Constraints on sterile neutrinos in the MeV to GeV mass range

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A detailed discussion is given of the analysis of recent data to obtain improved upper bounds on the couplings $|U_{e4}|^2$ and $|U_{\mu 4}|^2$ for a mainly sterile neutrino mass eigenstate ν_4. Using the excellent agreement among the $\mathcal{F}t$ values (products of kinematic rate factors times half-lives with radiative corrections included) for superallowed nuclear beta decays, an improved upper limit is derived for emission of a ν_4. The agreement of the ratios of branching ratios $R^{(\pi)}_{e/\mu} = \mathrm{BR}(\pi^+ \to e^+\nu_e)/\mathrm{BR}(\pi^+ \to \mu^+\nu_\mu)$, $R^{(K)}_{e/\mu}$, $R^{(D_s)}_{e/\tau}$, $R^{(D_s)}_{\mu/\tau}$, and $R^{(D)}_{e/\tau}$, and the branching ratios $\mathrm{BR}(B^+ \to e^+\nu_e)$ and $\mathrm{BR}(B^+ \to \mu^+\nu_\mu)$ decays with predictions of the Standard Model is utilized to derive new constraints on ν_4 emission covering the ν_4 mass range from MeV to GeV. We also discuss constraints from peak search experiments probing for emission of a ν_4 via lepton mixing, as well as constraints from pion beta decay, CKM unitarity, μ decay, leptonic τ decay, and other experimental inputs.

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I. INTRODUCTION

In a recent paper [1], we presented improved upper bounds on the coupling $|U_{e4}|^2$ of an electron to a sterile neutrino ν_4 from analyses of data on nuclear and particle decays, for ν_4 masses in the MeV to GeV range, and pointed out new experiments that could improve these constraints. Here we give the details of our analysis that yielded these constraints and also present a number of additional bounds on sterile neutrino mixings, in particular, on the coupling $|U_{\mu 4}|^2$.

Neutrino oscillations and hence neutrino masses and lepton mixing have been established and are of great importance as physics beyond the original Standard Model (SM) [2–11]. Most of the data from experiments with solar, atmospheric, accelerator, and reactor (anti) neutrinos can be explained within the minimal framework of three neutrino mass eigenstates with values of $\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$ given approximately by $\Delta m^2_{21} = 0.74 \times 10^{-4}\ \mathrm{eV}^2$ and $|\Delta m^2_{32}| = 2.5 \times 10^{-3}\ \mathrm{eV}^2$, with normal mass ordering $m_{\nu_3} > m_{\nu_2}$ favored; furthermore, the lepton mixing angles θ_{23}, θ_{12}, and θ_{13} have been measured, with a tentative indication of a nonzero value of the CP-violating quantity $\sin(\delta_{CP})$ (for compilations and fits, see [12–18]).

The possible existence of light sterile neutrinos, in addition to the three known neutrino mass eigenstates, is a fundamental question in particle physics. These would have to be primarily electroweak-singlets (sterile), since the invisible width of the Z boson is consistent with being due to decays to $\bar{\nu}_\ell \nu_\ell$, where $\nu_\ell = \nu_e, \nu_\mu$, and ν_τ, corresponding to the known three SM fermion families [19]. In the presence of sterile neutrinos, the neutrino interaction eigenstates ν_e, ν_μ, and ν_τ are linear combinations that include these additional mass eigenstates. In a basis in which the charged leptons are simultaneously flavor and mass eigenstates, the charged weak current has the form $J_\lambda = \bar{\ell}_L \gamma_\lambda \nu_{\ell,L}$, where $\ell = e$, μ, τ and

$$\nu_\ell = \sum_{i=1}^{3+n_s} U_{\ell i} \nu_i, \tag{1.1}$$

where n_s denotes the number of additional mass eigenstates. The near sterility of the ν_i with $4 \le i \le n_s$ is reflected in small upper bounds on the corresponding $|U_{\ell i}|$. We will use the term "sterile neutrino" both in its precise sense as an electroweak-singlet interaction eigenstate and in a commonly used approximate sense as the corresponding, mainly sterile, mass eigenstate(s) in this neutrino interaction eigenstate. For technical simplicity, we will assume one heavy neutrino, $n_s = 1$, with $i = 4$; it is straightforward to generalize to $n_s \geq 2$. Since a ν_4 in the mass range of interest here decays on a time scale much shorter than the age of the universe, it is not excluded by the cosmological upper limit on the sum of stable neutrinos, $\sum_i m_{\nu_i} \lesssim 0.12$ eV [20].

Possible sterile neutrinos are subject to many constraints from neutrino oscillation experiments using solar and atmospheric neutrinos, accelerator and reactor (anti)neutrinos, and kinematic effects in particle and nuclear decays, as well as cosmological constraints. Bounds from the nonobservation of neutrinoless double beta decay are satisfied by assuming that ν_4 is a Dirac, rather than Majorana, neutrino. Although Majorana neutrino masses have often been regarded as more generic, many ultraviolet extensions of the SM contain additional gauge symmetries that forbid Majorana mass terms, so that in these models, neutrinos are Dirac fermions [21]. Much attention has been focused on possible sterile neutrinos with masses in the eV region because of results from the LSND [22] and Miniboone [23] experiments and possible anomalies in reactor antineutrino experiments (recent reviews and discussions include [24–26]). In addition to eV-scale sterile neutrinos, there has also been interest in possible keV-scale sterile neutrinos as warm dark matter, and in even heavier sterile neutrinos with masses extending to the GeV range, and cosmological constraints on these have been discussed [27–34]. These cosmological constraints involve assumptions about properties of the early universe. One valuable aspect of laboratory bounds on heavy neutrinos is that they are free of such assumptions about the early universe.

Since sterile neutrinos violate the conditions for the diagonality of the weak neutral current [35,36], ν_4 has invisible tree-level decays of the form $\nu_4 \to \nu_j \bar{\nu}_i \nu_i$ where $1 \leq i, j \leq 3$ with model-dependent branching ratios. Because our bounds are purely kinematic, they are complementary to bounds from searches for neutrino decays, which involve model-dependent assumptions on branching ratios into visible versus invisible final states.

This paper is organized as follows. In Sec. II we derive upper bounds on $|U_{e4}|^2$ from nuclear beta decay data. Section III discusses pion beta decay. Section IV considers connections of nuclear decay data with the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. In Sec. V we discuss peak search experiments. In Secs. VI and VII we derive upper bounds on lepton mixing matrix coefficients from two-body leptonic decays of π^+, K^+, D^+, D_s, and B^+ mesons. Sections VIII and IX are devoted to constraints from μ decay and leptonic τ decays. In Sec. X we briefly discuss other constraints on sterile neutrinos. Section XI contains our conclusions.

II. LIMIT ON EMISSION OF MASSIVE NEUTRINOS IN NUCLEAR BETA DECAY

The emission of a heavy neutrino ν_j via lepton mixing and the associated nonzero $|U_{ej}|^2$, with a mass in the keV-MeV region can be searched for in several ways using nuclear beta decays. If the ν_j mass is less than the energy release Q in a given beta decay, its emission produces a kink in the Kurie plot. Reference [37] suggested a search for such kinks and used a retroactive data analysis to set upper bounds on this type of emission via lepton mixing of neutrinos with kinematically non-negligible masses in nuclear beta decays. In standard notation, (Z, A) denotes a nucleus with Z protons and A nucleons. For a nuclear beta decay $(Z,A) \to (Z+1,A) + e^- + \bar{\nu}_e$ or $(Z,A) \to (Z-1,A) + e^+ + \nu_e$ into a set of neutrino mass eigenstates $\nu_i \in \nu_e$ with negligibly small masses relative to the energy release in the decay plus a mass eigenstate ν_4 in ν_e with non-negligible mass, the differential decay rate is

$$\frac{dN}{dE} = C[(1-|U_{e4}|^2)pE(E_0-E)^2 + |U_{e4}|^2 pE(E_0-E)[(E_0-E)^2 - m_{\nu_4}^2]^{1/2}\theta(E_0-E-m_{\nu_4})], \tag{2.1}$$

where $p \equiv |\mathbf{p}|$ and E denote the 3-momentum and (total) energy of the outgoing $e^\pm$ in the parent nucleus rest frame, E_0 denotes its maximum energy for the SM case, the Heaviside θ function is defined as $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x \leq 0$, and $C = G_F^2|V_{ud}|^2 F_F |\mathcal{M}|^2/(2\pi^3)$, where $\mathcal{M}$ denotes the nuclear transition matrix element, V is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, and F_F is the Fermi function, which takes account of the Coulomb interactions of the outgoing $e^\pm$. In general, there is also a shape correction factor, but this is not important for the superallowed decays considered here. It is understood that if the decay is to an excited state of the daughter nucleus rather than to its ground state, then there is a corresponding reduction in the maximal value of E_0 relative to its value for the decay to the ground state. The kink in the Kurie plot arises as E reaches the endpoint for the decay yielding a ν_4 and the second term in Eq. (2.1) vanishes.

Early bounds on $|U_{e4}|^2$ were set from searches for kinks in Kurie plots in [37] and analyses of particle decays [38–40]. Subsequently, dedicated experiments were conducted to search for kinks in the Kurie plots due to possible emission of a massive neutrino via lepton mixing for a number of nuclear beta decays over a wide range of neutrino masses from O(10) eV to the MeV range. For example, a search for kinks in the Kurie plot in ^{20}F beta decay reported in Ref. [41] yielded an upper bound on $|U_{e4}|^2$ decreasing from 5.9×10^{-3} for $m_{\nu_4} = 0.4$ MeV to

1.8×10^{-3} for $m_{\nu_4} = 2.8$ MeV. (These and other upper bounds discussed in this paper are at the 90% confidence level unless otherwise stated.) Some recent reviews of searches for sterile neutrinos in various mass ranges include [24,25], and [42–50].

A general effect of the emission of a heavy neutrino ν_4 in a nuclear beta decay is to reduce the rate in a manner dependent on its mass, due to phase space suppression of the decay, and, if it is too massive to be emitted, to reduce the rate of the given decay by the factor $(1 - |U_{e4}|^2)$. Hence, in addition to examination of Kurie plots for possible kinks, a powerful method to constrain heavy neutrino emission, via lepton mixing, in nuclear beta decays is to analyze the overall rates. The apparent (app) rate, assuming no emission of a heavy neutrino, can be succinctly expressed as

$$\left.\frac{dN}{dE}\right|_{\mathrm{app}} \propto G_{F,\mathrm{app}}^2 |V_{ud,\mathrm{app}}|^2 F_{\mathrm{app}}, \tag{2.2}$$

where $F_{\mathrm{app}} = pE(E_0 - E)^2$ is the SM kinematic function assuming no heavy neutrino emission. Since, in general, the heavy neutrino would also be emitted in μ decay, the measurement of the μ lifetime performed assuming the SM would yield an apparent (app) value of the Fermi constant, denoted $G_{F,\mathrm{app}}$, that would be smaller than the true value [38–40], G_F, given at tree level by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{g^2 + g'^2}{8m_Z^2}, \tag{2.3}$$

where g and g' are the weak SU(2) and $\mathrm{U}(1)_Y$ gauge couplings, and m_W and m_Z are the masses of the W and Z bosons. The apparent kinematic function F_{app} is larger than the true kinematic function indicated in the square brackets in Eq. (2.1), which depends on m_{ν_4} and $|U_{e4}|^2$. Since $G_{F,\mathrm{app}}$ would be smaller than the true value of G_F, while F_{app} would be larger than the true F, the apparent value, $|V_{ud,\mathrm{app}}|^2$, extracted from a particular nuclear beta decay in the context of the SM could be larger or smaller than the true value. To avoid this complication, we compare ratios of rates of different nuclear beta decays. In these ratios, the factor $G_{F,\mathrm{app}}^2$ cancels, so one can gain information about the kinematic factor and hence about $|U_{e4}|^2$ as a function of m_{ν_4}.

The integration of dN/dE over E gives the kinematic rate factor f. The combination of this with the half-life for the nuclear beta decay, $t \equiv t_{1/2}$, yields the product ft. Incorporation of nuclear and radiative corrections yields the corrected ft value for a given decay, denoted $\mathcal{F}t$. Conventionally, analyses of the most precisely measured superallowed $0^+ \to 0^+$ nuclear beta decays have been used for many years to infer a value of the weak mixing matrix element $|V_{ud}|$ [51,52]. (In our discussion of these fits, we will follow conventional notation and denote the CKM mixing matrix factor as V_{ud}, with the implicit understanding that in our present context with possible emission of a heavy neutrino ν_4, this is really $V_{ud,\mathrm{app}}$.) In turn, these values of $|V_{ud}|$ extracted from superallowed nuclear beta decays were used in early Cabibbo fits, e.g., [53], which were subsequently extended to the full CKM matrix [54–56]. The analyses of nuclear beta decay data have continued up to the present with significant recent progress in precision [57–66].

A first step in these analyses has been to establish the mutual consistency of the $\mathcal{F}t$ values for these superallowed $0^+ \to 0^+$ decays. The emission of a ν_4 with a mass m_{ν_4} of a few MeV would have a different effect on the kinematic functions and integrated rates for nuclear beta decays with different Q (energy release) values and would therefore upset this mutual consistency. Therefore, from this mutual agreement of $\mathcal{F}t$ values, an upper limit on $|U_{e4}|^2$ can be derived for values of m_{ν_4} in the MeV range, such that a ν_4 could be emitted in some of these superallowed decays. $\mathcal{F}t$ is conventionally written as [58–61,63,65,66]

$$\mathcal{F}t = \frac{K}{2G_V^2(1 + \Delta_R^V)}, \tag{2.4}$$

where $K = 2\pi^3 \ln 2/m_e^5 = 0.81202776(9) \times 10^{-6}\,\mathrm{GeV}^{-4}-\mathrm{sec}$, $G_V = G_F|V_{ud}|$ and the radiative correction factor Δ_R^V is transition-independent. Reference [63] obtains the average $\overline{\mathcal{F}t} = 3072.27 \pm 0.72$ sec.

The excellent mutual agreement between the $\mathcal{F}t$ values obtained from a set of the most precisely measured superallowed $0^+ \to 0^+$ nuclear beta decays, which involve only the vector part of the charged weak current, in comparison with the value of G_F obtained from muon decay, allows one to extract, in a self-consistent manner, a value of $|V_{ud}|$. In the 1990 study [58], this yielded the result $|V_{ud}| = 0.9740 \pm 0.001$. At present, using a set of the 14 most precisely measured superallowed $0^+ \to 0^+$ nuclear beta decays, Hardy and Towner have obtained the considerably more precise value [64,67] (denoted HT)

$$\mathrm{HT:}\ |V_{ud}| = 0.97420(21). \tag{2.5}$$

Another recent estimate, in agreement with these, is $|V_{ud}| = 0.97425(13)$ [68] (see also [69]). Using a different method for calculating Δ_R^V, Seng *et al.* [65] (denoted SGPRM) obtain the slightly lower value

$$\mathrm{SGPRM:}\ |V_{ud}| = 0.97370(14), \tag{2.6}$$

with a smaller reported uncertainty than in Eq. (2.5). As noted in [65], this lower value of $|V_{ud}|$ leads to tension with first-row CKM unitarity. Although the central values of $|V_{ud}|$ in Eqs. (2.5) and (2.6) differ, the bounds on $|U_{e4}|$ obtained below depend primarily on the precision in the

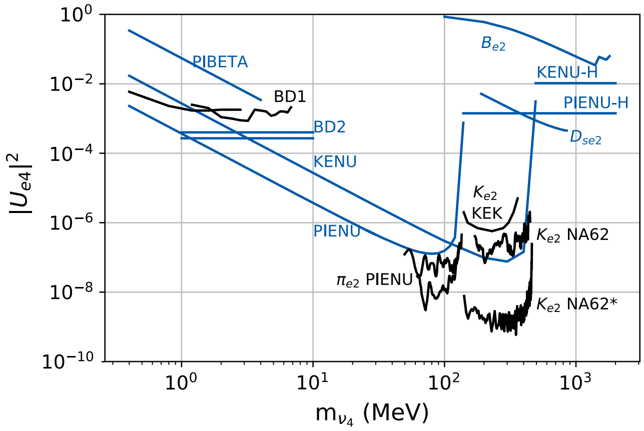

FIG. 1. 90% C.L. upper limits on $|U_{e4}|^2$ vs m_{ν_4} from various souces: PIBETA, pion beta decay (this work); BD1, previous limits from nuclear beta decay [41]; BD2, nuclear beta decay, based on our analysis using [64,65]; PIENU and PIENU-H, the ratio $\frac{BR(\pi^+\to e^+\nu_e)}{BR(\pi^+\to\mu^+\nu_\mu)}$ in the kinematically allowed and forbidden regions for ν_4 emission [90]; π_{e2} PIENU, $\pi^+ \to e^+\nu_4$ peak searches (upper and lower curves from [84,91], respectively); KENU and KENU-H, the ratio $\frac{BR(K^+\to e^+\nu_e)}{BR(K^+\to\mu^+\nu_\mu)}$ in the kinematically allowed and forbidden regions for ν_4 emission; K_{e2} KEK, $K^+ \to e^+\nu_4$ peak search [82]; K_{e2} NA62, $K^+ \to e^+\nu_4$ peak search [94]; and K_{e2} NA62*, the preliminary upper limit from a $K^+ \to e^+\nu_4$ peak search [95]. Other bounds are denoted D_{se2}, from our analysis of $\frac{BR(D_s^+\to e^+\nu_e)}{BR(D_s^+\to\tau^+\nu_\tau)}$, and B_{e2}, from our analysis of peak search data in $B^+ \to e^+\nu_4$ [125]. Our new bounds are colored blue, while previous bounds are colored black. See text for older bounds and further discussion.

mutual agreement of the $\mathcal{F}t$ values. The 14 parent nuclei in the set used in [64] are

$$^{10}\mathrm{C},\ ^{14}\mathrm{O},\ ^{22}\mathrm{Mg},\ ^{26m}\mathrm{Al},\ ^{34}\mathrm{Cl},\ ^{34}\mathrm{Ar},\ ^{38m}\mathrm{K},\ ^{38}\mathrm{Ca}$$
$$^{42}\mathrm{Sc},\ ^{46}\mathrm{V},\ ^{50}\mathrm{Mn},\ ^{54}\mathrm{Co},\ ^{62}\mathrm{Ga},\ \text{and}\ ^{74}\mathrm{Rb} \tag{2.7}$$

where the superscript m refers to a metastable excited state. The maximal Q value in this set is $Q = 9.4$ MeV (^{74}Rb) [63,70].

