W radiative decay to heavy-light mesons in HQET factorization through $\mathcal{O}(\alpha_s)$

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Analogous to the non-relativistic Quantum Chromodynamics (NRQCD) factorization for heavy quarkonium exclusive production, in this work we propose to employ the heavy quark effective theory (HQET) factorization, which has been predominantly applied to account for exclusive B decays, to study the exclusive production of the heavy-flavored mesons. We take $W \rightarrow B(D_s) + \gamma$ as a prototype process. The validity of the HQET factorization rests upon the presumed scale hierarchy: $m_W \sim m_b \gg \Lambda_{\text{QCD}}$. Through an explicit analysis at next-to-leading order (NLO) in α_s , yet at leading order in $1/m_b$, we verify that the decay form factors can indeed be expressed as the convolution between perturbatively calculable hard-scattering kernel and the B meson light-cone distribution amplitude (LCDA) defined in HQET. It is observed that the factorization scale dependence becomes reduced after incorporating the NLO perturbative correction. An interesting future investigation is to identify and resum large collinear logarithms of m_W/m_b that arise ubiquitously in the fixed-order expressions of the hard-scattering kernel in HQET factorization.

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I. INTRODUCTION

Heavy flavor physics has been one of the major forefronts of high energy physics in the past two decades, whose primary goal is to trace the origin of the CP violation, precisely pin down the Cabbibo-Kobayashi-Maskawa (CKM) matrix, as well as search for possible footprints of new physics. Much of the effort in this subject concentrates on the detailed study of B meson exclusive decays, which contains very rich phenomenology. To reliably describe uncountable decay channels, it is mandatory to develop a systematic and thorough understanding towards the underlying hadronization dynamics. By invoking the hierarchy $m_b \gg \Lambda_{\text{QCD}}$, the so-called QCD factorization (QCDF) approach has become the dominant theoretical arsenal which derives from the first principle [1]. The key concept is to express the decay amplitude into the convolution of the perturbatively

calculable, yet, process-dependent, hard-scattering kernel and the nonperturbative, yet, universal, B meson light-cone distribution amplitude (LCDA). It is important to emphasize that this factorization framework applies in the heavy quark limit, corroborated by the fact that the b quark field in the LCDA is defined in the heavy quark effective theory (HQET) rather than in full QCD. The simplest application of the QCD factorization is the radiative decay $B \rightarrow \gamma \ell \nu$ [2–4]. This factorization picture has also been extended to more sophisticated processes, e.g., $B \rightarrow \gamma \gamma$ [5], $B \rightarrow M_1 M_2$ [6,7], and so on.

In contrast to exclusive B decays, the hard exclusive production of heavy-flavored hadrons receives much fewer attention in literature. The main reason might be attributed to the highly suppressed production rate of such processes in high energy collision experiments. The D_s meson exclusive production from W decay, e.g., $W^+ \rightarrow D_s^+ + \gamma$, was among the early exploration in this topics [8,9], and an upper limit was placed by the Fermilab Tevatron in the late 1990s [10]. Motivated by a gigantic number of $W^\pm$, Z^0 bosons produced at LHC, i.e., about $\mathcal{O}(10^{11})$ with the integrated luminosity of 3000 fb^{-1} , a number of exclusive channels of W , Z radiative decays into heavy-light mesons has recently been investigated at next-to-leading order (NLO) in α_s [11]. The theoretical basis underneath [11] is the standard collinear factorization (or light-cone factorization) for hard exclusive reactions [12,13]. By

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exploiting the hierarchy $m_{W,Z} \gg m_{b,c} \sim \Lambda_{\text{QCD}}$, one expresses the amplitude as the convolution between the hard-scattering kernel and the universal LCDAs of heavy mesons. However, it is worth stressing that, the non-perturbative LCDAs encountered in this case are of the standard Brodsky-Lepage type [12], where the b quark field is defined in full QCD rather than in HQET, in sharp contrast with those arising in B exclusive decays. Unfortunately, our current constraints on the heavy meson QCD LCDAs are rather limited, which severely obstructs the predictive power of the standard collinear factorization approach. A further drawback of this conventional formalism is that, the LCDAs of heavy-flavor mesons cannot be genuinely nonperturbative, where the hard scale $m_{b,c}$ is still entangled with the hadronic scale Λ_{QCD} . It is desirable if asymptotic freedom can be invoked to separate some sort of short-distance effects from the heavy meson QCD LCDAs.

Inspired by non-relativistic Quantum Chromodynamics (NRQCD) factorization tailored for inclusive heavy quarkonium production [14], the *heavy-quark recombination* (HQR) mechanism [15] was developed in the beginning of this century, to supplement the single-parton fragmentation mechanism for the inclusive heavy-flavor hadron production with contribution subleading in $1/p_\perp$. The basic idea behind this mechanism is intuitively simple, after a hard scattering, the heavy quark would have some chance to combine with a spectator quark which is *soft* in its rest frame to form a heavy-light hadron. In the color-singlet channel, where the inclusive and exclusive production of heavy hadrons practically makes no difference at lowest order, the HQR formalism only involves a single non-perturbative factor, which is proportional to the first inverse moment of the B meson LCDA defined in HQET, $1/\lambda_B$, which has been intensively investigated in the field of B meson decays. A notable success of this mechanism is to economically account for the charm/anticharm hadron production asymmetry (leading particle effect) observed at numerous Fermilab fixed-target experiments [16–18].

The key idea behind HQR is to formalize the production rate of heavy-flavored hadrons in the heavy quark limit, separating the dynamics of order m_b or higher from the hadronic effects at $\mathcal{O}(\Lambda_{\text{QCD}})$, but no longer distinguishing the hard-scattering scale specific to the process, say, Q , and m_b . In this sense, one can tackle hard exclusive heavy hadron production in a fashion very much alike the B exclusive decay, in principle guaranteed by a factorization theorem valid to all orders in α_s . In this work, we choose not to stick to the old jargon HQR mechanism, rather we decide to term this factorization approach as the *HQET factorization*, in close analogy with the NRQCD factorization for heavy quarkonium production. To be specific, in this work we will take $W^+ \rightarrow B^+(D_s^+) + \gamma$ as the prototype processes for heavy meson exclusive production. To our knowledge, the HQR mechanism so far is only illustrated at leading order (LO) in α_s . It is the very goal of this work to

verify the validity of HQET factorization through NLO in α_s , which provides a much more informative revelation about nontrivial QCD dynamics.

The rest of the paper is structured as follows. In Sec. II, we first decompose the $W^+ \rightarrow B^+ + \gamma$ amplitude in terms of two Lorentz-invariant form factors, and introduce some light-cone kinematical variables. In Sec. III, we recap some essential features about the leading-twist B meson LCDA, which enters the factorization theorem in many B exclusive decay processes. In Sec. IV, in analogy with the NRQCD factorization for exclusive heavy quarkonium production, we propose the HQET factorization formalism for exclusive heavy-light hadron production, presumably valid to all orders in α_s . In Sec. V, we first determine the hard-scattering kernel for $W^+ \rightarrow B^+ + \gamma$ at tree level. We then proceed to compute the $\mathcal{O}(\alpha_s)$ correction to this process. We explicitly verify that the resulting soft IR pole can be properly factorized into the B meson LCDA, which establishes the correctness of HQET factorization at the first nontrivial order. The IR-finite hard-scattering kernel at $\mathcal{O}(\alpha_s)$ is also deduced. In Sec. VI, we present a comprehensive numerical prediction for the processes $W^+ \rightarrow B^+(D_s^+) + \gamma$, accurate through NLO in α_s . Assuming a simple exponential parametrization for B meson LCDA defined at some initial scale, we study its evolution behavior with different factorization scale. It is found that including the $\mathcal{O}(\alpha_s)$ correction would significantly reduce the LO decay rate. Finally, we present a summary and an outlook in Sec. VII.

