{14.01\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	5.19×10^3
2^3S_1	$\xrightarrow{26.76\%} 1^3S_1 \xrightarrow{100\%} B_c$	2.58×10^4
	$\xrightarrow{28.91\%} 1^3P_2 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	2.78×10^4
	$\xrightarrow{2.75\%} B_c$	2.65×10^3
	$\xrightarrow{9.59\%} 1P_1 \xrightarrow{68.78\%} B_c$	6.35×10^3
	$\xrightarrow{9.59\%} 1P_1 \xrightarrow{31.22\%} 1^3S_1 \xrightarrow{100\%} B_c$	2.88×10^3
	$\xrightarrow{15.76\%} 1P'_1 \xrightarrow{35.56\%} B_c$	5.4×10^3
	$\xrightarrow{15.76\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	9.78×10^3
	$\xrightarrow{16.23\%} 1^3P_0 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	1.56×10^4

TABLE XVII. Decay chains of $1P$ states and expected number of events at the LHC.

Initial state	Decay chain	Number of events
1^3P_2	$\xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	2.6×10^4
$1P'_1$	$\xrightarrow{35.56\%} B_c$	6.53×10^3
	$\xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	1.18×10^4
$1P_1$	$\xrightarrow{31.22\%} 1^3S_1 \xrightarrow{100\%} B_c$	5.73×10^3
	$\xrightarrow{68.78\%} B_c$	1.26×10^4
1^3P_0	$\xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	1.23×10^4

TABLE XVIII. Decay chains of 2^3P_2 states and expected number of events at the LHC.

Initial state	Decay chain	Number of events
2^3P_2	$\xrightarrow{27.03\%} 1^3S_1 \xrightarrow{100\%} B_c$	9.08×10^3
	$\xrightarrow{59.31\%} 2^3S_1 \xrightarrow{26.76\%} 1^3S_1 \xrightarrow{100\%} B_c$	5.33×10^3
	$\xrightarrow{59.31\%} 2^3S_1 \xrightarrow{28.91\%} 1^3P_2 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	5.76×10^3
	$\xrightarrow{59.31\%} 2^3S_1 \xrightarrow{9.59\%} 1P_1 \xrightarrow{68.78\%} B_c$	1.31×10^3
	$\xrightarrow{59.31\%} 2^3S_1 \xrightarrow{15.76\%} 1P'_1 \xrightarrow{35.56\%} B_c$	1.12×10^3
	$\xrightarrow{59.31\%} 2^3S_1 \xrightarrow{15.76\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	2.02×10^3
	$\xrightarrow{59.31\%} 2^3S_1 \xrightarrow{16.23\%} 1^3P_0 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	3.23×10^3
	$\xrightarrow{11.34\%} 1^3D_3 \xrightarrow{99.91\%} 1^3P_2 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	3.81×10^3
	$\xrightarrow{0.86\%} 1D_2 \xrightarrow{91.8\%} 1P_1 \xrightarrow{68.78\%} B_c$	1.82×10^2
	$\xrightarrow{1.29\%} 1D'_2 \xrightarrow{84.86\%} 1P'_1 \xrightarrow{35.56\%} B_c$	1.31×10^2
	$\xrightarrow{1.29\%} 1D'_2 \xrightarrow{84.86\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	2.37×10^2

model that incorporates scalar confinement and one gluon exchange spin-dependent interaction. These eigenfunctions were then used to obtain $E1$ and $M1$ radiative transitions. Strong decay amplitudes of excited B_c states above the BD threshold have been obtained using the modified 3P_0 pair creation model and fitted SHO wave functions. The hadronic transition rates for B_c mesons has been predicted using the Kuang-Yan approach. The total decay widths of excited B_c states have been predicted by summing the radiative, hadronic, and strong widths. The branching ratios of different final states are estimated by using the total widths. These branching ratios are then combined with production rates at the LHC to estimate the number of

TABLE XIX. Decay chains of $2P'_1$ states and expected number of events at the LHC.