The emission of a neutrino with a mass of order MeV in superallowed nuclear beta decays would cause kinematic suppression depending on the energy release Q and the neutrino mass m_{ν_4}, which would vary from nucleus to nucleus owing to the different values of the phase space factor in the third term, proportional to $|U_{e4}|^2$, in Eq. (2.1). Reference [57] set upper limits on $|U_{e4}|^2$ ranging from 3×10^{-2} to 4×10^{-3} for m_{ν_4} from 0.5 to 4.5 MeV, while Ref. [58] obtained an upper bound on $|U_{e4}|^2$ ranging from 10^{-2} down to 2×10^{-3} for m_{ν_4} from 0.5 to 2 MeV. Reference [41] incorporated the phase space integration for the massive-neutrino term proportional to $|U_{e4}|^2$ in Eq. (2.1) for eight available superallowed beta decays and then derived upper bounds on $|U_{e4}|^2$ from the consistency of corrected $\mathcal{F}t$ values, depending nonmonotonically on ν_4 masses from 1 to 7 MeV, with the results $|U_{e4}|^2 < 1\times10^{-3}$ to $|U_{e4}|^2 < 2\times10^{-3}$, shown as BD1 in Fig. 1.

A measure of the mutual agreement among $\mathcal{F}t$ values of the superallowed beta decays is the precision with which $|V_{ud}|^2$ is determined, so a reduction in the fractional uncertainty of the value of $|V_{ud}|^2$ results in an improved upper limit on $|U_{e4}|^2$. Let us denote this fractional uncertainty from the ith data analysis, as $[\delta^{(i)}|V_{ud,i}|^2]/|V_{ud,i}|^2$. Then it follows that

$$\frac{\delta^{(2)}|U_{e4}|^2}{\delta^{(1)}|U_{e4}|^2} = \frac{[\delta^{(2)}|V_{ud,2}|^2]/|V_{ud,2}|^2}{[\delta^{(1)}|V_{ud,1}|^2]/|V_{ud,1}|^2}. \tag{2.8}$$

The fractional uncertainties of $[\delta^{(2)}|V_{ud}|]/|V_{ud}| = 2\times10^{-4}$ and 1.4×10^{-4} in Refs. [63–65] are improvements by the respective factors of 5 and 7.5 relative to the inputs used in the 1990 studies [41,58].

We use these improvements to infer respective improved upper bounds on $|U_{e4}|^2$, following from the mutual agreement of the $\mathcal{F}t$ values among the fourteen superallowed beta decays [63–65]. Using the HT value in Eq. (2.5), we find the upper bound

$$|U_{e4}|^2 \lesssim 4\times10^{-4} \tag{2.9}$$

for ν_4 masses in the range from $m_{\nu_4} \simeq 1$ MeV to $m_{\nu_4} \simeq 9.4$ MeV, as indicated in Fig. 1 (BD2, upper line). Using the SGPRM value in Eq. (2.6), we find

$$|U_{e4}|^2 \lesssim 2.7\times10^{-4}, \tag{2.10}$$

also shown in Fig. 1 (BD2, lower line). Of course, the flat line segments shown are approximations; the actual upper limits on $|U_{e4}|^2$ from the nuclear beta decay data are not precisely constant as a function of m_{ν_4} over the range shown. If the uncertainties in the $\mathcal{F}t$ values for each of the superallowed nuclear beta decays used for the overall fit in [63–65] were equal, then one could extend this analysis to derive an upper bound on $|U_{e4}|^2$ as a function of m_{ν_4} in this range of 1 to 9.4 MeV. However, this condition, of equal precision for the measurement of the $\mathcal{F}t$ value of each individual nuclear beta decay in this set, has not yet been achieved. For this reason, we have conservatively presented our upper bounds (2.9) and (2.10) as applying uniformly throughout the specified range 1 MeV $< m_{\nu_4} <$ 9.4 MeV, i.e., as flat line segments in Fig. 1.

Since our bounds (2.9) and (2.10) above do not involve $|U_{\mu4}|^2$, they complement the upper limits on $|U_{e4}|^2$ derived from the measurement of the ratio of decay rates $R^{(\pi)}_{e/\mu} = \Gamma(\pi^+ \to e^+\nu_e)/\Gamma(\pi^+ \to \mu^+\nu_\mu)$ discussed in Sec. VI A in the subset of the range of ν_4 mass values where they overlap, namely $1 \lesssim m_{\nu_4} \lesssim 10$ MeV.

Other methods of determining $|V_{ud}|$ include pion beta decay (discussed in Sec. III) and the neutron lifetime (which also has the complication of involving the axial-vector part of the weak charged current), but these are not as accurate as the determination from the superallowed $0^+ \to 0^+$ beta decays.

III. LIMITS FROM $\pi^+ \to \pi^0 e^+ \nu_e$ DECAY

In this section we analyze limits on sterile neutrinos obtainable from pion beta decay, $\pi^+ \to \pi^0 e^+ \nu_e$. The mass difference between the charged and neutral pions is $\Delta_\pi = m_{\pi^+} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV [13]. It will be convenient to define

$$\epsilon_e = \frac{m_e^2}{\Delta_\pi^2} = 1.237 \times 10^{-2}. \tag{3.1}$$

If ν_e consists only of neutrino mass eigenstates with negligibly small masses, then the Standard-Model expression for the decay rate, denoted $\Gamma_{\pi\beta,\mathrm{SM}}$, is [71]

$$\Gamma_{\pi\beta,\mathrm{SM}} = \frac{G_F^2 |V_{ud}|^2 \Delta_\pi^5}{30\pi^3} \left(1 - \frac{\Delta_\pi}{2m_{\pi^+}}\right)^3 f(\epsilon_e)(1+\delta), \tag{3.2}$$

where

$$f(x) = (1-x)^{1/2}\left[1 - \frac{9}{2}x - 4x^2 + \frac{15}{2}x^2 \ln\left(\frac{1+\sqrt{1-x}}{\sqrt{x}}\right) - \frac{3\Delta_\pi^2}{7(m_{\pi^+}+m_{\pi^0})^2}\right] \tag{3.3}$$

and δ incorporates radiative corrections, calculated to be $\delta = 0.033$ [72,73]. Note that the last term in the square brackets in Eq. (3.3) is -1.20×10^{-4} and thus is much smaller than the leading-x terms. Neglecting this last term, the function $f(x)$ has the expansion

$$f(x) = 1 - 5x + \{O(x^2), O(x^2 \ln x)\}. \tag{3.4}$$

If ν_e contains the known three neutrinos with masses that are negligibly small for the kinematics here, together with an O(1) MeV ν_4, then the rate for pion beta decay has the form

$$\Gamma_{\pi\beta} = (1 - |U_{e4}|^2)\Gamma_{\pi\beta,\mathrm{SM}} + |U_{e4}|^2 \bar{\Gamma}_{\pi\beta,\nu_4} \theta(\Delta_\pi - m_e - m_{\nu_4}), \tag{3.5}$$

where $\Gamma_{\pi\beta,\nu_4} \equiv |U_{e4}|^2 \bar{\Gamma}_{\pi\beta,\nu_4}$ denotes the rate for the decay $\pi^+ \to \pi^0 e^+ \nu_4$. As in the case of nuclear beta decay, the emission of the ν_4 would produce a kink in the differential decay distribution $d\Gamma_{\pi\beta}/dE_e$, where E_e is the electron energy. In particular, while the maximum electron energy in the case of emission of neutrinos with negligibly small masses is

$$E_{e,\mathrm{max,SM}} = \frac{m_{\pi^+}^2 + m_e^2 - m_{\pi^0}^2}{2m_{\pi^+}} = 4.01 \text{ MeV}, \tag{3.6}$$

this is reduced to

$$E_{e,\mathrm{max},\nu_4} = \frac{m_{\pi^+}^2 + m_e^2 - (m_{\pi^0} + m_{\nu_4})^2}{2m_{\pi^+}} \tag{3.7}$$

in the $\pi^+ \to \pi^0 e^+ \nu_4$ decay. However, in contrast to nuclear beta decay, events ascribed to the decay $\pi^+ \to \pi^0 e^+ \nu_e$ were identified by the diphoton decay of the π^0, and the e^+ energy was not systematically measured, e.g., in the PIBETA experiment at PSI [74,75]. Hence, one could not do a kink search for this decay, which would be quite difficult anyway because of the very small branching ratio of 10^{-8} for pion beta decay.

However, one can use the comparison of the measured decay rate, or equivalently, branching ratio for pion beta decay with the SM prediction to obtain a limit on possible emission of a ν_4. We have

$$\begin{aligned}\overline{\mathrm{BR}}_{\pi\beta} &= \frac{\mathrm{BR}(\pi^+ \to \pi^0 e^+ \nu_e)}{\mathrm{BR}(\pi^+ \to \pi^0 e^+ \nu_e)_{\mathrm{SM}}} \\ &= (1 - |U_{e4}|^2) + |U_{e4}|^2 r_{\pi\beta,\nu_4},\end{aligned} \tag{3.8}$$

where $r_{\pi\beta,\nu_4}$ denotes the ratio of the kinematic factor for the $\pi^+ \to \pi^0 e^+ \nu_4$ decay divided by that for the decay into neutrinos of negligibly small mass, and, including radiative corrections [74,75],

$$\mathrm{BR}(\pi^+ \to \pi^0 e^+ \nu_e)_{\mathrm{SM}} = (1.039 \pm 0.001) \times 10^{-8}. \tag{3.9}$$

Defining $\epsilon_{\nu_4} = m_{\nu_4}^2/\Delta_\pi^2$, the function $r_{\pi\beta,\nu_4}$ can be approximated to leading order in ϵ_e and ϵ_{ν_4} as

$$r_{\pi\beta,\nu_4} \simeq \frac{1 - 5(\epsilon_e + \epsilon_{\nu_4})}{1 - 5\epsilon_e} \simeq 1 - 5\epsilon_{\nu_4}. \tag{3.10}$$

The current value listed by the Particle Data Group, dominated by the PIBETA measurement [74,75], is [13]

$$\mathrm{BR}(\pi^+ \to \pi^0 e^+ \nu_e) = (1.036 \pm 0.006) \times 10^{-8}. \tag{3.11}$$

This is in good agreement with the SM prediction (3.9), yielding

$$\overline{\mathrm{BR}}_{\pi/\beta} = 0.997 \pm 0.006. \tag{3.12}$$

From this we obtain the upper limit on $|U_{e4}|^2$ shown in Fig. 1 as PIBETA. As m_{ν_4} increases, and finally exceeds the value $m_{\pi^+} - m_{\pi^0} - m_e = 4.08$ MeV, the decay $\pi^+ \to \pi^0 e^+ \nu_4$ is kinematically forbidden, and hence the observed rate divided by the rate predicted in the SM with the usual mass eigenstates in ν_e of negligibly small masses is reduced to the first term in Eq. (3.8), namely $1 - |U_{e4}|^2$. The upper bounds on $|U_{e4}|^2$ from pion beta decay are less stringent than the bounds in Eqs. (2.9) and (2.10).

IV. CONSTRAINT FROM CKM UNITARITY

If the mass of ν_4 were sufficiently large so that it could not be emitted in any superallowed nuclear beta decays used in the determination of $|V_{ud}|$, then, although there would still be mutual consistency in this determination between the different superallowed nuclear decays, the result would be a spurious apparent value of $|V_{ud}|^2$, namely $|V_{ud,\mathrm{app}}|^2 = |V_{ud}|^2(1 - |U_{e4}|^2)$ (where we again assume just one heavy neutrino). In turn, this would reduce the apparent value of $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ used to check the first-row unitarity of the CKM matrix. If one uses the value of $|V_{ud}|$ in Eq. (2.5), then the sum $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ is equal to unity to within the stated theoretical and experimental uncertainties. Thus, this provides another constraint on possible massive neutrino emission in the decays involved. Numerically, using the value of $|V_{ud}|$ in Eq. (2.5), together with the values $|V_{us}| = 0.2243(5)$ and $|V_{ub}|^2 = (1.55 \pm 0.28) \times 10^{-5}$ from [13], Ref. [64] obtains

$$\Sigma \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99939(64). \tag{4.1}$$

The $|V_{ud}|^2$ term dominates both the sum and the uncertainty in (4.1). Thus, with the assumption of first-row CKM unitarity, this also yields an upper limit on $|U_{e4}|^2$, depending on m_{ν_4} and estimates of uncertainty in $|V_{us}|^2$. If, on the other hand, one uses the lower value of $|V_{ud}|$ in Eq. (2.6), then, as was observed in [65], there is tension with first-row CKM unitarity. However, since the difference between the analyses in [63–65] is in the value for the transition-independent correction term Δ_R^V, this does not upset the mutual agreement between the $\mathcal{F}t$ values, which was the key input for the bound (2.9).

V. CONSTRAINTS FROM PEAK SEARCH EXPERIMENTS

It is also of considerable interest to discuss correlated limits on sterile neutrinos from two-body leptonic decays of pseudoscalar mesons. Searches for subdominant peaks in charged lepton momenta in two-body leptonic decays of pseudoscalar mesons were suggested as a way to search for emission, via lepton mixing, of a possible heavy neutrino ν_h, and to set upper limits on the associated couplings $|U_{\ell h}|^2$, also including effects on ratios of branching ratios, in [37,38]. These observations were applied retroactively to existing data to derive such limits in [37–39]. In particular, the upper limit $|U_{e4}|^2 \lesssim 10^{-5}$ was obtained from retroactive analysis of data on $K^+ \to e^+\nu_e$ decays for $82 < m_{\nu_4} < 163$ MeV, and upper limits on $|U_{\mu4}|^2$ in the range 10^{-4}–10^{-5} were obtained from data on $\pi^+ \to \mu^+\nu_\mu(\pi_{\mu2})$ decay (Figs. 17, 22 in [38]). An analogous discussion of the emission of massive neutrino(s) in muon decay was given in [39,76], and an analysis of μ decay data was used in [39] to set upper limits on $|U_{e4}|^2$ and on $|U_{\mu4}|^2$ (see Sec. VIII).

Dedicated experiments have been carried out from 1981 to the present to search for the emission, via lepton mixing, of a heavy neutrino in two-body leptonic decays of $M^+ = \pi^+, K^+$ mesons and to search for effects of possible heavy neutrinos on the ratio $\mathrm{BR}(M^+ \to e^+\nu_e)/\mathrm{BR}(M^+ \to \mu^+\nu_\mu)$ [77–96]. These have set very stringent bounds. Data from the corresponding experiments with heavy-quark pseudoscalar mesons will be used below to derive new limits on sterile neutrinos. Some relevant properties of these experiments with two-body leptonic decays of charged pseudoscalar mesons will be discussed next. The peak search experiments are quite sensitive to massive neutrino emission because one is looking for a monochromatic signal and, furthermore, for a considerable range of m_{ν_4} masses, there is a kinematic enhancement of the decays $M^+ \to e^+\nu_4$ and $M^+ \to \mu^+\nu_4$ relative to the decays into neutrinos with negligibly small masses.

