Matter-coupled supersymmetric Kerr-Newman-AdS₄ black holes

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We present new analytic rotating four-dimensional anti-de Sitter space (AdS₄) black holes, found as solutions of gauged $\mathcal{N} = 2$ supergravity coupled to Abelian vector multiplets with a symmetric scalar manifold. These configurations preserve two real supercharges and have a smooth limit to the BPS Kerr-Newman-AdS₄ black hole. We spell out the solution of the *STU* model admitting an uplift to M-theory on S⁷. We identify an entropy function, which upon extremization gives the black hole entropy, to be holographically reproduced by the leading N contribution of the generalized superconformal index of the dual theory.

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I. INTRODUCTION

The AdS/CFT correspondence provides the natural setting for the statistical interpretation of black hole entropy in terms of a microscopic theory. The derivation of the Bekenstein-Hawking entropy of four-dimensional anti-de Sitter (AdS) black holes as a leading contribution to the Witten index of the boundary dual field theory [1] opened the way to new computations in this regard (see [2] for a review). The microstate description of rotating black holes is of particular importance, since the black holes in our Universe are often spinning close to extremality. Supersymmetry in AdS is compatible with the presence of angular momentum, therefore on the gravity side the first step is to find the relevant black hole solutions.

In this paper we extend the supersymmetric Kerr-Newman-AdS₄ (KN-AdS₄) solutions to supergravity models with vector multiplets. This corresponds to the last missing entry in Table I, summarizing all known spherical Bogomol'ny-Prasad-Sommerfield (BPS) black holes with AdS₄ asymptotics. The value of the gauged R-symmetry magnetic flux (given by $\langle G, \Gamma \rangle$ in the symplectically covariant notation we follow) distinguishes between two types of supersymmetry preserving asymptotics [3,4]. When the flux vanishes we have the asymptotically AdS₄ solution, while the case when the flux is fixed to -1 is an example of a particular asymptotically locally AdS space that was dubbed “magnetic AdS” in [3]. This dichotomy is well understood

on the AdS boundary for three-dimensional supersymmetric theories [5], where either full superconformal symmetry is preserved ($\langle G, \Gamma \rangle = 0$) or there is only a partial supersymmetry via the so-called topological twist ($\langle G, \Gamma \rangle = -1$). Static spherical BPS black holes in magnetic AdS₄ exist only after coupling gravity to additional matter [6–8] and these admit rotating generalizations found recently in [9]. Here instead we are after the other branch of rotating solutions, asymptotic to global AdS₄. The KN-AdS₄ black holes in Einstein-Maxwell theory with cosmological constant (embeddable in minimal gauged $\mathcal{N} = 2$ supergravity) were found in [10] and their supersymmetric limit was analyzed in [11,12]. Considering the $X^0 X^1$ model of one minimally coupled vector multiplet, [13,14] found electric supersymmetric KN solutions upon taking the BPS limit of thermal black hole solutions (hyperbolic horizons in the same model were found in [15]). In the present work we focus from the outset on supersymmetric black holes, constructed through the BPS equations of supergravity coupled to vector multiplets. Assuming that the associated scalar manifold is symmetric, we solve the BPS equations for any such model defined by an arbitrary gauging vector. From the point of view of holography, the main object of interest is the entropy function, which upon extremization with respect to a set of chemical potentials conjugate to the conserved charges gives the entropy of the newly discovered solutions. We find

$$S = -2 \frac{F(X)}{\omega} - F_I(X) P^I + X^I Q_I + \omega \left(\mathcal{J} + \frac{1}{2} P^I Q_I \right) + \lambda (2g_I X^I - \omega - 2\pi i), \quad (1)$$

where the X^I are conjugate to the electric charges Q_I , ω is conjugate to $\mathcal{J}$, g_I are the Fayet-Iliopoulos (FI) gauging

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TABLE I. Summary of known supersymmetric AdS_4 black holes with spherical topology. An asterisk denotes the absence of a regular horizon, i.e., a naked singularity.