II. DECOMPOSITION OF AMPLITUDE AND LIGHT-CONE KINEMATICS

Let us first specify the kinematics for the process $W^+ \rightarrow B^+ + \gamma$. The momenta of W^+ , B^+ , and γ are designated by Q , P , and q , respectively, with $Q = P + q$. They are subject to the on-shell conditions: $Q^2 = m_W^2$, $P^2 = m_B^2$, and $q^2 = 0$, with m_W , m_B standing for the masses of the W and B meson, respectively. For future usage, we also introduce a dimensionless four velocity v^μ via $P^\mu = m_B v^\mu$, obviously obeying $v^2 = 1$. The polarization vectors of the W and γ are denoted by ε_W and ε_γ .

In accordance with Lorentz invariance, the decay amplitude for $W^+ \rightarrow B^+ \gamma$ can be decomposed as [11]

$$\begin{aligned} \mathcal{M}(W^+ \rightarrow B^+ \gamma) &= \frac{e_u e^2 V_{ub}}{4\sqrt{2} \sin \theta_W} \left(\epsilon_{\mu\nu\alpha\beta} \frac{P^\mu q^\nu \varepsilon_W^\alpha \varepsilon_\gamma^{*\beta}}{P \cdot q} F_V + i \varepsilon_W \cdot \varepsilon_\gamma^* F_A \right), \end{aligned} \quad (1)$$

where e is the electric coupling constant, $e_u = +\frac{2}{3}$ is the electric charge of the u quark, θ_W is the weak mixing angle, V_{ub} denotes the CKM matrix element. $F_V(F_A)$ represent

the Lorentz scalar form factors, which are affiliated with vacuum-to- $B^+ + \gamma$ matrix element mediated by the vector (axial-vector) weak current. All the nontrivial QCD dynamics are encoded in these scalar form factors, which are functions of m_W , m_B , and Λ_{QCD} . Note that, since weak interaction violates the parity conservation, this exclusive process bears two independent helicity amplitudes, which can be expressed via the linear combination of these two form factors.

Squaring (1), averaging upon and summing over the polarizations of W and γ , one expresses the unpolarized decay rate in the W rest frame as

$$\Gamma(W^+ \rightarrow B^+ + \gamma) = \frac{e_u^2 \pi \alpha^2}{48 \sin^2 \theta_W m_W^3} |V_{ub}|^2 (m_W^2 - m_B^2) (|F_V|^2 + |F_A|^2), \quad (2)$$

with $\alpha \equiv e^2/4\pi$ denoting the QED fine structure constant.

To facilitate the discussion in the following sections, it is convenient to set up the light-cone representation for the kinematics. We first introduce two lightlike reference vectors $n_\pm^\mu \equiv \frac{1}{\sqrt{2}}(1, 0, 0, \pm 1)$, which obey $n_\pm^2 = 0$ and $n_+ \cdot n_- = 1$. In this light-cone basis, any four vector $a^\mu = (a^0, a^1, a^2, a^3)$ can then be decomposed into

$$a^\mu = (n_- \cdot a) n_+^\mu + (n_+ \cdot a) n_-^\mu + a_\perp^\mu \equiv a^+ n_+^\mu + a^- n_-^\mu + a_\perp^\mu, \quad (3)$$

where $a_\perp^\mu = (0, a^1, a^2, 0)$ is the transverse component of the four-vector. The scalar product of two four-vectors then becomes

$$a \cdot b = a^+ b^- + a^- b^+ + a_\perp \cdot b_\perp. \quad (4)$$

Were we interested in investigating the process $W^+ \rightarrow B^+ + \gamma$ within the standard collinear factorization approach, it would be most natural to stay with the rest frame of the W boson, where the B meson is energetic owing to $m_W \gg m_B$. In this reference frame, we presume that the B meson moves along the positive $\hat{z}$ axis, while the photon flies in the opposite direction. The light-cone representations for the momenta of the B^+ and γ then become

$$P^\mu|_{W \text{ rest frame}} = (P^+, P^-, \mathbf{P}_\perp) = \frac{1}{\sqrt{2}} \left(m_W, \frac{m_B^2}{m_W}, \mathbf{0}_\perp \right), \quad (5a)$$

$$q^\mu|_{W \text{ rest frame}} = (q^+, q^-, \mathbf{q}_\perp) = \frac{1}{\sqrt{2}} \left(0, \frac{m_W^2 - m_B^2}{m_W}, \mathbf{0}_\perp \right), \quad (5b)$$

where P^- is suppressed by a factor m_B^2/m_W^2 relative to P^+ .

Since the form factors $F_{V,A}$ themselves are Lorentz scalars, they can be computed in any reference frame. In order to make the picture of HQET factorization more transparent, as well as be closely connected with the exclusive B decay channel $B \rightarrow \gamma(W^* \rightarrow) l \nu$, it looks most natural to boost this process to the B meson rest frame. The corresponding momenta of B and photon in the light-cone basis then read

$$P^\mu|_{B \text{ rest frame}} = (P^+, P^-, \mathbf{P}_\perp) = \frac{1}{\sqrt{2}} (m_B, m_B, \mathbf{0}_\perp), \quad (6a)$$

$$q^\mu|_{B \text{ rest frame}} = (q^+, q^-, \mathbf{q}_\perp) = \frac{1}{\sqrt{2}} \left(0, \frac{m_W^2 - m_B^2}{m_B}, \mathbf{0}_\perp \right). \quad (6b)$$

Note the photon becomes enormously energetic in this frame, enhanced with respect to its energy in the W rest frame by a factor m_W/m_B .

III. REVIEW OF B MESON LCDA DEFINED IN HQET

B meson LCDA is a crucial nonperturbative entity that ubiquitously enters the various exclusive B decay processes. It is the same entity that also enters the B exclusive production process in the HQET factorization framework. In this section, we briefly recapitulate some of its essential features.

Let us consider the correlator composed of the light spectator quark and $\bar{b}$ quark separated by lightlike distance, sandwiched between the vacuum and the B meson with velocity v^μ (for simplicity, we actually will work in the B meson rest frame). Its most general parametrization may be cast into the following form [19,20]:¹

$$\begin{aligned} \langle B(v) | \bar{u}_\beta(z) [z, 0] h_{v,\alpha}(0) | 0 \rangle \\ = \frac{i \hat{f}_B m_B}{4} \left\{ \left[2 \tilde{\phi}_B^+(t) - \frac{\not{z}}{t} (\tilde{\phi}_B^-(t) - \tilde{\phi}_B^+(t)) \right] \frac{1 - \not{v}}{2} \gamma_5 \right\}_{\alpha\beta}, \end{aligned} \quad (7)$$

where $z^2 = 0$, $t = v \cdot z$, and $\tilde{\phi}_B^\pm$ are a pair of nonperturbative functions of t . Here u refers to the standard QCD field for u quark, and h_v signifies the $\bar{b}$ quark field with velocity label v introduced in HQET. α, β are spinor indices. $\hat{f}_B$ signifies the B meson decay constant defined in HQET as

¹Note that we have intentionally put the B meson in the bra rather than the ket, since we are interested in B production rather than decay in this work.

$$\langle B(v) | \bar{u} \gamma^\mu \gamma_5 h_v | 0 \rangle = i \hat{f}_B m_B v^\mu, \quad (8)$$

which can be converted from the QCD decay constant f_B through perturbative series [21,22]:

$$f_B = \hat{f}_B(\mu_F) \left[1 - \frac{\alpha_s C_F}{4\pi} \left(3 \ln \frac{\mu_F}{m_b} + 2 \right) \right] + \mathcal{O}(\alpha_s^2). \quad (9)$$

The lightlike gauge link,

$$[z, 0] = \mathcal{P} \exp \left[-ig_s \int_0^z d\xi^\mu A_\mu^a(\xi) t^a \right], \quad (10)$$

has been inserted in (7) to ensure gauge invariance of the nonlocal quark bilinear. Here $t^a (a = 1, \dots, 8)$ signify the $SU(3)$ generators in fundamental representation, and $\mathcal{P}$ indicates the path ordering.