In the SM, the rate for the decay $M^+ \to \ell^+\nu_\ell$ of a charged pseudoscalar M^+, where $M^+ = \pi^+, K^+$, etc., and ℓ is a charged lepton, is, to leading order,

$$\Gamma(M^+ \to \ell^+\nu_\ell)_{\mathrm{SM}} = \frac{G_F^2 |V_{ij}|^2 f_M^2 m_M m_\ell^2}{8\pi}\left(1 - \frac{m_\ell^2}{m_M^2}\right)^2, \tag{5.1}$$

where V_{ij} is the relevant CKM mixing matrix element, f_M is the corresponding pseudoscalar decay constant (normalized such that $f_\pi = 130$ MeV), and we have used the fact that the three known neutrino mass eigenstates ν_i, $i = 1, 2, 3$ in ν_ℓ are negligibly small compared with m_M for all pseudoscalar mesons M.

However, because of lepton mixing, other decay modes may also occur into some number of neutrinos with non-negligible masses. Focusing, as above, on the case of a single heavy neutrino ν_4, the SM rate is reduced by the factor $(1 - |U_{\ell4}|^2)$ and there is another decay yielding the heavy neutrino with rate,

$$\Gamma(M^+ \to \ell^+\nu_4) = \frac{G_F^2 |V_{ij}|^2 |U_{\ell4}|^2 f_M^2 m_M^3}{8\pi}\rho(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}), \tag{5.2}$$

in which the notation is as follows [37,38]:

$$\delta_\ell^{(M)} = \frac{m_\ell^2}{m_M^2}, \qquad \delta_{\nu_4}^{(M)} = \frac{m_{\nu_4}^2}{m_M^2}, \tag{5.3}$$

$$\rho(x,y) = f_{\mathcal{M}}(x,y)[\lambda(1,x,y)]^{1/2}, \tag{5.4}$$

where the factor $f_{\mathcal{M}}$ arises from the square of the matrix element $\mathcal{M}$, and

$$f_{\mathcal{M}}(x,y) = x + y - (x-y)^2. \tag{5.5}$$

In Eq. (5.4), $\lambda(1,x,y)$ arises from the final-state two-body phase space, with

$$\lambda(z,x,y) = x^2 + y^2 + z^2 - 2(xy + yz + zx). \tag{5.6}$$

Note that $\rho(x,y)$ has the symmetry property

$$\rho(x,y) = \rho(y,x). \tag{5.7}$$

In the SM case with zero or negligibly small neutrino masses, $\rho(x,0) = x(1-x)^2$. Here and below, it is implicitly understood that $\rho(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}) = 0$ if $m_{\nu_4} \geq m_M - m_\ell$, since in this case the decay $M^+ \to \ell^+ \nu_4$ is kinematically forbidden.

There is a clear signature for the decay $M^+ \to \ell^+ \nu_4$ into a heavy neutrino, namely the appearance of a monochromatic peak in the energy or momentum distribution of the charged lepton below the dominant peak associated with the emission of neutrino mass eigenstates of negligibly small mass. The energy and momentum of this additional peak, in the rest frame of the parent meson M, are

$$E_\ell = \frac{m_M^2 + m_\ell^2 - m_{\nu_4}^2}{2m_M} \tag{5.8}$$

and

$$p_\ell = |\mathbf{p}_\ell| = \frac{m_M}{2}\sqrt{\lambda(1, \delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})}. \tag{5.9}$$

An experiment on the two-body leptonic decay of a pseudoscalar meson $M^+ \to \ell^+ \nu_\ell$, searching for a subdominant peak in the charged lepton momentum or energy distribution due to the decay $M^+ \to \ell \nu_4$, is limited to the mass range $m_{\nu_4} < m_M - m_\ell$ for which the decay is kinematically allowed. It is also limited (i) to sufficiently small m_{ν_4} such that the momentum or energy of the outgoing ℓ^+ is large enough so that the event will not be rejected by the lower cut used in the event reconstruction and (ii) to sufficiently large m_{ν_4} so that the subdominant peak can be resolved from the dominant peak.

The function $f_{\mathcal{M}}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})$ increases from a minimum at $\delta_{\nu_4} = 0$ to a maximum at $\delta_{\nu_4}^{(M)} = (1/2) + \delta_\ell^{(M)}$, where it has the value $2\delta_\ell^{(M)} + (1/4)$. The maximum in $f_{\mathcal{M}}$ is in the physical region if $m_\ell < (m_M/4)$. The ratio of the value of $f_{\mathcal{M},\max}$ divided by $f_{\mathcal{M}}$ for emission of neutrinos of negligible mass is

$$\frac{f_{\mathcal{M},\max}}{f_{\mathcal{M}}(\delta_\ell^{(M)}, 0)} = \frac{2\delta_\ell^{(M)} + \frac{1}{4}}{\delta_\ell^{(M)}(1 - \delta_\ell^{(M)})}. \tag{5.10}$$

For decays in which $m_\ell \ll m_M$ and hence $\delta_\ell^{(M)} \ll 1$, this produces a large enhancement, since

$$\frac{f_{\mathcal{M},\max}}{f_{\mathcal{M}}(\delta_\ell^{(M)}, 0)} = \frac{1}{4\delta_\ell^{(M)}}[1 + O(\delta_\ell^{(M)})] \gg 1. \tag{5.11}$$

For example, for π_{e2}, K_{e2}, D_{e2}, $(D_s)_{e2}$, and B_{e2} decays, this ratio (5.10) has the very large values 1.87×10^4, 2.33×10^5, 3.35×10^6, 3.71×10^6, and 2.67×10^7, respectively. Physically, these large enhancement factors are due to the removal of the helicity suppression of the decay of the M^+ into a light ℓ^+ and neutrinos ν_i with negligibly small masses.

It is convenient to define the ratio

$$\bar\rho(x,y) \equiv \frac{\rho(x,y)}{\rho(x,0)} = \frac{\rho(x,y)}{x(1-x)^2}. \tag{5.12}$$

Thus,

$$\frac{\Gamma(M^+ \to \ell^+ \nu_4)}{\Gamma(M^+ \to \ell^+ \nu_\ell)_{\rm SM}} = \frac{|U_{\ell 4}|^2 \bar\rho(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})}{1 - |U_{\ell 4}|^2}. \tag{5.13}$$

Note that the dominant radiative corrections divide out between the numerator and denominator of Eq. (5.13). Since a given value of lepton momentum p_ℓ is uniquely determined by m_{ν_4} for a given pseudoscalar meson M, a null observation of an additional peak in an experiment and hence an upper limit on the ratio $\Gamma(M^+ \to \ell^+ \nu_4)/\Gamma(M^+ \to \ell^+ \nu_\ell)_{\rm SM}$ at a particular p_ℓ yields an upper limit on $|U_{\ell 4}|^2$ for the corresponding value of m_{ν_4}. Solving Eq. (5.13) for $|U_{\ell 4}|^2$ gives

$$|U_{\ell 4}|^2 = \frac{\frac{\Gamma(M^+ \to \ell^+ \nu_4)}{\Gamma(M^+ \to \ell^+ \nu_\ell)_{\rm SM}}}{\frac{\Gamma(M^+ \to \ell^+ \nu_4)}{\Gamma(M^+ \to \ell^+ \nu_\ell)_{\rm SM}} + \bar\rho(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})}. \tag{5.14}$$

Hence, denoting $\Gamma(M^+ \to \ell^+ \nu_4)_{ul}$ as the upper limit on $\Gamma(M^+ \to \ell^+ \nu_4)$, one has the resultant upper limit on $|U_{\ell 4}|^2$:

$$|U_{\ell 4}|^2 < \frac{\frac{\Gamma(M^+\to\ell^+\nu_4)_{ul}}{\Gamma(M^+\to\ell^+\nu_\ell)_{\rm SM}}}{\frac{\Gamma(M^+\to\ell^+\nu_4)}{\Gamma(M^+\to\ell^+\nu_\ell)_{\rm SM}} + \bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})}. \quad (5.15)$$

Provided that $|U_{\ell 4}|^2 \ll 1$, the right-hand side of Eq. (5.13) is, to a good approximation, equal to $|U_{\ell 4}|^2\bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})$, so that the upper limit (5.15) simplifies to

$$|U_{\ell 4}|^2 < \frac{\frac{\Gamma(M^+\to\ell^+\nu_4)_{ul}}{\Gamma(M^+\to\ell^+\nu_\ell)_{\rm SM}}}{\bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})}. \quad (5.16)$$

The large values of $\bar\rho(\delta_e^{(M)},\delta_{\nu_4}^{(M)})$ decays over a substantial part of the kinematically allowed range of m_{ν_4} mean that $M^+\to e^+\nu_4$ decays are quite sensitive to the possible emission of a heavy ν_4. With fixed x, the function $\bar\rho(x,y)$ has the following Taylor series expansion in y for small y:

$$\bar\rho(x,y) = 1 + \left[\frac{1-3x^2}{x(1-x)^2}\right]y + O(y^2). \quad (5.17)$$

The derivative of $\bar\rho(x,y)$ with respect to y is

$$\frac{d\bar\rho(x,y)}{dy} = \frac{1}{x(1-x)^2}\frac{d\rho(x,y)}{dy} = \frac{1-x-5y-3x^2+7y^2-4xy+9xy(y-x)+3(x^3-y^3)}{x(1-x)^2[\lambda(1,x,y)]^{1/2}}. \quad (5.18)$$

Hence,

$$\left.\frac{d\bar\rho(x,y)}{dy}\right|_{y=0} = \frac{1-3x^2}{x(1-x)^2}. \quad (5.19)$$

In our application,

$$x = \delta_\ell^{(M)} \quad \text{and} \quad y = \delta_{\nu_4}^{(M)}. \quad (5.20)$$

For $M^+\to\ell^+\nu_4$ decays such that $\delta_\ell^{(M)} \ll 1$, which include all of the $M^+\to e^+\nu_4$ decays, the derivative (5.19) is $[d\bar\rho(x,y)/dy]_{y=0} = x^{-1}[1+O(x)]$, i.e.,

$$\left.\frac{d\bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})}{d\delta_{\nu_4}}\right|_{\delta_{\nu_4}^{(M)}=0} = \frac{1}{\delta_\ell^{(M)}}[1+O(\delta_\ell^{(M)})] \gg 1. \quad (5.21)$$

Hence, in $M^+\to e^+\nu_4$ decays, as $\delta_{\nu_4}^{(M)}$ increases from 0, $\bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})$ increases very rapidly from unity to values $\gg 1$.

For a given x, the maximal value of $\bar\rho(x,y)$, as a function of y occurs where $d\rho(x,y)/dy = 0$, or equivalently, $d\bar\rho(x,y)/dy = 0$ in the physical region. The value of y at this maximum is given by the solution for y, of the equation

$$3y^3 - (9x+7)y^2 + (9x^2+4x+5)y + (-3x^3+3x^2+x-1) = 0. \quad (5.22)$$

In $M^+\to e^+\nu_4$ decays, $x = \delta_e^{(M)} \ll 1$, so that, to a very good approximation, Eq. (5.22) reduces to the equation $(3y-1)(y-1)^2 = 0$. The relevant solution of this equation, giving the value of y at which $\rho(x,y)$ and $\bar\rho(x,y)$ reach their respective maxima if $x \ll 1$, is

$$y_{\bar\rho_{\max}} = \frac{1}{3}, \quad \text{i.e.,} \quad m_{\nu_4} = \frac{m_M}{\sqrt{3}}. \quad (5.23)$$

Then (with $x \ll 1$),

$$\bar\rho(x,1/3) = \frac{4}{27x}\left[1 + \frac{13}{2}x + O(x^2)\right], \quad (5.24)$$

so

$$[\bar\rho(x,y)]_{\max} \simeq \bar\rho(x,1/3) = \frac{4}{27x}, \quad (5.25)$$

which is $\gg 1$. In Table I we list the maximal values of $\bar\rho(x,y) = \bar\rho(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})$ for the various pseudoscalar mesons M^+ considered here, and for $\ell = e,\mu$, together with the respective values of m_{ν_4} where these maxima occur. Particularly large maximal values of the $\bar\rho$ function occur for heavy-quark pseudoscalar mesons, including 1.98×10^6, 2.20×10^6, and 1.58×10^7 for the $D^+\to e^+\nu_4$, $D_s^+\to e^+\nu_4$, and $B^+\to e^+\nu_4$ decays, respectively. As is evident from this table, these maximal values are only slightly less than the maximal values of $\bar f_{\mathcal M}(\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})$ mentioned above. This is due to the slow falloff of the two-body phase space factor $[\lambda(1,x,y)]^{1/2} = [\lambda(1,\delta_\ell^{(M)},\delta_{\nu_4}^{(M)})]^{1/2}$ with increasing m_{ν_4}. To see this, let us define, as in [38], the ratio of the phase space factor divided by its value for zero neutrino mass,

$$[\bar\lambda(1,x,y)]^{1/2} \equiv \frac{[\lambda(1,x,y)]^{1/2}}{[\lambda(1,x,0)]^{1/2}} = \frac{[\lambda(1,x,y)]^{1/2}}{1-x}. \quad (5.26)$$

This has the Taylor series expansion

$$[\bar\lambda(1,x,y)]^{1/2} = 1 - \frac{(1+x)}{(1-x)^2}y + O(y^2) \quad (5.27)$$

TABLE I. Maximal values of the normalized kinematic rate factor $\bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})$ for the two-body leptonic decay $M^+ \to \ell^+ \nu_4$ of the pseudoscalar meson M^+, where $\ell = e, \mu$, together with the corresponding value of m_{ν_4}, denoted $(m_{\nu_4})_{\bar{\rho}_{\max}}$ (in MeV), where this maximum is reached.

Decay	$(m_{\nu_4})_{\bar{\rho}_{\max}}$	$\bar{\rho}_{\max}$
$\pi^+ \to e^+ \nu_4$	80.6	1.105×10^4
$K^+ \to e^+ \nu_4$	285	1.38×10^5
$D^+ \to e^+ \nu_4$	1.08×10^3	1.98×10^6
$D_s^+ \to e^+ \nu_4$	1.14×10^3	2.20×10^6
$B^+ \to e^+ \nu_4$	3.05×10^3	1.58×10^7
$\pi^+ \to \mu^+ \nu_4$	3.46	1.00
$K^+ \to \mu^+ \nu_4$	263	4.13
$D^+ \to \mu^+ \nu_4$	1.07×10^3	47.3
$D_s^+ \to \mu^+ \nu_4$	1.13×10^3	52.4
$B^+ \to \mu^+ \nu_4$	3.05×10^3	371

for small y. Hence, the phase space function normalized to its value for zero neutrino mass, i.e., $[\bar{\lambda}(1, \delta_\ell^{(M)}, \delta_{\nu_4}^{(M)})]^{1/2}$, decreases from unity rather slowly for small $\delta_{\nu_4}^{(M)}$. The maximal value of $y = \delta_{\nu_4}^{(M)}$ is

$$y_{\max} = (1 - \sqrt{x})^2, \quad \text{i.e.,}$$
$$(\delta_{\nu_4}^{(M)})_{\max} = \left(1 - \sqrt{\delta_\ell^{(M)}}\right)^2. \tag{5.28}$$

For fixed $x = \delta_\ell^{(M)}$, as $y = \delta_{\nu_4}^{(M)}$ approaches $y_{\max}$ from below, the phase space factor $[\lambda(1, x, y)]^{1/2} \to 0$, and hence so do $\rho(x, y)$ and $\bar{\rho}(x, y)$. From the factorized expression

$$\lambda(1, x, y) = (1 + \sqrt{x} + \sqrt{y})(1 + \sqrt{x} - \sqrt{y}) \times (1 - \sqrt{x} + \sqrt{y})(1 - \sqrt{x} - \sqrt{y}) \tag{5.29}$$

it follows that as $y \to y_{\max}$ from below, $[\lambda(1, x, y)]^{1/2}$ vanishes like $2x^{1/4}(y_{\max} - y)^{1/2}$. Hence,

$$\frac{d\bar{\rho}(x, y)}{dy} \to -\frac{2x^{3/4}}{[1 - \frac{y}{y_{\max}}]^{1/2}} \quad \text{as } y \to y_{\max}. \tag{5.30}$$

From Eq. (5.30), it follows that for any physical value of x, as $y \to y_{\max}$ from below, $\rho(x, y)$ and $\bar{\rho}(x, y)$ approach 0 with a negatively infinite slope.

For fixed x, over almost all of the kinematically allowed region in y, the reduced function $\bar{\rho}(x, y)$ is larger than 1. The fact that $\bar{\rho}(\delta_e^{(M)}, \delta_{\nu_4}^{(M)}) > 1$ up to values of m_ν extremely close to its upper endpoint is understandable in view of the property embodied in Eq. (5.30), that this function approaches zero with a slope that approaches $-\infty$, i.e., nearly vertically, as $\delta_{\nu_4}^{(M)} \to (\delta_{\nu_4}^{(M)})_{\max}$. For example, in the $M^+ \to e^+ \nu_4$ decay, with $M^+ = \pi^+$ or K^+, $\bar{\rho}(\delta_e^{(M)}, \delta_{\nu_4}^{(M)}) > 1$ for all $m_{\nu_4} > 0$ up to values that are within 0.015 MeV of the respective kinematic endpoints $m_{\pi^+} - m_e = 139.059$ MeV and $m_{K^+} - m_e = 493.156$ MeV. At these respective values of m_{ν_4}, the momentum of the e^+ is very small, namely 0.125 MeV, which would be below the lower cutoff for such an event to be accepted as a π_{e2} or K_{e2} event. Similar comments apply for the leptonic decays of heavy-quark pseudoscalar mesons. We will use this property in the limits that we derive below on $|U_{e4}|^2$.

