Exotic pentaquark states with the $qqQQ\bar{Q}$ configuration

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(Received 22 May 2019; published 10 September 2019)

In the framework of the color-magnetic interaction model, we systematically calculate the mass splittings for the S -wave triply heavy pentaquark states with the configuration $qqQQ\bar{Q}$ ($Q = c, b$; $q = u, d, s$). Their masses are estimated and their stabilities are discussed according to possible rearrangement decay patterns. Our results indicate that there may exist several stable or narrow such states. We hope the present study can help experimentalists search for exotic pentaquarks.

DOI: [10.1103/PhysRevD.100.056004](https://doi.org/10.1103/PhysRevD.100.056004)

I. INTRODUCTION

The possible existence of multiquark states beyond the ordinary hadrons was first proposed by Gell-Mann and Zweig [1,2]. Nowadays, it is still an important and interesting topic to look for such states [3]. With the experimental progress in recent decades, we are able to find heavy-quark multiquark candidates in various processes. In fact, experimentalists announced more exotic states in recent years since the Belle Collaboration reported the observation of $X(3872)$ in 2003 [4–17], which provides good opportunities to study the nonperturbative color interactions. Some of the states have been considered as good tetraquark candidates [18–26].

Recently, the LHCb Collaboration reported the observation of new pentaquark states at the Rencontres de Moriond QCD conference [27,28]. By analyzing the $J/\psi p$ invariant mass spectrum in the Λ_b^0 decay with updated data, a new pentaquark $P_c(4312)$ was discovered with 7.3σ significance. Meanwhile, the analysis shows two narrow subpeaks with 5.4 significance, $P_c(4440)$ and $P_c(4457)$, for the previously reported $P_c(4450)$ [29,30]. Although within the framework of the one-boson-exchange model, these three new states could be clearly identified as the loosely bound molecules $\Sigma_c \bar{D}$ with $I(J^P) = 1/2(1/2^-)$, $\Sigma_c \bar{D}^*$ with $I(J^P) = 1/2(1/2^-)$, and $\Sigma_c \bar{D}^*$ with

$I(J^P) = 1/2(3/2^-)$, respectively [31–38], Ramírez *et al.* [39] found evidence that the attractive interaction in the $\Sigma_c^+ \bar{D}$ channel is not strong enough to form a bound state.

Because of the complicated interactions between the internal quarks, it is generally hard to distinguish whether a hadron is a tightly bound tetraquark (pentaquark) state, a conventional meson (baryon), a molecular state, or a structure in other pictures. To understand the internal structures of $P_c(4380)$ and $P_c(4450)$ many interpretations have been proposed, such as the $\Sigma_c \bar{D}^*$, $\Sigma_c^* \bar{D}$, and $\Sigma_c^* \bar{D}^*$ molecules [40–51], compact pentaquark states [52–56], diquark-diquark-antiquark states, diquark-triquark states [57–64], $\bar{D}$ solitons, and kinematical effects from the triangle singularity or due to $\chi_{c1} p$ rescattering [65–69].

Studies of other possible pentaquarks were also stimulated by the observation of P_c states [70–74]. After the experimental confirmation of the doubly charmed baryon Ξ_{cc}^{++} [75,76], the multiquark states with two or more heavy quarks were studied in many works [70–74,77]. For example, two possible triple-charm molecular pentaquarks $\Xi_{cc} D_1$ and $\Xi_{cc} D_2^*$ were considered in Ref. [78]. In this paper, we systematically study the mass splittings of compact pentaquark states with the $qqQQ\bar{Q}$ configuration ($q = u, d, s$; $Q = c, b$). If a heavy quark-antiquark pair forms an unflavored state, such pentaquarks look like excited qqQ baryons. Otherwise, they are explicitly exotic states. At present, it is still not easy to dynamically solve the multibody problem. Here, we use the color-magnetic interaction (CMI) model to perform a preliminary calculation the mass splittings and investigate the mass spectrum of the $qqQQ\bar{Q}$ pentaquark states. One may consult relevant studies with other methods in Refs. [79–81].

The Hamiltonian of the quark potential model consists of the one-gluon-exchange interaction part and nonperturbative scalar confining part, which was proposed by de

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Rujula, Georgi, and Glashow in Ref. [82]. The mass splittings of ground-state hadrons with the same quark content (such as Δ and N) are mainly determined by the color-magnetic interaction [83]. When the spacial contributions are encoded in effective quark masses and coupling parameters, the Hamiltonian can be written in a form containing just the quark-mass term and the color-spin interaction term, and one gets the CMI model. There have been many studies about the mass spectrum for multi-quark systems within this model [84–94]. The qualitative properties of the obtained spectra would benefit searches for relevant exotic states. In earlier studies on the pentaquark properties, color-magnetic effects were intensively considered as the primary contribution in an attempt to explain the narrow hadronic resonances as well [95].

This paper is organized as follows. In Sec. II, we introduce the CMI model and construct the flavor $\otimes$ color $\otimes$ spin wave functions for the $qqQQ\bar{Q}$ pentaquark states. In Sec. III, we calculate the relevant Hamiltonian elements and present the corresponding results. In Sec. IV, we give numerical results for the masses of the pentaquark states, illustrate their possible rearrangement decay channels, and discuss the stability of the states. Finally, we present a summary in Sec. V.

II. THE COLOR-MAGNETIC INTERACTION AND THE WAVE FUNCTIONS

The Hamiltonian of the CMI model has the simple form

$$H = \sum_i^5 M_i + H_{\text{CMI}},$$

$$H_{\text{CMI}} = -\sum_{i<j} C_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j.$$

Here, M_i represents the effective quark mass for the i th quark or antiquark and it takes into account the effects from kinetic energy, color confinement, and other terms in realistic potential models. The effective constant $C_{ij} \sim \langle \delta^3(\vec{r}_{ij}) \rangle / (m_i m_j)$ reflects the coupling strength between the i th and j th quarks, which depends on the quark masses and the spatial wave functions of the ground states [89]. The Pauli matrix σ_i and Gell-Mann matrix $\lambda_i (-\lambda_i^*)$ act on the spin and color wave functions of the i th quark (antiquark), respectively.

To calculate the required matrix elements, we construct the wave functions of the ground $qqQQ\bar{Q}$ pentaquark states. They are the direct products of the $\text{SU}(3)_f$ flavor wave function, $\text{SU}(3)_c$ color wave function, and $\text{SU}(2)_s$ spin wave function. Here, we treat the heavy quark/antiquark as a flavor singlet state instead of constructing the wave function with flavor $\text{SU}(4)_f$ symmetry [96]. It is convenient to adopt the diquark-diquark-antiquark base in

FIG. 1. Flavor representations of the $qqQQ\bar{Q}$ pentaquark states. Here $Q = c, b$ and $q = n, s$, with $n = u, d$.

organizing the wave functions. Here “diquark” only means two quarks, and the meaning is different than that in the diquark model in Ref. [97] where the diquark was a strongly correlated quark-quark substructure with color = $\bar{3}_c$ and spin = 0. The constructed wave functions may also be used to study the properties of the $qqQQ\bar{Q}$ states in dynamical quark models.

In the $\text{SU}(3)_f$ flavor space, the $qqQQ\bar{Q}$ states belong to the flavor-symmetric 6_f and antisymmetric $\bar{3}_f$ representations (Fig. 1), which is similar to the situation for part of the $QQqq\bar{q}$ states [89]. For the $nnQQ\bar{Q}$ ($n = u, d$) case, the isovector states ($I = 1$) and the isoscalar states ($I = 0$) do not mix since we do not consider isospin-breaking effects. For the $nsQQ\bar{Q}$ case, the fact that $m_n \neq m_s$ leads to $\text{SU}(3)_f$ breaking and thus state mixing between 6_f and $\bar{3}_f$. As a result, we need to consider four cases of states: $nnQQ\bar{Q}$ ($I = 1$), $nnQQ\bar{Q}$ ($I = 0$), $nsQQ\bar{Q}$ ($I = 1/2$), and $ssQQ\bar{Q}$ ($I = 0$). Note that the isovector and isoscalar $nnQQ\bar{Q}$ states are not degenerate due to the Pauli principle.

In the color space, the wave functions can be analyzed using $\text{SU}(3)_c$ group theory [98]. The Young diagrams tell us that there are three color-singlet wave functions for the $qqQQ\bar{Q}$ states. With the diquark-diquark-antiquark base, they are

$$\begin{aligned} \phi_1 &\equiv \phi^{AA} = [(q_1 q_2)^{\bar{3}_c} (Q_3 Q_4)^{\bar{3}_c} \bar{Q}], \\ \phi_2 &\equiv \phi^{AS} = [(q_1 q_2)^{\bar{3}_c} (Q_3 Q_4)^{6_c} \bar{Q}], \\ \phi_3 &\equiv \phi^{SA} = [(q_1 q_2)^{6_c} (Q_3 Q_4)^{\bar{3}_c} \bar{Q}]. \end{aligned}$$

In the notation $[(q_1 q_2)^{\text{color}_1} (Q_3 Q_4)^{\text{color}_2} \bar{Q}]$, “color₁” and “color₂” stand for the color representations of the light diquark and heavy diquark, respectively. S (A) means “symmetric” (“antisymmetric”) with quark exchanges. The explicit wave functions are the same as those for the $QQqq\bar{q}$ states studied in Ref. [89].

One can also use the baryon-meson base ($qqQ - Q\bar{Q}$ or $qqQ - q\bar{Q}$) to construct the wave functions. The relevant decomposition is

TABLE I. The possible color-spin wave function bases.