Initial state	Decay chain	Number of events
$2P'_1$	$\xrightarrow{10.83\%} B_c$	3.64×10^3
	$\xrightarrow{20.08\%} 2^1S_0 \xrightarrow{51.42\%} B_c$	3.47×10^3
	$\xrightarrow{20.08\%} 2^1S_0 \xrightarrow{34.23\%} 1P_1 \xrightarrow{68.78\%} B_c$	1.59×10^3
	$\xrightarrow{20.08\%} 2^1S_0 \xrightarrow{34.23\%} 1P_1 \xrightarrow{31.22\%} 1^3S_1 \xrightarrow{100\%} B_c$	7.2×10^2
	$\xrightarrow{20.08\%} 2^1S_0 \xrightarrow{14.01\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	6.09×10^2
	$\xrightarrow{37.87\%} 2^3S_1 \xrightarrow{26.76\%} 1^3S_1 \xrightarrow{100\%} B_c$	3.41×10^3
	$\xrightarrow{37.87\%} 2^3S_1 \xrightarrow{28.91\%} 1^3P_2 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	3.68×10^3
	$\xrightarrow{37.87\%} 2^3S_1 \xrightarrow{9.59\%} 1P_1 \xrightarrow{68.78\%} B_c$	8.39×10^2
	$\xrightarrow{37.87\%} 2^3S_1 \xrightarrow{15.76\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	1.29×10^3
	$\xrightarrow{37.87\%} 2^3S_1 \xrightarrow{16.23\%} 1^3P_0 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	2.07×10^3
	$\xrightarrow{2.32\%} 1^3D_1 \xrightarrow{63.97\%} 1^3P_0 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	4.99×10^2
	$\xrightarrow{2.32\%} 1^3D_1 \xrightarrow{21.98\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	1.1×10^2
	$\xrightarrow{10.33\%} 1D'_2 \xrightarrow{13.32\%} 1^3P_2 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	4.62×10^2
	$\xrightarrow{10.33\%} 1D'_2 \xrightarrow{84.86\%} 1P'_1 \xrightarrow{35.56\%} B_c$	1.05×10^3
	$\xrightarrow{10.33\%} 1D'_2 \xrightarrow{84.86\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	1.9×10^3

TABLE XX. Decay chains of $2P_1$ states and expected number of events at the LHC.

Initial state	Decay chain	Number of events
$2P_1$	$\xrightarrow{23.16\%} B_c$	4.25×10^3
	$\xrightarrow{9.97\%} 1^3S_1 \xrightarrow{100\%} B_c$	1.83×10^3
	$\xrightarrow{39.02\%} 2^1S_0 \xrightarrow{51.42\%} B_c$	3.68×10^3
	$\xrightarrow{39.02\%} 2^1S_0 \xrightarrow{34.23\%} 1P_1 \xrightarrow{68.78\%} B_c$	1.69×10^3
	$\xrightarrow{39.02\%} 2^1S_0 \xrightarrow{34.23\%} 1P_1 \xrightarrow{31.22\%} 1^3S_1 \xrightarrow{100\%} B_c$	7.66×10^2
	$\xrightarrow{39.02\%} 2^1S_0 \xrightarrow{14.01\%} 1P'_1 \xrightarrow{64.44\%} 1^3S_1 \xrightarrow{100\%} B_c$	6.47×10^2
	$\xrightarrow{18.41\%} 2^3S_1 \xrightarrow{26.76\%} 1^3S_1 \xrightarrow{100\%} B_c$	9.04×10^2
	$\xrightarrow{18.41\%} 2^3S_1 \xrightarrow{28.91\%} 1^3P_2 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	9.77×10^2
	$\xrightarrow{9.27\%} 1D_2 \xrightarrow{8.12\%} 1^3P_2 \xrightarrow{100\%} 1^3S_1 \xrightarrow{100\%} B_c$	1.38×10^2
	$\xrightarrow{9.27\%} 1D_2 \xrightarrow{91.8\%} 1P_1 \xrightarrow{68.78\%} B_c$	1.07×10^3
	$\xrightarrow{9.27\%} 1D_2 \xrightarrow{91.8\%} 1P_1 \xrightarrow{31.22\%} 1^3S_1 \xrightarrow{100\%} B_c$	4.88×10^2

TABLE XXI. Decay chains of 2^3P_0 states and expected number of events at the LHC.

