Adjusted complexity equals action conjecture

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In this paper, we first use the original complexity equals action (CA) conjecture to discuss the complexity growth rate in both perturbative Einsteinian cubic gravity and nonperturbative Einstein-Weyl gravity. We find that the CA complexity growth rates in these cases are divergent. To avoid this divergence, we modify the original conjecture, in which we assume that the complexity of the boundary state equals the boundary actions contributed by the null segments as well as the joints of the Wheeler-DeWitt patch. Then, the late-time growth rate of this modified holographic complexity is given by entropy S times temperature T of the black holes, which is in agreement with the circuit analysis. Finally, to test its rationality, we also investigate the switchback effect by evaluating it in a Vaidya geometry and analyze the results in circuit models.

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I. INTRODUCTION

Recently, applications of quantum information concepts to high-energy physics and gravity have led to many promising results. Among all, it has become more clear that special properties of entanglement in holographic quantum field theories account for the emergence of smooth higher-dimensional geometries in gauge-gravity duality [1–4]. Inspired by the point of view that a black hole might be highly entangled with a system that is effectively infinitely far away, there is a further deeper idea that “ER = EPR” [3] when we consider a nontraversable wormhole created by an Einstein-Rosen (ER) [5] bridge and a pair of maximally entangled black holes. Here, “EPR” denotes the quantum entanglement (Einstein-Podolsky-Rosen paradox). However, recent developments pointed out that holographic entanglement is unable to describe the bulk spacetime far behind the event horizon of black holes [6]. Hence, there must exist some other quantities we do not yet know to describe the internal geometry of the black holes. And it has been suggested by Susskind and his collaborators that this quantity is the quantum computational complexity of the boundary state, which presents the minimum number of elementary operations or gates from a reference state to a target state. In the

context of holography, there are two related proposals conjectured to capture the quantum complexity of the boundary state from the bulk gravitational theory: “complexity equals volume” (CV) [7,8] and “complexity equals action” (CA) [9,10]. These conjectures have attracted many researchers to investigate the properties of both holographic complexity and circuit complexity in quantum field theory, e.g., Refs. [11–55].

In this paper, we only focus on the CA conjecture, which states that the complexity of a particular state $|\psi(t_L, t_R)\rangle$ on the anti-de Sitter (AdS) boundary is given by

$$\mathcal{C}_A(|\psi(t_L, t_R)\rangle) \equiv \frac{I_{\text{WDW}}}{\pi\hbar}, \quad (1.1)$$

where I_{WDW} is the on-shell action in the corresponding Wheeler-DeWitt (WDW) patch, which is enclosed by the past and future light sheets sent into the bulk spacetime from the time slices t_L and t_R . As suggested in Ref. [13], I_{WDW} should be the total action within the WDW patch which contains not only the bulk action but the null boundary terms and joint terms as well. Moreover, there is a bound of the complexity growth rate in a quantum system with energy E at the late times [56]

$$\frac{d\mathcal{C}}{dt} \leq \frac{2E}{\pi\hbar}, \quad (1.2)$$

which may be thought of as Lloyd’s bound on the quantum complexity. Although some recent developments have cast the exact prefactor of this bound into question, it is not difficult to believe that, for any system, the quantum complexity should always have a finite growth rate.

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Recently, Nally [57] investigated the CA conjecture in some stringy modes of higher-curvature gravity in which the general relativity is perturbed by higher powers of the Riemann tensor. The results suggest that, once stringy effects are taken into account, the neutral black hole will have a divergent complexity growth rate under the first leading-order correction. However, Lloyd's bound suggests that for any system the complexity growth rate should be finite. Therefore, the author claims that if the CA conjecture should prove correct in these perturbative gravitational theories the complexity growth rate must be a nonanalytic function of the small coupling parameter. To construct a better understanding of this feature, in Sec. II B, we investigated the CA conjecture in a nonperturbative theory and found that the rate is also divergent in the nonperturbative Einstein-Weyl gravity. Hence, we cannot simply attribute these divergences to the fact that it is a nonanalytic function. This apparent failure of the CA conjecture motivates us to redefine the holographic complexity such that its growth rate is convergent when the higher-curvature term is concerned.

Another motivation for the present paper is from the discussion of the quantum circuit model. In a simple class of systems known as random quantum circuits with N qubits, it has been shown that for generic circuits, after a short period of transient initial behavior, the complexity grows linearly in time and finally saturates at a maximum value. In the context of the AdS/CFT, we need to set N to be very large. Then, it can be generally argued that at the late times the quantum complexity should continue to grow with a rate given by [7,8]

$$\frac{dC}{dt} \sim TS, \quad (1.3)$$

where the entropy represents the width of the circuit and the temperature is an obvious choice for the local rate at which a particular qubit interacts. In the CA picture, this result is natural for high-temperature systems, but it is not universal for all of them. Motivated by this, we hope to find a new conjecture such that the complexity has this feature even at low-temperature regimes.

The remainder of our paper is organized as follows. In Sec. II, we first review the results in Ref. [57] and then study the original CA conjecture in both perturbative Einsteinian cubic gravity and nonperturbative Einstein-Weyl gravity. In Sec. III, in order to avoid these divergences mentioned in Sec. II, we modify the CA conjecture and investigate its time involution as well as the switchback effect for a general $F(R_{abcd})$ gravitational theory. Finally, we conclude our paper in Sec. IV.

II. CA COMPLEXITY FOR HIGHER-CURVATURE GRAVITY

In Ref. [57], Nally studied the holographic complexity in the CA picture for higher-curvature gravitational theories

which are directly motivated by string theory. In this case, string theory naturally comes equipped with an infinite tower of correction to the general relativity. And then, the corresponding bulk action can be expressed as

$$I = \int d^{d+2}x \sqrt{-g} \left(R - 2\Lambda + \sum_{n=1}^{\infty} \alpha^n \mathcal{L}_n \right), \quad (2.1)$$

where $\mathcal{L}_n$ is, in general, an n th-order polynomial in the Riemann tensor. After the calculation on a five-dimensional Gauss-Bonnet as well as Weyl gravitational theories, Nally found that the CA holographic complexity is extremely sensitive to singularity effects and its leading result is divergent at the late times. And this divergence comes from the singularity. However, the Lloyd bound suggests that any system, even if it has an infinite-dimensional Hilbert space, should have a finite complexity growth rate. Therefore, if the CA conjecture should prove correct in these perturbative gravitational theories, the complexity growth rate must be a nonanalytic function of the small parameter α . This means that when all of the corrections are concerned the CA complexity growth rate will be convergent. To understand these divergences better, it is also necessary for us to study whether these similar divergences would be found in a nonperturbative higher-curvature theory. Therefore, in this section, we will consider both the perturbative Einsteinian cubic gravity and nonperturbative Einstein-Weyl gravity.