$J^P = \frac{5}{2}^-:$		
$\phi_{1\chi_1} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{12}^{\frac{5}{2}}D_{12}^A]$	$\phi_{2\chi_1} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\frac{6}{2}}\bar{Q}_{12}^{\frac{5}{2}}D_{12}^A D_{34}]$	$\phi_{3\chi_1} = [(q_1 q_2)_1^6(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{12}^{\frac{5}{2}}D_{12}^S]$
$J^P = \frac{3}{2}^-:$		
$\phi_{1\chi_2} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{12}^{\frac{3}{2}}D_{12}^A]$	$\phi_{2\chi_2} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\frac{6}{2}}\bar{Q}_{12}^{\frac{3}{2}}D_{12}^A D_{34}]$	$\phi_{3\chi_2} = [(q_1 q_2)_1^6(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{12}^{\frac{3}{2}}D_{12}^S]$
$\phi_{1\chi_3} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^A]$	$\phi_{2\chi_3} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\frac{6}{2}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^A D_{34}]$	$\phi_{3\chi_3} = [(q_1 q_2)_1^6(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^S]$
$\phi_{1\chi_4} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_0^{\bar{3}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^A D_{34}]$	$\phi_{2\chi_4} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_0^{\frac{6}{2}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^A]$	$\phi_{3\chi_4} = [(q_1 q_2)_1^6(Q_3 Q_4)_0^{\bar{3}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^S D_{34}]$
$\phi_{1\chi_5} = [(q_1 q_2)_0^{\bar{3}}(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^S]$	$\phi_{2\chi_5} = [(q_1 q_2)_0^{\bar{3}}(Q_3 Q_4)_1^{\frac{6}{2}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^S D_{34}]$	$\phi_{3\chi_5} = [(q_1 q_2)_0^6(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{11}^{\frac{3}{2}}D_{12}^A]$
$J^P = \frac{1}{2}^-:$		
$\phi_{1\chi_6} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^A]$	$\phi_{2\chi_6} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\frac{6}{2}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^A D_{34}]$	$\phi_{3\chi_6} = [(q_1 q_2)_1^6(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^S]$
$\phi_{1\chi_7} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{10}^{\frac{1}{2}}D_{12}^A]$	$\phi_{2\chi_7} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_1^{\frac{6}{2}}\bar{Q}_{10}^{\frac{1}{2}}D_{12}^A D_{34}]$	$\phi_{3\chi_7} = [(q_1 q_2)_1^6(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{10}^{\frac{1}{2}}D_{12}^S]$
$\phi_{1\chi_8} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_0^{\bar{3}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^A D_{34}]$	$\phi_{2\chi_8} = [(q_1 q_2)_1^{\bar{3}}(Q_3 Q_4)_0^{\frac{6}{2}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^A]$	$\phi_{3\chi_8} = [(q_1 q_2)_1^6(Q_3 Q_4)_0^{\bar{3}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^S D_{34}]$
$\phi_{1\chi_9} = [(q_1 q_2)_0^{\bar{3}}(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^S]$	$\phi_{2\chi_9} = [(q_1 q_2)_0^{\bar{3}}(Q_3 Q_4)_1^{\frac{6}{2}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^S D_{34}]$	$\phi_{3\chi_9} = [(q_1 q_2)_0^6(Q_3 Q_4)_1^{\bar{3}}\bar{Q}_{11}^{\frac{1}{2}}D_{12}^A]$
$\phi_{1\chi_{10}} = [(q_1 q_2)_0^{\bar{3}}(Q_3 Q_4)_0^{\bar{3}}\bar{Q}_{10}^{\frac{1}{2}}D_{12}^S D_{34}]$	$\phi_{2\chi_{10}} = [(q_1 q_2)_0^{\bar{3}}(Q_3 Q_4)_0^{\frac{6}{2}}\bar{Q}_{10}^{\frac{1}{2}}D_{12}^S]$	$\phi_{3\chi_{10}} = [(q_1 q_2)_0^6(Q_3 Q_4)_0^{\bar{3}}\bar{Q}_{10}^{\frac{1}{2}}D_{12}^A D_{34}]$

$$\begin{aligned}
& (3_c \otimes 3_c \otimes 3_c) \otimes (3_c \otimes \bar{3}_c) \\
& = (1_c \oplus 8_c \oplus 8_c \oplus 10_c) \otimes (1_c \oplus 8_c) \\
& \rightarrow (1_c \otimes 1_c) \oplus (8_c \otimes 8_c) \oplus (8_c \otimes 8_c). \quad (1)
\end{aligned}$$

Reference [87] adopted this base in studying the hidden-charm pentaquark states. Although the final Hamiltonians are different for these two bases, the eigenvalues and mass spectra would be identical after diagonalization. However, the baryon-meson base is not suitable for the present systems since two pairs of identical quarks may exist in a state like $nncc\bar{Q}$.

In the spin space, the possible wave functions for the considered states in the diquark-diquark-antiquark base are

$$\begin{aligned}
J^P = \frac{5}{2}^- : \chi_1 &= [(q_1 q_2)_1(Q_3 Q_4)_1\bar{Q}_{12}^{\frac{5}{2}}], \\
J^P = \frac{3}{2}^- : \begin{cases} \chi_2 &= [(q_1 q_2)_1(Q_3 Q_4)_1\bar{Q}_{12}^{\frac{3}{2}}], \\ \chi_3 &= [(q_1 q_2)_1(Q_3 Q_4)_1\bar{Q}_{11}^{\frac{3}{2}}], \\ \chi_4 &= [(q_1 q_2)_1(Q_3 Q_4)_0\bar{Q}_{11}^{\frac{3}{2}}], \\ \chi_5 &= [(q_1 q_2)_0(Q_3 Q_4)_1\bar{Q}_{11}^{\frac{3}{2}}], \end{cases} \\
J^P = \frac{1}{2}^- : \begin{cases} \chi_6 &= [(q_1 q_2)_1(Q_3 Q_4)_1\bar{Q}_{11}^{\frac{1}{2}}], \\ \chi_7 &= [(q_1 q_2)_1(Q_3 Q_4)_1\bar{Q}_{10}^{\frac{1}{2}}], \\ \chi_8 &= [(q_1 q_2)_1(Q_3 Q_4)_0\bar{Q}_{11}^{\frac{1}{2}}], \\ \chi_9 &= [(q_1 q_2)_0(Q_3 Q_4)_1\bar{Q}_{11}^{\frac{1}{2}}], \\ \chi_{10} &= [(q_1 q_2)_0(Q_3 Q_4)_0\bar{Q}_{10}^{\frac{1}{2}}]. \end{cases}
\end{aligned}$$

In the notation $[(q_1 q_2)_{\text{spin}_1}(Q_3 Q_4)_{\text{spin}_2}\bar{Q}_{\text{spin}_3}]^{\text{spin}_4}$, spin_1 and spin_2 represent the spins of the light and heavy diquarks,

respectively, spin_3 represents the total spin of the four quarks, and spin_4 represents the total spin of the pentaquark. The diquark is symmetric (antisymmetric) when $\text{spin}_{1,2}$ is 1 (0).

Combining the spin and color wave functions, we obtain 30 possible bases which are shown in Table I with the notation $[(q_1 q_2)_{\text{spin}_1}^{\text{color}_1}(Q_3 Q_4)_{\text{spin}_2}^{\text{color}_2}\bar{Q}_{\text{spin}_3}]^{\text{spin}_4}$. Not all of them are allowed for a given set of quantum numbers. To reflect the constraint from the Pauli principle, we have inserted the three symbols D_{12}^A , D_{12}^S , and D_{34} into the wave functions. When the light diquark is symmetric (or antisymmetric) in flavor space, $D_{12}^S = 0$ (or $D_{12}^A = 0$); otherwise, $D_{12}^S = 1$ (or $D_{12}^A = 1$). When the two heavy quarks are the same, $D_{34} = 0$; otherwise, $D_{34} = 1$. Considering all possible configurations, we need to analyze 12 $qqQ\bar{Q}\bar{Q}$ systems. They can be divided into six classes:

- (1) $nncc\bar{Q}$ ($I = 1$), $nnbb\bar{Q}$ ($I = 1$), $sscc\bar{Q}$, $ssbb\bar{Q}$;
- (2) $nncc\bar{Q}$ ($I = 0$), $nnbb\bar{Q}$ ($I = 0$);
- (3) $nncb\bar{Q}$ ($I = 1$), $sscb\bar{Q}$;
- (4) $nncb\bar{Q}$ ($I = 0$);
- (5) $nscc\bar{Q}$, $nsbb\bar{Q}$;
- (6) $ns cb\bar{Q}$.

Each class has a similar structure and the same CMI Hamiltonian expressions.

III. THE CMI HAMILTONIAN EXPRESSIONS

With the constructed wave functions, we can calculate the CMI Hamiltonian matrix elements. To simplify the expressions we define combinations of the effective couplings, shown in Table II.

For the pentaquark states without constraints from the Pauli principle, e.g., $ns cb\bar{Q}$, all the color-spin wave function bases in Table I are involved. In the Appendix,

TABLE II. Variables defined to simplify the CMI Hamiltonian matrix elements.

Variable	Definition	Variable	Definition
α	$C_{12} + C_{34}$	ζ	$C_{13} + C_{23} + C_{14} + C_{24}$
β	$C_{34} - 2C_{12}$	η	$C_{13} + C_{23} - C_{14} - C_{24}$
γ	$2C_{34} - C_{12}$	θ	$C_{13} - C_{23} + C_{14} - C_{24}$
δ	$3C_{34} - C_{12}$	κ	$C_{13} - C_{23} - C_{14} + C_{24}$
ϵ	$C_{34} - 3C_{12}$		
ν	$C_{15} + C_{25}$	ξ	$C_{15} - C_{25}$
λ	$C_{35} + C_{45}$	μ	$C_{35} - C_{45}$

we show the obtained CMI matrices for the cases $J^P = 5/2^-, 3/2^-,$ and $1/2^-$ in Tables VII, VIII, and IX, respectively. The relevant matrices for the pentaquark states with constraints from the Pauli principle can be extracted from these tables. Here, we take the $nncc\bar{Q}$ case as an example. When one considers the $I(J^P) = 1(5/2^-)$ state, one has $D_{12}^S = 0$, $D_{12}^A = 1$, and $D_{34} = 0$ and only the base $\phi_1\chi_1$ is allowed. It is easy to read out the CMI Hamiltonian from Table VII,

$$\langle H_{\text{CMI}} \rangle = \frac{1}{3} [8\alpha + 2\zeta + 4(\nu + \lambda)]. \quad (2)$$

Similarly, when one considers the $I(J^P) = 0(5/2^-)$ state, only the wave function base $\phi_3\chi_1$ is allowed because $D_{12}^A = 0$, $D_{12}^S = 1$, and $D_{34} = 0$. The extracted CMI Hamiltonian from Table VII is

$$\langle H_{\text{CMI}} \rangle = \frac{1}{3} (4\gamma + 5\zeta - 2\lambda + 10\nu). \quad (3)$$