Recent bounds from $\pi_{\ell 2}$ and $K_{\ell 2}$ peak search experiments include those from the searches for $\pi^+ \to e^+ \nu_h$ and $\pi^+ \to \mu^+ \nu_h$ decays by the PIENU experiment at TRIUMF [90,91,96], for $K^+ \to \mu^+ \nu_h$ decay in the E949 experiment at BNL [89], and for the $K^+ \to \mu^+ \nu_h$ and $K^+ \to e^+ \nu_h$ decays in the NA62 experiment at CERN [93–95], where $\nu_h \equiv \nu_4$ in our notation. From the various π_{e2}, $\pi_{\mu 2}$, K_{e2}, and $K_{\mu 2}$ peak search experiments, some upper bounds include

(i) $|U_{e4}|^2 \lesssim 10^{-7}$–$10^{-8}$ for 50 MeV $< m_{\nu_4} <$ 135 MeV [90,91];
(ii) $|U_{e4}|^2 \lesssim 10^{-6}$–$10^{-7}$ for 170 MeV $< m_{\nu_4} <$ 450 MeV [94];
(iii) $|U_{\mu 4}|^2 \lesssim 10^{-2}$ to 10^{-5} for 5 MeV $< m_{\nu_4} <$ 30 MeV [77];
(iv) $|U_{\mu 4}|^2 \lesssim 10^{-4}$ for 3 MeV $< m_{\nu_4} <$ 19.5 MeV [83];
(v) $|U_{\mu 4}|^2 \lesssim 0.6 \times 10^{-5}$ for 16 MeV $< m_{\nu_4} <$ 29 MeV and $|U_{\mu 4}|^2 \lesssim 1 \times 10^{-5}$ for 29 MeV $< m_{\nu_4} <$ 32 MeV [96];
(vi) $|U_{\mu 4}|^2 \lesssim 10^{-8}$–$10^{-9}$ for 200 MeV $< m_{\nu_4} <$ 300 MeV [89]; and,
(vii) $|U_{\mu 4}|^2 \lesssim (1-4) \times 10^{-7}$ for 300 MeV $< m_{\nu_4} <$ 450 MeV [94].

Recently, the NA62 experiment at CERN reported more stringent preliminary upper limits on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$:

(i) $|U_{e4}|^2 \lesssim (1-3) \times 10^{-9}$ for 150 MeV $< m_{\nu_4} <$ 400 MeV, increasing to $|U_{e4}|^2 \lesssim (0.3$–$2) \times 10^{-8}$ for 400 MeV $< m_{\nu_4} <$ 450 MeV, and
(ii) $|U_{\mu 4}|^2 \lesssim (1$–$3) \times 10^{-8}$ for 220 MeV $< m_{\nu_4} <$ 380 MeV [95].

Peak search experiments have also been conducted very near to the kinematic endpoint in $\pi^+ \to \mu^+ \nu_4$ decay, which occurs at $m_{\nu_4} = 33.9122$ MeV [85–87]. For $m_{\nu_4} = 33.905$ MeV, a PSI experiment obtained an upper bound $\mathrm{BR}(\pi^+ \to \mu^+ \nu_4) < 6.0 \times 10^{-10}$ (95% C.L.) [87]. From Eq. (5.16), we estimate an upper limit

$$|U_{\mu 4}|^2 < 1.7 \times 10^{-8} (90\% \text{ C.L.}) \quad \text{at } m_{\nu_4} = 33.905 \text{ MeV}, \tag{5.31}$$

which is shown in Fig. 2. An analysis of data on the μ capture reaction $\mu^- + {}^3\mathrm{He} \to \bar{\nu}_\mu + {}^3\mathrm{H}$ yielded upper limits on $|U_{\mu 4}|^2$ from ~ 0.1 to $\lesssim 10^{-2}$ for m_{ν_4} in the interval from

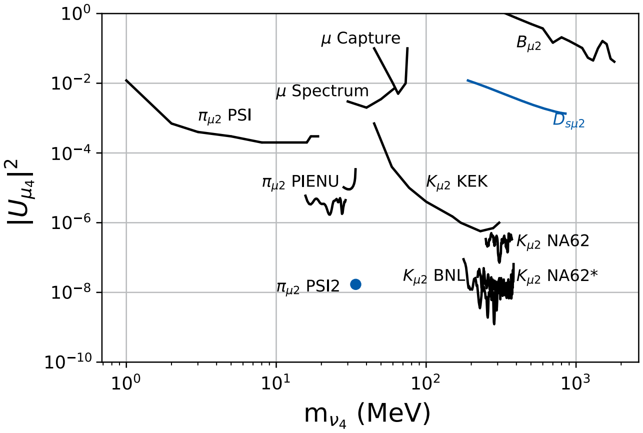

FIG. 2. Best 90% C.L. upper limits on $|U_{\mu4}|^2$ vs m_{ν_4} from various experiments: $\pi^+ \to \mu^+\nu_4$ peak searches, labeled as follows: $\pi_{\mu2}$ PSI [83], $\pi_{\mu2}$ PSI2 [87], $\pi_{\mu2}$ PIENU [96]; $K^+ \to \mu^+\nu_4$ peak searches: $K_{\mu2}$ KEK [79,82], $K_{\mu2}$ BNL [89], $K_{\mu2}$ NA62 [94], and the preliminary limit $K_{\mu2}$ NA62* [95]. Other limits include μ spectrum [39]; μ capture [97]; a $B^+ \to \mu^+\nu_4$ peak search denoted $B_{\mu2}$ [125]; and our analysis of $\frac{\mathrm{BR}(D_s^+\to\mu^+\nu_\mu)}{\mathrm{BR}(D_s^+\to\tau^+\tau_\mu)}$, labeled $D_{s\mu2}$, Our new bounds are colored blue while previous bounds are colored black. See text for previous bounds and further discussion.

62 to 72 MeV [97]. See [13] for further limits and references to the literature.

Upper limits on $|U_{e4}|^2$ vs $m_{\nu4}$ from π_{e2} and K_{e2} peak searches are shown in Fig. 1, labeled as π_{e2} PIENU, K_{e2} KEK, K_{e2} NA62, and K_{e2} NA62*, as well as the B_{e2} limit presented in [1], which will be discussed further below. Upper limits on $|U_{\mu4}|^2$ vs $m_{\nu4}$ from $\pi_{\mu2}$ and $K_{\mu2}$ peak searches are shown in Fig. 2, labeled as $\pi_{\mu2}$ PSI, $\pi_{\mu2}$ PSI2, $\pi_{\mu2}$ PIENU, $K_{\mu2}$ KEK, $K_{\mu2}$ BNL, $K_{\mu2}$ NA62, and $K_{\mu2}$ NA62*.

VI. CONSTRAINTS FROM DATA ON $e-\mu$ UNIVERSALITY

A. General formalism

In addition to producing a subdominant peak in the charged lepton momentum p_ℓ at the value (5.9), the emission of a massive neutrino in the two-body leptonic decay of a pseudoscalar meson M^+ would cause an apparent deviation from the SM prediction for the ratio of decay rates or branching ratios,

$$R^{(M)}_{\ell/\ell'} \equiv \frac{\mathrm{BR}(M^+ \to \ell^+\nu_\ell)}{\mathrm{BR}(M^+ \to \ell'^+\nu_{\ell'})}. \tag{6.1}$$

The experimental measurements of $M^+ \to \ell^+\nu_\ell$ include events with very soft photons; this is to be understood implicitly below. By convention, we take $m_{\ell'} > m_\ell$. This deviation would constitute an apparent violation of $e-\mu$ universality for the case $\ell = e$, $\ell' = \mu$. In contrast to a peak search experiment with the decay $M^+ \to \ell^+\nu_\ell$, which places an upper bound on $|U_{\ell4}|^2$ as a function of m_{ν_4}, a deviation in $R^{(M)}_{\ell/\ell'}$ depends, in general, on both $|U_{\ell4}|^2$ and $|U_{\ell'4}|^2$, as well as m_{ν_4} [see Eqs. (6.11) and (6.12) below]. The nonobservation of any additional peak in the dN/dp_ℓ spectrum in two-body leptonic decays of π^+ and K^+ was used via a retroactive data analysis in [37,38] and in a series of dedicated peak-search experiments to set stringent upper limits on $|U_{e4}|^2$ and $|U_{\mu4}|^2$ (individually) as functions of m_{ν_4}. Furthermore, the nonobservation of any deviation from $e-\mu$ universality in the ratio $R^{(M)}_{e/\mu}$ was used in [38,80,84] to obtain upper limits on lepton mixing, as will be discussed further below. As was the case with peak search experiments, in deriving a constraint from a comparison of a measured value of $R^{(M)}_{\ell/\ell'}$ with the SM prediction for this ratio, one must take into account that even if m_{ν_4} is small enough to be kinematically allowed to occur in either or both of these decays, an experiment might reject events involving emission of a ν_4 if the momentum or energy of the outgoing ℓ^+ or ℓ'^+ were below the cuts used in the event reconstruction and data analysis. We comment further on this below.

In Sec. V we reviewed the general formalism describing effects of possible massive neutrino emission in $M_{\ell2}$ decays, i.e., the decays $M^+ \to \ell^+\nu_\ell$, where $\ell = e$, μ, and, where allowed kinematically, also $\ell = \tau$ [37,38]. Although the actual decays and branching ratios depend on the pseudoscalar decay constants f_M and the CKM mixing matrix elements, these cancel in ratios of branching ratios, which can thus be calculated to high precision and compared with experimental measurements. Let us, then, consider the ratio of branching ratios (6.1). In the Standard Model, since the neutrino mass eigenstates ν_i, $i = 1, 2, 3$ have negligible masses, this ratio is

$$R^{(M)}_{\ell/\ell',\mathrm{SM}} = \frac{m_\ell^2}{m_{\ell'}^2}\left[\frac{1-\frac{m_\ell^2}{m_M^2}}{1-\frac{m_{\ell'}^2}{m_M^2}}\right]^2(1+\delta_{\mathrm{RC}}), \tag{6.2}$$

where δ_{RC} is the radiative correction [98–104], which takes into account soft photon emission, matching experimental conditions. We define the ratio of the measured ratio of branching fractions, $R^{(M)}_{\ell/\ell'}$ to the SM prediction for this ratio, $R^{(M)}_{\ell/\ell',\mathrm{SM}}$, as

$$\bar{R}^{(M)}_{\ell/\ell'} \equiv \frac{R^{(M)}_{\ell/\ell'}}{R^{(M)}_{\ell/\ell',\mathrm{SM}}}. \tag{6.3}$$

Including the radiative correction δ_{RC}, one has the following SM prediction for $R^{(\pi)}_{e/\mu,\mathrm{SM}}$ [100–103]

$$R^{(\pi)}_{e/\mu,\mathrm{SM}} = (1.2352 \pm 0.0002) \times 10^{-4}. \tag{6.4}$$

The most recent and precise experimental measurement of this ratio of branching ratios was carried out by the PIENU experiment at TRIUMF, with the result [90]

$$R^{(\pi)}_{e/\mu} = (1.2344 \pm 0.0023_{\mathrm{stat}} \pm 0.0019_{\mathrm{syst}}) \times 10^{-4}. \tag{6.5}$$

Combined with earlier data from lower-statistics experiments, this yields the current weighted average listed by the Particle Data Group (PDG) for this ratio, namely [13]

$$R^{(\pi)}_{e/\mu} = (1.2327 \pm 0.0023) \times 10^{-4}. \tag{6.6}$$

Using the PDG value of $R^{(\pi)}_{e/\mu}$, one finds

$$\bar{R}^{(\pi)}_{e/\mu} = 0.9980 \pm 0.0019. \tag{6.7}$$

For $R^{(K)}_{e/\mu}$, a similar analysis including radiative corrections [100–103] gives the SM prediction

$$R^{(K)}_{e/\mu,\mathrm{SM}} = (2.477 \pm 0.001) \times 10^{-5}. \tag{6.8}$$

The current average experimental value which is dominated by the measurement from the NA62 experiment [105] is [13]

$$R^{(K)}_{e/\mu} = (2.488 \pm 0.009) \times 10^{-5}. \tag{6.9}$$

To within the joint theoretical and experimental uncertainties, the measured value of $R^{(K)}_{e/\mu}$ is in agreement with the SM prediction, as shown by the ratio

$$\bar{R}^{(K)}_{e/\mu} = 1.0044 \pm 0.0037. \tag{6.10}$$

If a ν_4 is emitted, then the ratio $R^{(M)}_{\ell/\ell',\mathrm{SM}}$ changes to the following [37,38]:

$$R^{(M)}_{\ell/\ell'} = \left[\frac{(1-|U_{\ell 4}|^2)\rho(\delta^{(M)}_{\ell},0) + |U_{\ell 4}|^2\rho(\delta^{(M)}_{\ell},\delta^{(M)}_{\nu_4})}{(1-|U_{\ell' 4}|^2)\rho(\delta^{(M)}_{\ell'},0) + |U_{\ell' 4}|^2\rho(\delta^{(M)}_{\ell'},\delta^{(M)}_{\nu_4})}\right](1+\delta_{\mathrm{RC}}), \tag{6.11}$$

where $\delta^{(M)}_{\ell}$ and $\delta^{(M)}_{\nu_4}$ were defined in Eq. (5.3). In Eq. (6.11) we have used the fact that the leading order radiative correction is independent of m_{ν_4} [104]. As noted above, in analyzing experimental data, one must take account of the fact that unless an experiment is specifically searching for effects of possible heavy neutrino emission, it would normally set cuts on the energy and/or momentum of the outgoing charged lepton near to the value for the SM decay. It would thus reject events due to a sufficiently massive ν_4 and would thus measure an apparent total rate that would be reduced from the actual rate by the factor $(1-|U_{e4}|^2)$.

Combining Eqs. (6.2) and (6.11), we have, for the ratio of branching ratios divided by the SM prediction, $\bar{R}^{(M)}_{\ell/\ell'}$,

$$\bar{R}^{(M)}_{\ell/\ell'} = \frac{1-|U_{\ell 4}|^2 + |U_{\ell 4}|^2\bar{\rho}(\delta^{(M)}_{\ell},\delta^{(M)}_{\nu_4})}{1-|U_{\ell' 4}|^2 + |U_{\ell' 4}|^2\bar{\rho}(\delta^{(M)}_{\ell'},\delta^{(M)}_{\nu_4})}. \tag{6.12}$$

With a given M, one can distinguish three different intervals for m_{ν_4}:

(i) $I^{(M)}_1$: $m_{\nu_4} < m_M - m_{\ell'}$,

(ii) $I^{(M)}_2$: $m_M - m_{\ell'} < m_{\nu_4} < m_M - m_{\ell}$, and

(iii) $I^{(M)}_3$: $m_{\nu_4} > m_M - m_{\ell}$.

Thus, (i) if $m_{\nu_4} \in I^{(M)}_1$, then both the $M^+ \to \ell^+\nu_4$ and $M^+ \to \ell'^+\nu_4$ decays can occur; (ii) if $m_{\nu_4} \in I^{(M)}_2$, then the $M^+ \to \ell^+\nu_4$ can occur, but the $M^+ \to \ell'^+\nu_4$ decay is kinematically forbidden; and finally, (iii) if $m_{\nu_4} \in I^{(M)}_3$, then both of the decays $M^+ \to \ell^+\nu_4$ and $M^+ \to \ell'^+\nu_4$ are kinematically forbidden. We recall the values of these intervals for the comparison of the branching ratios for M_{e2} and $M_{\mu 2}$ decays with $M = \pi^+$ and $M = K^+$ (where we use the standard notation $M_{\ell 2}$ for the decay $M^+ \to \ell^+\nu_\ell$). Here, the mass intervals are

(i) $I^{(\pi)}_1$: $m_{\nu_4} < 33.91$ MeV,

(ii) $I^{(\pi)}_2$: $33.91 \text{ MeV} < m_{\nu_4} < 139.1$ MeV,

(iii) $I^{(\pi)}_3$: $m_{\nu_4} > 139.1$ MeV.

(iv) $I^{(K)}_1$: $m_{\nu_4} < 388.0$ MeV,

(v) $I^{(K)}_2$: $388.0 \text{ MeV} < m_{\nu_4} < 493.2$ MeV, and

(vi) $I^{(K)}_3$: $m_{\nu_4} > 493.2$ MeV.