A. Perturbative Einsteinian cubic gravity

In this subsection, we consider a four-dimensional Einsteinian cubic gravity. As suggested in Ref. [13] for Einstein gravity and developed in Ref. [58] for a general $F(R_{abcd})$ gravity, the total action in the CA conjecture is composed of the usual bulk action as well as various possible boundary terms

$$I_{\text{tot}} = I_{\text{bulk}} + I_{\text{surf}} + I_{\text{joint}}, \quad (2.2)$$

where the first term is integrated over the bulk of the manifold of interest [59]

$$I_{\text{bulk}} = \int_{\mathcal{M}} d^4x \sqrt{-g} \left(R + \frac{6}{L^2} + \mu \mathcal{P} \right) \quad (2.3)$$

with

$$\begin{aligned} \mathcal{P} = & 12R_{acbd}R^{cedf}R_e^a{}_f{}^b + R_{ab}{}^{cd}R_{cd}{}^{ef}R_{ef}{}^{ab} \\ & - 12R_{abcd}R^{ac}R^{bd} + 8R_a{}^b R_b{}^c R_c{}^a \end{aligned} \quad (2.4)$$

for the Einsteinian cubic gravity. The second term I_{surf} contains the generalized Gibbons-Hawking-York terms for timelike and spacelike boundary segments and the null surface terms and counterterms for the null segments [58]; i.e., we have

$$I_{\text{surf}} = I_{\text{GHY}} + I_{\text{null}} \quad (2.5)$$

with $I_{\text{null}} = I_{\text{null-surf}} + I_{\text{ct}}$. Here, null surface terms $I_{\text{null-surf}}$ are introduced to make the variational principle well defined on the null surfaces, and the counterterms are to ensure that all of the boundary terms are reparametrization invariant. According to Ref. [58], in the Einsteinian cubic gravity, these terms can be expressed as

$$\begin{aligned} I_{\text{GHY}} &= 4 \sum_s \left(\int_{B_s} d^3x \sqrt{|h|} \Psi^{ab} K_{ab} \right), \\ I_{\text{null-surf}} &= - \int_{\mathcal{N}} \hat{\Psi} \kappa d\lambda dS, \\ I_{\text{ct}} &= - \int_{\mathcal{N}} d\lambda d^2x \sqrt{\sigma} \hat{\Theta} \ln(l_{\text{ct}} \Theta) d\lambda, \end{aligned} \quad (2.6)$$

where l_{ct} is an arbitrary length scale, $\Psi_{ab} = \psi_{abcd} n^c n^d$, $\hat{\Psi} = \psi^{abcd} \epsilon_{ab} \epsilon_{cd}$ is the Wald entropy density, $\hat{\Theta} = \nabla_a (k^a \hat{\Psi}) = \frac{1}{\sqrt{\sigma}} \partial_\lambda (\hat{\Psi} \sqrt{\sigma})$ is the change rate of the Wald entropy density along the null generator $k^a = (\partial/\partial\lambda)^a$, $\Theta = \nabla_a k^a = \frac{1}{\sqrt{\sigma}} \partial_\lambda (\sqrt{\sigma})$ is the expansion scalar, and κ measures the failure of λ to be an affine parameter which is derived from $k^a \nabla_a k^b = \kappa k^b$. Here, n^a is the outward-directed normal vector for the corresponding surface, and ϵ_{ab} is the binormal of the transverse surface on the null segments or the joints of the segments. For the cubic Einsteinian gravity, we have

$$\begin{aligned} \psi_{abcd} &= \frac{\partial L}{\partial R^{abcd}} \\ &= \frac{1}{2} (g_{ac} g_{bd} - g_{ad} g_{bc}) + 6\mu (R_{ad} R_{bc} - R_{ac} R_{bd} \\ &\quad + g_{bd} R_a^e R_{ce} - g_{ad} R_b^e R_{ce} - g_{bc} R_a^e R_{de} \\ &\quad + g_{ac} R_b^e R_{de} - g_{bd} R^e R_{aecf} + g_{bc} R^e R_{aedf} \\ &\quad + g_{ad} R^e R_{becf} - g_{ac} R^e R_{bedf} - 3R_a^e R_d^f R_{becf} \\ &\quad + 3R_a^e R_c^f R_{bedf} + \frac{1}{2} R_{ab}^e R_{cdef}). \end{aligned} \quad (2.7)$$

The third term I_{joint} is also the boundary term, which is introduced to make the variational principle well defined on the intersections of these segments [58]. And it can be written as

$$I_{\text{joint}} = \int_{\mathcal{C}_\lambda} d^2x \sqrt{\sigma} \hat{\Psi} \eta_\lambda \quad (2.8)$$

with some parameters η_λ determined by these joints [58]. According to the bulk action, the equation of motion can be written as

$$\psi_{acde} R_b^{cde} - \frac{1}{2} g_{ab} L - 2 \nabla^c \nabla^d \psi_{acdb} = 0. \quad (2.9)$$

With similar consideration as in Ref. [57], we will also treat μ as a small parameter; i.e., the cubed terms are regarded as a small correction to Einstein gravity. Then, we can obtain the Schwarzschild-AdS (SAdS)-type black hole solution at the first subleading, the line element of which can be shown as [60]

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (2.10)$$

with the blackening factor

$$\begin{aligned} f(r) &= 1 - \frac{2M}{r} + \frac{r^2}{L^2} \\ &\quad + \mu \left(-\frac{368M^3}{r^7} + \frac{216M^2}{r^6} + \frac{336M^2}{L^2 r^4} + \frac{b}{2r} - \frac{8r^2}{L^6} \right). \end{aligned} \quad (2.11)$$

Next, we evaluate the complexity growth rate with the CA conjecture. As illustrated in the Penrose diagram of the SAdS-type spacetime, Fig. 1, $I(t_L, t_R)$, denoted as the action for the WDW patch determined by the time slice on the left and right AdS boundaries, is invariant under the time translation $I(t_L + \delta t, t_R - \delta t) = I(t_L, t_R)$. Thus, the action growth can be computed as $\delta I = I(t_0 + \delta t, t_1) - I(t_0, t_1)$, where the time on the right boundary has been fixed. To regulate the divergence near the AdS boundary, a cutoff surface $r = r_\Lambda$ is introduced. In addition, we also introduce a spacelike surface $r = \epsilon$ to avoid running into the spacelike singularity inside of this neutral black hole. Moreover, for simplicity, we shall adopt the affine parameter for the null generator of null segments such that the

FIG. 1. Wheeler-DeWitt patches of a Schwarzschild-AdS-type black hole.

