Fully-heavy tetraquarks: $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$

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In the framework of a nonrelativistic chiral quark model, we continue to study the mass spectra of the fully-heavy $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ tetraquarks. In the present calculations, two structures, meson-meson $[\bar{Q}Q]$ $[\bar{Q}Q]$ and diquark-antidiquark $[QQ]$ $[\bar{Q}\bar{Q}]$ ($Q = c$ or b), and their mixing, along with all possible color, spin configurations are considered. The calculations suggest that no bound state can be formed for $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ systems. However, resonances are possible because of the color structure. Several resonances are predicted and their stabilities are checked using the real scaling method.

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I. INTRODUCTION

In the past years, experimental searches for the exotic states beyond the conventional quark model have made great progress. The observation of XYZ states, such as $X(3872)$ [1], $Y(4260)$ [2,3], $Z_c(3900)$ [4–7], $Z_b(10610)$ [8], $P_c^+(4380)$, $P_c^+(4450)$ [9], and so on, provided us a good opportunity to extend our knowledge of the heavy flavor spectroscopy. Especially those charged quarkoniumlike states with heavy flavor mesons as the decay products, make them the best candidates for the exotic hadrons.

Recently, the tetraquarks composed of four heavy quarks $QQ\bar{Q}\bar{Q}$ ($Q = c$ or b) have received great attention. Experimentally, the heavy-flavor states provide some advantages, because they can be explored with the help of the efficient triggers such as J/ψ . LHCb Collaboration is hunting for the $bb\bar{b}\bar{b}$ tetraquark state and the existence information needs the further confirmation [10]. In the theoretical aspect, there are also many studies on the fully-heavy tetraquarks. For example, in some work [11–18], it is suggested that there exist stable bound $bb\bar{b}\bar{b}$ and $cc\bar{c}\bar{c}$ states with relatively smaller masses below the thresholds of the corresponding meson pairs. But some other work argues to the contrary that there should be no bound $bb\bar{b}\bar{b}$ or $cc\bar{c}\bar{c}$ tetraquark states because of the larger masses than the thresholds to decay [19–24]. References [25,26] also studied the weak decay properties about tetraquarks $bb\bar{c}\bar{c}$ and $b\bar{c}q\bar{q}$. Although some of the opinions are quite

different from each other, the researches on the exotic states are quite important for our understanding the underlying dynamics of the exotic states and the nature of strong interactions of QCD.

If the $bb\bar{b}\bar{b}$ and $cc\bar{c}\bar{c}$ tetraquark states do exist in nature, we have strong reason to believe that there exist more other heavy-flavor tetraquark states. In our previous work [24], we focused on the full-bottom tetraquarks $bb\bar{b}\bar{b}$ in the framework of the chiral quark model. In the present work, we would like to extend the study to the tetraquarks $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$. Although they are still missing in experiment, the study of the mass spectra of these two systems will offer information for the further experimental explorations. Theoretically, in Ref. [27], Liu *et al.* studied the mass spectra of the fully-heavy tetraquark systems including $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ within a potential model and no bound states with masses below the corresponding thresholds were found. Recent studies by Wu *et al.* showed that $bc\bar{b}\bar{c}$ bound state was found to be possible, but $bb\bar{c}\bar{c}$ state was not a bound state [22]. In Ref. [28], Richard *et al.* also observed that bound $bc\bar{b}\bar{c}$ state might be more favorable than $bb\bar{b}\bar{b}$ and $cc\bar{c}\bar{c}$. Reference [15] showed that tensor tetraquark $bc\bar{b}\bar{c}(2^{++})$ can be observed in both $B_c B_c$ and $J/\psi \Upsilon(1S)$ modes. It is well known that the color magnetic interaction (CMI) of the one-gluon exchange plays an important role in the hadron spectrum and hadron-hadron interactions. Compared with $bb\bar{b}\bar{b}$ tetraquark state, CMI is beneficial to form compact tetraquarks for $bc\bar{b}\bar{c}$. Considering the higher thresholds of $b\bar{c} + b\bar{c}$, $bb\bar{c}\bar{c}$ may also a possible tetraquark state. Our purpose is first to check whether there are stable bound states in the $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ systems, if not, second, we aim to look for the possible resonances.

In this work, we calculated the mass spectra of the $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ systems in a nonrelativistic chiral quark model

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systematically. For $bb\bar{c}\bar{c}$ state, the possible quantum numbers are $I(J^P) = 0(0^+), 0(1^+)$, and $0(2^+)$. For $bc\bar{b}\bar{c}$ state, it should have definite C -parity, and the allowed quantum numbers are $I(J^{PC}) = 0(0^{++}), 0(1^{+-}), 0(1^{++})$, and $0(2^{++})$. For the interaction between the heavy quarks, the short-distance one-gluon-exchange effects play an important role now. In the calculations, the meson-meson $[\bar{Q}Q]$ $[\bar{Q}Q]$ and diquark-antidiquark $[QQ]$ $[\bar{Q}\bar{Q}]$ structures, and the mixing of them are considered, respectively, along with all possible color, spin configurations. To distinguish genuine resonances, we employ the Gaussian expansion method [29] supplemented by the real scaling method (stabilization) [30,31]. The real scaling method was often used for analyzing electron-atom and electron-molecule scattering [32] and was applied in the quark model calculation recently [24,30,31].