IV. THE $qqQQ\bar{Q}$ PENTAQUARK MASS SPECTRA

A. The determination of parameters and estimation strategy

Now, we determine the values of the 17 coupling parameters ($C_{nn}, C_{ns}, C_{ss}, C_{cn}, C_{bn}, C_{cs}, C_{bs}, C_{bc}, C_{cc}, C_{bb}, C_{n\bar{c}}, C_{n\bar{b}}, C_{s\bar{c}}, C_{s\bar{b}}, C_{c\bar{c}}, C_{b\bar{c}}, C_{b\bar{b}},$ and $C_{c\bar{b}}$) in order to estimate the pentaquark masses. Most of them can be extracted from the measured masses of the conventional hadrons (see Table III). The related CMI expressions are

$$\begin{aligned} H_{\text{CMI}}^{J=1}(q_1\bar{q}_2) &= \frac{16}{3}C_{12}, \\ H_{\text{CMI}}^{J=0}(q_1\bar{q}_2) &= -16C_{12}, \\ H_{\text{CMI}}^{J=\frac{3}{2}}(q_1q_2q_3) &= \frac{8}{3}(C_{12} + C_{23} + C_{13}), \\ H_{\text{CMI}}^{J=\frac{1}{2}}(q_1q_2q_3) &= \frac{8}{3} \begin{pmatrix} C_{12} - 2[C_{23} + C_{13}] & \sqrt{3}[C_{23} - C_{13}] \\ \sqrt{3}[C_{23} - C_{13}] & -3C_{12} \end{pmatrix}, \end{aligned}$$

where the two bases for the last matrix correspond to the cases of $J_{q_1q_2} = 1$ and $J_{q_1q_2} = 0$. The obtained coupling parameters are listed in Table I of Ref. [3] and we just use these coupling parameters to calculate the CMI Hamiltonian.

Using the mass formula $M = \sum_i M_i + \langle H_{\text{CMI}} \rangle$ and the obtained parameters, one sees that the estimated masses of conventional hadrons are in general higher than the measured values, which is illustrated in Table IV. The reason is that the adopted model and parameters cannot account for the necessary attractions for all of the hadrons. Overestimated masses with this approach were also

TABLE III. Masses of the conventional hadrons used in our calculations (in units of MeV) [99]. The adopted masses of the not-yet-observed doubly heavy baryons are taken from Ref. [100]. The values in parentheses are obtained with the parameters in Ref. [90].

Mesons ($J = 0$)		Mesons ($J = 1$)		Baryons ($J = \frac{1}{2}$)		Baryons ($J = \frac{3}{2}$)	
D	1869.7	D^*	2010.3	Σ_c	2454.0	Σ_c^*	2518.4
D_s	1968.3	D_s^*	2112.2	Ξ'_c	2577.9	Ξ_c^*	2645.5
				Ω_c	2695.2	Ω_c^*	2765.9
B	5279.3	B^*	5324.7	Σ_b	5811.3	Σ_b^*	5832.1
B_s	5366.9	B_s^*	5415.4	Ξ'_b	5935.0	Ξ_b^*	5955.3
				Ω_b	6046.4	Ω_b^*	6090.0[101]
η_c	2983.9	J/ψ	3096.9	Ξ_{cc}	3621.4	Ξ_{cc}^*	(3685.4)
				Ξ_{cc}	(3557.4)	Ξ_{cc}^*	3621.4
				Ω_{cc}	(3730.4)	Ω_{cc}^*	3802.4[102]
η_b	9399.0	Υ	9460.3	Ξ_{bb}	10093.0	Ξ_{bb}^*	(10113.8)
				Ω_{bb}	10193.0	Ω_{bb}^*	(10212.2)
B_c	6275.1	B_c^*	6331.0[103]	Ξ_{bc}	6820.0		
				Ξ'_{bc}	(6845.9)	Ξ_{bc}^*	(6878.8)
				Ω_{bc}	6920.0		
				Ω'_{bc}	(6950.9)	Ω_{bc}^*	(6983.4)

TABLE IV. Mass differences ($\Delta M = M_{\text{Th.}} - M_{\text{Ex.}}$) between the calculated values (Th.) and experimental values (Ex.) for conventional hadrons in units of MeV. $M_{\text{Th.}}$ is obtained with $M = \sum_i M_i + \langle H_{\text{CMI}} \rangle$ and $M_n = 361.7$, $M_s = 540.3$, $M_c = 1724.6$, and $M_b = 5052.8$ MeV [90].

Hadron	ΔM	Hadron	ΔM	Hadron	ΔM	Hadron	ΔM
π	109.5	ρ	107.2	N	0	Δ	0
K	110.6	K^*	105.3	Σ	-12.4	Σ^*	-5.4
ω	99.8	ϕ	96.0	Ξ	9.4	Ξ^*	-7.3
D	112.2	D^*	113.7	Λ	1.1	Ω	0
D_s	189.7	D_s^*	188.7	Σ_c	0	Σ_c^*	0
B	101.6	B^*	101.2	Λ_c	14.7	Ξ_c	58.4
B_s	189.6	B_s^*	190.2	Ξ'_c	35	Ξ_c^*	37.6
η_c	380.9	J/ψ	381.0	Ω_c	76.5	Ω_c^*	82.6
η_b	660.0	Υ	661.0	Σ_b	0	Σ_b^*	0
B_c	450.0			Λ_b	9.7	Ξ_b	62.7
				Ξ'_b	39.8	Ξ_b^*	45.0
				Ω_b	92.1	Ξ_{cc}	161.5

obtained in various tetraquark and pentaquark states [84–91]. To make a more reasonable estimation, we use the improved mass formula by replacing $\sum_i M_i$ in Eq. (1) with $M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}}$, where M_{ref} is a reference mass scale and $\langle H_{\text{CMI}} \rangle$ is the corresponding CMI matrix element. Then

$$M = M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}} + \langle H_{\text{CMI}} \rangle. \quad (4)$$

In the present study of pentaquark states, we choose the baryon-meson thresholds as the mass scales, where the reference baryon-meson system should have the same constituent quarks as the considered system. The attraction not incorporated into the original approach is somehow phenomenologically compensated in this procedure [89].

Before discussing the $qqQQ\bar{Q}$ pentaquark states, we emphasize that our results are only rough estimations. They should be updated once a $qqQQ\bar{Q}$ pentaquark state is observed in future experiments and its mass can be chosen as a reference scale. Although the pentaquark masses may change drastically, the mass splittings should not be significantly affected.

In the following subsections we only present the numerical values obtained with Eq. (4). The masses of the relevant reference baryons and mesons are listed in Table III. To understand the decay properties in the following discussions, we also show some masses of the not-yet-observed doubly heavy baryons in the table, which are obtained from several theoretical calculations. Since the spin of the Ξ_{cc} observed by the LHCb Collaboration may be 1/2 or 3/2, we show results for both cases in Table III.

B. The $nncc\bar{Q}$, $sscc\bar{Q}$, $nnbb\bar{Q}$, and $ssbb\bar{Q}$ pentaquark states

By substituting the parameters into the CMI matrices and diagonalizing them, we obtain the pentaquark masses.

We present the masses with their corresponding reference systems for the $nncc\bar{Q}$ states in Table V. As for the $sscc\bar{Q}$, $nnbb\bar{Q}$, and $ssbb\bar{Q}$ states, we only present the obtained eigenvalues in Tables XI and XIII and the values of $M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}}$ in Table X in the Appendix. We obtain the values of their mass spectra using Eq. (4). In these systems, a state is explicitly exotic if the flavor of Q is different from that of the heavy quarks.

For the $nncc\bar{c}$ states, there are two types of reference systems that we can adopt: $(c\bar{c})(cnn)$ and $(\bar{c}n)(ccn)$. The mass $M_{\Xi_{cc}} = 3621.4$ MeV measured by the LHCb Collaboration is used in the latter case. We assume that the spin of Ξ_{cc} is 1/2 although this has not been determined yet. If the spin is 3/2, the pentaquark masses estimated with the threshold related to $M_{\Xi_{cc}}$ would be shifted downward by 64 MeV according to Ref. [90], but the gaps would be the same. As for the $nnbb\bar{Q}$, $sscc\bar{Q}$, and $ssbb\bar{Q}$ systems, we can similarly adopt two types of reference systems: $(Q\bar{Q})(Qqq)$ and $(\bar{Q}q)(QQq)$. Because other doubly heavy baryons except for the Ξ_{cc} have not been observed yet in experiments, the theoretical values $\Omega_{cc} = 3730.4$, $\Xi_{bb} = 10093.0$, and $\Omega_{bb} = 10193.0$ MeV in Table III are used. In Ref. [101], similar theoretical values for Ω_{cc} , Ξ_{bb} , and Ω_{bb} were obtained using a confining, symmetry-preserving regularization of a vector $\times$ vector contact interaction.

From Table V, it is obvious that the pentaquark masses will change when one adopts different reference systems, which indicates that the estimation method (4) should be further improved. If the adopted model can accurately reproduce all of the hadron masses, different reference thresholds should lead to the same result.

Table V shows us that the obtained $nncc\bar{c}$ ($nncc\bar{b}$) masses with the reference threshold $\Sigma_c J/\psi$ ($\Sigma_c B_c$) are lower than those with $\Xi_{cc} \bar{D}$ ($\Xi_{cc} B$). This feature is consistent with our anticipation since $\Delta_{\Sigma_c} + \Delta_{J/\psi} > \Delta_{\Xi_{cc}} + \Delta_D$ and $\Delta_{\Sigma_c} + \Delta_{B_c} > \Delta_{\Xi_{cc}} + \Delta_B$ from Table IV. At present, it is not clear which type of threshold gives more reasonable masses. For a pentaquark, the effective attraction is probably not strong and maybe a higher mass would be more reasonable [89]. However, the choice of reference scale does not affect the mass splittings.

In Figs. 2 and 3 we use the higher pentaquark masses, although relevant estimations rely on the masses of the not-yet-observed QQq states. The diagrams in Figs. 2 and 3 illustrate the relative positions of the $nncc\bar{c}$, $nnbb\bar{b}$, $nnbb\bar{c}$, $nncc\bar{b}$, $sscc\bar{c}$, $ssbb\bar{b}$, $ssbb\bar{c}$, and $sscc\bar{b}$ states in order. The selected masses are obtained with the reference systems $\Xi_{cc} \bar{D}$, $\Xi_{bb} B$, $\Sigma_b B_c^-$, $\Xi_{cc} B$, $\Omega_{cc} D_s^-$, $\Omega_{bb} B_s^0$, $\Omega_b B_c^-$, and $\Omega_{cc} B_s^0$, respectively. The thresholds for the relevant rearrangement decay patterns are also displayed.