FIG. 2. Penrose-like diagram for one shock wave on an SAdS-type black hole geometry.

surface term vanishes on the null segments. With these in mind, the change of the total action within WDW patch can be divided into

$$\delta I_{\text{tot}} = I_{\delta\mathcal{M}} + I_{\Sigma} + I_{\delta\mathcal{C}} + \delta I_{\text{ct}} \quad (2.12)$$

with $I_{\delta\mathcal{M}} = I_{\mathcal{M}_1} - I_{\mathcal{M}_2}$ and $I_{\delta\mathcal{C}} = I_{\mathcal{C}_1} - I_{\mathcal{C}_2}$. Here, $\mathcal{M}_1$ is bounded by $u = t_0$, $u = t_0 + \delta t$, $v = t_0 + \delta t$, and $r = r_{\min}$; $\mathcal{M}_2$ is bounded by $u = t_0$, $v = t_0$, $v = v_0 + \delta t$, and $u = t_1$. The null coordinates are defined as $u = t + r^*(r)$ and $v = t - r^*(r)$ with

$$r^*(r) = - \int_r^\infty \frac{dr}{f(r)}. \quad (2.13)$$

With the above preparation, we first calculate the change of the action which is contributed by the bulk region. Through straight calculation, we have

$$I_{\delta\mathcal{M}} = \Omega_2 \delta t \left[-\frac{2r^3}{L^2} + \mu \left(-\frac{1952M^3}{r^6} + \frac{864M^2}{r^5} + \frac{480M^2}{L^2 r^4} + \frac{64r^3}{L^6} \right) \right] \Big|_\epsilon^{r_1}, \quad (2.14)$$

where we denote $\Omega_2 = 4\pi$. We can see that when the cutoff surface $r = \epsilon$ approaches the singularity, i.e., $\epsilon \rightarrow 0$, the bulk contributions are divergent. Then, we proceed to the surface contribution from the spacelike surface $r = \epsilon$. Through straight calculation, we have

$$\begin{aligned} I_{\Sigma} &= 4 \int_S d^3x \sqrt{h} \psi^{abcd} n_c n_d \nabla_a n_b, \\ &= \Omega_2 \delta t \left[6M + \mu \left(-\frac{3b}{2} - \frac{144M}{L^4} - \frac{2400M^3}{\epsilon^6} + \frac{960M^2}{\epsilon^5} - \frac{288M^2}{L^2 \epsilon^3} + \frac{192M}{L^2 \epsilon^2} \right) \right], \end{aligned} \quad (2.15)$$

where $n^a = (dr)^a / \sqrt{|f|}$ is the outward-directed normal vector of $\mathcal{S}$. We can see that this term is also divergent under the limit $\epsilon \rightarrow 0$.

To evaluate the joint contributions from $\mathcal{C}_1$ and $\mathcal{C}_2$, we first choose $k_{1a} = \nabla_a u$ and $k_{2a} = -\nabla_a v$ as the null generator of the past right and past left null boundaries separately. Using these expressions, we can obtain $k_1 \cdot k_2 = -2/f$ and

$$\hat{\Psi} = -2 + \mu \left(\frac{48}{L^4} - \frac{240M^2}{r^6} - \frac{192b}{L^2 r^3} \right). \quad (2.16)$$

Then, we have

$$\begin{aligned} I_{\delta\mathcal{C}} &= \Omega_2 \delta t f'(r_1) \left[r_1^2 - \mu \left(\frac{24r_1^2}{L^2} - \frac{120M^2}{r_1^4} - \frac{96b}{L^2 r_1} \right) \right] \\ &\quad + \Omega_2 \delta t \left[2r_1 - \mu \left(\frac{48r_1}{L^2} + \frac{480M^2}{r_1^5} + \frac{96b}{L^2 r_1^2} \right) \right] \\ &\quad \times f(r_1) \ln[-f(r_1)], \end{aligned} \quad (2.17)$$

where we have used

$$\delta r = r_1 - r_2 = -\frac{1}{2} f(r_1) \delta t \quad (2.18)$$

with r_2 the r coordinate of $\mathcal{C}_2$.

Finally, we consider the counterterm contributions. According to the translation symmetry of this spacetime, there are only two null segments contributing to the action growth. The first one comes from the null segment $u = t_1$ with r as the affine parameter, i.e., $k_1^a = (\frac{\partial}{\partial r})^a$, which gives rise to the expansion $\Theta = 2/r$ and

$$\hat{\Theta} = -\frac{4}{r} + \mu \left(\frac{960M^2}{r^7} + \frac{192M}{L^2 r^4} + \frac{96}{L^4 r} \right). \quad (2.19)$$

Then, we can obtain

$$\begin{aligned}
I_{\text{ct}}^{(1)} &= -\Omega_2 \int_{r_1}^{r_{\text{max}}} dr r^2 \hat{\Theta} \ln(l_{\text{ct}} \Theta) \\
&= \text{“UV term”} + \Omega_2 \left[r_1^2 - \mu \left(\frac{60M^2}{r_1^4} + \frac{192M}{L^2 r_1} + \frac{24r_1^2}{L^4} \right) \right] \\
&\quad + \Omega_2 \left[2r_1^2 + \mu \left(\frac{240M^2}{r_1^4} + \frac{192M}{L^2 r_1} - \frac{48r_1^2}{L^4} \right) \right] \ln \left(\frac{2l_{\text{ct}}}{r_1} \right). \tag{2.20}
\end{aligned}$$

With similar calculation, we have $I_{\text{ct}}^{(2)} = I_{\text{ct}}^{(1)}$. Then, the counterterm contributions are given by

$$\begin{aligned}
\delta I_{\text{ct}} &= 2\Omega_2 \delta t \left[2r_1 - \mu \left(\frac{48r_1}{L^2} + \frac{480M^2}{r_1^5} + \frac{96b}{L^2 r_1^2} \right) \right] \\
&\quad \times f(r_1) \ln \left(\frac{2l_{\text{ct}}}{r_1} \right). \tag{2.21}
\end{aligned}$$