Initial state	Decay chain	Number of events
2^3P_0	$\xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 35.87% 100%	5.47×10^3
	$\xrightarrow{\gamma} 2^3S_1 \xrightarrow{\pi\pi} 1^3S_1 \xrightarrow{\gamma} B_c$ 55.38% 26.76% 100%	2.26×10^3
	$\xrightarrow{\gamma} 2^3S_1 \xrightarrow{\gamma} 1^3P_2 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 55.38% 28.91% 100% 100%	2.44×10^3
	$\xrightarrow{\gamma} 2^3S_1 \xrightarrow{\gamma} 1P'_1 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 55.38% 15.76% 64.44% 100%	8.57×10^2
	$\xrightarrow{\gamma} 2^3S_1 \xrightarrow{\gamma} 1^3P_0 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 55.38% 16.23% 100% 100%	1.37×10^3
	$\xrightarrow{\gamma} 1^3D_1 \xrightarrow{\gamma} 1^3P_0 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 8.74% 63.97% 100% 100%	8.52×10^2
	$\xrightarrow{\gamma} 1^3D_1 \xrightarrow{\gamma} 1P_1 \xrightarrow{\gamma} B_c$ 8.74% 12.1% 68.78%	1.61×10^2
	$\xrightarrow{\gamma} 1^3D_1 \xrightarrow{\gamma} 1P'_1 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 8.74% 21.98% 64.44% 100%	1.89×10^2

TABLE XXII. Decay chains of $1D$ states and expected number of events at the LHC.

Initial state	Decay chain	Number of events
1^3D_3	$\xrightarrow{\gamma} 1^3P_2 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 99.91% 100% 100%	2.92×10^3
$1D'_2$	$\xrightarrow{\gamma} 1^3P_2 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 13.32% 100% 100%	3.9×10^2
	$\xrightarrow{\gamma} 1P'_1 \xrightarrow{\gamma} B_c$ 84.86% 35.56%	8.83×10^2
	$\xrightarrow{\gamma} 1P'_1 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 84.86% 64.44% 100%	1.6×10^3
$1D_2$	$\xrightarrow{\gamma} 1^3P_2 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 8.12% 100% 100%	2.38×10^2
	$\xrightarrow{\gamma} 1P_1 \xrightarrow{\gamma} B_c$ 91.8% 68.78%	1.85×10^3
	$\xrightarrow{\gamma} 1P_1 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 91.8% 31.22% 100%	8.39×10^2
1^3D_1	$\xrightarrow{\gamma} 1^3P_0 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 63.97% 100% 100%	1.87×10^3
	$\xrightarrow{\gamma} 1P_1 \xrightarrow{\gamma} B_c$ 12.1% 68.78%	2.45×10^2
	$\xrightarrow{\gamma} 1P_1 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 12.1% 31.22% 100%	1.11×10^2
	$\xrightarrow{\gamma} 1P'_1 \xrightarrow{\gamma} B_c$ 21.98% 35.56%	2.29×10^2
	$\xrightarrow{\gamma} 1P'_1 \xrightarrow{\gamma} 1^3S_1 \xrightarrow{\gamma} B_c$ 21.98% 64.44% 100%	4.15×10^2

events of various decay chains of excited B_c states. We expect that the predictions presented in this work will help experimentalists find the excited B_c states at the LHC and measure their properties.

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