Type	$\text{AdS}_4(\langle G, \Gamma \rangle = 0)$		$\text{mAdS}_4(\langle G, \Gamma \rangle = -1)$	
	Gravity	+ matter	Gravity	+ matter
$J = 0$	[21]*	[22]*	[21]*	[6–8]
$J \neq 0$	[11]	[13] + here	[12]*	[9]

parameters setting the length scale of AdS_4 , and the prepotential $F(X)$ and its derivatives $F_I(X)$ are model-dependent functions of the X^I . λ is a Lagrange multiplier imposing a constraint among the chemical potentials such that upon extremization with respect to the independent set of ω , X^I one recovers the entropy. In the absence of magnetic charges P^I , the above entropy function was introduced in [16] and further elaborated on in [17], based on the previously known example in [13]. We confirm the conjecture of [16] for the full STU model with electric charges and its extension in (1) for a new BPS solution to the $X^0 X^1$ model including a magnetic charge. The Legendre transform of the entropy function presented above is expected to match the saddle point evaluation of the partition function of the holographically dual theory on (Euclidean) $S^1 \times S^2$, which in this case is the generalized superconformal index [18–20].

II. THE REAL FORMULATION OF SUPERGRAVITY

Our starting point is the action for Abelian gauged $\mathcal{N} = 2$ supergravity with n_V vector multiplets. Our conventions and initial steps coincide with those followed in Sec. 2 of [9]. The bosonic fields are the metric $g_{\mu\nu}$, $(n_V + 1)$ Abelian gauge fields A_μ^I ($I = 0, \dots, n_V$) and n_V complex scalars z^i ($i = 1, \dots, n_V$). The Lagrangian and supersymmetry transformation rules are uniquely specified by a choice of the so-called prepotential $F(X^I)$ and the symplectic vector of FI parameters $G = \{g^I, g_I\}$ defining a combination of Abelian gauge fields gauging the R-symmetry.

The BPS equations for solutions with a timelike Killing vector were given in [23–25], with a metric

$$ds_4^2 = -e^{2U}(dt + \omega d\phi)^2 + e^{-2U}ds_3^2, \quad (2)$$

where ds_3^2 is the metric of a three-dimensional base space, on which all quantities are defined.

We express the original complex scalars z^i and scale factor e^U in terms of a rescaled symplectic section

$$\mathcal{R} + i\mathcal{I} = e^{-U}\{X^I, F_I\}, \quad (3)$$

where $F_I \equiv \partial F / \partial X^I$. We choose to only involve $\mathcal{I}$ in our explicit *Ansatz*, noting that one can further use

$$\mathcal{R} = -\frac{1}{2I_4(\mathcal{I})}I_4'(\mathcal{I}) = -\frac{1}{2}e^{4U}I_4'(\mathcal{I}), \quad (4)$$

and then by the special choice of coordinates $z^i = (\mathcal{R} + i\mathcal{I})^i / (\mathcal{R} + i\mathcal{I})^0$ recover the physical scalars. In writing (4) we already assumed that the special Kähler manifold parametrized by the scalar fields is a symmetric space, such that we can use the quartic invariant formalism reviewed in Sec. 2.2 of [9]. The quartic form I_4 is invariant under symplectic transformations, while its derivative I_4' is a symplectic vector and is therefore covariant. One can explicitly evaluate I_4 and its derivatives for any given symmetric model. We are especially interested in the so-called magnetic STU model,

$$F^{mSTU} = -2i\sqrt{X^0 X^1 X^2 X^3}, \quad (5)$$

and purely electric gauging $G = \{0, g_I\}$, because the resulting Lagrangian can be embedded in 11D supergravity compactified on S^7 [22,26]. The I_4 invariant in this case was explicitly spelled out in Sec. 2.2 in [9].