Summing all the previous results, one can obtain

$$\begin{aligned}
\frac{d\mathcal{C}_A}{dt} &= \frac{\Omega_2}{\pi\hbar} \left[6M - \frac{2r_1^3}{L^2} + r_1^2 f'(r_1) + \left[2r_1 - \mu \left(\frac{48r_1}{L^2} + \frac{480M^2}{r_1^5} + \frac{96b}{L^2 r_1^2} \right) \right] f(r_1) \ln \left(\frac{2l_{\text{ct}}}{r_1} \right) \right] \\
&\quad + \frac{\Omega_2 \mu}{\pi\hbar} \left[\frac{192M}{L^2 \epsilon^2} - \frac{768M^2}{L^2 \epsilon^3} + \frac{96M^2}{\epsilon^5} - \frac{448M^3}{\epsilon^6} - \frac{144M}{L^4} - \frac{3b}{2} - \frac{1952M^3}{r_1^6} \right] \\
&\quad + \frac{\Omega_2 \mu}{\pi\hbar} \left[\frac{864M^2}{r_1^5} + \frac{480M^2}{L^2 r_1^4} + \frac{64r_1^3}{L^6} - f'(r_1) \left(\frac{24r_1^2}{L^2} - \frac{120M^2}{r_1^4} - \frac{96b}{L^2 r_1} \right) \right]. \tag{2.22}
\end{aligned}$$

From this result, we can see that the CA complexity growth rate is divergent when the cutoff surface approaches singularity at the first leading-order approximation of λ .

B. Nonperturbative Einstein-Weyl gravity

In this subsection, we investigate the CA conjecture for a nonperturbative four-dimensional Einstein-Weyl theory. The total action is given by [58]

$$\begin{aligned}
I_{\text{tot}} &= \int_{\mathcal{M}} d^4x \sqrt{-g} C_{abcd} C^{abcd} \\
&\quad + 4 \sum_s \left(\int_{B_s} d^3x \sqrt{|h|} \Psi^{ab} K_{ab} \right) - \int_{\mathcal{N}} \hat{\Psi} \kappa d\lambda dS \\
&\quad - \int_{\mathcal{N}} d\lambda d^2x \sqrt{\sigma} \hat{\Theta} \ln(l_{\text{ct}} \Theta) + \int_{\mathcal{C}_\lambda} d^2x \sqrt{\sigma} \hat{\Psi} \eta_\lambda. \tag{2.23}
\end{aligned}$$

According to the bulk action, the equation of motion can be written as

$$(2\nabla^b \nabla^c + R^{bc}) C_{abcd} = 0. \tag{2.24}$$

One can verify that the conformal gravity admits a Schwarzschild-AdS black hole solution, the line element of which can be shown as

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2.25}$$

with the blackening factor

$$f(r) = 1 - \frac{2M}{r} + \frac{r^2}{L^2}. \tag{2.26}$$

Using the line element (2.25), one can further obtain

$$C_{abcd} C^{abcd} = \frac{48M^2}{r^6}, \quad \hat{\Psi} = -\frac{16M}{r^3}. \tag{2.27}$$

Then, with the similar calculation, one can obtain the CA complexity growth rate,

$$\begin{aligned}
\frac{d\mathcal{C}_A}{dt} &= \frac{16\Omega_2 M}{\pi\hbar} \left[\frac{1}{L^2} + \frac{1}{\epsilon^2} - \frac{2M}{\epsilon^3} \right. \\
&\quad \left. - \frac{1}{2r_1^2} f(r_1) \ln \left(-\frac{4f(r_1) l_{\text{ct}}^2}{r_1^2} \right) \right]. \tag{2.28}
\end{aligned}$$

It is noteworthy that for this nonperturbative theory there also exists some similar divergences in the CA complexity growth rate. This implies that these divergences can also be found in the nonperturbative theory, and we cannot simply attribute them to the fact that the holographic complexity is a nonanalytic function.

III. MODIFIED CA CONJECTURE

According to the above calculation as well as the discussion in Ref. [57], we can see that these divergences come from the bulk as well as the spacelike surface S , which will approach the singularity. Therefore, if we neglect the bulk and these spacelike surface contributions to the total action, our result will be convergent. Hence, with rough consideration, we can modify the CA duality as

$$\mathcal{C}_{\text{mod}} = \frac{2d}{(d+1)\pi\hbar} (I_{\text{joint}} + I_{\text{null}}) \tag{3.1}$$

for a $(d+2)$ -dimensional spacetime. Here, the coefficient ensures that the late-time limit of the complexity growth

rate approaches $2M/\pi\hbar$. Then, this new conjecture will give a convergent holographic complexity growth rate for the planer SAdS black hole in Einstein gravity. Differing from the original CA conjecture, the modified version does not contain the bulk action I_{bulk} as well as the non-null surface actions I_{GHY} . To make our final result be reparametrization invariant, here I_{null} not only contains null surface terms $I_{\text{null-surf}}$ but also the counterterms I_{ct} .

Moreover, we also need to check whether this new conjecture would have some expected properties such that it can be regarded as complexity of boundary conformal field theory (CFT). To show this, we will first evaluate the growth rate of this modified complexity and then check its switchback effect of this modified conjecture in a SAdS-type spacetime for a general higher-curvature gravity.

A. Complexity growth rate

In this subsection, we consider a $(d+2)$ -dimensional static black hole in a general $F(R_{abcd})$ gravitational theory. For simplification, here we only restrict our discussion to the SAdS-type black hole in which $g_{tt}g_{rr} = -1$; i.e., the line element can be described by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + d\Sigma_{k,d}^2. \quad (3.2)$$

Although this is not the case in general, there are some important examples of theories for which this property actually holds, such as the Lovelock theories [61,62], Einsteinian cubic gravity in $d=2$ [59], quasitopological gravities [63,64], the so-called generalized quasitopological gravities [65,66], and so on.