The phenomenologically relevant B meson LCDAs are usually referred in momentum space, which can be inferred by Fourier transforming the coordinate-space correlators in (7) [15,19]:

$$\begin{aligned} \Phi_B^\pm(\omega) &\equiv i \hat{f}_B m_B \phi_B^\pm(\omega) \\ &= \frac{1}{v^\pm} \int \frac{dt}{2\pi} e^{i\omega t} \langle B(v) | \bar{u}(z) [z, 0] \not{v} \gamma_5 h_v(0) | 0 \rangle |_{z^+, z^\perp = 0}, \end{aligned} \quad (11)$$

where a pair of B meson LCDAs are defined through

$$\phi_B^\pm(\omega) = \int_0^\infty \frac{dt}{2\pi} e^{i\omega t} \tilde{\phi}_B^\pm(t). \quad (12)$$

Here ω indicates the “+”-momentum carried by the spectator quark in the B rest frame, whose typical value is $\sim \Lambda_{\text{QCD}}$. By construction, $\phi_B^\pm(\omega)$ has nonvanishing support only when $\omega \in (0, \infty)$. General principle constrains that as $\omega \rightarrow 0$, $\phi_B^+(\omega) \propto \omega$, whereas $\phi_B^-(\omega) \propto 1$.

Note the light-cone correlator in (11) in general entails UV divergences, and is subject to the renormalization involving the mixing among an infinite number of light-ray operators. As a consequence, $\phi_B^\pm(\omega)$ become scale-dependent quantities. Practically speaking, provided that we are only interested in the leading-power contribution in $1/m_b$ expansion, we are justified to concentrate on the B meson LCDA $\phi_B^+(\omega)$ and discard $\phi_B^-(\omega)$. The evolution equation governing $\phi_B^+(\omega, \mu)$ was first correctly written down by Lange and Neubert in 2003 [23]:

$$\begin{aligned} \frac{d}{d \ln \mu} \phi_B^+(\omega, \mu) &= -\frac{\alpha_s C_F}{4\pi} \int_0^\infty d\omega' \left\{ \left(4 \ln \frac{\mu}{\omega} - 2 \right) \delta(\omega - \omega') \right. \\ &\quad \left. - 4\omega \left[\frac{\theta(\omega' - \omega)}{\omega'(\omega' - \omega)} + \frac{\theta(\omega - \omega')}{\omega(\omega - \omega')} \right]_+ \right\} \\ &\quad \times \phi_B^+(\omega', \mu), \end{aligned} \quad (13)$$

with μ the renormalization scale. Note this renormalization group equation looks rather different from the celebrated Efremov-Radyushkin-Brodsky-Lepage (ERBL) equation [24–26], which controls the scale dependence of the meson LCDA defined in full QCD.

Once the profile of $\phi_B^+(\omega)$ is determined at some initial scale (say, $\mu_0 = 1$ GeV), one can utilize (13) to obtain its profile at any other scale (usually $1 \text{ GeV} \leq \mu \leq m_b$). There exists some model-dependent studies on the properties of the B meson LCDAs using QCD sum rules [27]. The model-independent features of $\phi_B^+(\omega)$ have also been abstracted by applying the operator-product-expansion technique [28].

It turns out that for hard heavy hadron exclusive production, only $\phi_B^+(\omega)$ survives in the factorization theorem in the heavy quark limit. As was mentioned before, of central phenomenological relevance is the first inverse moment of $\phi_B^+(\omega)$, usually referred to as λ_B^{-1} :

$$\lambda_B^{-1}(\mu) \equiv \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu). \quad (14)$$

Intuitively, one expects $\lambda_B^{-1} \sim \Lambda_{\text{QCD}}^{-1}$. Note this inverse moment is also scale-dependent.

We also plan to study the NLO perturbative corrections to $W \rightarrow B + \gamma$. To this purpose, it is also necessary to introduce the first and second logarithmic inverse moments by

$$\lambda_B^{-1} \sigma_{B,n}(\mu) \equiv - \int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{\omega}{\mu} \phi_B^+(\omega, \mu), \quad n = 1, 2 \quad (15)$$

which are scale dependent as well.

As a side remark, we finally mention that the positive Mellin moments of $\phi_B^+(\omega)$, that is, $\int_0^\infty d\omega \omega^{N-1} \phi_B^+(\omega)$ ($N > 0$), are generally UV divergent, thus become ill-defined. Usually one imposes a hard cutoff in the upper end of the integral to regularize the UV divergence [28].

IV. HQET FACTORIZATION FOR EXCLUSIVE HEAVY-LIGHT MESON PRODUCTION

In this section we shall state the exact form of HQET factorization for heavy-light meson exclusive production. Since this factorization picture naturally evolves from the previously developed heavy quark recombination mechanism, especially for the color-singlet channel, it might be beneficial to first elaborate on the underlying physical picture, by taking the $W^+ \rightarrow B^+ + \gamma$ channel as a concrete example. Viewed in the rest frame of the B^+ meson, after a hard scattering the $\bar{b}$ quark has a considerable chance to pick up the *soft* spectator u quark to hadronize into the B^+ meson, with the recombination probability proportional to the square of the nonperturbative factor λ_B^{-1} defined in (14). By emitting a highly energetic photon, the soft u quark is necessarily transformed into a hard-collinear one, which endorses the usage of the B meson LCDA.

Let k^μ signify the momentum carried by the spectator u quark inside the B^+ meson. According to the recipe of the HQR mechanism [20], one may obtain the $W^+ \rightarrow B^+ + \gamma$ amplitude through making the following substitution in the quark amplitude $W^+ \rightarrow [\bar{b}(P-k)u(k)]^{(1)} + \gamma$:²

$$v_i(P-k)\bar{u}_j(k) \rightarrow \frac{\delta_{ij}}{N_c} \frac{i\hat{f}_B m_b}{4} \left\{ \frac{1-\not{k}}{2} \left[\phi_B^+(\omega) \frac{\not{k}_+}{\sqrt{2}} + \phi_B^-(\omega) \frac{\not{k}_-}{\sqrt{2}} - \omega \phi_B^-(\omega) \gamma_\perp^\mu \frac{\partial}{\partial k_\perp^\mu} \right] \gamma_5 \right\} \Big|_{k=\omega v}, \quad (16)$$

where $i, j = 1, 2, \dots, N_c$ are color indices and $N_c = 3$. The first Kronecker symbol serves the color-singlet projector. After taking the momentum derivative on the quark amplitude, one then makes the substitution $k \rightarrow \omega v$ and retains the most singular piece in the $\omega \rightarrow 0$ limit, which is usually $\propto 1/\omega$. Curiously, in the heavy quark limit the $\phi_B^-(\omega)$ turns out not to contribute, and only $\phi_B^+(\omega)$ yields a nonvanishing contribution, whose effect is simply encoded in the first inverse moment λ_B^{-1} .