For the $nncc\bar{c}$ system, the $I = 0$ states generally have lower masses than the $I = 1$ states. The quantum numbers for both the lowest and the highest states are $J^P = 1/2^-$.

TABLE V. The estimated masses for the $nncc\bar{Q}$ ($Q = c, b$) ($I = 1, 0$) system in units of MeV. The values in the second column in each case are eigenvalues of the CMI Hamiltonian, and those after this column are determined with the relevant reference systems.

$nncc\bar{c}$ ($I = 1$)				$nncc\bar{c}$ ($I = 0$)		
J^P	Eigenvalue	$(\Sigma_c J/\psi)$	$(\Xi_{cc}\bar{D})$	Eigenvalue	$(\Sigma_c J/\psi)$	$(\Xi_{cc}\bar{D})$
$\frac{5}{2}^-$	100.5	5616.7	5732.7	48.5	5564.7	5680.7
$\frac{3}{2}^-$	$\begin{pmatrix} 117.3 \\ 81.2 \\ 56.9 \\ -43.0 \end{pmatrix}$	$\begin{pmatrix} 5633.5 \\ 5597.4 \\ 5573.2 \\ 5473.2 \end{pmatrix}$	$\begin{pmatrix} 5744.1 \\ 5708.0 \\ 5683.8 \\ 5583.8 \end{pmatrix}$	$\begin{pmatrix} 36.5 \\ -68.8 \\ -161.0 \end{pmatrix}$	$\begin{pmatrix} 5552.7 \\ 5447.4 \\ 5355.2 \end{pmatrix}$	$\begin{pmatrix} 5668.7 \\ 5563.4 \\ 5471.2 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} 182.8 \\ 104.4 \\ 3.9 \\ -62.7 \end{pmatrix}$	$\begin{pmatrix} 5699.1 \\ 5620.6 \\ 5520.1 \\ 5453.5 \end{pmatrix}$	$\begin{pmatrix} 5815.0 \\ 5736.5 \\ 5636.0 \\ 5569.5 \end{pmatrix}$	$\begin{pmatrix} 12.8 \\ -71.0 \\ -124.3 \\ -267.2 \end{pmatrix}$	$\begin{pmatrix} 5529.0 \\ 5445.2 \\ 5391.9 \\ 5249.0 \end{pmatrix}$	$\begin{pmatrix} 5644.9 \\ 5561.2 \\ 5507.9 \\ 5364.9 \end{pmatrix}$
$nncc\bar{b}$ ($I = 1$)				$nncc\bar{b}$ ($I = 0$)		
J^P	Eigenvalue	$(\Sigma_c B_c)$	$(\Xi_{cc}B)$	Eigenvalue	$(\Sigma_c B_c)$	$(\Xi_{cc}B)$
$\frac{5}{2}^-$	82.9	8858.4	9051.1	20.5	8796.0	8988.7
$\frac{3}{2}^-$	$\begin{pmatrix} 105.1 \\ 84.1 \\ 36.1 \\ 13.6 \end{pmatrix}$	$\begin{pmatrix} 8880.5 \\ 8859.5 \\ 8811.6 \\ 8789.1 \end{pmatrix}$	$\begin{pmatrix} 9073.2 \\ 9052.2 \\ 9004.3 \\ 8981.8 \end{pmatrix}$	$\begin{pmatrix} 11.5 \\ -45.4 \\ -136.7 \end{pmatrix}$	$\begin{pmatrix} 8787.0 \\ 8730.0 \\ 8638.8 \end{pmatrix}$	$\begin{pmatrix} 8979.7 \\ 8922.7 \\ 8831.5 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} 146.6 \\ 75.8 \\ 27.3 \\ -21.3 \end{pmatrix}$	$\begin{pmatrix} 8922.1 \\ 8851.3 \\ 8802.8 \\ 8754.1 \end{pmatrix}$	$\begin{pmatrix} 9114.8 \\ 9044.0 \\ 8995.5 \\ 8946.8 \end{pmatrix}$	$\begin{pmatrix} -19.4 \\ -71.0 \\ -121.9 \\ -198.8 \end{pmatrix}$	$\begin{pmatrix} 8756.1 \\ 8704.4 \\ 8653.6 \\ 8576.7 \end{pmatrix}$	$\begin{pmatrix} 8948.9 \\ 8897.2 \\ 8846.3 \\ 8769.4 \end{pmatrix}$

From Fig. 2(b)–2(d), one sees similar features for the $nnbb\bar{b}$, $nnbb\bar{c}$, and $nncc\bar{b}$ systems.

As for the stability of the pentaquark states, their dominant decay modes should be related to the rearrangement mechanism. Now we move on to such decays. One has to consider the constraints from angular momentum conservation, isospin conservation, parity conservation, and so on when discussing allowed decay channels. For convenience, we mark the spin and isospin of the baryon-meson channels in the superscripts and subscripts of their symbols in Fig. 2, respectively. For the $sscc\bar{Q}$ and $ssbb\bar{Q}$ states, only one isospin is possible and no explicit label is given. Of course, whether the decay can happen or not is also kinematically constrained by the pentaquark mass, which depends on the models. In the following discussions, we assume that the obtained masses shown in the figures are all reasonable.

The $nncc\bar{c}$ states look like excited nnc baryons. Because only orbital or radial excitation energy cannot explain their high masses, the states (once observed) would be good candidates of compact $nncc\bar{c}$ pentaquark states or hadronic molecules. The decay properties would be helpful to distinguish these two configurations. Here we just discuss the relevant rearrangement decay patterns. In the case of $I(J^P) = 1(5/2^-)$, the possible S -wave decay channels are $\Sigma_c^* J/\psi$ and $\Xi_{cc}^* \bar{D}^*$. In the case of $I(J^P) = 0(5/2^-)$, the

possible S -wave decay channel is only $\Xi_{cc}^* \bar{D}^*$. The $I(J^P) = 0(5/2^-)$ isoscalar pentaquark is a candidate for a stable state. We mark it in Fig. 2(a) with a dagger. In the case of $I(J^P) = 1(3/2^-)$, the possible S -wave channels are $\Xi_{cc}^* \bar{D}^*$, $\Xi_{cc} \bar{D}^*$, $\Sigma_c^* J/\psi$, $\Xi_{cc} \bar{D}$, $\Sigma_c J/\psi$, and $\Sigma_c^* \eta_c$. In the case of $I(J^P) = 0(3/2^-)$, the possible S -wave channels are $\Xi_{cc}^* \bar{D}^*$, $\Xi_{cc} \bar{D}^*$, and $\Lambda_c J/\psi$. In the case of $I(J^P) = 1(1/2^-)$, the possible S -wave channels are $\Xi_{cc}^* \bar{D}^*$, $\Xi_{cc} \bar{D}^*$, $\Sigma_c^* J/\psi$, $\Sigma_c J/\psi$, $\Xi_{cc} \bar{D}$, and $\Sigma_c \eta_c$. In the case of $I(J^P) = 0(1/2^-)$, the possible S -wave channels are $\Xi_{cc}^* \bar{D}^*$, $\Xi_{cc} \bar{D}^*$, $\Xi_{cc} \bar{D}$, $\Xi_{cc}^* \bar{D}$, $\Lambda_c J/\psi$, and $\Lambda_c \eta_c$. The observation of any one of the mentioned decay patterns could provide hints for the existence of an $nncc\bar{c}$ pentaquark state. Because the lowest $I(J^P) = 0(1/2^-)$ state is much lower than the $\Xi_{cc} \bar{D}$ threshold, if an observed state in $\Lambda_c \eta_c$ (or $\Lambda_c J/\psi$) is around 5.4 GeV this state would be more likely to be a compact pentaquark than a $\Xi_{cc} \bar{D}$ molecule. If the spin of the observed Ξ_{cc} by the LHCb Collaboration is $3/2$, $\Xi_{cc} \rightarrow \Xi_{cc}^*$ and the estimated pentaquark masses will be reduced by 64 MeV. The stability of the pentaquark states is not affected.

The $nnbb\bar{c}$ states shown in Fig. 2(c) are explicitly exotic. Since the nbb states and the excited B_c^* have not yet been observed in experiments, we use the theoretical masses of

FIG. 2. Relative positions (in units of MeV) for the $nncc\bar{c}$, $nnbb\bar{b}$, $nnbb\bar{c}$, and $nncc\bar{b}$ pentaquark states. The dotted lines indicate various baryon-meson thresholds. When the isospin (spin) of an initial pentaquark state is equal to a number in the subscript (superscript) of a baryon-meson state, its decay into that baryon-meson channel through the S wave is allowed by isospin (angular momentum) conservation. We have adopted the masses estimated with the reference thresholds of (a) $\Xi_{cc}\bar{D}$, (b) $\Xi_{bb}B$, (c) $\Sigma_b B_c^-$, and (d) $\Xi_{cc}B$. The relatively stable states, which cannot decay to the observed hadrons through strong interaction in our model, have been marked with a dagger.

B_c^* , Ξ_{bb} , and Ξ_{bb}^* in Table III to check the pentaquark stability. Now, it is easy to see that the lowest-lying states with $I(J^P) = 0(1/2^-)$ and $I(J^P) = 0(3/2^-)$ are both stable. The situation for the $nncc\bar{b}$ ($nnbb\bar{b}$) states can be analyzed similarly to the $nncc\bar{c}$ ($nnbb\bar{c}$) case, but now all of them are explicitly (implicitly) exotic.

