One pion exchange and the quantum numbers of the $P_c(4440)$ and $P_c(4457)$ pentaquarks

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The LHCb collaboration has recently discovered three pentaquark-like states—the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$—close to the $\bar{D}\Sigma_c$ and the $\bar{D}^*\Sigma_c$ meson-baryon thresholds. The standard interpretation is that they are heavy antimeson-baryon molecules. Their quantum numbers have not been determined yet, which implies two possibilities for the $P_c(4440)$ and $P_c(4457)$: $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$. The preferred interpretation within a contact-range effective field theory is that the $P_c(4440)$ is the $J^P = \frac{1}{2}^-$ molecule, while the $P_c(4457)$ is the $J^P = \frac{3}{2}^-$ one. Here we show that when the one pion exchange potential between the heavy antimeson and heavy baryon is taken into account, this conclusion changes, with the contrary identification being as likely as the original one. The identification is however cutoff dependent, which suggests that improvements of the present description (e.g., the inclusion of subleading order corrections, like two-pion exchanges) are necessary in order to disambiguate the spectroscopy of the molecular pentaquarks.

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I. INTRODUCTION

The $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ are three hidden-charm pentaquark-like states recently discovered by the LHCb collaboration [1]. Owing to their closeness to the $\bar{D}\Sigma_c$ and $\bar{D}^*\Sigma_c$ thresholds, they have been theorized to be S-wave meson-baryon bound states [2–8] (other explanations include hadrocharmonium [9] or a compact pentaquark [10,11]). The most natural identification is that the $P_c(4312)$ is a $\bar{D}\Sigma_c$ molecule and the $P_c(4440)$ and $P_c(4457)$ are $\bar{D}^*\Sigma_c$ molecules. This interpretation unambiguously predicts the quantum numbers of the $P_c(4312)$ to be $J^P = \frac{1}{2}^-$. In contrast there are two possibilities for the quantum numbers of the $P_c(4440)$ and $P_c(4457)$: $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$. That is, the identification is ambiguous. Yet checking which quantum number corresponds to each one of these two pentaquarks is important to clarify their nature, in particular when confronted with future experimental measurements of their properties. From the recent theoretical models for the spectroscopy and decays of these two molecules, the preferred identification so far seems to be that the $P_c(4440)$ and $P_c(4457)$ are the $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ molecules [3,4,6,12,13], respectively. On the other hand, from the seminal predictions of molecular hidden-charm pentaquarks we expect the $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ molecules to be degenerate [14–16] or for the $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ state to be lighter than the $\frac{1}{2}^-$ one [17].

The present manuscript considers this problem from the point of view of spectroscopy within the effective field theory (EFT) framework. Specifically we investigate the effect of including pion exchanges in the masses of the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ pentaquarks. Previously Ref. [18] proposed a contact-range EFT to describe the $\bar{D}\Sigma_c$ molecular states, which was used to predict a $J^P = \frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ molecular pentaquark from the old $P_c(4450)$ peak [19] (where we note that this state was first predicted in Ref. [16]). This EFT has been recently used in Ref. [4] to analyze the LHCb pentaquark trio, where the following two conclusions were reached: (i) the molecular pentaquarks belong to a multiplet with seven members (among which we count the aforementioned $\frac{5}{2}^-$ state of Refs. [16,18]) and (ii) the preferred quantum numbers for the $P_c(4440)$ and $P_c(4457)$ are $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$, respectively. The first of these conclusions is relatively robust and has been independently confirmed by other theoretical works [6,20,21], while the second is not so stringent, as originally discussed in Ref. [4]. Here we review these conclusions from the point of view of a

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pionful EFT, i.e., a theory that besides contact interactions also incorporates pions. As we will see the inclusion of pions will be able to change the preferred quantum number identification of the $P_c(4440)$ and $P_c(4457)$ pentaquarks (in agreement with the recent work of Ref. [21] which also considers the effects of pion exchanges).

The central idea of the present manuscript can be summarized as follows. Heavy-quark spin symmetry (HQSS) [22,23] when applied to hadronic molecules indicates that the interaction among heavy hadrons is independent of the spin of the heavy quarks within the aforementioned heavy hadrons [24–28]. For the case of $\bar{D}\Sigma_c$ and $\bar{D}^*\Sigma_c$ molecules, this symmetry implies that their S-wave potential takes the form [18]

$$V\left(\bar{D}\Sigma_c,\frac{1}{2}\right) = V_a, \tag{1}$$

$$V\left(\bar{D}^*\Sigma_c,\frac{1}{2}\right) = V_a - \frac{4}{3}V_b, \tag{2}$$

$$V\left(\bar{D}^*\Sigma_c,\frac{3}{2}\right) = V_a + \frac{2}{3}V_b, \tag{3}$$

with V_a and V_b a central and spin-spin contribution that are in principle unknown. If the particles are heavy enough we can assume that the binding energies are proportional to the potential ($E \sim \langle V \rangle$), in which case we find that the choice

$$E_a \sim -10 \text{ MeV} \quad \text{and} \quad \frac{2}{3}E_b \sim +5 \text{ MeV} \tag{4}$$

indeed fits the spectrum of the pentaquark trio. The inclusion of pion exchanges can potentially change this conclusion though. One pion exchange (OPE) contains a spin-spin and a tensor piece: while the spin-spin piece can be easily subsumed into the term V_b of the S-wave potential, the tensor piece will effectively generate a central contribution to the $\bar{D}^*\Sigma_c$ molecules that is not present in the $\bar{D}\Sigma_c$ system. In practice we can modify the previous relations to

$$V\left(\bar{D}^*\Sigma_c,\frac{1}{2}\right) = V_a - \frac{4}{3}V_b + \delta V_a^T, \tag{5}$$

$$V\left(\bar{D}^*\Sigma_c,\frac{3}{2}\right) = V_a + \frac{2}{3}V_b + \delta V_a^T, \tag{6}$$

where δV_a^T is the contribution to the tensor force.[1] If the effective contribution to the binding energy is $\delta V_a^T \sim 5$ MeV, the preferred quantum numbers of the pentaquark trio will change. In fact the following identification,

$$E_a \sim -10\,\text{MeV}, \quad \frac{2}{3}E_b \sim -5\,\text{MeV}, \quad \delta E_a^T \sim -5\,\text{MeV}, \tag{7}$$

also fits the spectrum of the pentaquark trio.