The paper is organized as follows. In Sec. II, the chiral quark model and the wave functions of the four-body system will be introduced briefly. In Sec. III, the numerical results and discussion are presented. A short summary is given in Sec. IV.

II. QUARK MODEL AND WAVE FUNCTIONS

The chiral quark model has been successful both in describing the hadron spectra and hadron-hadron interactions. The details of the model can be found in Ref. [33,34]. For $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ full-heavy system, the Hamiltonian of the chiral quark model consists of three parts: quark rest mass, kinetic energy, and potential energy.

$$H = \sum_{i=1}^4 m_i + \frac{p_{12}^2}{2\mu_{12}} + \frac{p_{34}^2}{2\mu_{34}} + \frac{p_{1234}^2}{2\mu_{1234}} + \sum_{i<j=1}^4 (V_{ij}^C + V_{ij}^G). \quad (1)$$

The potential energy consists of pieces describing quark confinement (C), one-gluon exchange (G). The detailed forms of potentials are shown below (only central parts are presented) [33].

$$V_{ij}^C = (-a_c r_{ij}^2 - \Delta) \lambda_i^c \cdot \lambda_j^c, \quad (2a)$$

$$V_{ij}^G = \frac{\alpha_s}{4} \lambda_i^c \cdot \lambda_j^c \left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j} \sigma_i \cdot \sigma_j \delta(r_{ij}) \right], \quad (2b)$$

$$\delta(r_{ij}) = \frac{e^{-r_{ij}/r_0(\mu_{ij})}}{4\pi r_{ij} r_0^2(\mu_{ij})}. \quad (2c)$$

m_i is the constituent mass of quark/antiquark, and μ_{ij} is the reduced mass of two interacting quarks and

TABLE I. Model parameters, determined by fitting the meson spectrum from light to heavy.

Quark masses (MeV)	$m_u = m_d$	313
	m_s	536
	m_c	1728
	m_b	5112
Confinement	a_c (MeV fm ⁻²)	101
	Δ (MeV)	-78.3
OGE	α_0	3.67
	Λ_0 (fm ⁻¹)	0.033
	μ_0 (MeV)	36.98
	s_0 (MeV)	28.17

$$\mu_{1234} = \frac{(m_1 + m_2)(m_3 + m_4)}{m_1 + m_2 + m_3 + m_4}, \quad (3)$$

$\mathbf{p}_{ij} = (\mathbf{p}_i - \mathbf{p}_j)/2$, $\mathbf{p}_{1234} = (\mathbf{p}_{12} - \mathbf{p}_{34})/2$; $r_0(\mu_{ij}) = s_0/\mu_{ij}$; σ are the $SU(2)$ Pauli matrices; λ, λ^c are $SU(3)$ flavor, color Gell-Mann matrices, respectively; and α_s is an effective scale-dependent running coupling [34],

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln[(\mu_{ij}^2 + \mu_0^2)/\Lambda_0^2]}. \quad (4)$$

All the parameters are determined by fitting the meson spectrum, from light to heavy, and the resulting values are listed in Table I. Table II gives the masses of some heavy mesons in the chiral quark model.

The wave functions of four-quark states for the two structures, diquark-antidiquark and meson-meson, can be constructed in two steps. For each degree of freedom, first we construct the wave functions for two-body subclusters, then couple the wave functions of two subclusters to obtain the wave functions of four-quark states.

(i) Diquark-antidiquark structure.

For the spin part, the wave functions for two-body subclusters are

$$\begin{aligned} \chi_{11} &= \alpha\alpha, & \chi_{10} &= \frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha), & \chi_{1-1} &= \beta\beta, \\ \chi_{00} &= \frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha), \end{aligned} \quad (5)$$

TABLE II. The masses of some heavy mesons (in units of MeV). M_{cal} and M_{exp} represent the theoretical and the experimental masses, respectively.