III. BASE SPACE ANSATZ

The BPS equations of [23–25] are conveniently cast using the following metric on the 3D base

$$ds_3^2 = d\rho^2 + e^{2\varphi}(dx^2 + dy^2), \quad (6)$$

for a general function $\varphi(\rho, x, y)$. For stationary black hole solutions, one further assumes that $\partial/\partial y$ is also an isometry, leaving us with $\varphi(\rho, x)$. There are two major classes of black hole solutions in the literature, depending on the separability of φ . The restricted choice

$$e^{2\varphi} = \Phi(x)e^{2\psi(\rho)} \quad (7)$$

leads to Cacciatori-Klemm-type solutions [6–8] and their rotating generalizations [9]. Here instead we focus on another class, leading to Kerr-Newman-type solutions [27] (and even more generally to the Plebanski-Damianski solution). In this case $e^{2\varphi}$ is separable in terms of new coordinates q and p , such that

$$e^{2\varphi} = Q(q)P(p), \quad \rho = qp, \quad x = \alpha(q) + \beta(p), \quad (8)$$

with arbitrary functions $Q(q)$, $P(p)$, while the functions $\alpha(q)$ and $\beta(p)$ are conventionally chosen as

$$\alpha'(q) = -\frac{q}{Q(q)}, \quad \beta'(p) = \frac{p}{P(p)}, \quad (9)$$

in order to bring the base metric in the diagonal form

$$ds_3^2 = e^{2\sigma} \left(\frac{dp^2}{P(p)} + \frac{dq^2}{Q(q)} \right) + Q(q)P(p)dy^2, \quad (10)$$

where we defined

$$e^{2\sigma} \equiv q^2 P(p) + p^2 Q(q). \quad (11)$$

The standard form of the base metric for supersymmetric Kerr-Newman is reached upon setting $\{q, p, y\} \sim \{r, \cos \theta, \phi\}$ where r is a radial coordinate and θ, ϕ are coordinates on a sphere.

IV. BPS EQUATIONS

We can massage the set of equations in Sec. 2.3 of [9] using the *Ansatz* for the base space discussed above. The symmetries imposed guarantee that the superalgebra of the resulting solution is given by $U(1|1)$, i.e., a quarter-BPS configuration. We find that the vielbein BPS equation (2.34) in [9] for the choice $\hat{G} = Gd(qp)$ leads to

$$\begin{aligned} \langle G, \mathcal{I} \rangle &= \frac{1}{4} e^{-2\sigma} \frac{\partial^2 e^{2\sigma}}{\partial q \partial p}, \\ \langle G, \mathcal{A} \rangle &= \frac{1}{2} e^{-2\sigma} (pQ(q)P'(p) - qP(p)Q'(q))dy, \end{aligned} \quad (12)$$

where $\mathcal{A}$ is the spatial part of the symplectic vector of electric/magnetic vector fields. The equation for the rotation one-form ω can be compactly written as

$$\star d\omega = \langle d\mathcal{I}, \mathcal{I} \rangle + \langle G, I'_4(\mathcal{I}) \rangle d(qp). \quad (13)$$

Finally, we need to solve the BPS equation for the scalars and electromagnetic fields,

$$\begin{aligned} \mathcal{F} &= qpGd\omega - \star d\mathcal{I} \\ &- \star d(qp) \left(\langle G, \mathcal{I} \rangle \mathcal{I} - \frac{1}{4} I'_4(\mathcal{I}, \mathcal{I}, G) \right). \end{aligned} \quad (14)$$

The equations of motion in this case are implied by the above equations and the requirement that the symplectic vector $\mathcal{F}$ of field strengths be closed, $d\mathcal{F} = 0$. Similar to [9], a rescaling of the symplectic section is convenient for expressing the BPS equations. For the class of solutions based on Eqs. (10) and (11) we use

$$\mathcal{H} = e^{2\sigma} \mathcal{I}, \quad I_4(\mathcal{H}) = e^{8\sigma} e^{-4U}. \quad (15)$$

This change of variable in Eqs. (12)–(14) brings the BPS equations to a form that can be solved in terms of polynomial *Ansätze* for the variables $e^{2\sigma}$ and $\mathcal{H}$, as will be shown explicitly below.