A shortcut can be invoked to quickly reproduce the heavy meson production amplitude from the HQR mechanism, with much less effort [20]. Rather than consider B^+ meson production, one simply starts with the flavored quarkonium, B_c , production. Assume the momenta are partitioned by two constituents of the B_c as $p_c = \kappa P$ and $p_b = (1-\kappa)P$, where P is the B_c momentum and $\kappa = m_c/(m_c + m_b)$. One can then employ the familiar covariant spin projector for quarkonium production at LO in velocity expansion:

$$v_i(p_b)\bar{u}_j(p_c) \rightarrow \delta_{ij} \frac{f_{B_c}}{12} (\not{P} - m_{B_c}) \gamma_5. \quad (17)$$

Consequently, the amplitude for B^+ production through the $[\bar{b}u](^1S_0^{(1)})$ channel can be deduced by taking the $\kappa \rightarrow 0$ limit of that for B_c production and replacing $f_{B_c}/(4\kappa) \rightarrow \hat{f}_B/(4\lambda_B)$. In general, many Feynman diagrams do not contribute for their lack of the $1/\kappa$ singularity. Notice that this shortcut has been utilized to ascertain the NLO perturbative corrections to the B electromagnetic form factor and $W^+ \rightarrow B^+ + \gamma$ from their B_c counterparts in NRQCD factorization [29,30].

To the best of our knowledge, the HQR mechanism thus far has not yet been extended to NLO in α_s . It is not straightforward to achieve this goal from the projector approach specified in (16), or from the shortcut of extracting the hard-scattering kernel from B_c production, as described in (17). One reason is that ϕ_B^+ develops UV divergence at NLO

in α_s , therefore the hard-scattering kernel also acquires an explicit factorization scale dependence. This is in sharp contrast with the one-loop correction to exclusive quarkonium production process, since the local NRQCD bilinear operator is one-loop UV finite. It is not *a priori* obvious why the aforementioned projector approach can lead to the IR-finite hard-scattering kernel once beyond α_s .

Since we treat m_W as the same order as m_b , the argument that leads to the factorization theorem for $B \rightarrow \gamma l \nu$ can be transplanted to $W^+ \rightarrow B^+ + \gamma$ without modification. We thus propose a factorization theorem for $W^+ \rightarrow B^+ + \gamma$, which is valid in the heavy quark limit, albeit to all orders in α_s :

$$\mathcal{M}(W^+ \rightarrow B^+ + \gamma) = \int_0^\infty d\omega T(\omega, m_b, \mu_F) \Phi_B^+(\omega, \mu_F) + \mathcal{O}(m_b^{-1}), \quad (18)$$

where $T(\omega, m_b, \mu_F)$ is referred to as the hard-scattering kernel, which can be computed in perturbation theory. Note the hard-scattering kernel is explicitly dependent on m_W and m_b as well as the factorization scale μ_F . Note only the leading-twist B meson LCDA, $\phi_B^+(\omega)$, explicitly enters the formula. The μ_F dependence of the hard-scattering kernel should counteract that of the $\hat{f}_B$ and ϕ_B^+ so that the physical amplitude becomes insensitive to the artificial scale μ_F . An important characteristic of this process is $T(\omega) \propto 1/\omega$, so the convolution integral is UV finite and well-defined.

Equation (18) lays down the foundation for the HQET factorization. In some sense, (18) offers a systematic realization of the HQR mechanism. The virtue of this factorization framework is to allow us to systematically investigate the higher-order perturbative corrections for the hard-scattering kernel, to ensure its IR finiteness.

V. FORM FACTORS FOR $W^+ \rightarrow B^+ + \gamma$ THROUGH NLO IN α_s

This section reports the central results of this work, where the hard-scattering kernel is computed through NLO in α_s . Rather than employ the projector approach given in (17), we choose to employ the perturbative matching method to determine the hard-scattering kernel. The calculation presented in this section closely follows the analogous NLO calculation for $B \rightarrow \gamma l \nu$ [3]. Unless otherwise specified, the calculation of the form factors $F_{V,A}$ is performed in the rest frame of the B meson.

The hard-scattering kernel is insensitive to the long-distance physics. For the sake of extracting it using perturbation theory, it is legitimate to replace the physical B^+ meson in (18) with a fictitious B^+ meson composed of a pair of free quarks $[\bar{b}(P-k)u(k)]$. One can impose the following projector on the quark amplitude:

$$v_i(P-k)\bar{u}_j(k) \rightarrow \frac{\delta_{ij}}{N_c} \frac{1-\not{k}}{4} \gamma_5, \quad (19)$$

²This form may look superficially different from the projectors adopted in [15]. Nevertheless, once the equation of motion and Wandzura-Wilczek approximation are invoked, they can be proven to be equivalent [20].

just similar to the projector (17) which guarantees the fictitious “ B^+ meson” being the color and spin singlet. The momentum of the spectator u quark is assumed to be soft, i.e., which scales as $k^\mu \sim \Lambda_{\text{QCD}}$. The LCDA in (11) for such a fictitious B^+ meson then becomes

$$\Phi_{[bu]}^\pm(\omega) = \frac{1}{v^\pm} \int \frac{dt}{2\pi} e^{i\omega t} \langle [\bar{b}u](P) | \bar{u}(z)[z, 0] \not{t} \gamma_5 h_v(0) | 0 \rangle |_{z^+, z^\perp = 0}. \quad (20)$$

Both of the ingredients in the right side of (18) can then be expanded in perturbation theory:

$$\begin{aligned} \Phi_{[bu]}^+ &= \Phi_{[bu]}^{+(0)} + \Phi_{[bu]}^{+(1)} + \mathcal{O}(\alpha_s^2), \\ T &= T^{(0)} + T^{(1)} + \mathcal{O}(\alpha_s^2), \end{aligned} \quad (21)$$

with the superscript indicating the powers of α_s . At LO, the LCDAs for the fictitious B^+ meson (20) look exceedingly simple

$$\begin{aligned} \Phi_{[bu]}^{\pm(0)}(\omega) &= \frac{1}{v^\pm} \delta(k^+/v^+ - \omega) \text{Tr} \left[\frac{1 - \not{v}}{4} \gamma_5 \not{t} \gamma_5 \right] \\ &= \delta(k^+/v^+ - \omega). \end{aligned} \quad (22)$$

The QCD amplitude on the left side of (18) can also be computed perturbatively. Through NLO in α_s , it reads

$$\mathcal{M} = \mathcal{M}^{(0)} + \mathcal{M}^{(1)} + \mathcal{O}(\alpha_s^2), \quad (23)$$

where

$$\mathcal{M}^{(0)} = \Phi_{[bu]}^{+(0)} \otimes T^{(0)}, \quad (24a)$$

$$\mathcal{M}^{(1)} = \Phi_{[bu]}^{+(0)} \otimes T^{(1)} + \Phi_{[bu]}^{+(1)} \otimes T^{(0)}, \quad (24b)$$

with $\otimes$ signifying the convolution integral in ω . Since both $\mathcal{M}$ and $\Phi_{[bu]}^+$ are perturbatively calculable for such a fictitious B^+ meson, one can solve (24) iteratively, to ascertain T order by order in α_s .

A. Tree level

As depicted in Fig. 1, there arise three electroweak diagrams at lowest order that contribute to the quark-level process $W \rightarrow [\bar{b}(P-k)u(k)]^{(1)} + \gamma$. Recall the momentum of the outgoing photon in (6b) scales as $(q^+, q^-, |\mathbf{q}_\perp|) \sim (0, m_b, 0)$. The u propagator in Fig. 1(a) becomes hard-collinear, and contribute a $1/q \cdot k \sim 1/k^+ q^-$ singularity to the amplitude. One can readily convince oneself that the other two diagrams, the one with photon emitted from the $\bar{b}$ quark [Fig. 1(b)] and the one emitted via a $WW\gamma$ vertex [Fig. 1(c)] do not possess such a $1/k^+$ enhancement, thus can be safely dropped.