However the previous is merely a heuristic argument which has to be supported by concrete calculations. HQSS for heavy hadron molecules does not directly apply to the binding energies, but rather to the potential between heavy hadrons. As a consequence, HQSS will in general not translate into the type of clean relations derived in the previous paragraph. For instance, in analogy to the discussion around Eq. (4) the predictions of pionless EFT prefers indeed the identification of the $P_c(4440)$ with the $J^P = \frac{1}{2}^-$ $\bar{D}^*\Sigma_c$ molecule, but there is room for the opposite identification to be possible [4]. In this manuscript we will investigate how the inclusion of pions modifies the previous conclusion. In pionful EFT the opposite identification—the $P_c(4440)$ is the $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ molecule—is preferred, yet the conclusion is not particularly strong at leading order. Uncertainties both within pionless and pionful EFT make it not possible to make a strong point based solely on spectroscopy. Yet they suggest a preference.

The manuscript is organized as follows: in Sec. II we review how HQSS applies to heavy baryon-meson molecules, in which we advocate the use of a particular notation—the light-quark notation [29]—for the description of the contact range and the OPE potential within EFT. In Sec. III we derive the one pion exchange potential for the heavy antimeson-baryon system. In Sec. IV we study the bound state spectrum for the heavy antimeson-baryon system within the pionful EFT and discuss their impact on the quantum numbers of the known hidden-charm pentaquarks. Finally, we present our conclusions in Sec. V.

II. HEAVY-QUARK SPIN SYMMETRY

In this section we briefly explain how HQSS constrains the interaction between a heavy meson and a heavy baryon. For this, we will use two different notations. The first is the standard heavy superfield notation, in which we define a superfield that groups together the heavy hadrons belonging to the same HQSS multiplet. The second is the light-quark notation, which is based on the quark model and in which we simply write down the light-quark subfield of the heavy hadrons, see Ref. [29] for a detailed exposition and Refs. [17,30] for previous examples of its use.

A. Heavy superfield notation

We begin by defining the superfields that are commonly used for the description of heavy meson and heavy baryons. The quark content of the S-wave heavy mesons is $Q\bar{q}$ with Q and q a heavy quark and a light quark, respectively. If the

[1]This contribution is not necessarily the same in the spin-$\frac{1}{2}$ and -$\frac{3}{2}$ molecules (see Ref. [21]), but the spin dependence can be reabsorbed in V_b leaving an effective tensor contribution which is spin independent.

spin of the $Q\bar{q}$ pair couples to $S=0$ we have the ground state heavy meson P and if it couples to $S=1$ we have the excited heavy meson P^*, where P and P^* are degenerate in the limit in which the heavy-quark mass goes to infinity. For the P and P^* heavy mesons the nonrelativistic superfield is

$$H_Q = \frac{1}{\sqrt{2}}[P + \vec{P}^* \cdot \vec{\sigma}], \tag{8}$$

which is adapted from the relativistic definition of Ref. [31]. H_Q is a 2×2 matrix and $\vec{\sigma}$ are the Pauli matrices.

For the S-wave heavy baryons the quark content is Qqq. If the light-quark pair is in the sextet configuration of the SU(3)-flavor symmetry group (the case we will be considering here), the spin of the light-quark pair couples to $S_L = 1$. This implies that the total spin of the heavy baryon is $S=\frac{1}{2}$ for the ground state Σ_Q and $S=\frac{3}{2}$ for the excited state Σ_Q^*, where Σ_Q and Σ_Q^* are degenerate in the heavy-quark limit. With this we define the nonrelativistic superfield as

$$\vec{S}_Q = \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_Q + \vec{\Sigma}_Q^*, \tag{9}$$

which again corresponds to the nonrelativistic limit of the superfield originally defined in Ref. [32].

From the $H_{\bar{Q}}$ and $\vec{S}_Q$ superfields, the most general contact-range Lagrangian with no derivatives we can construct is [18]

$$\mathcal{L} = C_a \vec{S}_Q^\dagger \cdot \vec{S}_Q \mathrm{Tr}[\bar{H}_{\bar{Q}}^\dagger \bar{H}_{\bar{Q}}] + C_b \sum_{i=1}^{3} \vec{S}_Q^\dagger \cdot (J_i \vec{S}_Q)\mathrm{Tr}[\bar{H}_{\bar{Q}}^\dagger \sigma_i \bar{H}_{\bar{Q}}], \tag{10}$$

where J_i with $i=1,\ 2,\ 3$ refers to the spin-1 angular momentum matrices and with C_a and C_b coupling constants. Note that the $H_{\bar{Q}}$ superfield refers to the heavy antimeson. If we particularize for the $\bar{D}\Sigma_c$ family of molecules, we obtain the contact-range potential of Table I.

TABLE I. The leading order contact-range potential for the charmed antimeson–charmed baryon system, i.e., the molecular hidden-charm pentaquarks. We show the potential for each particle and spin channel (the "Molecule" and "J^P" columns), where the potential depends on two independent couplings C_a and C_b. We do not explicitly show the isospin dependence of the couplings, but merely mention that the couplings in the $I=\frac{1}{2}$ and $\frac{3}{2}$ isospin configurations are different.