First, we consider the case in which the gravitational theory contains the higher-order equation of motion,¹ in which the total action can be shown as [58]

$$\begin{aligned} I_{\text{tot}} &= I_{\text{bulk}} + I_{\text{GHY}} + I_{\text{null-surf}} + I_{\text{ct}} + I_{\text{joint}} \\ &= \int_{\mathcal{M}} d^{d+2}x \sqrt{-g} F(R_{abcd}) \\ &\quad + 4 \sum_s \left(\int_{B_s} d^{d+1}x \sqrt{|h|} \Psi^{ab} K_{ab} \right) - \int_{\mathcal{N}} \hat{\Psi} \kappa d\lambda dS \\ &\quad - \int_{\mathcal{N}} d\lambda d^d x \sqrt{\sigma} \hat{\Theta} \ln(l_{\text{ct}} \Theta) + \int_{\mathcal{C}_i} d^d x \sqrt{\sigma} \hat{\Psi} \eta_{\lambda} \end{aligned} \quad (3.3)$$

with

$$\psi^{abcd} = \frac{\partial F(R_{abcd})}{\partial R_{abcd}}. \quad (3.4)$$

¹It is noteworthy that Lanczos-Lovelock theory does not satisfy this requirement since it only contains a second-order equation of motion and the boundary terms (without the counterterms) are given by Refs. [67,68] for this theory.

Here, $k = \{-1, 0, 1\}$ denotes the d -dimensional spherical, planar, and hyperbolic geometry, individually. For the joint contributions, with a similar calculation as in Sec. II, we can further obtain

$$\begin{aligned} \delta I_{\text{joint}} &= \Omega_{k,d} \delta r \frac{\partial}{\partial r} [\Phi(r) \ln(-f(r))] \Big|_{r=r_1} \\ &= -\frac{\Omega_{k,d} \delta t}{2} [\Phi(r_1) f'(r_1) + \Phi'(r_1) f(r_1) \ln(-f(r_1))], \end{aligned} \quad (3.5)$$

where we denoted

$$\Phi(r) = r^d \hat{\Psi}(r) \quad (3.6)$$

and used the relation (2.18). Then, we turn to the null surface contributions. If we adopt the affine parameter for the null generator of the null segments, the null surface term will vanish. Therefore, we only need to consider the counterterm contributions on the null segments. Considering the translation symmetry, here only two null segments will contribute to action growth. According to the line element (3.2), one can further obtain $\Theta = d/r$. For the past right null segment, its counterterm can be shown as

$$I_{\text{ct}}^{(1)} = -\Omega_{k,d} \int_{r_1}^{r_\Lambda} dr \Phi'(r) \ln\left(\frac{dl_{\text{ct}}}{r}\right). \quad (3.7)$$

Then, we have

$$\delta I_{\text{ct}}^{(1)} = -\frac{\Omega_{k,d} \delta t}{2} \Phi'(r_1) f(r_1) \ln\left(\frac{dl_{\text{ct}}}{r_1}\right). \quad (3.8)$$

Using the fact $\delta I_{\text{ct}}^{(1)} = \delta I_{\text{ct}}^{(2)}$, one can further obtain

$$\begin{aligned} \frac{dC_{\text{mod}}}{dt} &= -\frac{d\Omega_{k,d}}{(d+1)\pi\hbar} \left[\Phi(r_1) f'(r_1) \right. \\ &\quad \left. + \Phi'(r_1) f(r_1) \ln\left(\frac{-d^2 l_{\text{ct}}^2 f(r_1)}{r_1^2}\right) \right]. \end{aligned} \quad (3.9)$$

At the late-time limit $r_1 \rightarrow r_h$, we have

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{dC_{\text{mod}}}{dt} &= -\frac{d\Omega_{k,d}}{(d+1)\pi\hbar} r_h^d \hat{\Psi}(r_h) f'(r_h) \\ &= \frac{2dTS}{(d+1)\pi\hbar}, \end{aligned} \quad (3.10)$$

where we used the expressions of the entropy and temperature of this black hole,

$$S = -2\pi\Omega_{k,d} r_h^d \hat{\Psi}(r_h), \quad T = \frac{1}{4\pi} f'(r_h). \quad (3.11)$$

For Einstein gravity, the result can be directly obtained by setting $F(R_{abcd}) = R - 2\Lambda$, i.e., $\hat{\Psi} = -2$. Then, the growth rate of this modified complexity can be read off,

$$\frac{d\mathcal{C}_{\text{mod}}}{dt} = \frac{2d\Omega_{k,d}}{(d+1)\pi\hbar} \left[r_1^d f'(r_1) + dr_1^{d-1} f(r_1) \ln \left(\frac{-d^2 l_{\text{ct}}^2 f(r_1)}{r_1^2} \right) \right], \quad (3.12)$$

which gives a similar expression with the original CA conjecture. Therefore, our new conjecture will give time dependent behavior similar to that of the original conjecture in Einstein gravity.

Finally, we consider the complexity growth rate of this new conjecture in the SAdS-type black hole for Lovelock theories. Since the counterterms of the null segment are still unknown in Lovelock gravity, here we only evaluate its late-time result. By considering the symmetries of this spacetime, the null boundary terms (null surface term and its counterterm) can always be expressed as

$$I_{\text{null}}^{(1)} = \int_{r_1}^{r_\Lambda} dr \mathcal{Q}(r). \quad (3.13)$$

It is not hard to believe that $\mathcal{Q}$ can be expressed as some polynomial functions of intrinsic and extrinsic curvatures as well as the parameter κ ; therefore, $\mathcal{Q}(r_1)$ will be finite as $r_1 \rightarrow r_h$. With a calculation similar to (3.7), we have

$$I_{\text{null}} = 2I_{\text{null}}^{(1)} = f(r_1)\mathcal{Q}(r_1) \simeq 0 \quad (3.14)$$

at the late times. As shown in Ref. [67], the joint terms for intersections of null boundaries can be expressed as

$$I_{\text{joint}} = \int_{\mathcal{C}_\lambda} d^d x \sqrt{\sigma} \frac{\rho_{\text{JM}}}{2\pi} \eta_\lambda, \quad (3.15)$$

where ρ_{JM} is the Jacobson-Myers entropy [69] associated with the joint $\mathcal{C}$. Through the straight calculation, one can find that the late-time growth rate of modified CA complexity shares the same expression as (3.10).

We can note that, at the late times, the growth rate of this modified holographic complexity is always proportional to TS . As pointed out in Ref. [8], this is expected based on the quantum circuit model of complexity; the entropy represents the width of the circuit, and the temperature is an obvious choice for the local rate at which a particular qubit interacts. According to the above calculation, we can see that all of the late-time contributions come from the joints. Therefore, some research also assumes that the complexity of the boundary state only equals the joint actions [15]. However, as stated in Ref. [38], these joint actions depend on the choice of the null generators of the null boundaries which intersect these corners. Therefore, to eliminate ambiguities, we also necessarily introduce the counterterms into the conjecture. Moreover, in the next subsection, we will show that the counterterms also play an important role in the switchback effect.