Figure 1(a) yields the following tree-level QCD amplitude in the heavy quark limit:

$$\begin{aligned} \mathcal{M}^{(0)}(W^+ \rightarrow [\bar{b}u] + \gamma) &= \frac{eV_{ub}}{2\sqrt{2} \sin \theta_W} \langle [\bar{b}u](P) | \gamma(q, \epsilon_\gamma) | \bar{u} \not{t} (1 - \gamma_5) b | 0 \rangle \\ &\approx \frac{e_u e^2 V_{ub}}{4\sqrt{2} \sin \theta_W q^- k^+} \text{Tr} \left[\frac{1 - \not{v}}{4} \gamma_5 \not{t} \gamma_5 \right] \\ &= \frac{e_u e^2 V_{ub}}{4\sqrt{2} \sin \theta_W} \left(-i \frac{\epsilon_{\mu\nu\alpha\beta} v^\mu n^\nu \epsilon_W^\alpha \epsilon_\gamma^{*\beta}}{v^+} + \epsilon_W \cdot \epsilon_\gamma^* \right) \\ &\quad \times \int_0^\infty \frac{d\omega}{\omega} \delta(k^+/v^+ - \omega). \end{aligned} \quad (25)$$

With the aid of (22) and (24a), it is then straightforward to solve for the tree-level hard-scattering kernel:

$$T^{(0)}(\omega) = \frac{e_u e^2 V_{ub}}{4\sqrt{2} \sin \theta_W} \left(-i \frac{\epsilon_{\mu\nu\alpha\beta} P^\mu q^\nu \epsilon_W^\alpha \epsilon_\gamma^{*\beta}}{P \cdot q} + \epsilon_W \cdot \epsilon_\gamma^* \right) \frac{1}{\omega}. \quad (26)$$

Substituting into (18) and comparing with the Lorentz decomposition specified in (1), one can deduce the final expressions for the vector/axial-vector form factors at tree level:

$$F_V^{(0)} = F_A^{(0)} = f_B m_B \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega) = \frac{f_B m_B}{\lambda_B}. \quad (27)$$

We stress that the $\Phi_{[bu]}^-(\omega)$ indeed does not enter the factorization formula. Starting from (16), inspecting the spinor structure of (25), one can prove the $\Phi_{[bu]}^-$ -dependent terms vanish due to some specific identities such as $n_\perp^2 = 0$ and $\gamma_\perp^\mu \not{t} \gamma_\perp^\mu = (4-d) \not{t}$, with $d=4$ signifying the space-time dimension.

FIG. 1. The Feynman diagrams for $W^+ \rightarrow [\bar{b}u] + \gamma$ at tree level. The bold line represents the $\bar{b}$ quark.

B. One-loop level

In this subsection, we proceed to extract the hard-scattering kernel through order- α_s . Following the ansatz given in (24b), the one-loop hard-scattering kernel $T^{(1)}$ can be extracted via

$$\Phi^{(0)} \otimes T^{(1)} = \mathcal{M}^{(1)} - \Phi^{(1)} \otimes T^{(0)}, \quad (28)$$

The one-loop diagrams for $\Phi^{(1)}$ and $\mathcal{M}^{(1)}$ are depicted in Fig. 2 and Fig. 3, respectively.

By the general principle of effective field theory, both entities in the right-hand side of (28) must possess the *identical* IR divergences, so that upon subtraction, $T^{(1)}$ must be infrared finite. It appears instructive to compute the difference in (28) on a diagram-by-diagram basis.

We employ dimensional regularization (with spacetime dimensions $d = 4 - 2\epsilon$) to regularize UV divergences. We treat γ_5 in the naive dimensional regularization (NDR) scheme in which γ_5 anti-commutes with γ^μ ($\mu = 0, 1, \dots, d-1$). We affiliate the 't Hooft unit mass

FIG. 2. One-loop QCD correction to LCD for a fictitious B meson. The double line represents the $\bar{b}$ field in HQET, dashed line represents the gauge link.

FIG. 3. One-loop QCD correction to the amplitude for $W^+ \rightarrow [\bar{b}u]^{(1)} + \gamma$. We just retain those diagrams with photon emitted from the spectator u quark, which yield leading contribution in the heavy quark limit.

μ_R when calculating the QCD amplitude $\mathcal{M}^{(1)}$. We also affiliate a different 't Hooft unit mass μ_F in computing $\Phi^{(1)}$. When evaluating loop integrals, we have redefined the 't Hooft unit mass through $\mu^2 \rightarrow \mu^2 \frac{e'^E}{4\pi}$, which expedites the renormalization of the LCDA according to the $\overline{\text{MS}}$ subtraction scheme.

Moreover, a nonzero mass m_u is retained for the spectator u quark to regularize the mass (collinear) singularity. For simplicity we adopt the Feynman gauge. The Feynman rules for eikonal vertex and propagator are given by $-ig_s T^a n_-^\mu$ and $1/p^+$, respectively, with p denoting the momentum flowing into the gauge link [31].

The one-loop contributions to the perturbative “ B^+ meson” LCDA, as indicated in Fig. 2(a), 2(b), and 2(e) can also be extracted from the *soft* loop region of the electromagnetic vertex correction, weak vertex correction, and light quark propagator correction for their QCD counterparts as shown in Fig. 3. The contributions from these one-loop bare diagrams turn out to be [3]

$$\Phi_{+\text{em}}^{(1)} \otimes T^{(0)} = \frac{\alpha_s C_F}{4\pi} \left(\frac{2}{\epsilon} - 4 \ln \frac{m_u}{\mu_F} + 4 \right) \mathcal{M}^{(0)}, \quad (29a)$$

$$\begin{aligned} \Phi_{+\text{wk}}^{(1)} \otimes T^{(0)} &= \frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{k^+}{v^+ \mu_F} + 2 \ln^2 \frac{k^+}{v^+ \mu_F} + \frac{3\pi^2}{4} \right) \mathcal{M}^{(0)}, \\ &\quad (29b) \end{aligned}$$

$$\Phi_{+\delta Z_u}^{(1)} \otimes T^{(0)} = \frac{1}{2} \delta Z_u(\mu_F) \mathcal{M}^{(0)}, \quad (29c)$$

where δZ_u is the standard one-loop quark wave function renormalization constant in QCD. Note that the occurrence of the double UV pole in (29b) is a peculiar trait of the HQET LCDA, from which one can infer the cusp anomalous dimension.

The gauge link self-energy diagram Fig. 2(d) does not contribute to $\Phi_+^{(1)} \otimes T^{(0)}$ because its contribution is proportional to $n_-^2 = 0$.

To extract $T^{(1)}(\omega)$ following the recipe outlined in (28), we need also calculate $\mathcal{M}^{(1)}$ indicated by those one-loop QCD diagrams in Fig. 3. At leading power in $1/m_b$, we are only interested in retaining the contribution to $\mathcal{M}^{(1)}$ of order $\Lambda_{\text{QCD}}^{-1}$. It is for this reason that we have excluded those diagrams where the photon is emitted off the $\bar{b}$ quark or the W^+ boson.