Molecule	J^P	V_C
$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	C_a
$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	C_a
$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	$C_a - \frac{4}{3}C_b$
$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	$C_a + \frac{2}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$C_a - \frac{5}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$C_a - \frac{2}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$C_a + C_b$

B. Light-quark notation

Actually there is an easier and more direct method to write the heavy-quark symmetric interactions, in which the idea is to consider the heavy quark as a spectator, see Ref. [29] for a detailed explanation. Instead of writing superfields, we can write the interactions in terms of the light-quark subfields. For the P and P^* heavy mesons we consider the light-quark field within the heavy mesons: q_L. Equivalently, for the Σ_Q and Σ_Q^* heavy baryons we use the light-diquark field within them: d_L. With these q_L and d_L subfields, the lowest order contact-range Lagrangian can be written as

$$\mathcal{L} = C_a(q_L^\dagger q_L)(d_L^\dagger d_L) + C_b(q_L^\dagger \vec{\sigma}_L q_L)\cdot(d_L^\dagger \vec{S}_L d_L), \tag{11}$$

where $\vec{\sigma}_L$ refers to the Pauli matrices as applied to the q_L field and $\vec{S}_L$ to the light-spin operators of the d_L field. This Lagrangian leads to the contact-range potential

$$V(q_L d_L) = C_a + C_b \vec{\sigma}_{L1}\cdot\vec{S}_{L2}, \tag{12}$$

where the subscripts 1 and 2 refer to the heavy meson and baryon, respectively. Now the contact-range potential is written in terms of the light-quark subfields, i.e., in terms of the light-quark spin. To rewrite the interactions in terms of the heavy hadron degrees of freedom we apply a series of rules for translating the light-quark spin operators into the heavy hadron spin operators. For the heavy mesons the translation rules are

$$\langle P|\vec{\sigma}_L|P\rangle = 0, \tag{13}$$

$$\langle P^*|\vec{\sigma}_L|P^*\rangle = \vec{S}_1, \tag{14}$$

where $\vec{S}_1$ refers to the spin-1 matrices as applied to the heavy vector meson. For the heavy baryons we have instead

$$\langle \Sigma_Q|\vec{J}_L|\Sigma_Q\rangle = \frac{2}{3}\vec{\sigma}_2, \tag{15}$$

$$\langle \Sigma_Q^*|\vec{J}_L|\Sigma_Q^*\rangle = \frac{2}{3}\vec{S}_2, \tag{16}$$

where $\vec{\sigma}_2$ are the Pauli matrices (applied to the spin-$\frac{1}{2}$ heavy baryon fields) and $\vec{S}_2$ are the spin-$\frac{3}{2}$ angular momentum matrices (applied to the spin-$\frac{3}{2}$ heavy baryon fields). If we apply these substitution rules to the contact-range potential of Eq. (12) for the light-quark subfields, we arrive to the contact-range potential of Table I written in the particle

basis. However the light-quark notation is much more compact and convenient, as it reduces the seven possible heavy antimeson-baryon potentials to a single formula.

III. THE ONE PION EXCHANGE POTENTIAL

In this section we derive the OPE potential as applied to the charmed antimeson and charmed baryon two-body system. The derivation employs the light-quark notation presented in Sec. II B. We discuss the coordinate and momentum space versions of the OPE potential and its partial wave projection.

A. Derivation of the potential

For the pion interactions, we begin by writing the following Lagrangians written in terms of the superfields H_Q and $\vec{S}_Q$:

$$\mathcal{L}_{HH\pi} = \frac{g_1}{\sqrt{2}f_\pi}\mathrm{Tr}[H_{\bar{Q}}^\dagger \tau_a \vec{\sigma}\cdot\vec{\nabla}\pi_a H_{\bar{Q}}], \tag{17}$$

$$\mathcal{L}_{SS\pi} = \frac{ig_2}{\sqrt{2}f_\pi}\vec{S}_Q^\dagger \cdot (T_a \vec{\nabla}\pi_a \times \vec{S}_Q), \tag{18}$$

with g_1, g_2 the axial couplings of the pion to the heavy meson and heavy baryons, respectively, $f_\pi = 132$ MeV the pion decay constant, τ_a the Pauli matrices in isospin space, T_a the $I = 1$ isospin matrices and where the latin index a refers to the isospin. For the axial couplings we choose

$$g_1 = 0.60 \quad \text{and} \quad g_2 = 0.84, \tag{19}$$

where g_1 is taken from the $D^* \to D\pi$ decays [33,34] ($g_1 = 0.59 \pm 0.01 \pm 0.07$) and g_2 from the lattice QCD calculation of Ref. [35]. We notice that there are several conventions for g_2, which are discussed in Ref. [35] (from which one can also find the relations among them). The convention we use here differs by a sign of the one by Cho [32], i.e., $g_2 = -g_{2,\mathrm{Cho}}$. From this Lagrangian we can write the OPE potential as

$$V_{\mathrm{OPE}} = \frac{\mathcal{A}_1(\vec{q})\mathcal{A}_2(-\vec{q})}{q^2 + m_\pi^2}, \tag{20}$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ refer to the nonrelativistic amplitudes

$$\mathcal{A}_1 = \mathcal{A}(H_Q \to H'_Q\pi), \tag{21}$$

$$\mathcal{A}_2 = \mathcal{A}(S_Q \to S'_Q\pi), \tag{22}$$

in the nonrelativistic normalization of the amplitudes used in Refs. [26,28] (but notice that Ref. [28] uses the normalization of Cho [32] for the axial coupling of the heavy baryon). By specifying $\mathcal{A}_1$ and $\mathcal{A}_2$ for the particular heavy meson and heavy baryon of interest, we can obtain the potential for any of the cases. The procedure ends in seven possible potentials, one for each of the possible S-wave molecules, which we will not write here in detail.

Alternatively, we can write the Lagrangians of Eqs. (17) and (18) in terms of the light-quark fields within the heavy hadrons:

$$\mathcal{L}_{q_L q_L \pi} = \frac{g_1}{\sqrt{2}f_\pi} q_L^\dagger \vec{\sigma}_L \cdot \vec{\nabla}(\tau_a \pi_a) q_L, \tag{23}$$

$$\mathcal{L}_{d_L d_L \pi} = \frac{g_2}{\sqrt{2}f_\pi} d_L^\dagger \vec{S}_L \cdot \vec{\nabla}(T_a \pi_a) d_L. \tag{24}$$

From this, the OPE potential can be written in momentum space as

$$V_{\mathrm{OPE}}(\vec{q}) = -\frac{g_1 g_2}{2f_\pi^2}\vec{\tau}_1\cdot\vec{T}_2 \frac{\vec{\sigma}_{L1}\cdot\vec{q}\,\vec{S}_{L2}\cdot\vec{q}}{\vec{q}^2 + m_\pi^2}. \tag{25}$$