V. NEAR-HORIZON SOLUTION

We first want to solve the BPS equations (12)–(14) near the horizon. We impose an *Ansatz* compatible with the $SU(1,1)$ isometry group of AdS_2 such that the superalgebra is further enhanced to $SU(1,1|1)$, a half-BPS configuration. We choose here $q \equiv r$ and the function $Q(r)$ as

$$Q(r) = R_0^2 r^2, \quad (16)$$

where r is a radial coordinate and R_0 is a constant, so that $e^{2\sigma}$ is also separable,

$$e^{2\sigma} = r^2 e^{2\sigma_0}, \quad e^{2\sigma_0} = P(p) + R_0^2 p^2. \quad (17)$$

The conical structure of the base space implies the scaling behavior

$$e^{-2U} = \frac{1}{r^2} e^{-2U_0}, \quad \omega = \frac{1}{r} \omega_0, \quad \mathcal{H} = r \mathcal{H}_0, \quad (18)$$

for the components of the metric and the scalars, where the functions U_0, ω_0 , and $\mathcal{H}_0$ depend only on p .

With this *Ansatz*, we are left with solving the BPS equations in order to determine the dependence on the coordinate p , i.e., along the sphere. We first combine Eqs. (13) and (14) to solve for the rotation one-form as

$$\omega_0 = \nu P(p) e^{-2\sigma_0} = \nu \frac{P(p)}{P(p) + R_0^2 p^2}, \quad (19)$$

where ν is a constant to be fixed in due course. The remaining equations can be written in terms of $e^{2\sigma_0}$, the symplectic vector $\mathcal{H}_0$, and its contractions with the vector of parameters G . A polynomial *Ansatz* for $\mathcal{H}_0$,

$$\mathcal{H}_0 = \mathcal{C}_3 p^3 + \mathcal{C}_2 p^2 + \mathcal{C}_1 p + \mathcal{C}_0, \quad (20)$$

allows us to integrate Eq. (12) for e^{σ_0} as

$$\begin{aligned} e^{2\sigma_0} &= \frac{1}{2} \langle G, \mathcal{C}_3 \rangle p^4 + \frac{2}{3} \langle G, \mathcal{C}_2 \rangle p^3 \\ &+ \langle G, \mathcal{C}_1 \rangle p^2 + 2 \langle G, \mathcal{C}_0 \rangle p + \Xi^{-1}, \end{aligned} \quad (21)$$

with Ξ an integration constant. The BPS equations (14) are then solved order by order in p , remaining with a single constant symplectic vector $\mathcal{C}$,

$$\begin{aligned} \mathcal{C}_0 &= \frac{1}{\Xi} \mathcal{C}, \quad \mathcal{C}_3 = \frac{1}{2\Xi} I_4(\mathcal{C}) I'_4(G), \\ \mathcal{C}_1 &= \frac{1}{\Xi} \left(\langle G, \mathcal{C} \rangle \mathcal{C} + \frac{1}{4} I'_4(\mathcal{C}, \mathcal{C}, G) \right), \\ \mathcal{C}_2 &= -\frac{1}{2\Xi} \left(\langle G, I'_4(\mathcal{C}) \rangle G + \frac{1}{4} I'_4(I'_4(\mathcal{C}), G, G) \right). \end{aligned} \quad (22)$$

The rotation parameter ν is fixed in terms of $\mathcal{C}$ from

$$\nu = -\frac{1}{2\Xi} \langle G, I'_4(\mathcal{C}) \rangle. \quad (23)$$

Finally, the gauge field strengths are given by

$$\mathcal{F} = R_0^2 d(e^{-2\sigma_0} p(\mathcal{H}_0 - \nu p^2 G) dy), \quad (24)$$

and can be seen to satisfy automatically the second condition in Eq. (12).

We have presented a complete supersymmetric solution, which in general may allow for various horizon topologies and features nonvanishing Newman-Unti-Tamburino (NUT) charge. Although such solutions are interesting in their own right, here we focus on compact horizons, requiring that the space spanned by p and y is of spherical topology without any NUT charge. This translates to conditions on the function $P(p)$, as discussed in detail in Sec. 2.5 of [28], in particular that $P(p)$ must have two roots and be an even function, which by Eqs. (17) and (21) imply

$$\langle G, \mathcal{C} \rangle = 0, \quad \langle I'_4(G), I'_4(\mathcal{C}) \rangle = 0, \quad (25)$$