Meson	η_c	J/ψ	η_b	Υ	B_c	B_c^*
M_{cal}	2986.3	3096.4	9334.7	9463.9	6341.8	6395.1
M_{exp}	2983.6	3096.9	9399.1	9460.3	6275.6	...

then the wave functions for four-quark states are obtained,

$$\chi_{00}^{\sigma_1} = \chi_{00}\chi_{00}, \quad (6a)$$

$$\chi_{00}^{\sigma_2} = \sqrt{\frac{1}{3}}(\chi_{11}\chi_{1-1} - \chi_{10}\chi_{10} + \chi_{1-1}\chi_{11}), \quad (6b)$$

$$\chi_{11}^{\sigma_3} = \chi_{00}\chi_{11}, \quad (6c)$$

$$\chi_{11}^{\sigma_4} = \chi_{11}\chi_{00}, \quad (6d)$$

$$\chi_{11}^{\sigma_5} = \frac{1}{\sqrt{2}}(\chi_{11}\chi_{10} - \chi_{10}\chi_{11}), \quad (6e)$$

$$\chi_{22}^{\sigma_6} = \chi_{11}\chi_{11}, \quad (6f)$$

where the superscript $\sigma i (i = 1 \sim 6)$ of χ represents the index of the spin wave functions of four-quark states. The subscripts of χ are SM_S , the total spin and the third projection of total spin of the system. $S = 0, 1, 2$, and only one component ($M_S = S$) is shown for a given total spin S .

For the flavor part, the configurations of $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ states are demonstrated in Fig. 1 in diquark-antidiquark structure, and the wave functions for $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ systems take

$$\chi_{d0}^{f1} = (bb)(\bar{c}\bar{c}), \quad (7)$$

$$\chi_{d0}^{f2} = (bc)(\bar{b}\bar{c}), \quad (8)$$

respectively. The subscript $d0$ of χ represents the diquark-antidiquark structure and isospin ($I = 0$).

FIG. 1. Configurations of the $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ tetraquark states in pure diquark-antidiquark structure. (a) represents the configuration of $bb\bar{c}\bar{c}$ state, $[bb][\bar{c}\bar{c}]$ and (b) represents the configuration of $bc\bar{b}\bar{c}$ state, $[bc][\bar{b}\bar{c}]$.

For the color part, the wave functions of four-quark states must be color singlet [222] and it is obtained as below

$$\chi_d^{c1} = \frac{\sqrt{3}}{6}(rg\bar{r}\bar{g} - rg\bar{g}\bar{r} + gr\bar{g}\bar{r} - gr\bar{r}\bar{g} + rb\bar{r}\bar{b} - rb\bar{b}\bar{r} + br\bar{b}\bar{r} - br\bar{r}\bar{b} + gb\bar{g}\bar{b} - gb\bar{b}\bar{g} + bg\bar{b}\bar{g} - bg\bar{g}\bar{b}). \quad (9a)$$

$$\chi_d^{c2} = \frac{\sqrt{6}}{12}(2rr\bar{r}\bar{r} + 2gg\bar{g}\bar{g} + 2bb\bar{b}\bar{b} + rg\bar{r}\bar{g} + rg\bar{g}\bar{r} + gr\bar{g}\bar{r} + gr\bar{r}\bar{g} + rb\bar{r}\bar{b} + rb\bar{b}\bar{r} + br\bar{b}\bar{r} + br\bar{r}\bar{b} + gb\bar{g}\bar{b} + gb\bar{b}\bar{g} + bg\bar{b}\bar{g} + bg\bar{g}\bar{b}), \quad (9b)$$

where χ_d^{c1} and χ_d^{c2} represent the color antitriplet-triplet ($\bar{3} \times 3$) and sextet-antisextet ($6 \times \bar{6}$) coupling, respectively. The detailed coupling process for the color wave functions can refer to our previous work [35].

(ii) Meson-meson structure.

For the spin part, the wave functions are the same as those of the diquark-antidiquark structure, Eq. (6).

For the flavor part, there are three wave functions, one function for $bb\bar{c}\bar{c}$ system,

$$\chi_{m0}^{f1} = (\bar{c}b)(\bar{c}b), \quad (10)$$

and two functions for $bc\bar{b}\bar{c}$ system,

$$\chi_{m0}^{f2} = (\bar{b}b)(\bar{c}c), \quad (11)$$

$$\chi_{m0}^{f3} = (\bar{c}b)(\bar{b}c). \quad (12)$$

The subscript $m0$ of χ represents the meson-meson structure and isospin equals zero. Figure 2 shows the meson-meson structure of $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ systems.