We can Fourier transform the OPE potential into coordinate space,

$$V_{\mathrm{OPE}}(\vec{r}) = -\frac{g_1 g_2}{6f_\pi^2}\vec{\tau}_1\cdot\vec{T}_2 \vec{\sigma}_{L1}\cdot\vec{S}_{L2}\delta^{(3)}(\vec{r}) \tag{26}$$

$$+\vec{\tau}_1\cdot\vec{T}_2[\vec{\sigma}_{L1}\cdot\vec{S}_{L2}W_C(r) + S_{L12}(\hat{r})W_C(r)], \tag{27}$$

where W_C and W_T are defined as

$$W_C(r) = \frac{g_1 g_2 m_\pi^3}{24\pi f_\pi^2}\frac{e^{-m_\pi r}}{m_\pi r}, \tag{28}$$

$$W_T(r) = \frac{g_1 g_2 m_\pi^3}{24\pi f_\pi^2}\frac{e^{-m_\pi r}}{m_\pi r}\left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right). \tag{29}$$

B. Partial wave projection

Strong interactions preserve the total angular momentum $\vec{J} = \vec{L} + \vec{S}$, but not the orbital angular momentum or spin separately. As a consequence the OPE potential will mix partial waves with the same quantum number J, but different quantum numbers L and S. If we use the spectroscopic notation $^{2S+1}L_J$, the partial waves comprising the three pentaquark-like $\bar{D}\Sigma_c$ and $\bar{D}^*\Sigma_c$ molecular candidates are

$$\left|\bar{D}\Sigma_c\left(\frac{1}{2}^-\right)\right\rangle = \{^2S_{\frac{1}{2}}\}, \tag{30}$$

$$\left|\bar{D}^*\Sigma_c\left(\frac{1}{2}^-\right)\right\rangle = \{^2S_{\frac{1}{2}}, ^4D_{\frac{1}{2}}\}, \tag{31}$$

$$\left|\bar{D}^*\Sigma_c\left(\frac{3}{2}^-\right)\right\rangle = \{^2D_{\frac{3}{2}}, ^4S_{\frac{3}{2}}, ^4D_{\frac{3}{2}}\}, \tag{32}$$

plus the corresponding decomposition for the other four $\bar{D}\Sigma_c^*$ and $\bar{D}^*\Sigma_c^*$ molecular configurations containing S-waves.

The partial wave projection is done by defining a generalized spherical harmonic for the $^{2S+1}L_J$ wave,

$$\mathcal{Y}_{JM}^{LS}(\Omega) = \sum_{M_L M_S} Y_{LM_L}(\Omega)|SM_S\rangle\langle LM_L SM_S|JM\rangle, \quad (33)$$

where Ω is the solid angle and which can be used to project the potential into the partial wave basis. For the momentum space potential this is done as follows:

$$\langle k, JLS|V|k', JL'S'\rangle = \frac{1}{4\pi}\int d\hat{k}d\hat{k}'\, \mathcal{Y}_{JM}^{LS*}(\hat{k})V(\vec{k}'-\vec{k})\mathcal{Y}_{JM}^{L'S'}(\hat{k}'), \quad (34)$$

while for the coordinate space potential we have

$$\langle JLS|V(r)|JL'S'\rangle = \int d\hat{r}\, \mathcal{Y}_{JM}^{LS*}(\hat{r})V(\vec{r})\mathcal{Y}_{JM}^{L'S'}(\hat{r}), \quad (35)$$

where the projection is independent of the third component of the total angular momentum M. In coordinate space a further simplification is possible by noticing that the partial wave projection only involves writing the spin-spin and tensor operators as matrices in the space of the partial waves comprising a particular state:

$$\vec{\sigma}_{L1}\cdot\vec{S}_{L2} \to f_{12}\mathbf{C}_{12} \quad \text{and} \quad S_{L12} \to f_{12}\mathbf{S}_{12}, \quad (36)$$

where f_{12} is a conversion factor ($f_{12} = \frac{2}{3}$ in all cases) and the matrices $\mathbf{C}_{12}$ and $\mathbf{S}_{12}$ can be consulted in Table V.

IV. THE MOLECULAR PENTAQUARK SPECTRUM

In this section we discuss the description of the LHCb pentaquark trio—the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$—within the molecular picture in a pionful EFT. We will consider the $P_c(4312)$ as a $\bar{D}\Sigma_c$ bound state and the $P_c(4440)$ and $P_c(4457)$ as $\bar{D}^*\Sigma_c$ ones. The consistent description of the pentaquark trio suggests a slight preference for the quantum numbers $J^P = \frac{3}{2}^-$ and $\frac{1}{2}^-$ for the $P_c(4440)$ and $P_c(4457)$, respectively. The pionful EFT will also lead to the prediction of the other four molecular pentaquarks.

A. Bound state equations

We calculate the binding energies of a heavy baryon-antibaryon bound state by plugging the EFT potential into the Lippmann-Schwinger or Schrödinger equation, depending on whether the EFT potential has been written in momentum or coordinate space. For momentum space, the bound state equation takes the form

$$\phi_{LS}^J(p) = \sum_{L'S'}\int \frac{d^3q}{(2\pi)^3}\frac{\langle p, JLS|V|q, JL'S'\rangle}{E-\frac{q^2}{2\mu}}\phi_{L'S'}^J(q), \quad (37)$$

where L, S and J are the orbital, intrinsic and total angular momentum, with ϕ_{LS}^J the vertex function. This bound state equation can be solved by discretizing this integral equation and finding the eigenvalues of the ensuing linear equations. For coordinate space, we use the reduced Schrödinger equation,

$$-u_{LS}^J{}'' + 2\mu\sum_{L'S'}V_{LS,L'S'}^J(r)u_{L'S'}^J(r) + \frac{L(L+1)}{r^2}u_{LS}^J(r) = -\gamma^2 u_{LS}^J(r), \quad (38)$$

which is a system of coupled ordinary differential equations that can be solved by standard means.