The properties of the $sscc\bar{c}$, $ssbb\bar{b}$, $sscc\bar{b}$, and $ssbb\bar{c}$ states are similar to those of $nncc\bar{Q}$ ($I = 1$) and $nnbb\bar{Q}$ ($I = 1$). Here, we also use the theoretical masses of B_c^* , Ω_{cc} , Ω_{cc}^* , Ω_{bb} , and Ω_{bb}^* to discuss the possible decay channels. In the $sscc\bar{c}$ case shown in Fig. 3(a), any possible pentaquark is above their allowed rearrangement decay

FIG. 3. Relative positions (in units of MeV) for the $sccc$, $ssbb$, $ssbb$, and $sscc$ pentaquark states. The dotted lines indicate various baryon-meson thresholds. When the spin of an initial pentaquark state is equal to a number in the superscript of a baryon-meson state, its decay into that baryon-meson channel through the S wave is allowed by angular momentum conservation. We adopt the masses estimated with the reference thresholds of (a) $\Omega_{cc}D_s^-$, (b) $\Omega_{bb}B_s^0$, (c) $\Omega_b B_c^-$, and (d) $\Omega_{cc}B_s^0$.

channels and thus there is no stable state. One does not find stable states in the $ssbb$ and $sscc$ cases, either. In the $ssbb$ system, the lowest-lying (J^P) = $(3/2^-)$ pentaquark is slightly above its decay channel $\Omega_{bb}^*D_s^-$. It is probably not a broad state.

C. The $nnbc\bar{Q}$ and $sscb\bar{Q}$ pentaquark states

All of these $nnbc\bar{Q}$ and $sscb\bar{Q}$ states are implicitly exotic. To estimate their masses, we can use three types of reference systems: $(qqc) - (b\bar{Q})$, $(qqb) - (c\bar{Q})$, and $(qbc) - (q\bar{Q})$. We list the obtained eigenvalues in Tables XI and XII and the values of $M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}}$ in Table X in the Appendix for the $nnbc\bar{Q}$ and $sscb\bar{Q}$ states, respectively. From the relevant calculations, we can see that the results with these three types of reference systems are slightly different.

The masses we use are obtained with the reference thresholds of the $\Sigma_b J/\psi$, $\Xi_{bc} B$, $\Omega_b J/\psi$, and $\Omega_{bc} B_s$ channels for the $nnbc\bar{c}$, $nnbc\bar{b}$, $ssbc\bar{c}$, and $sscb\bar{b}$ states, respectively. Moreover, 16 rearrangement decay channels

are involved for the $nnbc\bar{c}$ and $nnbc\bar{b}$ states and 12 channels are involved for the $ssbc\bar{c}$ and $sscb\bar{b}$ states.

We first check for possible stable pentaquarks in the $nnbc\bar{c}$ case. The lowest $J^P = 1/2^-$ and $J^P = 3/2^-$ states both have rearrangement decay channels and should not be very narrow. On the contrary, the lowest $I(J^P) = 0(5/2^-)$ state is below the possible decay channel $\Xi_{cb}^* \bar{D}_s^*$ and is considered a relative state. Similarly, the only possible stable $nncb\bar{b}$ pentaquark has the quantum numbers

TABLE VI. The stable states in the $qqQQ\bar{Q}$ systems from relevant calculations.

Systems	$I(J^P)$	Masses	Systems	$I(J^P)$	Masses
$nncc\bar{c}$	$0(5/2^-)$	5680.7	$nnbb\bar{c}$	$0(3/2^-)$	11922.9
$nnbb\bar{b}$	$0(5/2^-)$	15414.0		$0(1/2^-)$	11876.3
$nncc\bar{b}$	$0(5/2^-)$	8988.7	$nsbb\bar{c}$	$1/2(5/2^-)$	12214.8
$nnbc\bar{c}$	$0(5/2^-)$	8882.1		$1/2(3/2^-)$	12026.3
$nncb\bar{b}$	$0(5/2^-)$	12164.2		$1/2(1/2^-)$	11983.6

$I(J^P) = 0(5/2^-)$ if the mass of Ξ_{bc}^* is larger than 6870 MeV. Last, it seems that there is no stable $ssbc\bar{c}$ or $sscbb\bar{c}$ pentaquark state. The possible stable pentaquarks are shown in Table VI.

D. The $nscc\bar{Q}$ and $nsbb\bar{Q}$ pentaquark states

For the $nscc\bar{Q}$ and $nsbb\bar{Q}$ states, there are also three types of reference systems we can use to estimate the masses: $(nsQ) - (Q\bar{Q})$, $(nQQ) - (s\bar{Q})$, and $(sQQ) - (n\bar{Q})$. For example, we estimate the $nscc\bar{c}$ masses with the thresholds of the $\Xi_c' J/\psi$, $\Xi_{cc} D_s^-$, and $\Omega_{cc} \bar{D}$ channels. Similarly, we use the reference systems of $\Xi_b' B_c^-$, $\Xi_{bb} D_s^-$, and $\Omega_{bb} \bar{D}$ to estimate the $nsbb\bar{c}$ masses. The eigenvalues and the values of $M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}}$ for the $nscc\bar{Q}$ and $nsbb\bar{Q}$ systems are presented in Tables XI and X in the Appendix, respectively.

Of these states, the $nscc\bar{b}$ and $nsbb\bar{c}$ pentaquarks are explicitly exotic. The observation of such a state in future measurements would be an important finding, in particular when the state is narrow. From the relevant calculations, we

can see that the $nscc\bar{c}$ pentaquark masses estimated with the $(nsQ) - (Q\bar{Q})$ type reference systems are lower than those calculated with other types of thresholds. It is easy to identify such states as five-quark states because of their high masses, although they are implicitly exotic. The situation is different for the $nscc\bar{n}$ or $nscc\bar{s}$ states studied in Ref. [89]. It is not easy to distinguish such a pentaquark state from a 3q baryon once it is observed.

From Table XII and the rough values of the doubly heavy baryons in Table III, it seems that no stable pentaquark states exist in the $nscc\bar{c}$, $nsbb\bar{b}$, and $nscc\bar{b}$ systems. However, the lowest $J^P = 1/2^-$, $J^P = 3/2^-$, and $J^P = 5/2^-$ states are below any possible rearrangement decay channels and they are possibly stable. Of course, if their masses are underestimated, they may also decay into $\Xi_{bb} D_s^-$ (and probably $\Omega_{bb} \bar{D}$), $\Omega_{bb}^* \bar{D}$, and $\Omega_{bb}^* \bar{D}^*$, respectively.

E. The $nsbc\bar{Q}$ pentaquark states

The $nsbc\bar{Q}$ states are implicitly exotic. Their wave functions are not constrained by the Pauli principle.

FIG. 4. Relative positions (in units of MeV) of the $nsbc\bar{c}$ and $nsbc\bar{b}$ pentaquark states. The dotted lines indicate various baryon-meson thresholds. When the spin of an initial pentaquark state is equal to a number in the superscript of a baryon-meson state, its decay into that baryon-meson channel through the S wave is allowed by angular momentum conservation. We adopt the masses estimated with the reference thresholds of (a) $\Xi_b' J/\psi$ and (b) $\Omega_{bc} B$.

The number of wave function bases for a pentaquark with given quantum numbers is larger than that for other states. After diagonalizing the Hamiltonian, one gets a number of possible pentaquark states. To estimate the $nsbc\bar{c}$ ($nsb\bar{c}\bar{b}$) masses, we use four different types of reference systems: $\Xi_c B_c^-$ ($\Xi_c \Upsilon$), $\Xi_b' J/\psi$ ($\Xi_b' B_c^+$), $\Xi_{bc} D_s^-$ ($\Xi_{bc} B_s^0$), and $\Omega_{bc} \bar{D}$ ($\Omega_{bc} B$). We present the obtained eigenvalues and the values of $M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}}$ in Tables XII and X in the Appendix for the $nsbc\bar{Q}$ states, respectively. Meanwhile, the results with these four types of reference systems are slightly different, and we use the highest values in Fig. 4.

The spectra for the $nsbc\bar{Q}$ pentaquark states with the $\Xi_b' J/\psi$ or $\Omega_{bc} B$ threshold are shown in Fig. 4. From the figure and the masses given in Table III, it is difficult to find stable pentaquarks in these systems. Only the lowest $nsbc\bar{c}$ pentaquark is slightly above the $\Xi_c B_c^-$ threshold and is possibly a state without a broad width. Of course, the $nsbc\bar{c}$ states can be searched for in the $\Xi_c B_c^-$ or $\Xi_b \eta_c$ channel in future experiments. If such a state is observed, its exotic nature could be easily identified, contrary to the situation in the $nsbc\bar{q}$ case [89].

V. DISCUSSIONS AND SUMMARY

Recently, the observation of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ states at LHCb [28] provided significant evidence for the existence of pentaquark states, which motivated us to study the ground compact $qqQQ\bar{Q}$ ($q = u, d, s$ and $Q = b, c$) pentaquark states within the CMI model. In the considered pentaquark systems, the $qqbb\bar{c}$ and $qqcc\bar{b}$ states are explicitly exotic and are easy to identify. Other states can also be easily identified as exotic baryons because their large masses could not be explained without an excited $Q\bar{Q}$ pair.

In this work, we first constructed the flavor-color-spin wave functions for the $qqQQ\bar{Q}$ pentaquark states from the SU(3) and SU(2) symmetries and the Pauli principle. We extracted the effective coupling constants from the mass splittings between conventional hadrons. Based on these, we systematically calculated the color-magnetic interaction for these pentaquark states and obtained the corresponding mass gaps. Then, various reference thresholds were used to estimate the masses of these states. Some theoretical results for the masses of the doubly heavy 3q baryons were

adopted in our estimation. At last, we analyzed the stability and possible rearrangement decay channels of the $qqQQ\bar{Q}$ pentaquark states.

We have shown the mass spectra and rearrangement decay patterns in various figures. Following Figs. 2 and 3 and Tables XI, XII, and XIII, we can see that ten stable states are possible, which are also collected in Table VI. However, not all of them are really stable states. The reason is that the predicted masses of the pentaquarks in the current model may deviate from the experimental values.

As a general feature, the high-spin $J^P = (5/2)^-$ pentaquark states should usually be narrow since they have many D -wave decay modes but one or two S -wave decay modes. This feature is similar to the $\bar{Q}Qqqq$ and $QQqq\bar{q}$ cases [87,89]. Now, the narrow $\bar{Q}Qqqq$ pentaquark states have been observed. It is also worthwhile to search for the exotic narrow $qqQQ\bar{Q}$ pentaquark states in future experiments.

In the study of the pentaquark states, the number of color-spin structures may be more than ten. Mixing or channel-coupling effects could be important. Such effects should be carefully considered in detail in further studies.