For the color part, the wave functions of four-quark states in the meson-meson structure are

FIG. 2. Three configurations of the $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ tetraquark states in meson-meson structure. (a) The only configuration of $bb\bar{c}\bar{c}$ system $(\bar{c}b)(\bar{c}b)$; (b) and (c) two configurations of $bc\bar{b}\bar{c}$ system, $(\bar{b}b)(\bar{c}c)$ and $(\bar{c}b)(\bar{b}c)$, respectively.

$$\chi_m^{c1} = \frac{1}{3}(\bar{r}r + \bar{g}g + \bar{b}b)(\bar{r}r + \bar{g}g + \bar{b}b), \quad (13a)$$

$$\begin{aligned} \chi_m^{c2} = & \frac{\sqrt{2}}{12}(3\bar{b}r\bar{r}b + 3\bar{g}r\bar{r}g + 3\bar{b}g\bar{g}b + 3\bar{g}b\bar{b}g + 3\bar{r}g\bar{g}r \\ & + 3\bar{r}b\bar{b}r + 2\bar{r}r\bar{r}r + 2\bar{g}g\bar{g}g + 2\bar{b}b\bar{b}b - \bar{r}r\bar{g}g \\ & - \bar{g}g\bar{r}r - \bar{b}b\bar{g}g - \bar{b}b\bar{r}r - \bar{g}g\bar{b}b - \bar{r}r\bar{b}b), \end{aligned} \quad (13b)$$

where χ_m^{c1} and χ_m^{c2} represent the color singlet-singlet (1×1) and color octet-octet (8×8) coupling, respectively. The details refer to our previous work [35].

As for the orbital wave functions, they can be constructed by coupling the orbital wave function for each relative motion of the system,

$$\Psi_L^{M_L} = [[\Psi_{l_1}(\mathbf{r}_{12})\Psi_{l_2}(\mathbf{r}_{34})]_{l_{12}}\Psi_{L_r}(\mathbf{r}_{1234})]_L^{M_L}, \quad (14)$$

where l_1 and l_2 are the angular momentum of two subclusters, respectively. $\Psi_{L_r}(\mathbf{r}_{1234})$ is the wave function of the relative motion between two subclusters with orbital angular momentum L_r . L is the total orbital angular momentum of four-quark states. Here for the low-lying $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ state, all angular momentum (l_1, l_2, L_r, L) are taken as zero. The used Jacobi coordinates are defined as

$$\begin{aligned} \mathbf{r}_{12} &= \mathbf{r}_1 - \mathbf{r}_2, \\ \mathbf{r}_{34} &= \mathbf{r}_3 - \mathbf{r}_4, \\ \mathbf{r}_{1234} &= \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3 + m_4}. \end{aligned} \quad (15)$$

For diquark-antidiquark structure, the quarks are numbered as 1,2, and the antiquarks are numbered as 3,4; for meson-meson structure, the antiquark and quark in one cluster are marked as 1,2, the other antiquark and quark are marked as 3,4. In the two structure coupling calculation, the indices of quarks, antiquarks in diquark-antidiquark structure will be changed to be consistent with the numbering scheme in meson-meson structure. In Gaussian expansion method (GEM), the spatial wave function is expanded by Gaussians [29],

$$\Psi_l^m(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n \Psi_{nlm}^G(\mathbf{r}), \quad (16a)$$

$$\Psi_{nlm}^G(\mathbf{r}) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}), \quad (16b)$$

where N_{nl} are normalization constants,

$$N_{nl} = \left[\frac{2^{l+2} (2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi} (2l+1)} \right]^{\frac{1}{2}}. \quad (17)$$

c_n are the variational parameters, which are determined dynamically. The Gaussian size parameters are chosen according to the following geometric progression:

$$\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_{n_{\max}}}{r_1} \right)^{\frac{1}{n_{\max}-1}}. \quad (18)$$

This procedure enables optimization of the expansion using just a small numbers of Gaussians. Finally, the complete channel wave function for the four-quark system for diquark-antidiquark structure is written as

$$\begin{aligned} \Psi_{IJ,i,j,k}^{M_I M_J} &= \mathcal{A}_1 [\Psi_L^{M_L} \chi_{SM_S}^{\sigma i}]_J^{M_J} \chi_{d0}^{fj} \chi_d^{ck}, \\ (i &= 1 \sim 6; j = 1, 2; k = 1, 2; S = 0, 1, 2), \end{aligned} \quad (19)$$

where $\mathcal{A}_1$ is the antisymmetrization operator, for $bb\bar{c}\bar{c}$ system,

$$\mathcal{A}_1 = \frac{1}{2}(1 - P_{12} - P_{34} + P_{12}P_{34}). \quad (20)$$

For meson-meson structure, the complete wave function is written as

$$\begin{aligned} \Psi_{IJ,i,j,k}^{M_I M_J} &= \mathcal{A}_2 [\Psi_L^{M_L} \chi_{SM_S}^{\sigma i}]_J^{M_J} \chi_{m0}^{fj} \chi_m^{ck}, \\ (i &= 1 \sim 6; j = 1, 2, 3; k = 1, 2; S = 0, 1, 2), \end{aligned} \quad (21)$$

where $\mathcal{A}_2$ is the antisymmetrization operator, for $bb\bar{c}\bar{c}$ system,

$$\mathcal{A}_2 = \frac{1}{2}(1 - P_{13} - P_{24} + P_{13}P_{24}). \quad (22)$$

Last, the eigenenergies of the four-quark system are obtained by solving a Schrödinger equation

$$H\Psi_{IJ}^{M_I M_J} = E^{IJ}\Psi_{IJ}^{M_I M_J}, \quad (23)$$

where $\Psi_{IJ}^{M_I M_J}$ is the wave function of the four-quark states, which is the linear combinations of the above channel wave functions, Eq. (19) in the diquark-anti-diquark structure or Eq. (21) in the meson-meson structure, or both wave functions of Eqs. (19) and (21), respectively.