The general forms of Eq. (6.12) are

$$|U_{\ell 4}|^2 < \frac{[1+|U_{\ell' 4}|^2(\bar{\rho}(\delta^{(M}_{\ell'},\delta^{(M)}_{\nu_4}) - 1)]\bar{R}^{(M)}_{\ell/\ell'} - 1}{\bar{\rho}(\delta^{(M}_{\ell},\delta^{(M)}_{\nu_4}) - 1} \quad \text{for } m_{\nu_4} \in I^{(M)}_1 \quad \text{and} \tag{6.13}$$

$$\bar{R}^{(M)}_{\ell/\ell'} = \frac{1-|U_{\ell 4}|^2 + |U_{\ell 4}|^2\bar{\rho}(\delta^{(M)}_{\ell},\delta^{(M)}_{\nu_4})}{1-|U_{\ell' 4}|^2} \quad \text{for } m_{\nu_4} \in I^{(M)}_2. \tag{6.14}$$

Consequently, for $m_{\nu_4} \in I^{(M)}_2$, from the upper limit on the deviation of $\mathrm{BR}(M^+ \to \ell^+\nu_\ell)/\mathrm{BR}(M^+ \to \ell'^+\nu_{\ell'})$ from its

SM value, i.e., the upper limit on the deviation of $\bar{R}^{(M)}_{\ell/\ell'}$ from 1, an upper bound on $|U_{\ell 4}|^2$ can be obtained. Then,

$$|U_{\ell 4}|^2 < \frac{(1-|U_{\ell' 4}|^2)\bar{R}_{M;\ell/\ell'} - 1}{\bar{\rho}(\delta_\ell^{(M}, \delta_{\nu_4}^{(M)}) - 1} \quad \text{for } m_{\nu_4} \in I_2^{(M)}. \tag{6.15}$$

If $m_{\nu_4} \in I_3^{(M)}$, then Eq. (6.12) takes the still simpler form

$$\bar{R}^{(M)}_{\ell/\ell'} = \frac{1-|U_{\ell 4}|^2}{1-|U_{\ell' 4}|^2} \quad \text{for } m_{\nu_4} \in I_3^{(M)}. \tag{6.16}$$

In general, if for a given m_{ν_4}, one knows, e.g., from peak-search experiments, that $|U_{\ell' 4}|^2$ is sufficiently small that the denominator of (6.12) can be approximated well by 1, then an upper bound on the deviation of $\bar{R}^{(M)}_{\ell/\ell'}$ from 1 yields an upper bound on $|U_{\ell 4}|^2$:

$$|U_{\ell 4}|^2 < \frac{\bar{R}^{(M)}_{\ell/\ell'} - 1}{\bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}) - 1}. \tag{6.17}$$

For cases in which $\ell = e$, this gives very stringent upper limits on $|U_{\ell 4}|^2$ because $\bar{\rho}(\delta_e^{(M}, \delta_{\nu_4}^{(M)}) \gg 1$ over much of the intervals $I_1^{(M)}$ and $I_2^{(M)}$, as can be seen from Figs. 3–5 in [38]. For $m_{\nu_4} \in I_3^{(M)}$, the inequality (6.17) takes a simpler form, since $\bar{\rho}(\delta_\ell^{(M)}, \delta_{\nu_4}^{(M)}) = 0$, namely

$$|U_{\ell 4}|^2 < 1 - \bar{R}^{(M)}_{\ell/\ell'} \quad \text{for } m_{\nu_4} \in I_3^{(M)}. \tag{6.18}$$

We now apply this analysis to $R^{(\pi)}_{e/\mu}$, using (6.17) and (6.18) with $M = \pi^+$, $\ell = e$, and $\ell' = \mu$. From previous $\pi_{\mu 2}$ peak search experiments [77,83] and the recent [96], and the calculation of $\bar{\rho}(\delta_\mu^{(\pi)}, \delta_{\nu_4}^{(\pi)})$, it follows that $|U_{\mu 4}|^2$ is sufficiently small for $m_{\nu_4} \in I_2^{(\pi)}$ that we can approximate the denominator of Eq. (6.12) by 1. From $\bar{R}^{(\pi)}_{e/\mu}$ in Eq. (6.7), using the procedure from [106], we obtain the limit $\bar{R}^{(\pi)}_{e/\mu} < 1.0014$. Then, for $\nu_4 \in I_2^{(\pi)}$, we find

$$|U_{e4}|^2 < \frac{\bar{R}^{(\pi)}_{e/\mu} - 1}{\bar{\rho}(\delta_e^{(\pi)}, \delta_{\nu_4}^{(\pi)}) - 1} < \frac{0.0014}{\bar{\rho}(\delta_e^{(\pi)}, \delta_{\nu_4}^{(\pi)}) - 1}. \tag{6.19}$$

This bound is labeled as PIENU in Fig. 1. If $m_{\nu_4} \in I_3^{(\pi)}$, i.e., $m_{\nu_4} > 139$ MeV, then, using (6.18), we obtain the upper bound on $|U_{e4}|^2$ given by the flat line labeled PIENU-H in Fig. 1.

We next obtain a bound on $|U_{e4}|^2$ by applying the same type of analysis to $R^{(K)}_{e/\mu}$. From $K_{\mu 2}$ peak search experiments [78,89,94,95] and the calculation of $\bar{\rho}(\delta_\mu^{(K)}, \delta_{\nu_4}^{(K)})$, $|U_{\mu 4}|^2$ is sufficiently small that we can approximate the denominator of Eq. (6.12) well by 1. Using Eq. (6.10) for $\nu_4 \in I_2^{(K)}$, we find

$$|U_{e4}|^2 < \frac{\bar{R}^{(K)}_{e/\mu} - 1}{\bar{\rho}(\delta_e^{(K)}, \delta_{\nu_4}^{(K)}) - 1} < \frac{0.010}{\bar{\rho}(\delta_e^{(K)}, \delta_{\nu_4}^{(K)}) - 1}. \tag{6.20}$$

This upper limit on $|U_{e4}|^2$ is labeled KENU in Fig. 1. For $m_{\nu_4} \in I_3^{(K)}$, i.e., $m_{\nu_4} > 493$ MeV, using ((6.18), we obtain the flat upper bound labeled KENU-H in Fig. 1.

VII. BOUNDS FROM LEPTONIC DECAYS OF HEAVY-QUARK MESONS

Two-body leptonic decays of heavy-quark pseudoscalar mesons [37,38] are also valuable sources of information on sterile neutrinos. We discuss the available bounds in this section.

A. Bounds from $D_s \to \ell^+ \nu_\ell$ decays

The two-body leptonic decays of the $D_s^+ = (c\bar{s})$ involve a large CKM mixing matrix factor $|V_{cs}|^2$. Two of these have been measured by the CLEO [107], *BABAR* [108], Belle [109], and BES III [110–112] experiments, yielding the current values

$$\mathrm{BR}(D_s^+ \to \mu^+ \nu_\mu) = (5.49 \pm 0.17) \times 10^{-3} \tag{7.1}$$

and

$$\mathrm{BR}(D_s^+ \to \tau^+ \nu_\tau) = (5.48 \pm 0.23) \times 10^{-2}. \tag{7.2}$$

Equation (7.1) is a weighted average of CLEO, *BABAR*, Belle, and earlier BES III measurements, combined with the most recent BES III result, $\mathrm{BR}(D_s^+ \to \mu^+ \nu_\mu) = (5.49 \pm 0.16_{\mathrm{stat}} \pm 0.15_{\mathrm{syst}}) \times 10^{-3}$ (both this new result and the weighted average (7.1) are reported in Ref. [111]).

Searches for $D_s^+ \to e^+ \nu_e$ have been carried out by CLEO [107], *BABAR* [108], and Belle [109], giving the current upper bound

$$\mathrm{BR}(D_s^+ \to e^+ \nu_e) < 0.83 \times 10^{-4}. \tag{7.3}$$

Hence, for the ratio of the e and τ branching ratios, one has the resultant upper limit

$$R^{(D_s)}_{e/\tau} = \frac{\mathrm{BR}(D_s^+ \to e^+ \nu_e)}{\mathrm{BR}(D_s^+ \to \tau^+ \nu_\tau)}\bigg|_{\mathrm{exp}} < 1.6 \times 10^{-3}. \tag{7.4}$$

For this ratio, from [100,104] we calculate the radiative correction $1 + \delta_{\mathrm{RC}} = 0.948$. Substituting this in Eq. (6.2) with $M = D_s$, $\ell = e$, and $\ell' = \tau$, we find that in the SM, this ratio of branching ratios is

$$R^{(D_s)}_{e/\tau,\mathrm{SM}} = \frac{\mathrm{BR}(D_s^+ \to e^+\nu_e)_{\mathrm{SM}}}{\mathrm{BR}(D_s^+ \to \tau^+\nu_\tau)_{\mathrm{SM}}} = 2.29 \times 10^{-6}. \quad (7.5)$$

Hence, the current experimental upper limit on $\mathrm{BR}(D_s^+ \to e^+\nu_e)$ yields the upper limit $\bar{R}^{(D_s)}_{e/\tau} < 7.0 \times 10^2$. Note that $m_{D_s} - m_\tau = 191$ MeV.

For $R^{(D_s)}_{e/\tau}$, the interval $I_2^{(D_s)}$ is 191 MeV $< m_{\nu_4} <$ 1.457 GeV. We restrict m_{ν_4} to a lower-mass subset of this full interval, for the following reason. In the $D_s^+ \to e^+\nu_4$ decay, as m_{ν_4} increases from small values to its kinematic limit, the momentum of the outgoing e^+ in the rest frame of the parent D_s decreases from its SM value, $p_e \equiv |\mathbf{p}_e| =$ 0.984 GeV. In order for the event reconstruction procedure in a given experiment to count such a decay as a $D_s^+ \to e^+\nu_e$ decay, it is necessary that $p_e > p_{e,cut}$, where $p_{e,cut}$ denotes a lower experimental cut on p_e. A representative value of this cut is the value $p_{e,cut} = 0.8$ GeV used in the BES III experiment [112]. The e^+ momentum decreases to $p_e = 0.8$ GeV as m_{ν_4} reaches the value $m_{\nu_4} = 0.85$ GeV. Thus, we consider the interval 0.191 GeV $< m_{\nu_4} <$ 0.85 GeV. For m_{ν_4} in this interval, the ratio of branching ratios of the observed $D_s^+ \to e^+\nu_e$ and $D_s^+ \to \tau^+\nu_\tau$ decays is given by Eq. (6.12) with $M = D_s^+$, $\ell = e$, and $\ell' = \tau$. Hence, from Eq. (6.14), this ratio of branching ratios, divided by the value in the SM, is

$$\bar{R}^{(D_s)}_{e/\tau} = \frac{1 - |U_{e4}|^2 + |U_{e4}|^2 \bar{\rho}(\delta_e^{(D_s)}, \delta_{\nu_4}^{(D_s)})}{1 - |U_{\tau 4}|^2}. \quad (7.6)$$

Requiring that the emission of the ν_4 should not alter the experimentally observed upper limit on $\bar{R}^{(D_s)}_{e/\tau}$ given above, we obtain the following upper bound on $|U_{e4}|^2$ for m_{ν_4} in this mass range, which is the special case of (6.15) with $M = D_s$, $\ell = e$, and $\ell' = \tau$:

$$|U_{e4}|^2 < \frac{(1 - |U_{\tau 4}|^2)\bar{R}^{(D_s)}_{e/\tau,ul} - 1}{\bar{\rho}(\delta_e^{(D_s)}, \delta_{\nu_4}^{(D_s)}) - 1}. \quad (7.7)$$

This limit is largely independent of the $|U_{\tau 4}|^2$ term, since $|U_{\tau 4}|^2$ is constrained to be less than upper bounds ranging from ∼0.1 to ∼0.01 for m_{ν_4} in this mass range [42,44,48]. For the minimal value of m_{ν_4} taken here, namely $m_{\nu_4} = 0.191$ GeV, the $\bar{\rho}$ function in Eq. (7.7) is already quite large, having the value 1.37×10^5. As m_{ν_4} increases to 0.85 GeV, this $\bar{\rho}$ function increases to 1.83×10^6. Thus, over this range of m_{ν_4}, the upper limit on $|U_{e4}|^2$ in (7.7) decreases from $|U_{e4}|^2 < 5.1 \times 10^{-3}$ to $|U_{e4}|^2 < 3.8 \times 10^{-4}$. We thus obtain the upper bound on $|U_{e4}|^2$ labeled D_{se2} in Fig. 1. In the interval 450 MeV $< m_{\nu_4} <$ 850 MeV, these upper bounds on $|U_{e4}|^2$ (denoted as D_{se2} in Fig. 1) are the best available. As was pointed out in [1], dedicated peak-search experiments to search for the heavy-neutrino decays $D_s^+ \to e^+\nu_4$ and $D^+ \to e^+\nu_4$ would be worthwhile and could improve our upper bound on $|U_{e4}|^2$.

In addition to the comparison of the branching ratios $\mathrm{BR}(D_s^+ \to e^+\nu_e)$ and $\mathrm{BR}(D_s^+ \to \tau^+\nu_\tau)$, it is also useful to comment on the comparison of $\mathrm{BR}(D_s^+ \to \mu^+\nu_\mu)$ and $\mathrm{BR}(D_s^+ \to \tau^+\nu_\tau)$, both of which have been measured. From the experimental results (7.1) and (7.2), the resultant measured ratio of branching ratios is

$$R^{(D_s)}_{\mu/\tau} = 0.100 \pm 0.005. \quad (7.8)$$

Substituting our calculated $1 + \delta_{\mathrm{RC}} = 0.985$ for this decay in the general formula (6.2), we find that the SM prediction for the branching ratio is

$$R^{(D_s)}_{\mu/\tau,\mathrm{SM}} = 0.101, \quad (7.9)$$

so to this order,

$$\bar{R}^{(D_s)}_{\mu/\tau} = 0.990 \pm 0.05. \quad (7.10)$$

This yields the upper limit $\bar{R}^{(D_s)}_{\mu/\tau} < \bar{R}^{(D_s)}_{\mu/\tau,ul}$, where

$$\bar{R}^{(D_s)}_{\mu/\tau,ul} = 1.05. \quad (7.11)$$

Emission of a ν_4 with non-negligible mass would change the ratio (7.10) to the expression in Eq. (6.12) with $M = D_s$, $\ell = \mu$, and $\ell' = \tau$. The interval $I_2^{(D_s)}$ for this decay is 191 MeV $< m_{\nu_4} <$ 1.863 GeV, and for m_{ν_4} in this interval, Eq. (6.12) reduces to the expression in (6.14) with $M = D_s$, $\ell = \mu$, and $\ell' = \tau$. The maximum value of m_{ν_4} to enable a large enough p_μ to satisfy an experimental lower momentum cut of 0.8 GeV is $m_{\nu_4} = 0.84$ GeV, which is almost the same as for the $D_s^+ \to e^+\nu_4$ decay. We thus obtain an upper limit on $|U_{\mu 4}|^2$ which is the special case of (6.15) with $M = D_s$, $\ell = \mu$, and $\ell' = \tau$, namely

$$|U_{\mu 4}|^2 < \frac{(1 - |U_{\tau 4}|^2)\bar{R}^{(D_s)}_{\mu/\tau,ul} - 1}{\bar{\rho}(\delta_\mu^{(D_s)}, \delta_{\nu_4}^{(D_s)}) - 1}. \quad (7.12)$$

Given that $|U_{\tau 4}|^2 \ll 1$, this reduces to the special case of Eq. (6.17) with $M = D_s$, $\ell = \mu$, and $\ell' = \tau$. For $m_{\nu_4} = 0.191$ GeV, the $\bar{\rho}$ function in Eq. (7.12) has the value 4.22. As m_{ν_4} increases to 0.84 GeV, this $\bar{\rho}$ function increases to 43.4. With $|U_{\tau 4}|^2 \ll 1$, the resultant upper bound on $|U_{\mu 4}|^2$ is shown in Fig. 2. This bound decreases from $\sim 10^{-2}$ to $\sim 10^{-3}$ over this range of m_{ν_4}. In the lower part of this interval, 0.22 GeV $< m_{\nu_4} <$ 0.38 GeV, the BNL E949 and NA62 peak search experiments with $K_{\mu 2}$ decay have set more stringent upper bounds, but in the upper part of the interval, between $m_{\nu_4} = 0.46$ GeV and $m_{\nu_4} = 0.84$ GeV, our upper bound on $|U_{\mu 4}|^2$ from this analysis of D_s decays is the best current direct laboratory upper bound.