In short summary, we have systematically studied the exotic states with the structure $qqQQ\bar{Q}$ ($q = u, d, s$ and $Q = c, b$). They can be easily identified as explicitly exotic or implicitly exotic pentaquark states once they are observed. We hope that the present study may stimulate further theoretical and experimental investigations into the properties of the $qqQQ\bar{Q}$ pentaquark states.

ACKNOWLEDGMENTS

This project is supported by the National Natural Science Foundation of China under Grant Nos. 11705072 and 11775132, and the China National Funds for Distinguished Young Scientists under Grant No. 11825503.

APPENDIX

In this appendix we show the CMI Hamiltonian matrix elements with $J^P = 5/2^-$, $J^P = 3/2^-$, and $J^P = 1/2^-$ in Tables VII, VIII, and IX, respectively.

Then, we show the eigenvalues of the $nncc(bb)\bar{Q}$, $sscc(bb)\bar{Q}$, $nnbc\bar{Q}$, $ssbc\bar{Q}$, $nscc(bb)\bar{Q}$, and $nsbc\bar{Q}$

TABLE VII. The H_{CMI} matrix elements for the case $J^P = \frac{5}{2}^-$.

Bases	Elements		
	Bases		
	$\phi_{1\chi_1}$	$\phi_{2\chi_1}$	$\phi_{3\chi_1}$
$\phi_{1\chi_1}$	$\frac{1}{3}[8\alpha + 2\zeta + 4(\nu + \lambda)]$	$\sqrt{2}\eta - 2\sqrt{2}\mu$	$-\sqrt{2}\theta - 2\sqrt{2}\xi$
$\phi_{2\chi_1}$	$\sqrt{2}\eta - 2\sqrt{2}\mu$	$\frac{1}{3}(-4\beta + 5\zeta + 10\lambda - 2\nu)$	$-\kappa$
$\phi_{3\chi_1}$	$-\sqrt{2}\theta - 2\sqrt{2}\xi$	$-\kappa$	$\frac{1}{3}(4\gamma + 5\zeta - 2\lambda + 10\nu)$

TABLE VIII. The H_{CMI} matrix elements for the case $J^P = \frac{3}{2}^-$.

Elements												
Bases												
Bases	$\phi_1\chi_2$	$\phi_1\chi_3$	$\phi_1\chi_4$	$\phi_1\chi_5$	$\phi_2\chi_2$	$\phi_2\chi_3$	$\phi_2\chi_4$	$\phi_2\chi_5$	$\phi_3\chi_2$	$\phi_3\chi_3$	$\phi_3\chi_4$	$\phi_3\chi_5$
$\phi_1\chi_2$	$\frac{1}{3}[-\frac{8\alpha+2\xi}{6}(\nu+\lambda)]$	$\frac{2\sqrt{5}}{3}(\nu-\lambda)$	$-\frac{2\sqrt{10}}{3}\xi$	$-\frac{2\sqrt{10}}{3}\mu$	$\sqrt{2}(\eta+3\xi)$	$-\sqrt{10}\xi$	$2\sqrt{5}\nu$	0	$\sqrt{2}(\eta+3\mu)$	$\sqrt{10}\mu$	0	$2\sqrt{5}\lambda$
$\phi_1\chi_3$	$\frac{2\sqrt{5}}{3}(\nu-\lambda)$	$\frac{1}{3}[\frac{8\alpha-2\xi}{2}(\nu+\lambda)]$	$\frac{2\sqrt{2}}{3}(\theta+\xi)$	$-\frac{2\sqrt{2}}{3}(\eta+\mu)$	$\sqrt{10}\xi$	$-\sqrt{2}(\theta+\xi)$	$2(\xi-\nu)$	-2 κ	$\sqrt{10}\mu$	$-\sqrt{2}(\eta+\mu)$	2 κ	$-2(\xi-\lambda)$
$\phi_1\chi_4$	$-\frac{2\sqrt{10}}{3}\xi$	$\frac{2\sqrt{2}}{3}(\theta+\xi)$	$\frac{1}{3}(8\epsilon+4\lambda)$	$\frac{2}{3}\kappa$	$2\sqrt{5}\nu$	$2(\xi-\nu)$	0	$\sqrt{2}\eta$	0	2 κ	$-2\sqrt{2}\mu$	$\sqrt{2}\theta$
$\phi_1\chi_5$	$-\frac{2\sqrt{10}}{3}\mu$	$-\frac{2\sqrt{2}}{3}(\eta+\mu)$	$\frac{2}{3}\kappa$	$\frac{1}{3}(-8\delta+4\nu)$	0	-2 κ	$\sqrt{2}\eta$	$-2\sqrt{2}\xi$	$2\sqrt{5}\lambda$	$-2(\xi-\lambda)$	$\sqrt{2}\theta$	0
$\phi_2\chi_2$	$\sqrt{2}(\theta+3\xi)$	$\sqrt{10}\xi$	$2\sqrt{5}\nu$	0	$\frac{1}{3}(-\frac{4\gamma+5\xi}{15\nu+3\lambda})$	$\frac{\sqrt{5}}{3}(5\nu+\lambda)$	$-\frac{5\sqrt{10}}{3}\xi$	$\frac{\sqrt{10}}{3}\mu$	- κ	0	0	0
$\phi_2\chi_3$	$-\sqrt{10}\xi$	$-\sqrt{2}(\theta+\xi)$	$2(\xi-\nu)$	-2 κ	$\frac{\sqrt{5}}{3}(5\nu+\lambda)$	$\frac{1}{3}(\frac{4\gamma+5\xi}{5\nu-\lambda})$	$\frac{5\sqrt{2}}{3}(\theta+\xi)$	$\frac{\sqrt{2}}{3}(\nu-5\eta)$	0	κ	$-\sqrt{2}\eta$	$\sqrt{2}\theta$
$\phi_2\chi_4$	$2\sqrt{5}\nu$	$2(\xi-\nu)$	0	$\sqrt{2}\eta$	$-\frac{5\sqrt{10}}{3}\xi$	$\frac{5\sqrt{2}}{3}(\theta+\xi)$	$\frac{1}{3}(9\alpha-5\delta-2\lambda)$	$\frac{5}{3}\kappa$	0	$-\sqrt{2}\eta$	0	- ξ
$\phi_2\chi_5$	0	-2 κ	$\sqrt{2}\eta$	$-2\sqrt{2}\xi$	$\frac{\sqrt{10}}{3}\mu$	$\frac{\sqrt{2}}{3}(\mu-5\eta)$	$\frac{5}{3}\kappa$	$\frac{1}{3}(-\frac{9\alpha-5\delta}{10\nu})$	0	$\sqrt{2}\theta$	- ξ	0
$\phi_3\chi_2$	$\sqrt{2}(\eta+3\mu)$	$\sqrt{10}\mu$	0	$2\sqrt{5}\lambda$	- κ	0	0	0	$\frac{1}{3}(-\frac{4\theta+5\xi}{3\nu-15\lambda})$	$-\frac{\sqrt{5}}{3}(5\lambda+\nu)$	$\frac{\sqrt{10}}{3}\xi$	$-\frac{5\sqrt{10}}{3}\mu$
$\phi_3\chi_3$	$\sqrt{10}\mu$	$-\sqrt{2}(\eta+\mu)$	2 κ	$-2(\xi-\lambda)$	0	κ	$-\sqrt{2}\eta$	$\sqrt{2}\theta$	$-\frac{\sqrt{5}}{3}(5\lambda+\nu)$	$\frac{1}{3}(\frac{-4\theta-5\xi}{\nu+5\lambda})$	$\frac{\sqrt{2}}{3}(5\theta-\xi)$	$-\frac{5\sqrt{2}}{3}(\eta+\mu)$
$\phi_3\chi_4$	0	2 κ	$-2\sqrt{2}\mu$	$\sqrt{2}\theta$	0	$-\sqrt{2}\eta$	0	- ξ	$\frac{\sqrt{10}}{3}\xi$	$\frac{1}{3}(-\frac{9\alpha+5\epsilon}{10\lambda})$	$\frac{5}{3}\kappa$	$\frac{5}{3}\kappa$
$\phi_3\chi_5$	$2\sqrt{5}\lambda$	$-2(\xi-\lambda)$	$\sqrt{2}\eta$	0	0	$\sqrt{2}\theta$	- ξ	0	$-\frac{5\sqrt{10}}{3}\mu$	$-\frac{5\sqrt{2}}{3}(\eta+\mu)$	$\frac{5}{3}\kappa$	$\frac{1}{3}(9\alpha+\delta-2\nu)$

TABLE IX. The H_{CMI} matrix elements for the case $J^P = \frac{1}{2}^-$.