B. Switchback effect

As suggested in Ref. [70], the eternal black hole geometry is dual to a thermofield double (TFD) state, which can be constructed by entangling two copies of CFT,

$$|\text{TFD}(t_L, t_R)\rangle = Z^{-1} \sum_{n=0}^{\infty} e^{-\beta E_n/2} |E_n\rangle_L |E_n\rangle_R, \quad (3.16)$$

where L and R denote the left and right systems. Tracing out either copy induces the thermal density matrix at inverse temperature $\beta > 0$. Its time evolution can be obtained by the left and right evolution operators $U_L(t_L) = e^{-iH_L t_L}$ and $U_R(t_R) = e^{-iH_R t_R}$; i.e., we have

$$\begin{aligned} |\text{TFD}(t_L, t_R)\rangle &= U_L(t_L) U_R(t_R) |\text{TFD}\rangle \\ &= Z^{-1} \sum_{n=0}^{\infty} e^{-\beta E_n/2} |E_n\rangle_L |E_n\rangle_R. \end{aligned} \quad (3.17)$$

We can see that this state is invariant under the shift transformation

$$t_L \rightarrow t_L + \Delta t, \quad t_R \rightarrow t_R - \Delta t. \quad (3.18)$$

In the following, we consider a perturbation of the TFD state

$$|\text{TFD}\rangle_{\text{pert}} = \mathcal{O}_R(-t_w) |\text{TFD}\rangle = U(t_w) \mathcal{O}_R U(-t_w) |\text{TFD}\rangle, \quad (3.19)$$

where $\mathcal{O}_R(-t_w)$ is an operator inserted in the right CFT at a time $-t_w$. This precursor can be expressed as $\mathcal{O}_R(-t_w) = U(t_w) \mathcal{O}_R U(-t_w)$ with the localized simple perturbed operator $\mathcal{O}_R$. Then, evolving the perturbed state independently in the left and right times can yield the expression

$$|\text{TFD}(t_L, t_R)\rangle_{\text{pert}} = U_R(t_R + t_w) \mathcal{O}_R U_R(t_L - t_w) |\text{TFD}\rangle. \quad (3.20)$$

This expression illustrates that to obtain the perturbed state we can first evolve the right degrees of freedom backward by a time $t_w - t_L$, insert the perturbed operator $\mathcal{O}_R$, and then evolve the right CFT by $t_R + t_w$. Moreover, for this localized operator, we have $U_R(t) \mathcal{O}_R U_R(-t) = I$ with the identity operator I when $t < t_{\text{scr}}^*$ with the scrambling time t_{scr}^* of the quantum chaos system. Therefore, there might be a trivial evolution of the perturbed TFD state. And these features are connected to the switchback effect of the complexity [8, 14, 71].

In the context of the holography, this perturbed state is dual to a Vaidya black hole with a thin shell of null fluid (shock wave) injected into the eternal black hole geometry as shown in Fig. 2 [72, 73]. Since the total action of the

Lovelock gravity is still unknown, here we only consider the $F(R_{abcd})$ gravitational theory with the total action (3.3). If we only focus on the case of a light shock wave, the black hole's event horizon will shift by a small amount; i.e., we have

$$\frac{r_{h,2}}{r_{h,1}} = 1 + \delta, \quad (3.21)$$

where the subscripts 1 and 2 denote the quantities before and after the shock wave, individually. Then, the scrambling time can be expressed as

$$t_{\text{scr}}^* = \frac{1}{2\pi T_1} \ln \frac{1}{\delta}. \quad (3.22)$$

For the dual quantum system of this black hole, there are two typical regions which are divided by the scrambling time $t = t_{\text{scr}}^*$ after a perturbation is introduced. For $t < t_{\text{scr}}^*$, the complexity remains essentially unchanged, while for $t > t_{\text{scr}}^*$, it begins to grow linearly. And our goal in this subsection is to investigate whether this feature can be preserved in our modified CA conjecture.

Following analysis and notation similar to those in Refs. [27,28], the line element of this shock wave geometry can be written as

$$ds^2 = -F(r, v)dv^2 + 2drdv + r^2 d\Sigma_{k,d}^2 \quad (3.23)$$

with the blackening factor

$$F(r, v) = f_2(r)\mathcal{H}(v + t_w) + f_1(r)[1 - \mathcal{H}(v + t_w)], \quad (3.24)$$

and here we set $r_{h,2} > r_{h,1}$. This line element describes an infinitely thin-shell collapse, which generates a shape transition from a SAdS-type black hole with horizon $r = r_{h,1}$ to another one with $r = r_{h,2}$.

For the convenience of later calculations, we would like to introduce the tortoise coordinates as

$$\begin{aligned} v_R < -t_w: \quad r_1^*(r) &= -\int_r^\infty \frac{dr}{f_1(r)}, \\ v_R > -t_w: \quad r_2^*(r) &= -\int_r^\infty \frac{dr}{f_2(r)}. \end{aligned} \quad (3.25)$$

We choose this range of integration to make both expressions satisfy the boundary condition $\lim_{r \rightarrow \infty} r_{1,2}^*(r) \rightarrow 0$. Using these coordinates, one can also define an “outgoing” null coordinate u and auxiliary time coordinate t as

$$u_i \equiv v - 2r_i^*(r), \quad t_i \equiv v - r_i^*(r). \quad (3.26)$$

The switchback effect is revealed in the “complexity of formation” of the boundary TFD state [28], which is the

extra complexity required to prepare the two copies of the fermionic field theory in the TFD state compared to simply preparing each of the copies in the vacuum state, i.e.,

$$\Delta\mathcal{C} = \mathcal{C}(|\text{TFD}\rangle) - \mathcal{C}(|0\rangle_L \otimes |0\rangle_R). \quad (3.27)$$

In the context of AdS/CFT, the holographic calculation is to evaluate the complexity at $t_L = t_R = 0$ for the black hole and subtract that for two copies of the AdS vacuum geometry. Therefore, in the following, it is sufficient to restrict our attention to the case $t_L = t_R = 0$. To show the switchback effect for the modified conjecture, next, we consider the derivative of the complexity of formation with respect to t_w (the slope of the complexity of formation). And since the holographic complexity for the AdS vacuum geometry is time independent, the slope of the complexity of formation can be expressed as

$$\frac{d\Delta\mathcal{C}_{\text{mod}}}{dt_w} = \frac{d\mathcal{C}_{\text{mod}}}{dt_w}. \quad (3.28)$$