It is straightforward to calculate the electromagnetic vertex correction, weak vertex correction and internal quark self-energy QCD diagrams, which have been depicted in Fig. 3(a), 3(b), and 3(d), respectively (We have also tacitly included those quark mass counterterm diagrams in order to obtain UV-finite results). All three diagrams possess the same Lorentz structure as $\mathcal{M}^{(0)}$ at order $\Lambda_{\text{QCD}}^{-1}$, hence the

corresponding contributions to $T^{(1)}(\omega)$ can be readily extracted by subtracting the respective contributions of type $\Phi_+^{(1)} \otimes T^{(0)}$ in Fig. 2. We then obtain³

$$T_{\text{em}}^{(1)}(\omega) = \frac{\alpha_s C_F}{4\pi} T^{(0)}(\omega) \left(\ln \frac{2q^- v^+ \omega}{\mu_F^2} + 2 \ln \frac{\mu_R}{\mu_F} - 4 - i\pi \right), \quad (30a)$$

$$\begin{aligned} T_{\text{wk}}^{(1)}(\omega) &= \frac{\alpha_s C_F}{4\pi} T^{(0)}(\omega) \left\{ \ln \frac{2q^- v^+ \omega}{(1-r)m_W^2} \right. \\ &\quad \times \left[\ln \frac{2(1-r)m_W^2 q^- v^+ \omega}{\mu_F^4} - 2 \right] \\ &\quad + 2 \left(\ln^2 \frac{m_b}{\mu_F} - \ln \frac{m_b}{\mu_R} \right) + 2 \text{Li}_2(r) + \ln^2(1-r) \\ &\quad + [r + 2 \ln(1-r) - 1] \ln \frac{r}{1-r} \\ &\quad \left. + \frac{\pi^2}{12} - i\pi \left(r + 2 \ln \frac{2q^- v^+ \omega}{m_W^2} - 1 \right) \right\}, \quad (30b) \end{aligned}$$

$$T_{\Sigma}^{(1)}(\omega) = \frac{\alpha_s C_F}{4\pi} T^{(0)}(\omega) \left(\ln \frac{2q^- v^+ \omega}{\mu_R^2} - 1 - i\pi \right), \quad (30c)$$

where we have introduced a dimensionless ratio for convenience:

$$r \equiv m_b^2/m_W^2. \quad (31)$$

The LSZ reduction formula requests us also to consider the wave function correction to the u quark, which are represented by Figs. 3(f) and 2(f). They contribute to $T^{(1)}$ through

$$\Phi_+^{(0)}(\omega) \otimes T_{\delta Z_u}^{(1)}(\omega) = \mathcal{M}_{\delta Z_u}^{(1)} - T^{(0)}(\omega) \otimes \Phi_{+\delta Z_u}^{(1)}(\omega). \quad (32)$$

Obviously, the u quark self-energy diagrams are identical between Figs. 3(e) and 2(e). Substituting $\mathcal{M}_{\delta Z_u}^{(1)} = \frac{1}{2} \delta Z_u(\mu_R) \mathcal{M}_{\delta Z_u}^{(0)}$ and $\Phi_{+\delta Z_u}^{(1)} = \frac{1}{2} \delta Z_u(\mu_F) \Phi_+^{(0)}$ in (32), one finds

$$\begin{aligned} T_{\delta Z_u}^{(1)}(\omega) &= \frac{1}{2} [\delta Z_u(\mu_R) - \delta Z_u(\mu_F)] T^{(0)}(\omega) \\ &= \frac{\alpha_s C_F}{4\pi} \ln \frac{\mu_F}{\mu_R} T^{(0)}(\omega), \quad (33) \end{aligned}$$

which simply vanishes once setting $\mu_R = \mu_F$.

³Note the matching procedure specified in (28) involves the difference between two *renormalized* quantities. Including the quark wave-function and mass renormalization, the one-loop QCD amplitude becomes UV finite and free from μ_R dependence; for the HQET LCDA, one tacitly utilizes the $\overline{\text{MS}}$ subtraction scheme to render it finite. For simplicity, in the following we will drop the UV poles in the hard-scattering kernel associated with each individual Feynman diagram.

In a similar vein, one needs also consider the external $\bar{b}$ leg correction in QCD amplitude and HQET LCDA, as depicted in Figs. 3(f) and 2(f). The on-shell b -quark wave function renormalization constant in QCD reads

$$\delta Z_b(\mu) = \frac{\alpha_s C_F}{4\pi} \left(-\frac{1}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}} - 3 \ln \frac{\mu^2}{m_b^2} - 4 \right). \quad (34)$$

When computing the $\bar{b}$ quark self-energy contribution to $\Phi_+^{(1)} \otimes T^{(0)}$, one encounters a vanishing result in DR. This is because the self-energy diagram of an on-shell b quark in HQET yields a scaleless integral. If we insist on distinguishing the UV and IR poles, the b quark on-shell wave function renormalization constant in HQET then reads

$$\delta \hat{Z}_b = \frac{\alpha_s C_F}{4\pi} \left(\frac{2}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}} \right), \quad (35)$$

which obviously has the same IR pole as Z_b in full QCD.

Discarding the UV poles, the external $\bar{b}$ leg corrections yield the following contribution to $T^{(1)}$:

$$\begin{aligned} T_{\delta Z_b}^{(1)}(\omega) &= \frac{1}{2} [\delta Z_b(\mu_R) - \delta \hat{Z}_b(\mu_F)] T^{(0)}(\omega) \\ &= \frac{\alpha_s C_F}{4\pi} \left(2 \ln \frac{m_b}{\mu_F} + \ln \frac{m_b}{\mu_R} - 2 \right) T^{(0)}(\omega). \end{aligned} \quad (36)$$

In the leading power of $1/m_b$, it turns out that, reassuringly, the box diagram in Fig. 3(c) makes vanishing contribution to the hard-scattering kernel. Because the loop momentum generates the leading contribution to $\mathcal{M}_{\text{box}}^{(1)}$ only in the *soft* region ($l^\mu \sim \Lambda_{\text{QCD}}$), which has already been fully captured by $\Phi_{\text{box}}^{(1)} \otimes T^{(0)}$. Therefore there arises no contribution to $T_{\text{box}}^{(1)}$, which exhibit the same pattern as what is observed in the case of $B \rightarrow \gamma l \nu$ [3].

Piecing all the relevant one-loop contributions in (30), (33), and (36) together, we can deduce the complete hard-scattering kernel at $\mathcal{O}(\alpha_s)$:

$$\begin{aligned} T^{(1)}(\omega, m_b, \mu_F) &= \frac{\alpha_s C_F}{4\pi} \left\{ \ln^2 \frac{2q^- v^+ \omega}{\mu_F^2} - 2 \ln^2 \frac{m_b}{\mu_F} + \left(5 - 4 \ln \frac{1-r}{r} \right) \ln \frac{m_b}{\mu_F} \right. \\ &\quad + 2 \text{Li}_2(r) + \ln^2 r - \left(2 \ln \frac{1-r}{r} - 3 + r \right) \ln \frac{1-r}{r} + \frac{\pi^2}{12} - 7 \\ &\quad - i\pi \left[2 \ln \frac{2q^- v^+ \omega}{\mu_F^2} - 4 \ln \frac{m_b}{\mu_F} - r - 4 \ln(1-r) \right. \\ &\quad \left. \left. + 2 \ln r + 3 \right] \right\} T^{(0)}(\omega). \end{aligned} \quad (37)$$

It should be noticed that the unit mass μ_R has disappeared in the final answer, as the consequence of the conserved (axial)vector current in QCD. Nevertheless, $T^{(1)}$ still explicitly depends on the factorization scale μ_F . One can readily check that, the μ_F dependence of $T^{(1)}$ reads

$$\mu_F \frac{d}{d\mu_F} T^{(1)}(\omega, \mu_F) = -\frac{\alpha_s C_F}{4\pi} \left(4 \ln \frac{\omega}{\mu_F} + 5 \right) T^{(0)}(\omega) + \mathcal{O}(\alpha_s^2). \quad (38)$$

Hearteningly, this specific μ_F dependence is exactly what we need. In junction with the μ_F dependence of $\hat{f}_B(\mu_F)$ specified in (9) and the scale dependence of $\phi_B^+(\omega, \mu_F)$ governed by (13), such a scale dependence of the hard-scattering kernel guarantees that the physical amplitude $\mathcal{M}^{(1)}$ in (18) is independent of the artificial scale μ_F .