$$\Xi R_0^2 = 1 + I_4(G)I_4(\mathcal{C}) + \frac{1}{4}I_4(\mathcal{C}, \mathcal{C}, G, G), \quad (26)$$

leading to

$$P(p) = \frac{1}{\Xi} (1 - I_4(G)I_4(\mathcal{C})p^2)(1 - p^2). \quad (27)$$

We also need to restrict the range of the coordinate p to only reach until the smaller of the two double roots, and thus arrive at the coordinate redefinition

$$p = \cos \theta, \quad y = \phi, \quad (28)$$

which brings the metric in a more conventional form in terms of the spherical coordinates $\{\theta, \phi\}$, upon the additional requirement that $I_4(G)I_4(\mathcal{C}) < 1$.

The final constraint on the spherical part of the metric comes from the requirement that near the poles $p = \pm 1$ ($\theta = 0, \pi$) we recover flat space (i.e., no conical singularities), which fixes

$$\Xi = 1 - I_4(G)I_4(\mathcal{C}). \quad (29)$$

To make the relation with previous literature more manifest, we can define

$$\Xi \equiv \left(1 - \frac{a^2}{l^2}\right), \quad a \equiv \frac{\sqrt{I_4(\mathcal{C})}}{l}, \quad (30)$$

where $l = (I_4(G))^{-1/4}$ sets the AdS_4 radius, as we show in the next section. The constraint that $\Xi > 0$ also translates

in the more familiar $a < l$ and the metric function P becomes

$$P(\theta) = \frac{\sin^2 \theta}{\Xi} \left(1 - \frac{a^2}{l^2} \cos^2 \theta\right). \quad (31)$$

The resulting charge vector (with the usual periodicity of θ, ϕ) is then computed through Eq. (24), as

$$\Gamma \equiv \frac{1}{4\pi} \int \mathcal{F} = \frac{1}{\Xi} \left(\mathcal{C} - \frac{1}{8} I'_4(I'_4(\mathcal{C}), G, G) \right). \quad (32)$$

This constitutes the main attractor equation, through which the scalars and metric functions encoded in the vector $\mathcal{C}$ can be solved for in terms of the charges $\Gamma = \{P^I, Q_I\}$. If we now contract Eq. (32) with the vector G and use the constraints (25), we find the anticipated constraint on the magnetic flux of the R-symmetry,

$$\langle G, \Gamma \rangle = 0. \quad (33)$$

We can also present the Bekenstein-Hawking entropy in the compact form

$$S = \frac{\pi}{\Xi} \left(\Xi R_0^2 I_4(\mathcal{C}) - \frac{1}{4} \langle G, I'_4(\mathcal{C}) \rangle^2 \right)^{1/2}, \quad (34)$$

where R_0 is given by Eq. (26) and we use units where the Newton constant is fixed as $G_N = 1$.

VI. FULL FLOW

We now use $q = r$ as a radial variable that runs between the horizon and the asymptotic AdS_4 spacetime. It is natural to extend the near-horizon behavior of $\mathcal{H}$ in Eq. (18) to the more general polynomial *Ansatz*

$$\mathcal{H} = r(\mathcal{H}_0(p) + (\mathcal{H}_1^{(0)} + \mathcal{H}_1^{(1)} p)r + \mathcal{H}_2^{(1)} p r^2), \quad (35)$$

where all vectors $\mathcal{H}_1^{(0,1)}$ and $\mathcal{H}_2^{(1)}$ are constant, while $\mathcal{H}_0$ is automatically identified with the one on the horizon. The highest power of r is dictated by the fact that the $e^{2\sigma}$ arising from Eq. (12) needs to be quartic in r to keep the desired AdS_4 asymptotics. With this *Ansatz* the expression for $e^{2\sigma}$ is

$$e^{2\sigma} = r^2 \left(e^{2\sigma_0} + \frac{2}{3} \langle G, \mathcal{H}_1^{(1)} r + \mathcal{H}_2^{(1)} r^2 \rangle p^2 \right), \quad (36)$$

upon disregarding integration constants and imposing $\langle G, \mathcal{H}_1^{(0)} \rangle = 0$, in order to keep the structure assumed in Eq. (11). We also make the following *Ansatz* for the rotation form ω ,

$$\omega = e^{-2\sigma} (\mu Q(r) + \nu r P(p)) - \mu, \quad (37)$$

where ν was fixed already on the horizon in Eq. (23), while μ is another integration constant.