III. RESULTS AND DISCUSSIONS

In the present work, we calculated the mass spectra of the $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ systems with allowed quantum numbers in the nonrelativistic quark model. Two structures of four-quark states, meson-meson and diquark-antidiquark, and the mixing of them are investigated, respectively. All possible color and spin configurations are also considered. For example, for meson-meson structure, two color configurations, color singlet-singlet (1×1), and octet-octet (8×8) are employed; for diquark-antidiquark structure, color antitriplet-triplet ($\bar{3} \times 3$) and sextet-antisextet ($6 \times \bar{6}$) are taken into account. For $bb\bar{c}\bar{c}$ state, the wave functions need to be antisymmetrized. All the allowed channels are demonstrated in Table III. For $bc\bar{b}\bar{c}$, there is no need to consider the antisymmetrization because of no identical quarks. Because the hamiltonian of the system is invariant

TABLE III. The allowed channels of $bb\bar{c}\bar{c}$ and $bcb\bar{c}$ systems in meson-meson (M-M) and diquark-antidiquark (D-A) structures. $\chi_m^{\sigma 34f3c1,2\mp} = \sqrt{\frac{1}{2}}(\chi_1^{\sigma 3}\chi_{m0}^f\chi_m^{c1,2} \pm \chi_1^{\sigma 4}\chi_{m0}^f\chi_m^{c1,2,C})$, $\chi_d^{\sigma 34f2c1,2\mp} = \sqrt{\frac{1}{2}}(\chi_1^{\sigma 3}\chi_{d0}^f\chi_d^{c1,2} \pm \chi_1^{\sigma 4}\chi_{d0}^f\chi_d^{c1,2,C})$. The wave functions with superscript ‘‘C,’’ for example, $\chi_{m0}^{f3,C}$, are the charge conjugate of the corresponding wave function without superscript ‘‘C’’.

$bb\bar{c}\bar{c}$				
$I(J^P)$	$0(0^+)$	$0(1^+)$	$0(2^+)$	
M-M	$\chi_0^{\sigma 1,2}\chi_{m0}^f\chi_m^{c1,2}$	$\chi_1^{\sigma 3,4}\chi_{m0}^f\chi_m^{c1,2}$	$\chi_2^{\sigma 6}\chi_{m0}^f\chi_m^{c1,2}$	
D-A	$\chi_0^{\sigma 1}\chi_{d0}^f\chi_d^{c2}$ $\chi_0^{\sigma 2}\chi_{d0}^f\chi_d^{c1}$	$\chi_1^{\sigma 5}\chi_{d0}^f\chi_d^{c1}$	$\chi_2^{\sigma 6}\chi_{d0}^f\chi_d^{c1}$	

$bcb\bar{c}$				
$I(J^{PC})$	$0(0^{++})$	$0(1^{+-})$	$0(1^{++})$	$0(2^{++})$
M-M	$\chi_0^{\sigma 1,2}\chi_{m0}^{f2,3}\chi_m^{c1,2}$	$\chi_1^{\sigma 3,4}\chi_{m0}^f\chi_m^{c1,2}$ $\chi_m^{\sigma 34f3c1,2-}$ $\chi_m^{\sigma 5}\chi_{m0}^f\chi_m^{c1,2}$	$\chi_1^{\sigma 5}\chi_{m0}^f\chi_m^{c1,2}$ $\chi_m^{\sigma 34f3c1,2+}$	$\chi_2^{\sigma 6}\chi_{m0}^{f2,3}\chi_m^{c1,2}$
D-A	$\chi_0^{\sigma 1,2}\chi_{d0}^f\chi_d^{c1,2}$	$\chi_d^{\sigma 34f2c1,2-}$ $\chi_1^{\sigma 5}\chi_{d0}^f\chi_d^{c1,2}$	$\chi_d^{\sigma 34f2c1,2+}$	$\chi_2^{\sigma 6}\chi_{d0}^f\chi_d^{c1,2}$

under the charge conjugate, the C -parity is a good quantum number and is shown in the table for $bcb\bar{c}$ system. All possible channels are also showed in Table III.