We have formally assigned $m_W \sim m_b$ in order to justify the HQET factorization. Nevertheless, we should admit that $m_W \gg m_b$ in the realistic world. The symptom of such practical hierarchy is reflected in the large logarithm of m_W/m_b in the hard-scattering kernel. To see this more transparently, let us expand $T^{(1)}(\omega, m_b, \mu_F)$ in (37) to the zeroth order of r :

$$\begin{aligned} T^{(1)}(\omega, m_b, \mu_F)|_{\text{expd}} &= \frac{\alpha_s C_F}{4\pi} \left[\ln^2 \frac{\omega}{\mu_F} - \ln^2 \frac{m_b}{\mu_F} + \ln \frac{m_b}{\mu_F} \left(5 + 2 \ln \frac{\omega}{\mu_F} \right) \right. \\ &\quad + \ln \frac{m_W^2}{m_b^2} \left(3 + 2 \ln \frac{\omega}{m_b} \right) \\ &\quad \left. + \frac{\pi^2}{12} - 7 - i\pi \left(3 + 2 \ln \frac{\omega}{m_b} \right) \right] T^{(0)}(\omega) + \mathcal{O}(r). \end{aligned} \quad (39)$$

Interestingly, we notice that the occurrence of the single collinear logarithm $\frac{\alpha_s C_F}{4\pi} \ln \frac{m_W}{m_b}$, which might be large and potentially spoil the convergence of perturbative expansion. We admit this is an inevitable drawback of the fixed-order calculation in the HQET factorization. In the future work, we will illustrate how to apply the ERBL equation to effectively resum these types of large logarithms through a refactorization program. The theoretical framework that renders such a resummation program feasible is based upon a recently proposed factorization theorem that links the LCDAs of B meson defined in full QCD and HQET [32].

With the $\mathcal{O}(\alpha_s)$ hard-scattering kernel at hand, we are then able to present the NLO perturbative corrections to the form factors $F_{V/A}$:

$$\begin{aligned}
F_V^{(1)} &= F_A^{(1)} = F_{V/A}^{(0)} \int_0^\infty \frac{d\omega}{\omega} \left[\frac{T^{(1)}(\omega)}{T^{(0)}(\omega)} + \frac{\hat{f}_B}{f_B} \right] \phi_B^+(\omega) \\
&= F_{V/A}^{(0)} \frac{\alpha_s C_F}{4\pi} \left\{ -\ln^2 \frac{m_b}{\mu_F} - \ln \frac{m_b}{\mu_F} \left(2 \ln \frac{1-r}{r} - 2 \right) + 2\text{Li}_2(r) - \ln^2(1-r) \right. \\
&\quad + 2 \ln r \ln(1-r) + (3-r) \ln \frac{1-r}{r} + \frac{\pi^2}{12} - 5 - 2\sigma_{B,1} \left(\ln \frac{1-r}{r} + \ln \frac{m_b}{\mu_F} \right) \\
&\quad \left. - \sigma_{B,2} + i\pi \left[2 \ln \frac{m_b}{\mu_F} - 3 + r + 2 \ln(1-r) + 2\sigma_{B,1} \right] \right\}, \tag{40}
\end{aligned}$$

where the first inverse moment λ_B^{-1} and the n th logarithmic moment $\sigma_n \lambda_B^{-1}$ of the B meson LCDA have been defined in (14) and (15). In the final expression, we have also utilized (9) to trade $\hat{f}_B$ in favor of the QCD decay constant f_B . One can check that, the NLO prediction $F_{V/A}^{(0)} + F_{V/A}^{(1)}$ becomes independent of the factorization scale μ_F , up to the error at $\mathcal{O}(\alpha_s^2)$.

Equation (40) constitutes the most important result of this work. The equality between F_V and F_A at NLO in α_s looks peculiar, which should not be simply viewed as a coincidence. The underlying reason may be likely attributed to the heavy quark spin symmetry, which is an exact symmetry at the lowest order in $1/m_b$.

It is instructive to compare (40), which entails the NLO corrections rigorously derived from HQET factorization, with the corresponding expressions of the form factors for a similar process with B^+ meson replaced with the flavored quarkonium B_c^+ , e.g., $W^+ \rightarrow B_c^+ + \gamma$ [30]. This exclusive quarkonium production process has recently been investigated through $\mathcal{O}(\alpha_s)$ within NRQCD factorization framework [30]. There the NRQCD short-distance coefficient for such a case contains three distinct scales: m_W , m_b , and m_c . Besides the standard light-cone limit $m_W \gg m_b \sim m_c$, the authors of [30] have also investigated the so-called ‘‘heavy quark limit,’’ where the NRQCD short-distance coefficient is expanded according to the scale hierarchy $m_W \sim m_b \gg m_c$. In such a limit, the expanded NRQCD short-distance coefficient reads [30]

$$\begin{aligned}
F_A^{(1)} &= F_A^{(0)} \frac{\alpha_s C_F}{4\pi} \left\{ -\ln^2 x_0 + \ln x_0 \left(2 \ln \frac{1-r}{r} - 5 \right) \right. \\
&\quad + 2\text{Li}_2(r) - \ln^2(1-r) + 2 \ln(1-r) \ln r \\
&\quad + (3-r) \ln \frac{1-r}{r} - \frac{2\pi^2}{3} - 9 \\
&\quad \left. + i\pi [-2 \ln x_0 - 3 + r + 2 \ln(1-r)] \right\}, \tag{41}
\end{aligned}$$

where $x_0 \equiv \frac{m_c}{m_b+m_c} \approx \frac{m_c}{m_b}$. It is amazing that (41) resembles (40) in many aspects, once the substitution $\mu_F = \omega \rightarrow m_c$ is made in (40). A simplification of this scale-fixing is that the logarithmic moments $\sigma_{B,n}$ ($n = 1, 2$) drops out in (40).

It looks peculiar that these two expressions agree on the bulk of (di-)logarithmic functions of r , but only differ in constant.⁴

Similar to (39), we can also expand the form factors $F_{V/A}^{(1)}$ to the zeroth order in r :

$$\begin{aligned}
F_{V/A}^{(1)}|_{\text{expd}} &= F_{V/A}^{(0)} \frac{\alpha_s C_F}{4\pi} \left[\ln \frac{m_b^2}{m_W^2} \left(2 \ln \frac{m_b}{\mu_F} + 2\sigma_{B,1} - 3 \right) \right. \\
&\quad - \ln^2 \frac{m_b}{\mu_F} + 2(1 - \sigma_{B,1}) \ln \frac{m_b}{\mu_F} \\
&\quad - \sigma_{B,2} + \frac{\pi^2}{12} - 5 + i\pi \left(2 \ln \frac{m_b}{\mu_F} + 2\sigma_{B,1} - 3 \right) \Big] \\
&\quad + \mathcal{O}(r). \tag{42}
\end{aligned}$$

Given the huge hierarchy between m_W and m_b , we expect that this approximate expression should be numerically quite close to the exact one in (40).

VI. NUMERICAL RESULTS

In this section, we carry out detailed numerical predictions for vector/axial-vector form factors related to W^+ radiative decay into B^+ and D_s^+ mesons, as well as the corresponding partial widths and branching fractions. The impact of the NLO perturbative corrections is also investigated.