B. Bounds from $D^+ \to \ell^+ \nu_\ell$ leptonic decays

In the case of the D^+ meson, the $c\bar{d}$ annihilation amplitude is suppressed by the CKM factor $|V_{cd}|^2$ relative to semileptonic and hadronic decay channels, which can proceed by $c \to s$ charged-current vertices and hence involve the much larger $|V_{cs}|^2$ factor in the rates. There is significant phase-space suppression of the $D^+ \to \tau^+ \nu_\tau$ channel, since $m_{D^+} - m_\tau$ is only 92.8 MeV. For one of these leptonic D decays, one has an upper limit, namely

$$\mathrm{BR}(D^+ \to e^+ \nu_e) < 0.88 \times 10^{-5}. \tag{7.13}$$

The branching ratio for $D^+ \to \mu^+ \nu_\mu$ has been measured by CLEO and BES III [13,113,114] as

$$\mathrm{BR}(D^+ \to \mu^+ \nu_\mu) = (3.74 \pm 0.17) \times 10^{-4}. \tag{7.14}$$

Recently, BES III has measured the branching ratio for $D^+ \to \tau^+ \nu_\tau$ [115] as

$$\mathrm{BR}(D^+ \to \tau^+ \nu_\tau) = (1.20 \pm 0.24_{\mathrm{stat}} \pm 0.12_{\mathrm{syst}}) \times 10^{-3}. \tag{7.15}$$

With the radiative correction $1 + \delta_{\mathrm{RC}} = 0.963$, the SM prediction for the ratio of these branching ratios is, from Eq. (6.1),

$$\bar{R}^{(D)}_{e/\mu,\mathrm{SM}}|_{\mathrm{SM}} = 2.27 \times 10^{-5}. \tag{7.16}$$

From the experimental limit (7.13) and measurement (7.14), we have the 90% C.L. upper limit

$$\bar{R}^{(D)}_{e/\mu}|_{\mathrm{exp}} < 2.5 \times 10^{-2}. \tag{7.17}$$

With ν_4 emission, this ratio would be changed to

$$\bar{R}^{(D)}_{e/\mu} = \frac{1 - |U_{e4}|^2 + |U_{e4}|^2 \bar{\rho}(\delta_e^{(D)}, \delta_{\nu_4}^{(D)})}{1 - |U_{\mu 4}|^2 + |U_{\mu 4}|^2 \bar{\rho}(\delta_\mu^{(D)}, \delta_{\nu_4}^{(D)})}. \tag{7.18}$$

Requiring that $\bar{R}^{(D)}_{e/\mu}$ not violate the upper bound (7.17) yields correlated upper limits on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$ as a function of m_{ν_4}.

C. Bounds from $B^+ \to \ell^+ \nu_\ell$ leptonic decays

Here we analyze constraints from two-body leptonic B^+ decays. These decays involve $u\bar{b}$ annihilation and hence are suppressed by the small CKM factor $|V_{ub}|^2$ relative to semileptonic and hadronic B^+ decays involving the larger CKM factor $|V_{cb}|^2$. Currently, there is an upper limit on one leptonic B^+ decay,

$$\mathrm{BR}(B^+ \to e^+ \nu_e) < 0.98 \times 10^{-6} \tag{7.19}$$

from Belle [116] and *BABAR* [117], and measurements of the other two, namely

$$\mathrm{BR}(B^+ \to \mu^+ \nu_\mu) = (6.46 \pm 2.22_{\mathrm{stat}} \pm 1.60_{\mathrm{syst}}) \times 10^{-7} \tag{7.20}$$

from Belle [118],

$$\mathrm{BR}(B^+ \to \mu^+ \nu_\mu) = (5.3 \pm 2.0_{\mathrm{stat}} \pm 0.9_{\mathrm{syst}}) \times 10^{-7} \tag{7.21}$$

from a Belle update [119,120], and

$$\mathrm{BR}(B^+ \to \tau^+ \nu_\tau) = (1.09 \pm 0.24) \times 10^{-4} \tag{7.22}$$

from *BABAR* [121] and Belle [122,123]. Both the published and preliminary updated values of the $\mathrm{BR}(B^+ \to \mu^+ \nu_\mu)$ are in agreement with the SM prediction [118]

$$\mathrm{BR}(B^+ \to \mu^+ \nu_\mu)_{\mathrm{SM}} = (3.80 \pm 0.31) \times 10^{-7}. \tag{7.23}$$

The measured value of $\mathrm{BR}(B^+ \to \tau^+ \nu_\tau)$ in (7.22) is also in agreement with the SM prediction [123,124]

$$\mathrm{BR}(B^+ \to \tau^+ \nu_\tau)_{\mathrm{SM}} = (0.75^{+0.10}_{-0.05}) \times 10^{-4}. \tag{7.24}$$

From [100] we calculate the radiative correction factor $1 + \delta_{\mathrm{RC}} = 0.942$ for $R^{(B)}_{e/\tau,\mathrm{SM}}$. Combining this with (7.22) and (6.2), we then obtain the SM prediction

$$\mathrm{BR}(B^+ \to e^+ \nu_e)_{\mathrm{SM}} = (1.08 \pm 0.24) \times 10^{-11}. \tag{7.25}$$

A recent experiment to search for $B^+ \to e^+ X^0$ and $B^+ \to \mu^+ X^0$ was carried out by Belle [125], where X^0 is a weakly interacting particle that does not decay in the detector. Assuming that $X^0 = \nu_4$, one can use the results of this experiment to set upper limits on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$. For m_{ν_4} in the range from 0.1 GeV to 1.4 GeV, this experiment obtained an upper limit on $\mathrm{BR}(B^+ \to e^+ \nu_4)$ of 2.5×10^{-6}, while in the interval of m_{ν_4} from 1.4 GeV to 1.8 GeV, this upper limit increased to 7×10^{-6}. In the range of m_{ν_4} from 0.1 to 1.3 GeV, the experiment obtained (nonmonotonic) upper limits on $\mathrm{BR}(B^+ \to \mu^+ \nu_4)$ of approximately 2×10^{-6} to 4×10^{-6}, and in the interval of m_{ν_4} from 1.3 GeV to 1.8 GeV, it obtained upper limits varying from 2×10^{-6} to 1.1×10^{-5}. These limits are less restrictive than the bounds (7.19) and (7.20), but have the advantage of being reported for specific values of m_{ν_4}.

Substituting the experimental upper limit on $\mathrm{BR}(B^+ \to e^+\nu_4)$ as a function of m_{ν_4} from [125] into the relevant special case of (5.15) with $M = B^+$ and $\ell = e$, we obtain the upper bound on $|U_{e4}|^2$ as a function of m_{ν_4} shown in Fig. 1. This upper bound decreases from 0.83 to 3.4×10^{-2} as m_{ν_4} increases from 0.1 to 1.2 GeV. Since the experimental upper limit on $\mathrm{BR}(B^+ \to e^+\nu_4)$ is less stringent as m_{ν_4} increases from 1.4 to 1.8 GeV, the same is true of the resultant upper limit on $|U_{e4}|^2$; for example, if $m_{\nu_4} = 1.6$ GeV, we get $|U_{e4}|^2 < 5.4 \times 10^{-2}$.

Carrying out the analogous procedure with the upper bound on $\mathrm{BR}(B^+ \to \mu^+\nu_4)$ from [125], we obtain an upper limit on $|U_{\mu4}|^2$ that decreases from 0.83 to 3.4×10^{-2} as m_ν increases from 0.1 GeV to 1.2 GeV. As m_{ν_4} increases from 1.2 to 1.5 GeV and then to 1.8 GeV, the upper limit on $\mathrm{BR}(B^+ \to \mu^+\nu_4)$ from [125] rises from approximately 3×10^{-6} to 1.1×10^{-5} and then decreases again to 3×10^{-6}. In this interval of m_{ν_4} masses, using the appropriate special case of (5.15), we obtain upper limits ranging from $|U_{\mu4}|^2$ of 0.12 for $m_{\nu_4} = 1.5$ GeV to $|U_{\mu4}|^2$ of 2.7×10^{-2} at $m_{\nu_4} = 1.8$ GeV. See also [48]. Further peak searches for $B^+ \to e^+\nu_4$ and $B^+ \to \mu^+\nu_4$ with Belle II would be valuable and could improve the limits from Ref. [125]. Moreover, when measurements of two-body leptonic decays of B_c^+ mesons become available, it would also be of interest to use them to constrain lepton mixing matrix coefficients.

As was true for the other decays, in obtaining these limits from leptonic B decays, it is assumed that the only new physics is the emission of the massive ν_4. However, in the B system there are currently several quantities whose experimental measurements are in possible tension with SM predictions, including, for example, the ratios of branching ratios $R(D^{(*)}) = \mathrm{BR}(B \to D^{(*)}\tau\bar{\nu}_\tau)/\mathrm{BR}(B \to D^{(*)}\ell\bar{\nu}_\ell)$, where $\ell = e, \mu$, and the ratio $R(K^{(*)}) = \mathrm{BR}(B \to K^{(*)}e^+e^-)/\mathrm{BR}(B \to K^{(*)}\mu^+\mu^-)$ (see, e.g., [126,127]).

VIII. CONSTRAINTS FROM μ DECAY

A. General analysis with massive neutrino emission

In this section we discuss constraints from μ decays. The lifetime of the μ^+ was measured to 0.5 ppm accuracy by the MuLan experiment at PSI [128], yielding the value $G_F = 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$ with the implicit assumption of decays only into the three known neutrino mass eigenstates. With this assumption, the uncertainty in this determination of G_F is mainly from the experimental measurement; it is estimated that the uncertainty due to radiative corrections [129–131] is approximately 0.14 ppm and the uncertainty from the measured value of m_μ is 0.08 ppm [128].

However, as was pointed out and analyzed in [37,39], in the presence of neutrino masses and lepton mixing, the decay $\mu \to \nu_\mu e\bar{\nu}_e$ would actually consist of the decays $\mu \to \nu_i e\bar{\nu}_j$ into the individual mass eigenstates ν_i and $\bar{\nu}_j$ in the interaction eigenstates ν_μ and $\bar{\nu}_e$, where $1 \le i, j \le 3 + n_s$, as allowed by phase space. The emission of massive neutrino(s) with non-negligible mass(es) in muon decay would produce several changes relative to the Standard Model. These include (i) kink(s) in the observed electron energy spectrum associated with the fact that the maximum electron energy in the rest frame of the parent μ is reduced from its SM value with neutrinos of negligibly small masses,

$$E_{e,\mathrm{max}} = \frac{m_\mu^2 + m_e^2}{2m_\mu} \tag{8.1}$$

to

$$E_{e,\mathrm{max},ij} = \frac{m_\mu^2 + m_e^2 - (m_{\nu_i} + m_{\nu_j})^2}{2m_\mu}; \tag{8.2}$$

(ii) reduction of the differential and total decay rate; (iii) a reduction in the apparent value of the Fermi coupling G_F, relative to its value in the Standard Model with neutrinos of negligibly small masses; and (iv) changes in the spectral parameters ρ and η, and, for a polarized muon, ξ, and δ, that have been used to fit the differential decay spectrum of the muon. Reference [39] calculated the changes in these spectral parameters that would be caused by emission of a massive (anti)neutrino in μ decay and used existing data to set upper limits on lepton mixing coefficients as functions of neutrino mass. From data on the ρ parameter describing the e^+ momentum distribution in unpolarized μ^+ decay, Ref. [39] derived an upper limit on $|U_{r4}|^2$, where $r = e, \mu$ in the interval m_{ν_4} up to 70 MeV, extending down to a few times 10^{-3} at $m_{\nu_4} = 30$ MeV. This constraint applies to both $|U_{e4}|^2$ and $|U_{\mu4}|^2$ since the ν_4 or $\bar{\nu}_4$ can be emitted at either the charged-current vertex with the μ or with the e. The upper bound on $|U_{e4}|^2$ from μ decay is not as restrictive as upper bounds from π_{e2} or K_{e2} decay. However, the upper bound on $|U_{\mu4}|^2$ from μ decay is valuable for an interval of m_{ν_4} that is not covered by peak search experiments, namely the interval above the kinematic endpoint for $\pi_{\mu2}$ decay at $m_{\nu_4} = 33.9$ MeV and below the value of $m_{\nu_4} \simeq 40$ MeV, which was the lowest value at which a $K_{\mu2}$ peak search experiment (at KEK [82]) obtained an upper limit on $|U_{\mu4}|^2$. In [40,132] it was pointed out that because, in the presence of massive neutrino emission in μ decay, the value of $G_{F,\mathrm{app}}$ extracted in the framework of the SM is smaller than the true value of G_F, this would lead to predictions of the masses of the W and Z, that would be larger than the true values, and these effects were calculated. Subsequent discussions of massive neutrino effects in μ decay include [32,48,133–135]. In particular, the TWIST experiment at TRIUMF measured ρ with greater accuracy [49]. Using an analysis similar to that in [39] applied to the

TWIST data, one obtains upper limits on $|U_{\mu 4}|^2$ extending down to 2×10^{-3} at $m_{\nu_4} = 30$ MeV (e.g., [48]).

Let us consider the change in the total rate as a consequence of muon decays to a neutrino mass eigenstate ν_4 with a non-negligible mass. In the SM with neutrinos of negligibly small mass, the rate for μ decay has the form

$$\Gamma_{\mu,\mathrm{SM}} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1+\delta_\alpha)\bar{\Gamma}_{\mu,\mathrm{SM}}. \tag{8.3}$$

Here we have separated off a rate factor

$$\bar{\Gamma}_{\mu,\mathrm{SM}} = f(a_e^{(\mu)}, 0, 0), \tag{8.4}$$

where f is a dimensionless kinematic function resulting from the integration over the three-body final-state phase space, which depends on three arguments, namely the (squares of the) ratios of each of the final-state particle masses to the muon mass. Finally, in Eq. (8.3), the δ_α term incorporates electroweak corrections and has the leading-order value $\delta_\alpha = -[\alpha_{em}/(2\pi)][\pi^2 - (25/4)] = -4.2 \times 10^{-3}$ [129]. For the SM with neutrinos of negligibly small mass, the kinematic function f is

$$f(a,0,0) = (1 - 8a + a^2)(1 - a^2) + 12a^2 \ln\left(\frac{1}{a}\right) \tag{8.5}$$

with

$$a_e^{(\mu)} = \frac{m_e^2}{m_\mu^2} = 2.339010 \times 10^{-5}. \tag{8.6}$$

Numerically, $f(a_e^{(\mu)}, 0, 0) = 1 - (1.87 \times 10^{-4})$. The SM kinematic function for μ decay has the series expansion

$$f(a,0,0) = 1 - 8a + O(\{a^2, a^2 \ln a\}) \tag{8.7}$$

with $a = a_e^{(\mu)}$. Because $a_e^{(\mu)} \ll 1$, $f(a_e^{(\mu)}, 0, 0)$ is very well approximated, to three-figure accuracy, by the first two terms in its series expansion, $1 - 8a_e^{(\mu)}$.

For our case, from the general formulas in [39], the μ decay rate is given by

$$\begin{aligned}\bar{\Gamma}_\mu &= (1-|U_{e4}|^2)(1-|U_{\mu 4}|^2) f(a_e^{(\mu)},0,0) + (1-|U_{e4}|^2)|U_{\mu 4}|^2 f(a_e^{(\mu)},0,a_{\nu_4}^{(\mu)}) \\ &\quad + |U_{e4}|^2(1-|U_{\mu 4}|^2) f(a_e^{(\mu)},a_{\nu_4}^{(\mu)},0) + |U_{e4}|^2|U_{\mu 4}|^2 f(a_e^{(\mu)},a_{\nu_4}^{(\mu)},a_{\nu_4}^{(\mu)}) \\ &= \Gamma_{\mu,\mathrm{SM}}[(1-|U_{e4}|^2)(1-|U_{\mu 4}|^2) + (1-|U_{e4}|^2)|U_{\mu 4}|^2 \bar{f}(a_e^{(\mu)},0,a_{\nu_4}^{(\mu)}) \\ &\quad + |U_{e4}|^2(1-|U_{\mu 4}|^2)\bar{f}(a_e^{(\mu)},a_{\nu_4}^{(\mu)},0) + |U_{e4}|^2|U_{\mu 4}|^2 \bar{f}(a_e^{(\mu)},a_{\nu_4}^{(\mu)},a_{\nu_4}^{(\mu)})],\end{aligned} \tag{8.8}$$

where $\bar{f}(x,y,z))$ is the ratio of the kinematic phase space integral for each of the decays divided by the kinematic integral for the SM decay (8.5):

$$\bar{f}(x,y,z) = \frac{f(x,y,z)}{f(x,0,0)} \tag{8.9}$$

with

$$a_{\nu_4}^{(\mu)} = \frac{m_{\nu_4}^2}{m_\mu^2}. \tag{8.10}$$

Here and below, the kinematic function $f(x,y,z) = 0$ if the decay is kinematically forbidden, i.e., if $\sqrt{x} + \sqrt{y} + \sqrt{z} \geq 1$. The four terms in Eq. (8.8) arise from the decays (a) $\mu \to \nu_i e \bar{\nu}_j$, (b) $\mu \to \nu_4 e \bar{\nu}_i$, (c) $\mu \to \nu_i e \bar{\nu}_4$, and (d) $\mu \to \nu_4 e \bar{\nu}_4$, where here ν_i and ν_j denote the known three neutrino mass eigenstates, whose masses are negligibly small in μ decay. Note that the second and third terms are present only if $m_\mu > m_e + \nu_4$, and the fourth term is present only if $m_\mu > m_e + 2m_{\nu_4}$. Furthermore, the fourth term is strongly suppressed because it involves the product of the squares of two small leptonic mixing matrix coefficients, $|U_{e4}|^2|U_{\mu 4}|^2$, and because of the smaller phase space if m_{ν_4}/m_μ is substantial. Hence, to evaluate Eq. (8.8) for $\bar{\Gamma}_\mu$, to a very good approximation, we may drop the last term, and hence we need only the kinematic function $f(x,y,0)$, which was calculated in Ref. [39]. A basic symmetry property of the kinematic function is that [39]

$$f(x,y,z) = f(x,z,y), \tag{8.11}$$

so the second and third terms in Eq. (8.8) have the same kinematic factor, $\bar{f}(a_e^{(\mu)},0,a_{\nu_4}^{(\mu)}) = \bar{f}(a_e^{(\mu)},a_{\nu_4}^{(\mu)},0)$.