For full-heavy flavor system, the large masses of b and c quark prevent the appearance of the Goldstone boson exchanges, only gluon exchanges are included. It is helpful to understand the numerical results by analyzing qualitatively the properties of the interactions between quarks in the system. Table IV gives the matrix elements of color operators and spin operators (only the results for total spin $S=0$ are given here). With the help of these matrix elements, we can estimate roughly the binding energy of the system. For single meson, the matrix element of color operator is

TABLE IV. The matrix elements of color and spin operators. $O_{ij} = \lambda_i \cdot \lambda_j$ for color and $O_{ij} = \sigma_i \cdot \sigma_j$ for spin.

	Color				Spin			
	D-A		M-M		D-A		M-M	
	$\bar{3} \otimes 3$	$6 \otimes \bar{6}$	$1 \otimes 1$	$8 \otimes 8$	$0 \otimes 0$	$1 \otimes 1$	$0 \otimes 0$	$1 \otimes 1$
$\langle O_{12} \rangle$	$-\frac{4}{3}$	$-\frac{10}{3}$	$-\frac{16}{3}$	$\frac{2}{3}$	0	-2	-3	1
$\langle O_{13} \rangle$	$-\frac{8}{3}$	$\frac{4}{3}$	0	$-\frac{4}{3}$	-3	1	0	-2
$\langle O_{14} \rangle$	$-\frac{4}{3}$	$-\frac{10}{3}$	0	$-\frac{14}{3}$	0	-2	0	-2
$\langle O_{23} \rangle$	$-\frac{4}{3}$	$-\frac{10}{3}$	0	$-\frac{14}{3}$	0	-2	0	-2
$\langle O_{24} \rangle$	$-\frac{8}{3}$	$\frac{4}{3}$	0	$-\frac{4}{3}$	-3	1	0	-2
$\langle O_{34} \rangle$	$-\frac{4}{3}$	$-\frac{10}{3}$	$-\frac{16}{3}$	$\frac{2}{3}$	0	-2	-3	1

$$\langle \lambda_1 \cdot \lambda_2 \rangle = -16/3. \quad (24)$$

For tetraquark states, the matrix element of color operator is

$$\sum_{i>j=1}^4 \langle \lambda_i \cdot \lambda_j \rangle = -32/3, \quad \text{for D-A and M-M.} \quad (25)$$

So the color matrix elements are exactly same for the tetraquark system and two-meson pairs, and the pure color interaction cannot contribute to the binding energy of the tetraquark system.

For CMI, all the matrix elements are given in Table V. From the table, we can see that the difference of the CMI matrix elements between tetraquark system and two-meson pairs is not smaller than 0, so CMI cannot lead to deep bound state. It is worth to note that Δ_{CMI} for color-spin configuration $\bar{3} \otimes 3$, $0 \otimes 0$, and $8 \otimes 8$, $1 \otimes 1$ may be negative if x is large enough, which means that the bound states are more possible in $QQ\bar{q}\bar{q}$ systems. Some previous work, e.g., Ref. [36], obtained several bound states in these systems.

The numerical results of $bb\bar{c}\bar{c}$ and $bcb\bar{c}$ systems are shown in Tables VI and VII, respectively. E_{cc} represents the ground state energy for each state after considering the all possible color and spin channels (refer to Table III). For $bcb\bar{c}$ system, the states with different C -parity are separated.

From Table VI, we found that the lowest energies of 0^+ , 1^+ , and 2^+ in the meson-meson structure are a little higher than the relevant thresholds. In the diquark-antidiquark structure, the energies are all much larger than those in the meson-meson structure. The effects of the two-structure mixing seem to be tiny. So we cannot find the bound states of $bb\bar{c}\bar{c}$ tetraquark in the present calculation. For $bcb\bar{c}$ system, with the lower threshold $b\bar{b} + c\bar{c}$ compared with $b\bar{c} + b\bar{c}$, it may be much harder to form a bound state. In Table VII, the lowest energies of the three structures of $bcb\bar{c}$ system are all larger than the corresponding thresholds. Situations are not changed in considering the mixing of the three quark structures. No bound states are found and recent study by Liu *et al.* [27] also draws the same conclusion with ours.

Because the colorful clusters cannot fall apart, there may be a resonance even with the higher eigenenergy. To find the genuine resonances, the dedicated real scaling (stabilization) method is employed. To realize the real scaling method in our calculation, the Gaussian size parameters r_n in Eq. (18) are multiplied by a factor α , αr_n just for the meson-meson structure with color singlet-singlet configuration. α takes the values between 0.9 and 1.6. With the increasing of α , all states will fall off toward its thresholds, but a compact resonance should be stable because it will not be affected by the boundary at a large distance. We

TABLE V. The matrix elements of CMI in the unit of $1/m_b^2$, $x = m_b/m_c$. Δ_{CMI} is the difference of the matrix elements between tetraquark system and two-meson pairs. The expressions with underline are the thresholds of the corresponding systems.