B. Regularization and renormalization

The EFT potential is not well behaved at distances below the pion Compton wavelength, a problem that is taken care of by means of a regularization and renormalization procedure. The regularization part is as follows: for the momentum space version of the potential, we use a separable regulator of the type

$$\langle p'|V_\Lambda|p\rangle = \langle p'|V|p\rangle f\left(\frac{p'}{\Lambda}\right)f\left(\frac{p}{\Lambda}\right), \quad (39)$$

where $f(x) = e^{-x^2}$, i.e., a Gaussian regulator. For the coordinate space potential we use a local regulator, which is different depending on whether it is a applied for the contact- or finite-range piece of the EFT potential. For the regularization of the contact-range potential, we use a Gaussian regulator of the type

$$\delta^{(3)}(\vec{r}) \to \frac{e^{-(r/R_c)^4}}{\frac{4}{3}\pi\Gamma(\frac{7}{4})R_c^3}, \quad (40)$$

while for the OPE potential we use

$$V_{\rm OPE}(r) \to V_{\rm OPE}(r)[1-e^{-(r/R_c)^4}]. \quad (41)$$

This type of local r-space regulators has been recently put in use in pionful EFT as applied to nuclear physics [36]. We choose the Gaussian exponent to be $n = 4$ as this is enough to suppress the divergence of the tensor force at short distances.

For the renormalization part, the idea is that the contact-range couplings, C_a and C_b in this case, will be able to absorb the cutoff dependence. Thus the predictions derived within the EFT framework are expected to be cutoff independent. For checking the cutoff independence hypothesis, we choose the following cutoff window in momentum space:

$$\Lambda = (0.75\text{–}1.5)\ \text{GeV}, \tag{42}$$

which roughly corresponds to $\{m_\rho, 2m_\rho\}$. This window is harder than the one we previously used in the contact-range EFT of Ref. [4], i.e., $\Lambda = 0.5$–1.0 GeV. The choice of a harder cutoff is driven by the experience from pionful EFT as applied to heavy meson-antimeson molecules [37,38], in which larger cutoffs than in a purely contact theory seemed to make a difference. For the coordinate space calculation we choose

$$R_c = 0.5\text{–}1.0\ \text{fm}, \tag{43}$$

which comes from rounding up the $\{\pi/2m_\rho, \pi/m_\rho\}$ cutoff window. This is approximately equivalent to the momentum space window if we consider the relation $R_c = \pi/\Lambda$ for the r- and p-space cutoffs. Unfortunately cutoff independence is not achieved at the accuracy level we will require to unambiguously distinguish the quantum numbers of the P_c pentaquarks.

C. The quantum numbers of the pentaquark trio

The couplings C_a and C_b are actually determined from observable quantities, for which we will use the binding energies of the $P_c(4440)$ and $P_c(4457)$ pentaquarks. The natural expectation in the molecular picture is that the $P_c(4440)$ and $P_c(4457)$ are $\bar{D}^*\Sigma_c$ bound states with isospin $I = \frac{1}{2}$, for which two possibilities exist for the total angular momentum: $J = \frac{1}{2}$ and $J = \frac{3}{2}$. We do not know which is the total angular momentum of each of the molecular pentaquark candidates, which means that we will consider two scenarios:

(i) scenario A: the $P_c(4440)$ is the $J = \frac{1}{2}$ molecule, while the $P_c(4457)$ is the $J = \frac{3}{2}$ molecule,
(ii) scenario B: the $P_c(4440)$ is the $J = \frac{3}{2}$ molecule, while the $P_c(4457)$ is the $J = \frac{1}{2}$ molecule,

which are the same two scenarios considered in Ref. [4]. The values of the couplings C_a and C_b that are obtained in each scenario can be consulted in Table II. Each of the scenarios predicts a different mass for the $P_c(4312)$ pentaquark. In momentum space, scenario A predicts

$$M_1^A = 4314 - (4319)^V\ \text{MeV}, \tag{44}$$

where the only uncertainty we have taken into account is the cutoff variation, with the V superscript standing for the fact that the bound state disappears and becomes a virtual state instead for $\Lambda = 1.5$ GeV. On the other hand scenario B predicts

$$M_1^B = 4308\text{–}4321\ \text{MeV}. \tag{45}$$

TABLE II. The contact-range couplings C_a and C_b from the condition of reproducing the mass of the $P_c(4440)$ and $P_c(4457)$ as molecular pentaquarks in p- and r-space (as indicated by type of cutoff: Λ and R_c). Scenario A corresponds to considering that the spin parities of the $P_c(4440)$ and $P_c(4457)$ are $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively, while scenario B corresponds to the opposite identification.

Scenario	Λ (MeV)	C_a (fm^2)	C_b (fm^2)
A	0.75	−1.1199	−0.1183
A	1.50	−0.3466	−0.1669
B	0.75	−1.2755	−0.5494
B	1.50	−0.4001	−0.3760
Scenario	**R_c (fm)**	**C_a (fm^2)**	**C_b (fm^2)**
A	0.5	−0.5741	+0.1345
A	1.0	−1.7447	+0.4074
B	0.5	−0.6494	−0.0400
B	1.0	−2.0142	−0.3503

This preliminary comparison indicates that scenario B is slightly favored over scenario A, but the conclusion is merely tentative at best.

The residual cutoff variation alone already indicates that the error of the pionful EFT at leading order is probably too large to distinguish between the two scenarios. Besides the cutoff uncertainty, there are two other error sources that we have not explicitly considered: the uncertainty (i) in HQSS and (ii) in the g_2 axial coupling constant of the pion with the sextet heavy baryons. Regarding (i), HQSS, the location of the $P_c(4312)$ is determined from the contact-range coupling C_a, but in doing so we are assuming that HQSS is exact for the hidden-charm molecular pentaquarks. This is not the case, with HQSS violations expected to have a size of $\Lambda_{\rm QCD}/m_c$, with $\Lambda_{\rm QCD} \sim 200$–$300$ MeV and m_c the charm quark mass, yielding a 15%–20% variation for the coupling C_a around the determination we have done. Regarding (ii), the g_2 axial coupling, the uncertainty in the lattice QCD calculation is sizable: $g_2 = 0.84 \pm 0.20$. Besides, this lattice QCD calculation applies to the heavy-quark limit ($m_Q \to \infty$, with m_Q the mass of the heavy quark). The g_2 axial coupling can be derived from the axial coupling involved in the sextet to antitriplet heavy baryon transitions, g_3, and a quark model relation (see Ref. [39] for a comprehensive review, which uses the normalization of Yan [40] for the axial couplings). In turn the g_3 axial coupling can be determined from the $\Sigma_c \to \Lambda_c \pi$ decay. This procedure yields $g_2 \sim 1.4$ [39], a value considerably larger than the one we have chosen (and which indeed makes a difference). If this were not enough, the location of the $P_c(4312)$ is not known with the required accuracy either. A recent theoretical exploration has proposed that the $P_c(4312)$ is a virtual state instead of a bound state [41]: if this is the case, scenario A should be the preferred one.