The apparent value of the Fermi coupling, $G_{F,\mathrm{app}}$, obtained from the measurement of the μ decay rate is given by

$$\frac{G_{F,\mathrm{app}}^2}{G_F^2} = \frac{\Gamma_\mu}{\Gamma_{\mu,\mathrm{SM}}} \equiv \kappa \tag{8.12}$$

and is less than unity if (anti)neutrinos with non-negligible masses are emitted in μ decay [39]. Explicitly,

$$\frac{G_{F,\mathrm{app}}^2}{G_F^2} = (1-|U_{e4}|^2)(1-|U_{\mu4}|^2) + (1-|U_{e4}|^2)|U_{\mu4}|^2\bar{f}(a_e^{(\mu)},0,a_{\nu_4}^{(\mu)}) + |U_{e4}|^2(1-|U_{\mu4}|^2)\bar{f}(a_e^{(\mu)},a_{\nu_4}^{(\mu)},0) + |U_{e4}|^2|U_{\mu4}|^2\bar{f}(a_e^{(\mu)},a_{\nu_4}^{(\mu)},a_{\nu_4}^{(\mu)}). \tag{8.13}$$

In the SM, the predicted mass of the Z is determined in terms of $\alpha = e^2/(4\pi)$, the weak mixing angle $\theta_W = \arctan(g'/g)$, and $G_{F,\mathrm{app}}$ by

$$m_{Z,\mathrm{pred}} = \left(\frac{\pi\alpha}{2^{1/2}G_{F,\mathrm{app}}}\right)^{1/2} \frac{1}{\sin\theta_W\cos\theta_W}(1+\delta_{Z,\mathrm{RC}}), \tag{8.14}$$

and $m_{W,\mathrm{pred}} = m_{Z,\mathrm{pred}}\cos\theta_W$, where $\delta_{Z,\mathrm{RC}}$ is the radiative correction [136]. As pointed out in [40,132], in the presence of massive neutrino emission in μ decay, these predicted values of m_Z and m_W would be larger than the true values, since $G_{F,\mathrm{app}} < G_F$:

$$m_{Z,\mathrm{true}} = \kappa^{1/2} m_{Z,\mathrm{pred}} < m_{Z,\mathrm{pred}} \tag{8.15}$$

and

$$m_{W,\mathrm{true}} = \kappa^{1/2} m_{W,\mathrm{pred}} < m_{W,\mathrm{pred}}. \tag{8.16}$$

The effects on the W and Z widths were also discussed in [40,132]. The agreement between the predicted and observed masses and widths of the W and Z thus yield constraints on leptonic mixing angles as functions of m_{ν_4}. With current values of m_W, m_Z, Γ_W, and Γ_Z, these imply $|U_{\ell4}|^2 \lesssim 10^{-2}$ (e.g., [48]).

As mentioned above, the test of relative agreement of $\mathcal{F}t$ values obtained from the set of 14 superallowed nuclear beta decays in [63,64] is independent of $G_{F,\mathrm{app}}$ since this divides out in the ratios of the $\mathcal{F}t$ values. However, depending on m_{ν_4}, $|U_{e4}|^2$, and $|U_{\mu4}|^2$, the result would generically be that the value of $|V_{ud}|$ obtained from these nuclear beta decays would not be equal to the true value, because of both the reduction of the rates for the various nuclear beta decays and the fact that the value of $G_{F,\mathrm{app}}$ used in Eq. (2.1) would be different from the true value. In turn, this would generically lead to a spurious apparent violation of the first-row CKM unitarity test. Whether the apparent value of $|V_{ud}|$ would be larger or smaller than the true value would depend on the values of m_{ν_4}, $|U_{e4}|^2$, and $|U_{\mu4}|^2$ and thus on the relative effects of massive neutrino emission in muon decay and in the nuclear beta decays used to obtain $|V_{ud}|$.

Since the determination of $|V_{ud}|$ from the superallowed nuclear beta decays depends on the input value of $G_{F,\mathrm{app}}$ from muon decay, an apparent violation of the first-row CKM unitarity relation $\Sigma = 1$ could indicate the presence of effects of new physics beyond the Standard Model (BSM) in muon decay. Although our discussion above has focused on the effect of the possible emission of neutrino(s) of non-neglible masses and couplings in muon decay, we note that there could also be exotic muon decays in BSM scenarios that would appear observationally to be the same as $\mu^+ \to \bar{\nu}_\mu e^+ \nu_e$, i.e., $\mu^+ \to e^+ +$ missing neutrals, where the additional neutral particles are weakly interacting. An explicit example studied in the context of supersymmetric extensions of the SM was the decay $\mu^+ \to e^+\tilde{\gamma}\tilde{\gamma}$, where $\tilde{\gamma}$ denotes the photino [137]. An analogous decay involving hadrons was $K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma}$ [138], which would appear observationally as $K^+ \to \pi^+ +$ missing neutrals and hence would be experimentally indistinguishable from the SM decay $K^+ \to \pi^+\nu\bar{\nu}$ [139]. (In modern notation, these decays would be denoted as $\mu^+ \to e^+\tilde{\chi}^0\tilde{\chi}^0$ and $K^+ \to \pi^+\tilde{\chi}^0\tilde{\chi}^0$, where $\tilde{\chi}^0$ is a neutralino.) As was noted in [137], the existence of the decay $\mu^+ \to e^+\tilde{\gamma}\tilde{\gamma}$ by itself would lead to an apparent value of $G_{F,\mathrm{app}}$ larger than the true value, opposite to the effect of massive neutrino emission. Another possibility for an exotic μ decay is $\mu \to e + x$, where x is a neutral, light, weakly interacting boson; upper limits on this were given in [140,141]. Another example of this type of additional exotic μ decay was studied in a model with dynamical electroweak symmetry breaking [142], in which the $\mu^+ \to e^+$ transition would be mediated by a neutral virtual massive generation-changing vector boson, which then would produce a final-state $\bar{\nu}_\mu\nu_e$ pair (see also [143]).

B. Limit on exotic μ decay modes

If there are no light sterile neutrinos relevant for μ decay, but there are additional exotic muon decays such as in the examples above, then, since the experimentally extracted value of $G_{F,\mathrm{app}}$ would be larger than the true G_F, the resultant apparent value of $|V_{ud}|$ obtained from the superallowed nuclear beta decays, denoted $|V'_{ud}|$, would be smaller than the true value. In turn, this would yield an apparent spurious violation of CKM unitarity in which the apparent value of Σ would be less than unity. Since an exotic BSM decay channel would increase Γ_μ relative to the SM value $\Gamma_{\mu,\mathrm{SM}}$, while emission of heavy neutrino(s) would decrease Γ_μ relative to $\Gamma_{\mu,\mathrm{SM}}$, it is possible, in principle, for both of these non-SM effects to be present and to tend to cancel each other, yielding a resultant Γ_μ close to $\Gamma_{\mu,\mathrm{SM}}$. However, in the absence of any symmetry reason, such a cancellation may be regarded as unlikely. Accordingly, in our analyses, we will treat each of these two cases individually.

If one considers the possibility that no heavy sterile (anti)neutrinos are emitted in μ decay but instead, there is an exotic extra decay channel (indicated with subscript ext) with rate $\Gamma_{\mu,\mathrm{ext}}$, then the total decay rate would be $\Gamma_\mu = \Gamma_{\mu,\mathrm{SM}} + \Gamma_{\mu,\mathrm{ext}}$. Let us denote $\Gamma_{\mu,\mathrm{SM}} \equiv G_F^2 \hat{\Gamma}_{\mu,\mathrm{SM}}$ and the branching ratio of the exotic decay mode as $\mathrm{BR}_{\mu,\mathrm{ext}} = \Gamma_{\mu,\mathrm{ext}}/\Gamma_\mu$. Experimentalists would then extract the apparent value $G_{F,\mathrm{app}}$ as

$$G_{F,\mathrm{app}}^2 \hat{\Gamma}_{\mu,\mathrm{SM}} = \Gamma_\mu = G_F^2 \hat{\Gamma}_{\mu,\mathrm{SM}} + \Gamma_{\mu,\mathrm{ext}}, \tag{8.17}$$

so

$$\begin{aligned} \frac{G_{F,\mathrm{app}}^2}{G_F^2} &= 1 + \frac{\Gamma_{\mu,\mathrm{ext}}}{\Gamma_{\mu,\mathrm{SM}}} = 1 + \frac{\Gamma_{\mu,\mathrm{ext}}}{\Gamma_\mu - \Gamma_{\mu,\mathrm{ext}}} \\ &= 1 + \frac{\mathrm{BR}_{\mu,\mathrm{ext}}}{1 - \mathrm{BR}_{\mu,\mathrm{ext}}}. \end{aligned} \tag{8.18}$$

Assuming that the BSM physics responsible for the additional contribution, $\Gamma_{\mu,\mathrm{ext}}$, to μ decay does not affect nuclear beta decays, then the resultant apparent value of $|V'_{ud}|^2$ obtained from the superallowed nuclear beta decays would be given by $G_{F,\mathrm{app}}^2 |V'_{ud}|^2 = G_F^2 |V_{ud}|^2$, i.e.,

$$|V'_{ud}|^2 = \frac{|V_{ud}|^2}{1 + \mathrm{BR}_{\mu,\mathrm{ext}}}. \tag{8.19}$$

For our present analysis, let us further assume that the BSM physics leading to this value would not affect the decays used to determine $|V_{us}|$ and $|V_{ub}|$. The apparent value of Σ, denoted Σ_{app}, would then be

$$\begin{aligned} \Sigma_{\mathrm{app}} &= |V'_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \\ &= -\mathrm{BR}_{\mu,\mathrm{ext}} |V'_{ud}|^2 + |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \\ &= -\mathrm{BR}_{\mu,\mathrm{ext}} |V'_{ud}|^2 + \Sigma. \end{aligned} \tag{8.20}$$

Assuming CKM unitarity, i.e., $\Sigma = 1$, we then have

$$\mathrm{BR}_{\mu,\mathrm{ext}} = \frac{1 - \Sigma_{\mathrm{app}}}{|V'_{ud}|^2}. \tag{8.21}$$

Presuming that this is responsible for Σ_{app} being less than unity and using the experimentally determined value and uncertainty in Eq. (4.1),

$$\mathrm{BR}_{\mu,\mathrm{ext}} < 1.3 \times 10^{-3}. \tag{8.22}$$

IX. CONSTRAINTS FROM LEPTONIC τ DECAYS

As with nuclear beta decay and the two-body leptonic decays of pseudoscalar mesons, semihadronic τ decays have the simplifying property of only involving a single leptonic charged-current vertex in their amplitudes, so one may define an effective mass $m_{\tau,\mathrm{eff}} = [\sum_i |U_{\tau,i}|^2 m_{\nu_i}^2]^{1/2}$. The best upper limit $m_{\nu_\tau,\mathrm{eff}} < 18.2$ MeV (95% C.L.) [144] comes from semihadronic τ decays.

As in the case of μ decay, one can analyze leptonic τ decays in the presence of possible sterile neutral emission; see Table II in [39] and also Ref. [135]. We denote the $\tau \to \nu_\tau e \bar{\nu}_e$ mode as $\tau \to e$ and the $\tau \to \nu_\tau \mu \bar{\nu}_\mu$ as $\tau \to \mu$ for short and define a reduced, dimensionless decay rate $\bar{\Gamma}_{\tau\to\ell}$ via the equation

$$\Gamma_{\tau\to\ell} = \frac{G_F^2 m_\tau^5}{192\pi^3}(1 + \delta_\alpha)\bar{\Gamma}_{\tau\to\ell} \tag{9.1}$$

where we have used the fact that the leading-order correction, δ_α, is mass-independent. In the Standard Model with neutrinos of negligible masses,

$$\Gamma_{\tau\to\ell,\mathrm{SM}} = f(a_\ell^{(\tau)}, 0, 0). \tag{9.2}$$

Using $a_e^{(\tau)} = 0.827 \times 10^{-7}$ and $a_\mu^{(\tau)} = 3.536 \times 10^{-3}$ in Eq. (8.5), one has $f(a_e^{(\tau)}, 0, 0) = 1 - (0.662 \times 10^{-6})$ and $f(a_\mu^{(\tau)}, 0, 0) = 0.9726$.

With massive (anti)neutrino emission, we calculate

$$\begin{aligned} \bar{\Gamma}_{\tau\to\ell} = {} & (1 - |U_{\ell 4}|^2)(1 - |U_{\tau 4}|^2) f(a_\ell^{(\tau)}, 0, 0) + (1 - |U_{\ell 4}|^2)|U_{\tau 4}|^2 f(a_\ell^{(\tau)}, 0, a_{\nu_4}^{(\tau)}) \\ & + |U_{\ell 4}|^2 (1 - |U_{\tau 4}|^2) f(a_\ell^{(\tau)}, a_{\nu_4}^{(\tau)}, 0) + |U_{\ell 4}|^2 |U_{\tau 4}|^2 f(a_\ell^{(\tau)}, a_{\nu_4}^{(\tau)}, a_{\nu_4}^{(\tau)}). \end{aligned} \tag{9.3}$$

Just as in Eq. (8.8) for μ, the term involving emission of $\nu_4 \bar{\nu}_4$ is negligibly small relative to the other terms because it involves the product of two small leptonic mixing matrix elements squared, $|U_{\ell 4}|^2 |U_{\tau 4}|^2$, and because of the greater kinematic suppression of the decay into $\nu_4 \bar{\nu}_4$ for substantial m_{ν_4}; one can therefore drop the final term in Eq. (9.3). The kinematic function $f(x, y, 0)$ was calculated in [39]. It is worthwhile to inquire what can be learned from a purely leptonic observable which can be calculated and measured to high precision, namely

$$\frac{\mathrm{BR}_{\tau\to e}}{\mathrm{BR}_{\tau\to\mu}} \equiv R^{(\tau)}_{e/\mu} \tag{9.4}$$

and the resultant ratio

$$\bar{R}^{(\tau)}_{e/\mu} = \frac{R^{(\tau)}_{e/\mu}}{R^{(\tau)}_{e/\mu,\mathrm{SM}}}. \tag{9.5}$$

We comment below on studies that also include semihadronic τ decays.

Measurements of the individual branching ratios for $\tau \to \nu_\tau e\bar{\nu}_e$ and $\tau \to \nu_\tau \mu\bar{\nu}_\mu$ have been carried out, with the results [13]

$$\mathrm{BR}_{\tau\to\nu_\tau e\bar{\nu}_e} = 0.1782 \pm 0.0004 \tag{9.6}$$

and

$$\mathrm{BR}_{\tau\to\nu_\tau \mu\bar{\nu}_\mu} = 0.1739 \pm 0.0004. \tag{9.7}$$

Experiments have also reported measurements of the ratio $R^{(\tau)}_{e/\mu}$; a global fit to the data yields the result [13,145]

$$R^{(\tau)}_{e/\mu} = 1.024 \pm 0.003. \tag{9.8}$$

This is consistent with the theoretical SM prediction

$$R^{(\tau)}_{e/\mu,\mathrm{SM}} \equiv \left(\frac{\mathrm{BR}_{\tau\to e}}{\mathrm{BR}_{\tau\to\mu}}\right)_{\mathrm{SM}} = \frac{f(a^{(\tau)}_e, 0, 0)}{f(a^{(\tau)}_\mu, 0, 0)} = 1.028. \tag{9.9}$$

The uncertainty in the theoretical prediction (9.9) is small compared with the uncertainty in the experimental measurement (9.8). Note that the leading-order radiative correction term $(1+\delta_\alpha)$ divides out in the ratio (9.9) since it is mass-independent. Thus,

$$\bar{R}^{(\tau)}_{e/\mu} = 0.996 \pm 0.003. \tag{9.10}$$

The simplest situation applies if m_{ν_4} is sufficiently large that all of the decays $\tau \to \nu_4 e\bar{\nu}_j$ and $\tau \to \nu_4 \mu\bar{\nu}_j$, where $1 \le j \le 4$, are kinematically forbidden. In this case,

$$\begin{aligned} R^{(\tau)}_{e/\mu} &= \frac{(1-|U_{e4}|^2) f(a^{(\tau)}_e, 0, 0)}{(1-|U_{\mu4}|^2) f(a^{(\tau)}_\mu, 0, 0)} \\ &= \frac{(1-|U_{e4}|^2)}{(1-|U_{\mu4}|^2)} R^{(\tau)}_{e/\mu,\mathrm{SM}} \quad (\text{no emission of } \nu_4), \end{aligned} \tag{9.11}$$

i.e., $\bar{R}^{(\tau)}_{e/\mu} = (1-|U_{e4}|^2)/(1-|U_{\mu4}|^2)$. Requiring that $\bar{R}^{(\tau)}_{e/\mu}$ not deviate excessively from 1 yields an upper bound on the magnitude of the difference $|U_{e4}|^2 - |U_{\mu4}|^2$, although this does not by itself provide separate upper bounds on $|U_{e4}|^2$ or $|U_{\mu4}|^2$.