Color	$\bar{3} \otimes 3$		$6 \otimes \bar{6}$		$1 \otimes 1$		$8 \otimes 8$	
Spin	$0 \otimes 0$	$1 \otimes 1$	$0 \otimes 0$	$1 \otimes 1$	$0 \otimes 0$	$1 \otimes 1$	$0 \otimes 0$	$1 \otimes 1$
$bb\bar{c}\bar{c}$	$-8 - 8x^2$	$-\frac{32x}{3} + \frac{8}{3} + \frac{8x^2}{3}$	$4 + 4x^2$	$-\frac{80x}{3} + \frac{4}{3} + \frac{4x^2}{3}$	$-32x$	$\frac{32x}{3}$	$4x$	$-\frac{60x}{3} - \frac{8}{3} - \frac{8x^2}{3}$
$b\bar{c} + b\bar{c}$					$-\underline{32x}$	$\underline{\frac{32x}{3}}$		
Δ_{CMI}	$16x - 8(x-1)^2$	$\frac{64x}{3} + \frac{8}{3} + \frac{8x^2}{3}$	$4 + 4x^2 + 32x$	$\frac{16x}{3} + \frac{4}{3} + \frac{4x^2}{3}$	0	$\frac{128x}{3}$	$36x$	$\frac{20x}{3} - \frac{8}{3}(x-1)^2$
$bc\bar{b}\bar{c}$	$-16x$	$-\frac{8}{3} - \frac{8x^2}{3}$	$4x$	$-\frac{32x}{3} - \frac{20}{3} - \frac{20x^2}{3}$	$-16 - 16x^2$	$\frac{16}{3} + \frac{16x^2}{3}$	$2 + 2x^2$	$-\frac{72x}{3} - \frac{2}{3} - \frac{2x^2}{3}$
$b\bar{c} + c\bar{b}$					$-\underline{32x}$	$\underline{\frac{32x}{3}}$		
$b\bar{b} + c\bar{c}$					$-\underline{16 - 16x^2}$	$\underline{\frac{16}{3} + \frac{16x^2}{3}}$		
Δ_{CMI}	$16 + 16x^2 - 16x$	$\frac{40}{3} + \frac{40x^2}{3}$	$16 + 16x^2 + 4x$	$\frac{28}{3} + \frac{28x^2}{3} - \frac{32x}{3}$	0	$\frac{64}{3} + \frac{64x^2}{3}$	$18 + 18x^2$	$\frac{46}{3} + \frac{46x^2}{3} - \frac{72x}{3}$

TABLE VI. The results of $bb\bar{c}\bar{c}$ state in pure meson-meson structure, diquark-antidiquark structure, and in considering the mixing of two structures, respectively. “ $E_{\text{th}}^{\text{theo}}$ ” represents the theoretical thresholds. The masses are all in units of MeV.

Structure	J^P	E_{cc}	Thresholds	$E_{\text{th}}^{\text{theo}}$
$[\bar{c}b][\bar{c}b]$	0^+	12683.9	$2B_c^-$	12683.6
	1^+	12737.4	$B_c^- B_c^{*-}$	12736.9
	2^+	12790.7	$2B_c^{*-}$	12790.2
$[bb][\bar{c}\bar{c}]$	0^+	12891.5	$2B_c^-$	12683.6
	1^+	12897.6	$B_c^- B_c^{*-}$	12736.9
	2^+	12904.5	$2B_c^{*-}$	12790.2
$[\bar{c}b][\bar{c}b] \otimes [bb][\bar{c}\bar{c}]$	0^+	12683.9	$2B_c^-$	12683.6
	1^+	12737.4	$B_c^- B_c^{*-}$	12736.9
	2^+	12790.7	$2B_c^{*-}$	12790.2

TABLE VII. The results of $bc\bar{b}\bar{c}$ state in pure meson-meson structure, diquark-antidiquark structure, and in considering the mixing of two structures, respectively. “ $E_{\text{th}}^{\text{theo}}$ ” represents the theoretical thresholds. The masses are all in units of MeV.