We recognize the following three factors influencing the preference over scenarios A and B:

(i) softer cutoffs ($\Lambda \sim 0.5$ GeV) favor scenario A, while harder ones ($\Lambda \gtrsim 1$ GeV) favor scenario B,
(ii) larger axial couplings ($g_2 \sim 1.4$) favor scenario B,
(iii) a less bound (or virtual) $P_c(4312)$ favors scenario A.

The first of these factors refers to the inner workings of the EFT and probably can be only dealt with by improving the current EFT description, e.g., calculating the subleading order corrections,[2] which will require new data as the next-to-leading order contact-range potential will involve new couplings. The second of these factors is difficult to settle experimentally—the g_2 axial coupling does not directly appear in decays or other quantities that are directly observable [39]—but can probably be determined by lattice QCD calculations that take into account the finite charm quark mass. The third factor can eventually be determined in future experiments with smaller uncertainties.

At this point it is important to comment about cutoff independence. In principle we expect cutoff independence to be achieved by means of the renormalization process, where the contact-range couplings—C_a and C_b in this case—are expected to absorb the divergences associated with the short-range quirks of the EFT potential. However this is not the case for the calculations presented here: the effects of the tensor force have not been completely reabsorbed in the couplings C_a and C_b. The manifestation of this problem is the binding energy prediction of the $P_c(4312)$ pentaquark. If we assume it to be a $\bar{D}\Sigma_c$ molecule this system cannot exchange pions. As the cutoff Λ grows, the effect of the tensor force will be increasingly attractive, forcing the C_a coupling to be less and less attractive. Eventually, for Λ hard enough, the $P_c(4312)$ will cease to be bound and will become a virtual state instead. In momentum space this indeed happens for scenario A and a cutoff of the order of 1.5 GeV. It also happens for scenario B, though in this case a harder cutoff is required (around 2.0 GeV, give or take).

This is bad news because it partially invalidates one of the expected advantages of the EFT framework over phenomenological models: systematic error estimations. In a properly renormalized EFT, where calculations do not strongly depend in the cutoff, the cutoff variation might be used as a proxy of the EFT uncertainty. However it is impossible to describe the LHCb pentaquark trio in a cutoff independent way: large cutoffs invariably lead to the disappearance of the $P_c(4312)$ member of the trio. Of course this happens for relatively hard cutoffs in the 1.5–2.0 GeV range, which means that this disappearance is not physically relevant but rather an artifact. Yet, despite being an artifact, it prevents the systematic estimation of the theoretical uncertainty. Basically, even if the experimental error in the determination of the $P_c(4312)$ mass was negligible, there will be no completely model independent way to distinguish both scenarios in the pionful EFT proposed here. Despite this drawback, pionful calculations are still useful even if they begin to show a sizable cutoff dependence at $\Lambda > 1.5$ GeV. It is interesting to notice that a similar cutoff dependency has been discussed for EFTs involving heavy flavor symmetry [43], which is a different manifestation of heavy-quark symmetry. Be it as it may, the degree of model dependence is probably smaller than for phenomenological models.

The conclusion is that there is a preference for scenario B. The fact that this preference is not particularly strong is in line with the early speculations about the existence of molecular pentaquarks, in which predictions showed a clear degeneracy in spin [14–16]. Later it was realized that the inclusion of pions in the hidden-gauge approach will break this degeneracy [44], leading to the conclusion that the $J^P = \frac{3}{2}^-$ pentaquark is lighter than the $J^P = \frac{1}{2}^-$ one, in agreement with scenario B. In turn this is compatible with the hypothesis of Karliner and Rosner [17], where the $J^P = \frac{3}{2}^-$ molecular pentaquark is expected to be more bound than its $J^P = \frac{1}{2}^-$ partner. In contrast in the traditional one boson exchange model this pattern is apparently inverted [2,11], with the lower spin molecules being more bound than the higher spin ones. However a recent work [45], which has revisited the application of the one boson exchange model to heavy antimeson-baryon molecules, suggests that this is not necessarily the case and that scenario B might be the most probable.

D. The pentaquark HQSS septuplet

The consistent description of the $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ pentaquark trio in the molecular picture fully determines the LO potential in pionful EFT. As a consequence we can compute the binding energies of all the S-wave molecular configurations. The results are summarized in Tables III and IV for the momentum and coordinate space versions[3] of the LO potential. As happened in the pionless EFT at LO [4], we predict the seven possible HQSS partners of the pentaquark trio, independently of

[2]We notice in passing that the subleading EFT potential has been calculated in Ref. [42], though with the aim of deducing the existence of the pentaquark trio from the two-nucleon system (by extrapolating the contact-range couplings from the two-nucleon system to the heavy antimeson-baryon system). That is, the use of pionful EFT in Ref. [42] is very different from the one in the present manuscript. Nonetheless we point out that it might be possible to combine the subleading potential of Ref. [42] with the ideas of Ref. [38] (properly adapted from the heavy meson-antimeson to the heavy baryon-antimeson case) to better pinpoint the quantum numbers of the pentaquark trio.

[3]The momentum space calculation contains all the partial waves, including the G-waves in the $J^P = \frac{3}{2}^-$ and $\frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ molecules (see Table V), the contribution of which can be checked to be negligible (less than 0.1 MeV). In view of this result, the coordinate space calculation ignores the G-waves, which greatly simplifies the required computations.