Elements															
Bases															
Bases	$\phi_1\chi_6$	$\phi_1\chi_7$	$\phi_1\chi_8$	$\phi_1\chi_9$	$\phi_1\chi_{10}$	$\phi_2\chi_6$	$\phi_2\chi_7$	$\phi_2\chi_8$	$\phi_2\chi_9$	$\phi_2\chi_{10}$	$\phi_3\chi_6$	$\phi_3\chi_7$	$\phi_3\chi_8$	$\phi_3\chi_9$	$\phi_3\chi_{10}$
$\phi_1\chi_6$	$\frac{1}{3}$ $(\frac{8\alpha-2\xi}{-4\lambda-4\nu})$	$\frac{4\sqrt{2}}{3}$ $(\nu-\lambda)$	$\frac{2\sqrt{2}}{3}$ $(\theta-2\xi)$	$\frac{2\sqrt{2}}{3}$ $(2\mu-\eta)$	0	$\sqrt{2}$ $(2\xi-\theta)$	-4ξ	$2(\xi+2\nu)$	-2κ	0	$\sqrt{2}(2\mu-\eta)$	4μ	2κ	$-2(\xi+2\lambda)$	0
$\phi_1\chi_7$	$\frac{4\sqrt{2}}{3}$ $(\nu-\lambda)$	$\frac{4}{3}$ $(2\alpha-\xi)$	$\frac{4\xi}{3}$	$\frac{4}{3}\mu$ $(\frac{2\nu\sqrt{3}}{3}\kappa)$	$-\frac{2\sqrt{3}}{3}\kappa$	-4ξ	$-2\sqrt{2}\theta$	$-2\sqrt{2}\nu$	0	$-\sqrt{6}\eta$	4μ	$-2\sqrt{2}\eta$	0	$-2\sqrt{2}\lambda$	$-\sqrt{6}\theta$
$\phi_1\chi_8$	$\frac{2\sqrt{2}}{3}$ $(\theta-2\xi)$	$\frac{4\xi}{3}$ $(2\alpha-\xi)$	$\frac{8}{3}(\epsilon-\lambda)$	$\frac{2}{3}\kappa$	$-\frac{4\sqrt{3}}{3}\mu$	2 $(\xi+2\nu)$	$-2\sqrt{2}\nu$	0	$\sqrt{2}\eta$	0	2κ	0	$4\sqrt{2}\mu$	$\sqrt{2}\theta$	$2\sqrt{6}\lambda$
$\phi_1\chi_9$	$\frac{2\sqrt{2}}{3}$ $(2\mu-\eta)$	$\frac{4}{3}\mu$	$\frac{2}{3}\kappa$	$-\frac{8}{3}(\nu+\delta)$	$-\frac{4\sqrt{3}}{3}\xi$	-2κ	0	$\sqrt{2}\eta$	$4\sqrt{2}\xi$	$2\sqrt{6}\nu$	$-2(\xi+2\lambda)$	$-2\sqrt{2}\lambda$	$\sqrt{2}\theta$	0	0
$\phi_1\chi_{10}$	0	$-\frac{2\sqrt{2}}{3}\kappa$	$-\frac{4\sqrt{3}}{3}\mu$	$-\frac{4\sqrt{3}}{3}\xi$	-8α	0	$-\sqrt{6}\eta$	0	$2\sqrt{6}\nu$	0	0	$-\sqrt{6}\theta$	$2\sqrt{6}\lambda$	0	0
$\phi_2\chi_6$	$\sqrt{2}(2\xi-\theta)$	-4ξ	$2(\xi+2\nu)$	-2κ	0	$\frac{1}{3}$ $(\frac{4\gamma-5\xi-\epsilon}{10\nu+2\lambda})$	$\frac{2\sqrt{2}}{3}$ $(\lambda+5\nu)$	$\frac{10\xi}{3}$ $(\theta-2\xi)$	$-\frac{2}{3}\mu$ $(5\eta+2\mu)$	0	κ	0	$-\sqrt{2}\eta$	$\sqrt{2}\theta$	0
$\phi_2\chi_7$	-4ξ	$-2\sqrt{2}\theta$	$-2\sqrt{2}\nu$	0	$-\sqrt{6}\eta$	$\frac{2\sqrt{2}}{3}$ $(\lambda+5\nu)$	$\frac{1}{3}$ $(4\gamma-10\xi)$	$\frac{10\xi}{3}$ $(9\alpha-\epsilon+4\lambda)$	$-\frac{2}{3}\mu$ $(5\eta+2\mu)$	$-\frac{2}{3}\mu$ $(5\eta+2\mu)$	0	2κ	0	0	$\sqrt{3}\xi$
$\phi_2\chi_8$	$2(\xi+2\nu)$	$-2\sqrt{2}\nu$	0	$\sqrt{2}\eta$	0	$\frac{5\sqrt{2}}{3}$ $(\theta-2\xi)$	$\frac{10\xi}{3}$ $(\theta-2\xi)$	$\frac{1}{3}(9\alpha-\epsilon+4\lambda)$	$\frac{5}{3}\kappa$	$\frac{2\sqrt{3}}{3}\mu$	$-\sqrt{2}\eta$	0	0	$-\xi$	0
$\phi_2\chi_9$	-2κ	0	$\sqrt{2}\eta$	$4\sqrt{2}\xi$	$2\sqrt{6}\nu$	$-\frac{\sqrt{2}}{3}$ $(5\eta+2\mu)$	$-\frac{2}{3}\mu$	$\frac{5}{3}\kappa$	$-\frac{1}{3}(9\alpha+5\delta-\frac{10\sqrt{3}}{3}\xi-20\nu)$	$\sqrt{2}\theta$	0	0	$-\xi$	0	0
$\phi_2\chi_{10}$	0	$-\sqrt{6}\eta$	0	$2\sqrt{6}\nu$	0	0	$-\frac{5\sqrt{3}}{3}\kappa$	$\frac{2\sqrt{3}}{3}\mu$	$-\frac{10\sqrt{3}}{3}\xi$	$\alpha-3\delta$	0	$\sqrt{3}\xi$	0	0	0
$\phi_3\chi_6$	$\sqrt{2}(2\mu-\eta)$	4μ	2κ	$-2(\xi+2\lambda)$	0	κ	0	$-\sqrt{2}\eta$	$\sqrt{2}\theta$	0	$\frac{1}{3}(-\alpha-3\epsilon-\frac{5\xi}{3})$	$-\frac{2\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	$\frac{5\sqrt{2}}{3}$	0
$\phi_3\chi_7$	4μ	$-2\sqrt{2}\eta$	0	$-2\sqrt{2}\lambda$	$-\sqrt{6}\theta$	0	2κ	0	0	$\sqrt{3}\xi$	$-\frac{2\sqrt{2}}{3}$	$-\frac{1}{3}(\alpha+3\epsilon-\frac{5\lambda+\nu}{3})$	$-\frac{2}{3}\xi$	$(2\mu-\eta)$	$-\frac{5\sqrt{3}}{3}\kappa$
$\phi_3\chi_8$	2κ	0	$4\sqrt{2}\mu$	$\sqrt{2}\theta$	$2\sqrt{6}\lambda$	$-\sqrt{2}\eta$	0	0	$-\xi$	0	$\frac{\sqrt{2}}{3}(5\theta+2\xi)$	$-\frac{2}{3}\xi$	$\frac{1}{3}(\frac{-9\alpha-5}{e-20\lambda})$	$\frac{5}{3}\kappa$	$-\frac{10\sqrt{3}}{3}\mu$
$\phi_3\chi_9$	$-2(\xi+2\lambda)$	$-2\sqrt{2}\lambda$	$\sqrt{2}\theta$	0	0	$\sqrt{2}\theta$	0	$-\xi$	0	0	$\frac{5\sqrt{2}}{3}(2\mu-\eta)$	$\frac{10}{3}\mu$	$\frac{5}{3}\kappa$	$\frac{1}{3}(9\alpha+\delta+4\nu)$	$\frac{2\sqrt{3}}{3}\xi$
$\phi_3\chi_{10}$	0	$-\sqrt{6}\theta$	$2\sqrt{6}\lambda$	0	0	0	$\sqrt{3}\xi$	0	0	0	0	$-\frac{5\sqrt{3}}{3}\kappa$	$-\frac{10\sqrt{3}}{3}\mu$	$\frac{2\sqrt{2}}{3}\xi$	$\alpha+3\epsilon$

TABLE X. The $M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}}$ values of different reference systems in units of MeV. The first rows represent the different reference systems for the relevant pentaquark states. The second rows represent the mass values for the relevant reference system.

	$nncc\bar{c}$	$nncc\bar{b}$	$nnbb\bar{c}$	$nnbb\bar{b}$	$sscc\bar{c}$	$sscc\bar{b}$	$ssbb\bar{c}$	$ssbb\bar{b}$
Reference	$\Sigma_c J/\psi$	$\Sigma_c B_c^+$	$\Sigma_b B_c^-$	$\Sigma_b \Upsilon$	$\Omega_c J/\psi$	$\Omega_c B_c^+$	$\Omega_b B_c^-$	$\Omega_b \Upsilon$
System 1	5516.2	8775.5	12104.0	15220.9	5797.7	9056.9	12369.8	15486.7
Reference	$\Xi_{cc}\bar{D}$	$\Xi_{cc}B$	$\Xi_{bb}\bar{D}$	$\Xi_{bb}B$	$\Omega_{cc}\bar{D}_s^-$	$\Omega_{cc}B_s^0$	$\Omega_{bb}\bar{D}_s^-$	$\Omega_{bb}B_s^0$
System 2	5632.2	8968.2	12079.0	15414.9	5848.3	9176.5	12276.5	15604.7

	$nnbc\bar{c}$	$nncb\bar{b}$	$ssbc\bar{c}$	$sscb\bar{b}$	$nscc\bar{c}$	$nscc\bar{b}$	$nsbb\bar{c}$	$nsbb\bar{b}$
Reference	$\Sigma_c B_c^-$	$\Sigma_c \Upsilon$	$\Omega_c B_c^-$	$\Omega_c \Upsilon$	$\Xi'_c J/\psi$	$\Xi'_c B_c^+$	$\Xi'_b B_c^-$	$\Xi'_b \Upsilon$
System 1	8775.5	11892.4	9057.0	12173.9	5660.4	8919.7	12243.1	15360.1
Reference	$\Sigma_b J/\psi$	$\Sigma_b B_c^+$	$\Omega_b J/\psi$	$\Omega_b B_c^+$	$\Xi_{cc}D_s^-$	$\Xi_{cc}B_s^0$	$\Xi_{bb}D_s^-$	$\Xi_{bb}B_s^0$
System 2	8844.8	12104.0	9110.5	12369.8	5730.8	9059.0	12177.5	15505.8
Reference	$\Xi_{bc}\bar{D}$	$\Xi_{bc}B$	$\Omega_{bc}D_s$	$\Omega_{bc}B_s$	$\Omega_{cc}\bar{D}$	$\Omega_{cc}B$	$\Omega_{bb}\bar{D}$	$\Omega_{bb}B$
System 3	8819.9	12155.9	9022.7	12381.3	5749.7	9085.8	12177.9	15513.9

	$nsbc\bar{c}$		$nsbc\bar{b}$		$nscc\bar{c}$		$nscc\bar{b}$		$nsbb\bar{c}$		$nsbb\bar{b}$
Reference	$\Xi'_c B_c^-$	Reference	$\Xi'_b J/\psi$	Reference	$\Xi'_c \Upsilon$	Reference	$\Xi'_c B_c^+$	Reference	$\Xi'_b B_c^-$	Reference	$\Xi'_b \Upsilon$
System 1	8917.4	System 2	8983.9	System 1	12034.4	System 2	12243.2	System 1	12034.4	System 2	12243.2
Reference	$\Xi_{bc}D_s^-$	Reference	$\Omega_{bc}\bar{D}$	Reference	$\Xi_{bc}B_s^0$	Reference	$\Omega_{bc}B$	Reference	$\Xi_{bb}D_s^-$	Reference	$\Omega_{bb}B$
System 3	8944.3	System 4	8954.4	System 3	12272.6	System 4	12290.5	System 3	12272.6	System 4	12290.5

TABLE XI. The eigenvalues for the $ssbc\bar{Q}$, $nscc(bb)\bar{Q}$, and $nncc(bb)\bar{Q}$ ($Q = c, b$) systems in units of MeV. In the $nncc(bb)\bar{Q}$ systems, the I represents isospin, and 0 and 1 represent the pentaquark states with $I = 0$ and $I = 1$, respectively.