Plugging in the full flow *Ansatz* for $\mathcal{H}$ and ω in the BPS Eqs. (12)–(14) results in an overconstrained system of equations from the various powers of p and r . The solution is eventually fully fixed in terms of the vectors $\mathcal{C}$ and G . We find

$$\begin{aligned}\mathcal{H}_2^{(1)} &= \frac{l^2}{2\Xi} I_4'(G), & \mathcal{H}_1^{(0)} &= \frac{l}{\Xi} G, \\ \mathcal{H}_1^{(1)} &= \frac{l}{4\Xi} I_4'(\mathcal{C}, G, G), & \mu &= -\frac{l}{\Xi}.\end{aligned}\quad (38)$$

The metric function Q therefore becomes

$$Q(r) = \frac{r^2}{\Xi} \left(\Xi R_0^2 + l \langle \mathcal{C}, I_4'(G) \rangle r + \frac{r^2}{l^2} \right), \quad (39)$$

where the first term in the brackets also depends explicitly on the vectors $\mathcal{C}$ and G via Eq. (26). The solutions found here asymptote to AdS₄ with boundary metric

$$\begin{aligned}ds^2 &= \frac{r^2}{\Xi} \left[-\frac{P(\theta)}{\sin^2\theta} \frac{\Xi}{l^2} dt^2 + \frac{\sin^2\theta}{P(\theta)} d\theta^2 \right. \\ &\quad \left. + \sin^2\theta \left(d\phi + \frac{1}{l} dt \right)^2 \right].\end{aligned}\quad (40)$$

The subleading terms of the metric near this boundary encode the mass M which we computed via the Ashtekar-Magnon-Das procedure [29,30], and the angular momentum $\mathcal{J}$,

$$\mathcal{J} = \frac{1}{2\Xi} \left(\frac{I_4(\mathcal{C})}{\Xi} \langle \mathcal{C}, I_4'(G) \rangle - \left(1 + \frac{I_4(\mathcal{C})}{l^4} \right) \nu \right), \quad (41)$$

computed through the Komar integral. Combined with the charges in Eq. (32), we find that the following BPS bound is obeyed:

$$M = \frac{\mathcal{J}}{l} + \frac{l^3}{4} \langle \Gamma, I_4'(G) \rangle = \frac{\mathcal{J}}{l} + \frac{1}{2\sqrt{2}} \sum_{I=1}^4 Q_I, \quad (42)$$

where the second term corresponds to the R-symmetry charge T [3,4] and in the second equality we evaluated explicitly for the *STU* model below. Finally, the roots of $Q(r)$ in Eq. (39) determine the location of the four horizons. The product of their areas, as expected, depends only on quantized charges [31–33]:

$$\prod_{\alpha=1}^4 A_\alpha = (4\pi)^4 l_{\text{AdS}}^4 (I_4(\Gamma) + 4\mathcal{J}^2). \quad (43)$$

VII. SOLUTIONS OF THE *STU* MODEL

We now look for explicit solutions of the *STU* model. We work in the standard electric gauging frame, with the prepotential (5) and the FI terms given by

$$G = \{0, 0, 0, 0; g, g, g, g\}, \quad (44)$$

resulting in AdS₄ length scale $l = (\sqrt{2}g)^{-1}$. To write down a solution, we need to find a symplectic vector $\mathcal{C}$ that satisfies the constraints (25), leaving us with up to six free parameters: four independent electric and two independent magnetic charges. These constraints are however nonlinear and for the sake of brevity we choose to give a configuration corresponding to four electric and only one independent magnetic charge,

$$\mathcal{C}^{STU} = \left\{ -\alpha, \alpha, -\alpha \frac{(\beta_0 - \beta_1)}{(\beta_2 - \beta_3)}, \alpha \frac{(\beta_0 - \beta_1)}{(\beta_2 - \beta_3)}; \beta_I \right\}, \quad (45)$$

for constant α and β_I for $I = 1 \dots 4$. The vector $\mathcal{C}$ then determines the conserved charges [through Eq. (32)] and physical properties of the black holes. The full expressions for the conserved charges are in general rather long, therefore in the following we restrict the parameters α , β_I in two different ways that provide a more accessible insight into the properties of the solutions.