TABLE III. Predictions for the S-wave HQSS molecular multiplet of heavy antimeson-baryon molecules, as derived from the lowest-order potential in pionful EFT (p-space). This potential contains a contact-range piece with two unknown couplings C_a and C_b and a finite-range piece, given by OPE. In all cases we assume that the isospin of the listed molecules is $I = \frac{1}{2}$. We determine the value of the C_a and C_b couplings from the condition of reproducing the location of the $P_c(4440)$ and $P_c(4457)$ resonances, which are known to be close to the $\bar{D}^*\Sigma_c$ threshold. We do not know however the quantum numbers of the $P_c(4440)$ and $P_c(4457)$, but consider two possibilities instead, scenario A and B, where in the first the $P_c(4440)$ is the $\frac{1}{2}^-$ molecule and in the second the $P_c(4457)$ is the $\frac{1}{2}^-$ molecule. If a molecular pentaquark becomes unbound but survives as a virtual state (a situation that happens for the $\bar{D}\Sigma_c$ and $\bar{D}\Sigma_c^*$ systems), we indicate this situation with the superscript V. Calculations are done in momentum space with the regularization described in Eq. (39) and a cutoff $\Lambda = 0.75$–1.5 GeV.

Scenario	Molecule	J^P	B (MeV)	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	$(2)^V-7$	$4314-(4319)^V$
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	$(1)^V-7$	$4378-(4384)^V$
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	Input	4440.3
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	Input	4457.3
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	27–44	4483–4500
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	16–20	4507–4512
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4–6	4520–4523
B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	0–12	4308–4321
B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	0–13	4372–4385
B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	Input	4457.3
B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	Input	4440.3
B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4–14	4513–4523
B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	11–16	4511–4516
B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	26–29	4497–4501

TABLE IV. Predictions for the S-wave HQSS molecular multiplet of heavy antimeson-baryon molecules, as derived from the lowest-order potential in pionful EFT (r-space). We refer to Table III for details. Calculations are done in coordinate space with the regularization described in Eqs. (40) and (41) and a cutoff $R_c = 0.5$–1.0 fm. The G-wave components are ignored for the $J^P = \frac{3}{2}^-$ and $\frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ molecules, as their contribution to the binding energy is negligible.

Scenario	Molecule	J^P	B (MeV)	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	1–8	4313–4320
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	1–8	4377–4384
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	Input	4440.3
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	Input	4457.3
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	28–36	4490–4499
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	17–20	4507–4510
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4–7	4520–4523
B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	5–14	4307–4315
B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	6–14	4371–4379
B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	Input	4457.3
B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	Input	4440.3
B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	3–8	4518–4523
B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	11–15	4512–4516
B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	28–33	4494–4499

whether we use scenario A or B for the $P_c(4440)$ and $P_c(4457)$ quantum numbers. The most important difference with the contact-range theory is that the predictions for the $\bar{D}\Sigma_c$ and $\bar{D}\Sigma_c^*$ molecules are less bound, leading to a marginal preference of scenario B over A. In every other respect, Tables III and IV only confirm the patterns already discovered in Ref. [4]: scenario A (B) leads to the higher spin states being more (less) massive. If this were not enough, further confirmation can be found in the recent pionless EFT calculation of Ref. [20], which also considers transitions among the $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$ and $\bar{D}^*\Sigma_c^*$ channels. In this regard, the eventual discovery of a $\frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ molecule will probably settle the question about the quantum numbers of the $P_c(4440)$ and $P_c(4457)$: the prediction of the location of this $\bar{D}^*\Sigma_c^*$ molecules varies by about 20–25 MeV depending on the scenario. However, owing to its angular momentum $J = \frac{5}{2}$, the experimental detection of a $\bar{D}^*\Sigma_c^*$ pentaquark state is not probable in the $J/\Psi p$ channel where the other pentaquarks have been discovered. The $J = \frac{5}{2}$ state might indeed be difficult to observe from its decays to a charmonium: all possible charmonium decays for this state are p- or d-wave, which indicates that they might be relatively suppressed.

Notice that other works lead to different predictions of the septuplet. In Ref. [6] the binding energy of the molecular pentaquarks is almost independent of the spin and the identification between scenarios A and B is done on the basis of the predicted decay widths. This approximate degeneracy of the binding energy is however a consequence of explicitly ignoring the coupling C_b: Ref. [6] determines the couplings from resonance saturation in the hidden gauge model, with C_b receiving its main contribution from OPE, which is assumed to be weak. Reference [7] also predicts a multiplet structure for the hidden charm pentaquarks, which relies on HQSS and OPE. But the multiplet structure of Ref. [7] is merely a subset of the septuplet of Refs. [4,6]. The reason for the difference is that Ref. [7] only considers the longest-range part of the heavy antimeson-baryon potential, i.e., OPE. More recently, Ref. [21] improves over the OPE calculation of Ref. [7] by explicitly including the $\bar{D}\Lambda_c$ and $\bar{D}^*\Lambda_c$ channels and a compact $c\bar{c}qqq$ core. These improvements lead Ref. [21] to predict the existence of the full pentaquark septuplet and to

TABLE V. Matrix elements of the spin-spin and tensor operator for the partial waves we are considering in this work.