J^P	$ssbc\bar{c}$	$sscb\bar{b}$	$nscc\bar{c}$	$nscc\bar{b}$	$nsbb\bar{c}$	$nsbb\bar{b}$	I	$nncc\bar{c}$	$nncc\bar{b}$	$nnbb\bar{c}$	$nnbb\bar{b}$
$\frac{5}{2}^-$	$\begin{pmatrix} 62.5 \\ 51.9 \end{pmatrix}$	$\begin{pmatrix} 68.3 \\ 31.8 \end{pmatrix}$	$\begin{pmatrix} 85.8 \\ 59.0 \end{pmatrix}$	$\begin{pmatrix} 68.3 \\ 31.8 \end{pmatrix}$	$\begin{pmatrix} 67.9 \\ 36.9 \end{pmatrix}$	$\begin{pmatrix} 54.8 \\ 7.4 \end{pmatrix}$	1	100.5	82.9	84.0	70.7
							0	48.5	20.5	29.2	-0.9
$\frac{3}{2}^-$	$\begin{pmatrix} 84.9 \\ 58.4 \\ 38.0 \\ 33.9 \\ -6.0 \\ -31.3 \\ -91.2 \end{pmatrix}$	$\begin{pmatrix} 63.5 \\ 50.3 \\ 34.2 \\ 18.3 \\ -7.4 \\ -16.7 \\ -42.7 \end{pmatrix}$	$\begin{pmatrix} 97.1 \\ 68.7 \\ 49.7 \\ 35.9 \\ -52.1 \\ -61.1 \\ -127.1 \end{pmatrix}$	$\begin{pmatrix} 90.1 \\ 66.3 \\ 23.7 \\ 14.9 \\ -3.9 \\ -37.9 \\ -94.1 \end{pmatrix}$	$\begin{pmatrix} 94.0 \\ 55.4 \\ 34.2 \\ 31.9 \\ -31.9 \\ -51.0 \\ -151.6 \end{pmatrix}$	$\begin{pmatrix} 63.9 \\ 49.5 \\ 41.5 \\ 5.9 \\ -4.9 \\ -34.6 \\ -92.3 \end{pmatrix}$	1	$\begin{pmatrix} 117.3 \\ 81.2 \\ 56.9 \\ -43.0 \end{pmatrix}$	$\begin{pmatrix} 105.1 \\ 84.1 \\ 36.1 \\ 13.6 \end{pmatrix}$	$\begin{pmatrix} 115.1 \\ 71.1 \\ 49.2 \\ -33.0 \end{pmatrix}$	$\begin{pmatrix} 83.9 \\ 68.7 \\ 57.7 \\ 12.6 \end{pmatrix}$
							0	$\begin{pmatrix} 36.5 \\ -68.8 \\ -161.0 \end{pmatrix}$	$\begin{pmatrix} 11.5 \\ -45.4 \\ -136.7 \end{pmatrix}$	$\begin{pmatrix} 25.3 \\ -57.5 \\ -181.1 \end{pmatrix}$	$\begin{pmatrix} -3.2 \\ -43.2 \\ -137.8 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} 125.0 \\ 94.5 \\ 52.1 \\ 11.7 \\ -8.9 \\ -39.2 \\ -90.1 \\ -138.0 \end{pmatrix}$	$\begin{pmatrix} 92.9 \\ 63.9 \\ 23.5 \\ 21.5 \\ -6.6 \\ -24.6 \\ -45.5 \\ -96.2 \end{pmatrix}$	$\begin{pmatrix} 168.3 \\ 85.0 \\ 27.4 \\ -14.6 \\ -48.2 \\ -83.6 \\ -86.7 \\ -240.3 \end{pmatrix}$	$\begin{pmatrix} 133.0 \\ 55.9 \\ 7.7 \\ -3.8 \\ -42.3 \\ -57.8 \\ -83.9 \\ -163.4 \end{pmatrix}$	$\begin{pmatrix} 118.3 \\ 80.3 \\ 28.6 \\ 12.0 \\ -34.9 \\ -50.1 \\ -92.2 \\ -194.3 \end{pmatrix}$	$\begin{pmatrix} 87.7 \\ 62.3 \\ 19.8 \\ -1.4 \\ -9.2 \\ -43.4 \\ -76.9 \\ -139.6 \end{pmatrix}$	1	$\begin{pmatrix} 182.8 \\ 104.4 \\ 3.9 \\ -62.7 \end{pmatrix}$	$\begin{pmatrix} 146.6 \\ 75.8 \\ 27.3 \\ -21.3 \end{pmatrix}$	$\begin{pmatrix} 139.6 \\ 96.9 \\ 28.0 \\ -32.5 \end{pmatrix}$	$\begin{pmatrix} 107.8 \\ 79.8 \\ 37.5 \\ 6.8 \end{pmatrix}$
							0	$\begin{pmatrix} 12.8 \\ -71.0 \\ -124.3 \\ -267.2 \end{pmatrix}$	$\begin{pmatrix} -19.4 \\ -71.0 \\ 121.9 \\ -198.8 \end{pmatrix}$	$\begin{pmatrix} 19.6 \\ -63.6 \\ -134.1 \\ -227.7 \end{pmatrix}$	$\begin{pmatrix} -11.1 \\ -55.7 \\ -122.1 \\ -184.6 \end{pmatrix}$

systems in Tables XI, XII, and XIII, respectively. The corresponding mass M can be obtained with the eigenvalue $\langle H_{\text{CMI}} \rangle$ from the equation

$$M = \langle H_{\text{CMI}} \rangle + (M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}}), \quad (5)$$

and we list $M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}}$ in Table X.

TABLE XII. The eigenvalues for the $nsbc\bar{Q}$ and $nncb\bar{Q}$ ($I = 1, 0$) ($Q = c, b$) systems in units of MeV. In the $nncb\bar{Q}$ systems, the I represents isospin, and 0 and 1 represent the pentaquark states with $I = 0$ and $I = 1$, respectively.

J^P	$nsbc\bar{c}$	$nscb\bar{b}$	I	$nncb\bar{c}$	$nncb\bar{b}$
$\frac{5}{2}^-$	$\begin{pmatrix} 76.7 \\ 67.7 \\ 46.5 \end{pmatrix}$	$\begin{pmatrix} 76.7 \\ 67.7 \\ 46.5 \end{pmatrix}$	1	$\begin{pmatrix} 91.3 \\ 83.3 \\ 37.4 \end{pmatrix}$	$\begin{pmatrix} 85.9 \\ 71.3 \\ 8.3 \end{pmatrix}$
$\frac{3}{2}^-$	$\begin{pmatrix} 101.1 \\ 77.2 \\ 54.6 \\ 48.5 \\ 40.3 \\ 16.4 \\ 13.6 \\ -17.1 \\ -45.3 \\ -72.3 \\ -75.7 \\ -141.4 \end{pmatrix}$	$\begin{pmatrix} 78.1 \\ 66.6 \\ 52.3 \\ 35.4 \\ 13.2 \\ 8.9 \\ 1.1 \\ -11.2 \\ -23.9 \\ -41.8 \\ -85.0 \\ -93.6 \end{pmatrix}$	1	$\begin{pmatrix} 118.8 \\ 95.2 \\ 71.2 \\ 63.2 \\ 33.1 \\ -2.7 \\ -54.1 \end{pmatrix}$	$\begin{pmatrix} 94.1 \\ 82.9 \\ 70.4 \\ 52.2 \\ 26.2 \\ 17.9 \\ -6.2 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} 142.2 \\ 105.7 \\ 73.0 \\ 31.9 \\ 26.2 \\ 5.7 \\ -18.9 \\ -36.0 \\ -54.8 \\ -70.0 \\ -88.5 \\ -119.1 \\ -138.6 \\ -192.6 \\ -238.5 \end{pmatrix}$	$\begin{pmatrix} 108.9 \\ 78.6 \\ 44.2 \\ 37.7 \\ 14.0 \\ -3.4 \\ -4.2 \\ -26.6 \\ -50.7 \\ -53.9 \\ -76.4 \\ -81.8 \\ -101.1 \\ -148.0 \\ -172.7 \end{pmatrix}$	0	$\begin{pmatrix} 29.5 \\ 6.0 \\ -65.4 \\ -121.9 \\ -173.1 \end{pmatrix}$	$\begin{pmatrix} 2.6 \\ -21.7 \\ -49.2 \\ -131.1 \\ -138.1 \end{pmatrix}$

TABLE XIII. The eigenvalues for the $sscc\bar{Q}$ and $ssbb\bar{Q}$ ($Q = c, b$) systems in units of MeV.

J^P	$sscc\bar{c}$	$sscc\bar{b}$	$ssbb\bar{c}$	$ssbb\bar{b}$
$\frac{5}{2}^-$	(70.9)	(53.9)	(52.0)	(39.2)
$\frac{3}{2}^-$	$\begin{pmatrix} 77.2 \\ 56.2 \\ 14.6 \\ -78.4 \end{pmatrix}$	$\begin{pmatrix} 76.1 \\ 48.4 \\ -6.4 \\ -22.9 \end{pmatrix}$	$\begin{pmatrix} 73.6 \\ 40.0 \\ 14.9 \\ -68.9 \end{pmatrix}$	$\begin{pmatrix} 45.7 \\ 29.6 \\ 25.8 \\ -22.3 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} 154.5 \\ 64.9 \\ -33.8 \\ -106.4 \end{pmatrix}$	$\begin{pmatrix} 120.1 \\ 35.4 \\ -12.6 \\ -63.7 \end{pmatrix}$	$\begin{pmatrix} 97.6 \\ 63.9 \\ -3.8 \\ -67.7 \end{pmatrix}$	$\begin{pmatrix} 68.6 \\ 44.8 \\ 2.0 \\ -25.3 \end{pmatrix}$

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