A. The T^3 model with electric charges

The T^3 truncation is achieved by setting equal the three different vector multiplets. In this case, Eqs. (25) do not allow for an independent magnetic charge, so we concentrate on the purely electric case, setting $\alpha = 0$, $\beta_1 = \beta_2 = \beta_3$ in $\mathcal{C}^{STU}$ to find

$$\mathcal{C}^{T^3} = \{0, 0, 0, 0; \beta_0, \beta_1, \beta_1, \beta_1\}. \quad (46)$$

We can find the electric charges via Eq. (32),

$$\begin{aligned}Q_0 &= \frac{1}{\Xi} (\beta_0 + 2g^2(\beta_1)^2(3\beta_0 - \beta_1)), \\ Q_1 &= Q_2 = Q_3 = \frac{1}{\Xi} (\beta_1 + 2g^2(\beta_1)^2(\beta_0 + \beta_1)),\end{aligned}\quad (47)$$

with

$$\Xi = 1 - 16g^4\beta_0(\beta_1)^3, \quad a = 2\sqrt{2}g\sqrt{\beta_0(\beta_1)^3}, \quad (48)$$

and the entropy via Eq. (34),

$$\begin{aligned}S^{T^3} &= \frac{2\pi}{\Xi} (\beta_0(\beta_1)^3(1 + 16g^4\beta_0(\beta_1)^3) \\ &\quad + g^2(\beta_1)^4(6\beta_0\beta_1 + 3(\beta_0)^2 - (\beta_1)^2))^{1/2}.\end{aligned}\quad (49)$$

The entropy as a function of electric charges follows the expected behavior from the entropy function (1), where the angular momentum is also fixed to be

$$\mathcal{J}^{T^3} = \frac{g(\beta_1)^2}{\Xi^2} ((3\beta_0 + \beta_1)(1 + 16g^4\beta_0(\beta_1)^3) + 8g^2\beta_0\beta_1(\beta_0 + 3\beta_1)). \quad (50)$$

We have also checked explicitly the validity of Eq. (1) for the full STU model with four independent electric charges.

B. The X^0X^1 model with dyonic charges

Here we specialize to the so-called X^0X^1 model, for which the parameters in Eq. (45) are identified pairwise, $\beta_0 = \beta_2$, $\beta_1 = \beta_3$ such that

$$\mathcal{C}^{X^0X^1} = \{-\alpha, \alpha, -\alpha, \alpha; \beta_0, \beta_1, \beta_0, \beta_1\}. \quad (51)$$

This corresponds to having two independent electric charges and one free magnetic charge and thus generalizes the already existing purely electric solution of [13]. The set of conserved charges is given by

$$P \equiv P^0 = -P^1 = -\frac{\alpha}{\Xi} (1 + 4g^2(\beta_0\beta_1 - \alpha^2)),$$

$$Q_{0,1} = \frac{\beta_{0,1}}{\Xi} (1 + 4g^2(\beta_0\beta_1 - \alpha^2)), \quad (52)$$

with

$$\Xi = 1 - 16g^4(\beta_0\beta_1 - \alpha^2)^2, \quad a = 2\sqrt{2}g(\beta_0\beta_1 - \alpha^2). \quad (53)$$

The entropy and angular momentum can be computed via Eqs. (34) and (41), respectively, as

$$S^{X^0X^1} = \frac{\pi}{4g^2} \left(-1 + \sqrt{1 + 16g^2(Q_0Q_1 - P^2)} \right),$$

$$= \frac{\pi \mathcal{J}^{X^0X^1}}{g(Q_0 + Q_1)}. \quad (54)$$

These quantities are again consistent with the entropy function (1) and coincide with those of [13] in the limit of vanishing magnetic charge.

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