Molecule	Partial waves	J^P	$\vec{a}_1\cdot\vec{a}_2$	$S_{12}=3\vec{a}_1\cdot\hat{r}\vec{a}_2\cdot\hat{r}-\vec{a}_1\cdot\vec{a}_2$
$\bar{D}\Sigma_c$	$^2S_{1/2}$	$\frac{1}{2}^-$	0	0
$\bar{D}\Sigma_c^*$	$^4S_{3/2}$-$^4D_{3/2}$	$\frac{3}{2}^-$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
$\bar{D}^*\Sigma_c$	$^2S_{1/2}$-$^4D_{1/2}$	$\frac{1}{2}^-$	$\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & -2 \end{pmatrix}$
$\bar{D}^*\Sigma_c$	$^2D_{3/2}$-$^4S_{1/2}$-$^4D_{1/2}$	$\frac{3}{2}^-$	$\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	$^2S_{1/2}$-$^4D_{1/2}$-$^6D_{1/2}$	$\frac{1}{2}^-$	$\begin{pmatrix} -\frac{5}{2} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \frac{3}{2} \end{pmatrix}$	$\begin{pmatrix} 0 & -\frac{7}{2\sqrt{5}} & -\frac{3}{\sqrt{5}} \\ -\frac{7}{2\sqrt{5}} & -\frac{8}{5} & -\frac{3}{10} \\ -\frac{3}{\sqrt{5}} & -\frac{3}{10} & -\frac{12}{5} \end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	$^2D_{3/2}$-$^4S_{3/2}$-$^4D_{3/2}$ -$^6D_{3/2}$-$^6G_{3/2}$	$\frac{3}{2}^-$	$\begin{pmatrix} -\frac{5}{2} & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{2} \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{7}{2\sqrt{10}} & -\frac{7}{2\sqrt{10}} & \frac{3}{\sqrt{35}} & -3\sqrt{\frac{6}{35}} \\ \frac{7}{2\sqrt{10}} & 0 & \frac{8}{5} & -\frac{3}{10}\sqrt{\frac{7}{2}} & 0 \\ -\frac{7}{2\sqrt{10}} & \frac{8}{5} & 0 & -\frac{3}{2\sqrt{14}} & -\frac{3}{5}\sqrt{\frac{3}{7}} \\ \frac{3}{\sqrt{35}} & -\frac{3}{10}\sqrt{\frac{7}{2}} & -\frac{3}{2\sqrt{14}} & -\frac{6}{7} & \frac{9\sqrt{6}}{35} \\ -3\sqrt{\frac{6}{35}} & 0 & -\frac{3}{5}\sqrt{\frac{3}{7}} & \frac{9\sqrt{6}}{35} & -\frac{15}{7} \end{pmatrix}$
$\bar{D}^*\Sigma_c^*$	$^2D_{5/2}$-$^4D_{5/2}$-$^4G_{5/2}$-$^6S_{5/2}$ -$^6D_{5/2}$-$^6G_{5/2}$	$\frac{5}{2}^-$	$\begin{pmatrix} -\frac{5}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{2} \end{pmatrix}$	$\begin{pmatrix} 0 & \frac{1}{2}\sqrt{\frac{7}{5}} & -\sqrt{\frac{21}{10}} & -\sqrt{\frac{3}{5}} & 2\sqrt{\frac{6}{35}} & -3\sqrt{\frac{2}{35}} \\ \frac{1}{2}\sqrt{\frac{7}{5}} & \frac{8}{7} & \frac{16\sqrt{6}}{35} & \frac{\sqrt{21}}{10} & -\frac{1}{7}\sqrt{\frac{3}{2}} & -\frac{12\sqrt{2}}{35} \\ -\sqrt{\frac{21}{10}} & \frac{16\sqrt{6}}{35} & -\frac{8}{7} & 0 & \frac{9}{70} & -\frac{3\sqrt{3}}{14} \\ -\sqrt{\frac{3}{5}} & \frac{\sqrt{21}}{10} & 0 & 0 & \frac{2\sqrt{14}}{5} & 0 \\ 2\sqrt{\frac{6}{35}} & -\frac{1}{7}\sqrt{\frac{3}{2}} & \frac{9}{70} & \frac{3\sqrt{14}}{5} & \frac{6}{7} & \frac{27\sqrt{3}}{35} \\ -3\sqrt{\frac{2}{35}} & -\frac{12\sqrt{2}}{35} & -\frac{3\sqrt{3}}{14} & 0 & \frac{27\sqrt{3}}{35} & -\frac{6}{7} \end{pmatrix}$

determine that the quantum numbers of the $P_c(4440)$ and $P_c(4457)$ are $J^P=\frac{3}{2}^-$ and $\frac{1}{2}^-$, i.e., scenario B. But there are two important differences between Ref. [21] and the calculations in the present manuscript: (i) Ref. [21] takes $g_2\sim 1.5$ (notice that they use the normalization of Yan [40] for the axial coupling, where $g_2=\frac{3}{2}g_{1,\mathrm{Yan}}$), (ii) the treatment of the short-range piece of the interaction is phenomenological and is modeled with a compact $c\bar{c}qqq$ core, which in turn leads to a short-range potential.

V. SUMMARY

In this manuscript we have described the impact that pion exchanges have in the description of the hidden-charm pentaquarks, provided they are indeed molecular. Pion exchanges are an important factor in the ordering of the pentaquark spectrum, a factor that might determine which quantum numbers are more/less bound.

If we try to describe consistently the LHCb pentaquark trio with a pionful EFT, the preliminary conclusion is that the $P_c(4440)$ and the $P_c(4457)$ are the $J^P=\frac{3}{2}^-$ and $\frac{1}{2}^-$ $\bar{D}^*\Sigma_c$ molecular pentaquarks, respectively. This conclusion agrees with the previous works of Uchino, Liang and Oset [17] and Karliner and Rosner [44], which is not surprising once we take into account that this is a consequence of OPE being attractive (repulsive) in the $\frac{3}{2}^-$ ($\frac{1}{2}^-$) channel. But this identification is only marginally preferred over the opposite one: the different uncertainties within the pionful EFT description we use make it impossible to reach a definite conclusion. This is further compounded with the uncertainties in the location of the $P_c(4312)$, $m=4311.9\pm 0.7^{+6.8}_{-0.7}$, where the systematic uncertainty (i.e., the $^{+6.8}_{-0.7}$ error) leans in the direction which results in a less bound molecular pentaquark. The recent amplitude analysis of Ref. [41], which claims that the $P_c(4312)$ could be a virtual state, cements this idea further. If this is the case, the preferences of both scenarios could likely change.

Besides the quantum numbers of the molecular pentaquarks, pion exchanges lead to the prediction of a total of

seven hidden-charm molecular pentaquarks in the isodoublet $I = \frac{1}{2}$ sector. This confirms the previous conclusions obtained in a pionless EFT [4], a more sophisticated pionless EFT including coupled channels [20], the hidden gauge model (as constrained by HQSS) [6] and a recent phenomenological pionful calculation [21]. In turn this points toward the idea that the existence of the HQSS multiplet is more a consequence of HQSS than of the explicit dynamics leading to binding. In particular the most important factor determining the details of the binding energy is the quantum numbers of the $P_c(4440)$ and $P_c(4457)$.

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