Let us investigate a hierarchical lepton mixing situation in which $|U_{\ell4}|^2 \ll |U_{\tau4}|^2$ for $\ell = e, \mu$. This is effectively equivalent to using the upper limits $m_{\nu_e,\mathrm{eff}} < 2$ eV and $m_{\nu_\mu,\mathrm{eff}} < 0.19$ MeV [13] to infer that these have a negligible effect on the ratio $R^{(\tau)}_{e/\mu}$. Then

$$\begin{aligned} R^{(\tau)}_{e/\mu} &= \frac{(1-|U_{\tau4}|^2) f(a^{(\tau)}_e, 0, 0) + |U_{\tau4}|^2 f(a^{(\tau)}_e, a^{(\tau)}_{\nu_4}, 0)}{(1-|U_{\tau4}|^2) f(a^{(\tau)}_\mu, 0, 0) + |U_{\tau4}|^2 f(a^{(\tau)}_\mu, 0, a^{(\tau)}_{\nu_4})} \\ &= R^{(\tau)}_{e/\mu,\mathrm{SM}} \left[\frac{(1-|U_{\tau4}|^2) + |U_{\tau4}|^2 \bar{f}(a^{(\tau)}_e, a^{(\tau)}_{\nu_4}, 0)}{(1-|U_{\tau4}|^2) + |U_{\tau4}|^2 \bar{f}(a^{(\tau)}_\mu, a^{(\tau)}_{\nu_4}, 0)}\right], \end{aligned} \tag{9.12}$$

where we have used the symmetry (8.11). Solving Eq. (9.12) for $|U_{\tau4}|^2$, we get

$$|U_{\tau4}|^2 = \frac{\bar{R}^{(\tau)}_{e/\mu} - 1}{\bar{R}^{(\tau)}_{e/\mu}[1-\bar{f}(a^{(\tau)}_\mu, a^{(\tau)}_{\nu_4}, 0)] - [1-\bar{f}(a^{(\tau)}_e, a^{(\tau)}_{\nu_4}, 0)]}. \tag{9.13}$$

With (9.10), we obtain a 95% C.L. upper bound on $|U_{\tau4}|^2$ that extends down to below 10^{-2} as m_{ν_4} increases to 1 GeV. More stringent constraints have been obtained from semihadronic decays [42–44,146].

One can also use the measured branching ratios (9.6) and (9.7) and the τ lifetime $\tau_\tau = (2.903 \pm 0.005) \times 10^{-13}$ s [13] in comparison with the decay rates calculated using the MuLan value for G_F to obtain limits on $m_{\nu_\tau,\mathrm{eff}}$. The definition of $m_{\nu_\tau,\mathrm{eff}}$ is more complicated here than in nuclear beta decays and two-body leptonic decays of pseudoscalar mesons, where only a single charged-current vertex is involved, so $m_{\nu_e,\mathrm{eff}} = [\sum_j |U_{ei}|^2 m^2_{\nu_j}]^{1/2}$ and $m_{\nu_\mu,\mathrm{eff}} = [\sum_j |U_{\mu i}|^2 m^2_{\nu_j}]^{1/2}$, where the sums include all neutrino mass eigenstates that lead to the respective outgoing charged lepton with an energy or momentum such that it is included in the cuts used by a given experiment in its event reconstruction. In contrast, for leptonic τ decays, the amplitudes involve two charged-current vertices and hence products of lepton mixing matrices. If one assumes that the ν_4 is emitted via the $\tau - \nu_\tau$ charged-current coupling, then only the $U_{\tau j}$ lepton mixing matrix element is relevant in the amplitude, and one can express $m_{\nu_\tau,\mathrm{eff}}$ in an analogous manner, as $m_{\nu_\tau,\mathrm{eff}} = [\sum_j |U_{\tau i}|^2 m^2_{\nu_j}]^{1/2}$. Then, using the formulation in [147], one finds calculated values for the branching ratios (denoted by superscript (c)) of $R^{(c)}_{\tau\to e} = 0.17781 \pm 0.0031$ and $\mathrm{BR}^{(c)}_{\tau\to\mu} = 0.17293 \pm 0.00030$. Then, the ratios of experimental to calculated branching ratios are

$$S_{\tau\to e} = \Gamma_{\tau\to e}/\Gamma_{\tau\to e,\mathrm{SM}} = 1.022 \pm 0.0028 \tag{9.14}$$

and

$$S_{\tau\to\mu} = \Gamma_{\tau\to\mu}/\Gamma_{\tau\to\mu,\mathrm{SM}} = 1.0056 \pm 0.0029. \tag{9.15}$$

Since the measured values exceed the calculated ones, we find the following 95% C.L. interval for the physical regions for massive neutrino emission i.e. $S_{\tau\to e} < 1$ and $S_{\tau\to\mu} < 1$:

$$0.9964 < S_{\tau\to e} < 1 \tag{9.16}$$

and

$$0.9982 < S_{\tau\to\mu} < 1. \tag{9.17}$$

Equations (9.16) and (9.17) correspond to limits $m_{\nu_\tau,\mathrm{eff}} <$ 38 MeV from $\tau \to \nu_\tau e\bar{\nu}_e$ and $m_{\nu_\tau,\mathrm{eff}} < 26.8$ MeV from $\tau \to \nu_\tau \mu\bar{\nu}_\mu$. These limits may be compared with the current limit, $m_{\nu_\tau,\mathrm{eff}} < 18.2$ MeV [144].

X. REMARKS ON SOME OTHER PARTICLE AND NUCLEAR PHYSICS CONSTRAINTS

Sterile neutrinos with masses in the range considered here are subject to a number of other constraints. We begin with a remark on $K_{\ell 3}$ decays as potential sources of constraints on sterile neutrinos. These decays include $K^+ \to \pi^0 \ell^+ \nu_\ell$, $K_L^0 \to \pi^+ \ell^- \bar{\nu}_\ell$, and $K_L^0 \to \pi^- \ell^+ \nu_\ell$, where $\ell = e,\ \mu$. Since these K_{e3} decays are not helicity-suppressed, in contrast to M_{e2} decays, where $M = \pi^+$, K^+, etc., there is no associated enhancement of $K_{\ell 3}$ decays into a massive ν_4 resulting from removal of helicity suppression, as is the case in M_{e2} decays. These $K_{\ell 3}$ decays into a massive (anti)neutrino are subject to the usual three-body phase space suppression. The maximum ν_4 masses in the K_{e3}, $(K_L^0)_{e3}$, $K_{\mu 3}$, and $(K_L^0)_{\mu 3}$ decays are 358, 362, 253, and 252 MeV, respectively. This mass range is already covered by the limits from peak search and branching ratio constraints from $\pi_{\ell 2}$ and $K_{\ell 2}$ experiments. Furthermore, the calculations of the rates for $K_{\ell 3}$ and $(K_L^0)_{\ell 3}$ decays involve more uncertainty than for $\pi_{\ell 2}$ and $K_{\ell 2}$ because the hadronic amplitudes contain form factors whose dependence on q^2 (where $q^\lambda = p^\lambda - p_\pi^\lambda$ is the four-momentum imparted to the outgoing $\ell^- \nu_i$ or $\bar{\ell}^+ \nu_i$ pair) cannot be calculated from first principles. (For a recent discussion of parametrizations of these form factors, see [13].) The resultant uncertainty is only partially cancelled in ratios such as $\mathrm{BR}((K_L^0)_{e3})/\mathrm{BR}((K_L^0)_{\mu 3})$, since the $(K_L^0)_{e3}$ and $(K_L^0)_{\mu 3}$ involve different momenta transfers to the outgoing lepton pairs.

Next, it may be recalled that quite restrictive upper limits on mixings of mainly sterile heavy neutrinos have also been obtained from time-of-flight searches [148,149] and for neutrino decays [13,38,150–152]. A recent search of this type is [153]. In the mass range of a few MeV, experiments have been performed to search for the decay $\bar{\nu}_4 \to e^+ e^- \nu_e$ using $\bar{\nu}_e$ beams from nuclear reactors [154–156]. These eventually obtained upper limits on $|U_{e4}|^2$ of 0.5×10^{-2} at $m_{\nu_4} = 1$ MeV down to 3×10^{-4} for $m_{\nu_4} = 4$ MeV, and then increasing to 0.6×10^{-2} for $m_{\nu_4} = 9.5$ MeV [156]. From observations of the solar neutrino flux, the Borexino experiment has set upper bounds $|U_{e4}|^2$ of 10^{-3} to 0.4×10^{-5} for m_{ν_4} from 1.5 to 14 MeV [157]. However, since the conditions for the diagonality of the neutral weak current are violated in the presence of sterile neutrinos [35,36], a sterile neutrino may decay invisibly, as $\nu_4 \to \nu_j \bar{\nu}_\ell \nu_\ell$. Other invisible neutrino decay modes occur in models in which neutrinos couple to a light scalar or pseudoscalar (for recent discussions and limits, see, e.g., [158–160] and references therein). Consequently, because of their model dependence, we do not use limits on lepton mixing from neutrino decays here.

One can also check that a ν_4 mass in the range considered here would make a negligible contribution to decays such as $\mu^+ \to e^+\gamma$. The branching ratio for $\mu^+ \to e^+\gamma$ is [161]

$$\mathrm{BR}(\mu^+ \to e^+\gamma) = \frac{3\alpha_{em}}{32\pi}\left(\frac{m_{\nu_4}}{m_W}\right)^4 |U_{\mu 4} U_{e4}|^2. \tag{10.1}$$

For $m_{\nu_4} = 100$ MeV, the factor multiplying $|U_{\mu 4} U_{e4}|^2$ is 0.52×10^{-15}. Given the experimental upper limit $\mathrm{BR}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}$ [162], it is clear that this does not yield a useful constraint on $|U_{\mu 4} U_{e4}|^2$.

A massive Dirac neutrino has a magnetic moment [163] (see also [164]; recent reviews include [165,166]). Limits on such a magnetic moment are commonly quoted for the interaction eigenstates, although they are really properties of the mass eigenstates. For one of the three SM neutrino mass eigenstates ν_i in an active neutrino interaction eigenstate ν_ℓ, where $\ell = e,\ \mu,\ \tau$, this is

$$\begin{aligned}\mu_{\nu_i} &= \frac{3eG_F m_{\nu_i}}{8\pi^2\sqrt{2}} \sum_\ell |U_{\ell i}|^2 \\ &= (3.2 \times 10^{-19})\left(\frac{m_{\nu_i}}{1\ \mathrm{eV}}\right)\left[\sum_\ell |U_{\ell i}|^2\right]\mu_B,\end{aligned} \tag{10.2}$$

where $\mu_B = e/(2m_e)$ is the Bohr magneton. For a heavy (mostly sterile) neutrino mass eigenstate with a mass in the range considered here, the expression for $\mu_{\nu_4 2}$ is given by Eq. (10.2) with $i = 4$. Thus, a Dirac ν_4 with a mass of 5 MeV would have $\mu_{\nu_4} = (1.6 \times 10^{-12})[\sum_\ell |U_{\ell 4}|^2]\mu_B$ [163].

The upper limits conventionally quoted for the neutrino interaction eigenstates are of order $(10^{-10}\text{–}10^{-11})\mu_B$ from reactor and accelerator experiments, 3×10^{-11} from the Borexino experiment [167], and of order $(10^{-11}\text{–}10^{-12})\mu_B$

from limits on stellar cooling rates [13]. Since $|U_{e1}|^2$, $|U_{\mu 2}|^2$, and $|U_{\tau 3}|^2$ are $O(1)$, there is not a large difference between the usually quoted upper limits on $\mu_{\nu_e}, \mu_{\nu_\mu}, \mu_{\nu_\tau}$, and the upper limits on μ_{ν_i} with $i = 1, 2, 3$.

The situation is different for a heavy ν_4 in the mass range considered here. When considering how these limits might apply to the ν_4, however, one must take into account the fact that there would be strong kinematic and mixing-angle suppression or exclusion of the initial emission of the heavy $\bar{\nu}_4$ in the beta decays that yield the $\bar{\nu}_e$ flux from a reactor, and a ν_4 with an MeV-scale mass would be kinematically forbidden from being emitted in the $pp \to \mathrm{D} + e^+ + \nu_e$ reaction and the electron-capture transition $e + {}^7\mathrm{Be} \to {}^7\mathrm{Li} + \nu_e$ in the sun, since these have maximum energy releases of only 0.42 and 0.86 MeV, respectively. Hence, one cannot necessarily apply the constraints on neutrino magnetic moments from reactor antineutrino and solar neutrino experiments to a heavy neutrino. Similarly, the constraint from stellar cooling is not directly relevant here because it only applies to neutrino mass eigenstates ν_j with masses $\lesssim 5$ keV so that a plasmon in the star would be kinematically able to produce the $\bar{\nu}_j \nu_j$ pair [13].

Finally, we comment on how a heavy neutrino could affect Higgs decays. Ref. [168] pointed out that the Higgs boson could have decays to invisible final states, and calculated rates for several of these, including decays to neutrinos. Currently, all of the decay branching ratios of the Higgs are in agreement with SM predictions, but these allow for a substantial branching ratio into invisible modes, $\mathrm{BR}(H \to \text{invisible}) \lesssim 20\%$ [13,169,170]. The way in which the diagonal and nondiagonal couplings of neutrinos to the Higgs boson are related to the couplings $U_{\ell 4}$ that enters in the weak charged current depends, in general, on details of a given model.

XI. CONCLUSIONS

One of the most important outstanding questions in nuclear and particle physics at present concerns whether light sterile neutrinos exist. In this paper we have presented a detailed analysis yielding new upper bounds on the squared lepton mixing matrix elements $|U_{e4}|^2$ and $|U_{\mu 4}|^2$ involved in the possible emission of a mostly sterile neutrino mass eigenstate, ν_4, from analyses of a number of nuclear and particle decays. A brief report on the upper bounds on $|U_{e4}|^2$ was given in [1]. We have used recent advances in the precision of measured $\mathcal{F}t$ values for a set of superallowed nuclear beta decays to improve the upper limits on $|U_{e4}|^2$ obtained from these beta decays for a ν_4 with a mass in the range of a few MeV. From analyses of the ratios of branching ratios $R^{(\pi)}_{e/\mu} = \mathrm{BR}(\pi^+ \to e^+ \nu_e)/\mathrm{BR}(\pi^+ \to \mu^+ \nu_\mu)$, $R^{(K)}_{e/\mu}$, $R^{(D_s)}_{e/\tau}$, $R^{(D_s)}_{\mu/\tau}$, and $R^{(D)}_{e/\tau}$, and from B_{e2} and $B_{\mu 2}$ decays, we have derived upper limits on couplings $|U_{e4}|^2$ and $|U_{\mu 4}|^2$. Our bounds on $|U_{e4}|^2$ cover most of the ν_4 mass range from approximately 1 MeV to 1 GeV, and in several parts of this range they are the best bounds for a Dirac neutrino that do not make use of model-dependent assumptions on visible neutrino decays. We have also obtained a new upper bound on $|U_{\mu 4}|^2$ from a $\pi_{\mu 2}$ peak search experiment searching for ν_4 emission via lepton mixing and have updated existing upper bounds on $|U_{\mu 4}|^2$ in the MeV to GeV mass range. New experiments to search for $D_s^+ \to e^+ \nu_4$ and $D^+ \to e^+ \nu_4$ are suggested. These, as well as a continued search for $B^+ \to e^+ \nu_4$ and $B^+ \to \mu^+ \nu_4$ decays, would be valuable and could further improve the bounds. In addition, we examined limits on $|U_{e4}|^2$ obtained from examining pion beta decay and showed that they are less stringent than those from superallowed beta decay in the same ν_4 mass range. As part of the analysis, we updated constraints from CKM unitarity on sterile neutrinos. In addition, we examined correlated constraints on lepton mixing matrix coefficients $|U_{e4}|^2$, $|U_{\mu 4}|^2$ and $|U_{\tau 4}|^2$ from analyses of leptonic decays of heavy-quark pseudoscalar mesons, from μ decay, and from leptonic τ decays.

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