To examine the switchback effect, next, we consider two limit regimes $t \ll t_{\text{scr}}^*$ and $t \gg t_{\text{scr}}^*$. For regime $t_w \ll t_{\text{scr}}^*$, considering the shift symmetry and the light shock wave condition $f_1 \simeq f_2$, the complexity of formation will vanish, i.e.,

$$\left. \frac{d\Delta\mathcal{C}_{\text{mod}}}{dt_w} \right|_{t_w \ll t_{\text{scr}}^*} = 0. \quad (3.29)$$

For another regime, $t_w \gg t_{\text{scr}}^*$, one can verify that all of the light sheets will be sent into the singularity. And the geometry of the WDW patch is characterized by the dynamical points: r_s and r_b , the point where the null shell crosses the past right and future left boundaries. Using the tortoise coordinates, these positions yield

$$t_w + 2r_2^*(r_s) = 0, \quad t_w + 2r_1^*(r_b) = 0. \quad (3.30)$$

Then, the derivatives of these dynamical points with respect to t_w are given by

$$\frac{dr_s}{dt_w} = -\frac{f_2(r_s)}{2}, \quad \frac{dr_b}{dt_w} = -\frac{f_1(r_b)}{2}. \quad (3.31)$$

Following the standard prescription proposed by Refs. [27,28], we shall apply the affine parameter for the null generator of null segments. As a consequence, the contributions from the joints at $r_{s/b}$ as well as all of the null segments will vanish. By considering the symmetries of this spacetime, the time-dependent contributions only come from the counterterms of the past right and future left null boundaries. We first consider the past right segment of the WDW patch; the relevant null normals to these boundaries can be defined as

$$k_a^{(p)} = H(r, v) \left[-(dv)_a + \frac{2}{F(r, v)} (dr)_a \right] \quad (3.32)$$

with affine parameters, where we denote

$$H(r, v) = \alpha_2 \mathcal{H}(r - r_s) + \alpha_1 [1 - \mathcal{H}(r - r_s)]. \quad (3.33)$$

Demanding that the null boundary is affinely parametrized across the shock wave, we have [27]

$$\frac{\alpha_1}{\alpha_2} = \frac{f_1(r_s)}{f_2(r_s)}. \quad (3.34)$$

Meanwhile, we will fix the parameter in infinity; i.e., we fix α_2 when we change the shock wave t_w . Because of

$k^a = (\frac{\partial}{\partial \lambda})^a$, one can obtain $dr/d\lambda = H(r, v)$. Then, using the expressions of Θ and $\hat{\Theta}$, one can further obtain

$$\Theta = \frac{dH(r, v)}{r}, \quad \hat{\Theta} = \hat{\Theta}_2 \mathcal{H}(r - r_s) + \hat{\Theta}_1 [1 - \mathcal{H}(r - r_s)] \quad (3.35)$$

with

$$\hat{\Theta}_i = \frac{\alpha_i}{r^d} \Phi'_i(r), \quad (3.36)$$

whence the counterterm contribution from the past right null boundary can be written as

$$\begin{aligned} I_{\text{ct}}^{(1)} &= -\Omega_{k,d} \int_0^{r_\Lambda} d\lambda r^d \hat{\Theta} \ln \left(\frac{dl_{\text{ct}} H(r, v)}{r} \right) \\ &= -\Omega_{k,d} \int_0^{r_\Lambda} dr \Phi'(r) \ln \left(\frac{dl_{\text{ct}} H(r, v)}{r} \right) \\ &= -\Omega_{k,d} \left[\int_0^{r_\Lambda} dr \Phi'(r) \ln \left(\frac{dl_{\text{ct}} \alpha_2}{r} \right) + \Phi_1(r_s) \ln \left(\frac{f_1(r_s)}{f_2(r_s)} \right) \right]. \end{aligned} \quad (3.37)$$

Then, we have

$$\frac{dI_{\text{ct}}^{(1)}}{dt_w} = \frac{\Omega_{k,d}}{2} \left[\Phi_1'(r_s) f_2(r_s) \ln \left(\frac{f_1(r_s)}{f_2(r_s)} \right) + \Phi_1(r_s) \left(\frac{f_2(r_s) f_1'(r_s)}{f_1(r_s)} - f_2'(r_s) \right) \right]. \quad (3.38)$$

With the same calculation, one can further obtain the counterterm contribution from the left future null segment as

$$\frac{dI_{\text{ct}}^{(2)}}{dt_w} = \frac{\Omega_{k,d}}{2} \left[\Phi_2'(r_b) f_1(r_b) \ln \left(\frac{f_2(r_b)}{f_1(r_b)} \right) + \Phi_2(r_b) \left(\frac{f_1(r_b) f_2'(r_b)}{f_2(r_b)} - f_1'(r_b) \right) \right]. \quad (3.39)$$

Summing the various expressions above, the slope of the complexity of formation can be written as

$$\begin{aligned} \frac{d\Delta\mathcal{C}_{\text{mod}}}{dt_w} &= \frac{d\Omega_{k,d}}{(d+1)\pi\hbar} \left[\Phi_1'(r_s) f_2(r_s) \ln \left(\frac{f_1(r_s)}{f_2(r_s)} \right) + \Phi_2'(r_b) f_1(r_b) \ln \left(\frac{f_2(r_b)}{f_1(r_b)} \right) \right. \\ &\quad \left. + \Phi_1(r_s) \left(\frac{f_2(r_s) f_1'(r_s)}{f_1(r_s)} - f_2'(r_s) \right) + \Phi_2(r_b) \left(\frac{f_1(r_b) f_2'(r_b)}{f_2(r_b)} - f_1'(r_b) \right) \right]. \end{aligned} \quad (3.40)$$

At the large time limit $t_w \rightarrow \infty$, we have $r_s \rightarrow r_{h,2}$, $r_b \rightarrow r_{h,1}$. The slope of the complexity of formation becomes

$$\left. \frac{d\Delta\mathcal{C}_{\text{mod}}}{dt_w} \right|_{t_w \gg t_{\text{scr}}^*} = -\frac{d\Omega_{k,d}}{(d+1)\pi\hbar} [\Phi_1(r_{h,2}) f_2'(r_{h,2}) + \Phi_2(r_{h,1}) f_1'(r_{h,1})]. \quad (3.41)$$

To study the switchback effect, next, we consider the light shock wave case; i.e., the null shell only injects a small amount of energy into the system. In this situation, we have $f_1 \simeq f_2$. Then, Eq. (3.41) becomes

$$\left. \frac{d\Delta\mathcal{C}_{\text{mod}}}{dt_w} \right|_{t_w \gg t_{\text{scr}}^*} = \frac{4dTS}{(d+1)\pi\hbar}. \quad (3.42)$$

FIG. 3. The derivative of the complexity of formation with respect to t_w for the Bañados-Teitelbiom-Zanelli geometry, where we set $L = 1$, $\omega_1 = 1$, and the dashed lines describe the corresponding scrambling time $t = t_{\text{scr}}^*$.