Structure	J^{PC}	E_{cc}	Thresholds	$E_{\text{th}}^{\text{theo}}$
$[\bar{b}b][\bar{c}c]$	0^{++}	12321.5	$\eta_b \eta_c$	12321.0
	1^{++}	12561.0	$\Upsilon J/\psi$	12560.3
	1^{+-}	12431.6	$\eta_b J/\psi$	12431.1
	2^{++}	12560.9	$\Upsilon J/\psi$	12560.3
$[\bar{c}b][\bar{b}c]$	0^{++}	12684.0	$B_c^+ B_c^-$	12683.6
	1^{++}	12737.3	$B_c^+ B_c^{*-}$	12736.9
	1^{+-}	12737.3	$B_c^+ B_c^{*-}$	12736.9
	2^{++}	12790.6	$B_c^{*+} B_c^{*-}$	12790.2
$[bc][\bar{b}\bar{c}]$	0^{++}	12746.2	$\eta_b \eta_c$	12321.0
	1^{++}	12804.2	$\Upsilon J/\psi$	12560.3
	1^{+-}	12776.4	$\eta_b J/\psi$	12431.1
	2^{++}	12809.3	$\Upsilon J/\psi$	12560.3
$[\bar{b}b][\bar{c}c] \otimes [\bar{c}b][\bar{b}c] \otimes [bc][\bar{b}\bar{c}]$	0^{++}	12321.5	$\eta_b \eta_c$	12321.0
	1^{++}	12561.0	$\Upsilon J/\psi$	12560.3
	1^{+-}	12431.6	$\eta_b J/\psi$	12431.1
	2^{++}	12561.0	$\Upsilon J/\psi$	12560.3

FIG. 3. The stabilization plots of the energies of $bb\bar{c}\bar{c}$ for $I(J^P) = 0(0^+)$ and $bc\bar{b}\bar{c}$ state for $I(J^{PC}) = 0(0^{++})$ with respect to the scaling factor α .

FIG. 4. The stabilization plots of the energies of $bb\bar{c}\bar{c}$ for $I(J^P) = 0(2^+)$ and $bc\bar{b}\bar{c}$ state for $I(J^{PC}) = 0(2^{++})$ with respect to the scaling factor α .

illustrate the results for $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ states with all possible quantum numbers (Figs. 3–6).

Figure 3(a) represents the $bb\bar{c}\bar{c}$ state for $0(0^+)$. The first two horizontal lines represent the thresholds of

$B_c^- + B_c^-(0 \otimes 0 \rightarrow 0)$ and $B_c^{*-} + B_c^{*-}(1 \otimes 1 \rightarrow 0)$ for $bb\bar{c}\bar{c}$ state. The horizontal line around 13 140 MeV is on behalf of a genuine resonance state, and its energy is stable against the variation of range. For $bc\bar{b}\bar{c}$ state with

FIG. 5. The stabilization plots of the energies of $bc\bar{b}\bar{c}$ state for $I(J^{PC}) = 0(1^{+-}), 0(1^{++})$ with respect to the scaling factor α .

FIG. 6. The stabilization plots of the energies of $bb\bar{c}\bar{c}$ state for $I(J^P) = 0(1^+)$ with respect to the scaling factor α .

quantum number $0(0^{++})$ in (b), four thresholds $\eta_b + \eta_c$, $\Upsilon + J/\psi$, $B_c^+ + B_c^-$, and $B_c^{*+} + B_c^{*-}$ are clearly showed, and the first resonance state with energy about 12 860 MeV stays stable.

From Fig. 4, we can see that the energy of the lowest resonance is about 13 230 and 13 020 MeV for $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ states, respectively.

For $bc\bar{b}\bar{c}$ states with $0(1^{++})$ and $0(1^{+-})$ in Fig. 5, the lowest possible resonance is at 12 910 MeV for C -parity negative and 12 780 MeV for C -parity positive. For $bb\bar{c}\bar{c}$ states with $I(J^P) = 0(1^+)$ in Fig. 6, two possible resonances stay very close to each other, with the stable energies 13 180 and 13 200 MeV, respectively.

From our calculation, we can see that there may be more resonance states with the higher energies, and these states may be too wide to be observed or too hard to be produced. We are interested in the genuine resonance state with as low

as possible energy, so we only give the lowest resonances for each quantum number set. We hope this information will be helpful for searching for the $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ states in experiment.

IV. SUMMARY

In the framework of the chiral quark model, we do a systematical calculation for the mass spectra of $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ systems with allowed quantum numbers using the Gaussian expansion method. The meson-meson structure, the diquark-antidiquark structure, and the mixing of them are investigated severally. In our calculation, all these states are found to have masses above the corresponding two meson decay thresholds, leaving no space for a bound state. These results are consistent with our qualitative analysis of properties of the interactions between quarks. With the help of the real scaling method, we try to look for the possible resonances in $bb\bar{c}\bar{c}$ and $bc\bar{b}\bar{c}$ systems. For $bb\bar{c}\bar{c}$ system, the energies of the possible resonances are 13 140, 13 180, and 13 230 MeV for $0(0^+)$, $0(1^+)$, and $0(2^+)$ states, respectively. For $bc\bar{b}\bar{c}$ system, the resonance energies are little lower than $bb\bar{c}\bar{c}$ state, which takes 12 860, 12 780, and 13 020 MeV for $0(0^{++})$, $0(1^{++})$, and $0(2^{++})$ states, and 12 910 MeV for $0(1^{+-})$, respectively. Hopefully, this information about the exotic tetraquark states composed of four heavy quarks may be useful for the search in experiments in the future.

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