This result is in agreement with the switchback effect, which has been illustrated in the previous literature [28,29] for the CA conjecture.

To be specific, in Fig. 3, we present the slope of the complexity of formation for a light shock wave in a Bañados-Teitelbiom-Zanelli black hole for Einstein gravity. We can see that there exists a scrambling time t_{scr}^* , which is characterized by the energy of the shock wave. And the slope is approximately zero until $t \simeq t_{\text{scr}}^*$, at which point it rapidly rises to the final constant value. This implies that in the order of the scrambling time t_{scr}^* the complexity of formation is the same as in the case of the unperturbed state, and in the regime of $t_w > t_{\text{scr}}^*$, it grows linearly with respect to the time t_w .

C. Circuit analogy

In this subsection, we would like to investigate the connection between the behaviors of our holographic results and the switchback effect of the circuit model. As illustrated in (3.19), the evolution of this perturbed state can be divided into two processes: before and after the perturbation. In the following, we denote the rate of the complexity to c_1 before the operator $\mathcal{O}_R$ is inserted and c_2 after it [28]. Under the limit of the light shock wave, according to (3.10), we have

$$c_1 \approx c_2 \approx \frac{2dTS}{(d+1)\pi\hbar}. \quad (3.43)$$

First of all, we consider the case $t_w < t_{\text{scr}}^*$. When $t_L < t_w$, the process in (3.20) can be illustrated in Fig. 4(b). In this situation, the switchback effect produces a cancellation for the process below the dashed line. Therefore, the complexity is given by

FIG. 4. A representation of the insertion of a perturbed operator $\mathcal{O}_R$ at the time $-t_w$ for the TFD state, in analogy to the construction in Fig. 25 of Ref. [28] as well as Fig. 6 of Ref. [8].

$$C_{\text{pert}} \approx \frac{2dTS}{(d+1)\pi\hbar}, \quad (3.44)$$

where we set $t_L = t_R = t/2$. One can note that this complexity is exactly the result of the eternal case in which the cancellation is always valid for the process below the dashed line. When $t_L > t_w$, the process can be illustrated by Fig. 4(c). We can see that there is no opportunity for the switchback effect. Hence, the complexity is also the result of the eternal case, which can be described by (3.44). For a summary, we find that when $t_w < t_{\text{scr}}^*$, by the switchback effect, the complexity is the same as that of the unperturbed state.

Then, we consider the case $t_w > t_{\text{scr}}^*$. When $t_L - t_w > -t_{\text{scr}}^*$, the complexity shares the same result with the case $t_w < t_{\text{scr}}^*$. When $t_w - t_L > t_{\text{scr}}^*$, the process can be illustrated by Fig. 4(a). In this case, the two time-evolution operators cancel out only during the scrambling time. Therefore, the complexity can be written as

$$C_{\text{pert}} \approx \frac{4dTS}{(d+1)\pi\hbar} (t_w - t_{\text{scr}}^*). \quad (3.45)$$

This result implies that the growth rate is very close to zero in the regime $t < 2(t_w - t_{\text{scr}}^*)$.

Next, we consider the complexity of formation. By setting $t = 0$ and using the above equations, one can obtain

$$\frac{dC_{\text{pert}}}{dt_w} \approx \frac{4dTS}{(d+1)\pi\hbar} \mathcal{H}(t_w - t_{\text{scr}}^*). \quad (3.46)$$

Again, this formula also matches our holographic result as illustrated in the last subsection in which when $t < t_{\text{scr}}^*$ the rate of the complexity of formation is close to zero and when $t > t_{\text{scr}}^*$ it remains constant.

IV. CONCLUSION

It has been found that the CA complexity growth rate is divergent when a higher-curvature correction is taken into account [57]. This immediately seems unphysical. The Lloyds bound suggests that any system, even if it has an infinite-dimensional Hilbert space, should have a finite complexity growth rate. Then, as suggested by Nally [57], if the CA conjecture should prove correct in these perturbative gravitational theories, the complexity growth rate must be a nonanalytic function of the small coupling parameters. This means that when all of the corrections are concerned the CA complexity growth rate will be convergent. However, through our calculations, we found that these divergences can also be found in the non-perturbative theory. Therefore, we can not simply treat it as a nonanalytic function. It seems clear that either the CA proposal does not work in general or the physical issues associated with the introduction of higher-derivative corrections are somehow polluting the calculation. There might be two strategies to eliminate this failure of the CA complexity: one is to directly modify the CA proposal, and another is to introduce some counterterms to subtract these divergences. And in this paper, in order to obtain expected properties of the holographic complexity, we modified the CA conjecture, in which we assume that the complexity of the boundary state equals the boundary

actions contributed by the null segments as well as the joints of the WDW patch. Then, we can see this modified holographic complexity will be convergent.

Since it is a candidate of the boundary complexity, it is also necessary for us to check whether it gives expected properties as the circuit complexity, such as the late-time growth rate and switchback effect. Therefore, in Sec. III A, we first calculate the complexity growth in a SAdS-type spacetime for a general $F(R_{abcd})$ gravity. We found that the late-time growth rate of this modified holographic complexity is proportional to TS , which is quite in agreement with circuit analysis, in which the entropy represents the width of the circuit and the temperature is an obvious choice for the local rate. Hence, at this point, our conjecture is suitable for describing the circuit complexity of the boundary state. Finally, we also investigate the switchback effect of this new conjecture in Vaidya geometry. And our result is actually in agreement with the switchback effect, which has been illustrated in the previous literature for the original CA conjecture, although here all of the contributions at large time come from the counterterms of the null boundary.

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