We specify various input parameters as follows [34,35]: We utilize the automated package

$$\begin{aligned}
\sin \theta_W &= 0.481, \quad \alpha(m_W/2) = 1/130, \quad m_W = 80.379 \text{ GeV}, \\
f_B &= 0.187 \text{ GeV}, \quad |V_{ub}| = 3.65 \times 10^{-3}, \quad m_b = 4.6 \text{ GeV}, \\
m_B &= 5.279 \text{ GeV}, \quad f_{D_s} = 0.249 \text{ GeV}, \quad |V_{cs}| = 0.997, \\
m_c &= 1.4 \text{ GeV}, \quad m_{D_s} = 1.968 \text{ GeV}.
\end{aligned}$$

⁴To ‘‘perfectly’’ match (41) with (40), one may attempt to substitute the analytic B^+ meson LCDA defined in HQET [33], where the B^+ meson is modeled as a free nonrelativistic $\bar{b}u$ pair. The μ_F dependence in (40) would cancel explicitly, after adding two convolution integrals $\Phi_{[\bar{b}u]}^{+(0)} \otimes T^{(1)}$ and $\Phi_{[\bar{b}u]}^{+(1)} \otimes T^{(0)}$. After the substitutions $\lambda_B \rightarrow m_c$, $\sigma_{B,1} \rightarrow -\ln m_c/\mu_F$, and $\sigma_{B,1} \rightarrow -\ln^2 m_c/\mu_F$ are made, (40) becomes almost identical to (41).

FIG. 4. Profiles of the LCDAs $\phi_{B/D_s}^+(\omega, \mu_F)$ at some typical values of the factorization scale.

HOPPET [36] to evaluate the QCD running coupling α_s with one-loop accuracy, which has appropriately incorporated the effect associated with crossing the flavor threshold.

For the LCDAs of the B^+ and D_s^+ mesons defined at the initial scale $\mu_{F0} = 1$ GeV, we employ the simple exponential ansatz, first introduced by Grozin and Neubert [19]:

$$\phi_M^+(\omega) = \frac{\omega}{\lambda_M^2} \exp\left(-\frac{\omega}{\lambda_M}\right), \quad (43)$$

with $\lambda_B = 0.360$ GeV [37] and $\lambda_{D_s} = 0.294$ GeV [38].

We would like to fathom out how the predicted decay rates depend on the artificial factorization scale μ_F . To this purpose, we first need know how the LCDAs, and the corresponding (logarithmic) inverse moments, vary with μ_F . The analytic solutions of the Lange-Neubert evolution equation in (13) have been recently available [28,39]. Nevertheless in this work, we are content with numerically solving the Lange-Neubert equation via the fourth order Runge-Kutta method, with the help of the package GNU SCIENTIFIC LIBRARY [40]. The profiles of the LCDAs for B and D_s at several factorization scales are depicted in Fig. 4.

In Fig. 5, we plot the scale dependence of the first inverse moment and logarithmic inverse moments for both B^+ and D_s^+ mesons. We observe that the moments λ_B^{-1} and $\lambda_M^{-1}\sigma_{M,1}$

exhibit a mild μ_F dependence between 1 GeV and twice meson mass, whereas the inverse logarithmic moment $\lambda_M^{-1}\sigma_{M,2}$ develops a relatively stronger μ_F dependence.

We define the LO and NLO predictions to form factors as $F_{V/A}^{\text{LO}} \equiv F_{V/A}^{(0)}$ and $F_{V/A}^{\text{NLO}} \equiv F_{V/A}^{(0)} + F_{V/A}^{(1)}$. At a given μ_F , we evaluate the form factors through $\mathcal{O}(\alpha_s)$ in accordance with (27) and (40). In Fig. 6, we plot the variation of the form factors with μ_F , at both LO and NLO accuracy. Clearly $\mathcal{O}(\alpha_s)$ correction is negative and significant for both $W^+ \rightarrow B^+\gamma$ and $W^+ \rightarrow D_s^+\gamma$, especially with relatively small μ_F . Notice the scale dependence of the form factors becomes significantly reduced after incorporating the $\mathcal{O}(\alpha_s)$ correction, in particular with relatively greater μ_F .

One can analytically prove that $F_{V/A}^{\text{NLO}}$ in (40) is independent of μ_F through $\mathcal{O}(\alpha_s)$. Notwithstanding considerable reduction of the μ_F dependence relative to the LO prediction, $F_{V/A}^{\text{NLO}}$ in Fig. 6 still bear notable factorization scale dependence, particularly in small μ_F . The residual scale dependence is clearly caused by neglected higher-order corrections. With small μ_F , the scale dependence may be amplified either by the large α_s or the large prefactors accompanying $\ln \mu_F$.

One source causing the residual μ_F dependence can be readily identified, that is, the $\ln m_b^2/m_W^2 \ln m_b/\mu_F$ term

FIG. 5. Scale dependence of the first inverse moments λ_M^{-1} and the logarithmic inverse moments $\lambda_M^{-1}\sigma_{M,1/2}$ for $M = B^+, D_s^+$. The factorization scale ranges from 1 GeV to twice meson mass.

FIG. 6. Factorization scale dependence of the vector/axial-vector form factors $F_{V/A}$ at LO and NLO in α_s , for the process $W^+ \rightarrow B^+ \gamma$ and $W^+ \rightarrow D_s^+ \gamma$, respectively. The range of μ_F lies between 1 GeV to twice meson mass.

FIG. 7. Dependence of $\text{Re}[F_{V/A}^{\text{NLO}}]$ on μ_F , chosen with some fictitious W mass. The factorization scale ranges from 1 GeV to twice the meson mass.

in (42). Numerically, the collinear logarithm is sizable, $\ln m_b^2/m_W^2 \approx -5.7$, which is largely responsible for the fall of the NLO prediction of the form factors with decreasing μ_F . In order to quantify our reasoning about the major source governing the residual scale dependence, in Fig. 7 we artificially tune the value of m_W from 80.38 GeV down to 10 GeV for $W^+ \rightarrow B^+ + \gamma$, and from physical mass down to 5 GeV for $W^+ \rightarrow D_s^+ + \gamma$. As can be clearly visualized from Fig. 7, the smaller m_W is chosen, the less scale dependence $F_{V/A}^{\text{NLO}}$ does have in small μ_F .

One might tend to conclude that, in order to reduce μ_F dependence in HQET factorization, it appears desirable to resum the large collinear logarithm $\ln m_W^2/m_b^2$ appearing in the hard-scattering kernel to all orders in α_s . Note LN equation can only serve to sum large logarithm

$\ln m_b/\Lambda_{\text{QCD}}$, which has already been included in our numerical analysis. The occurrence of large logarithm $\ln m_W^2/m_b^2$ is a weakness of the HQET factorization, since two distinct scales m_W and m_b have not been disentangled in this approach. Resumming collinear logarithms of this type can only be accomplished by appealing to the EBL equation in the collinear factorization framework. We recall that, for the analogous hard exclusive heavy quarkonium production processes, exemplified by $\gamma^* \rightarrow \eta_c + \gamma$ and $H \rightarrow J/\psi + \gamma$, the leading collinear logarithm of type $\ln Q/m_c$ has been identified and resummed to all orders in α_s , but the numerical effect turns out to be modest [41]. It is desirable if the similar goal can be achieved for exclusive heavy-light hadron production.

TABLE I. Numerical predictions to the partial widths and branching ratios for the processes $W^+ \rightarrow B^+(D_s^+) \gamma$. The uncertainty is estimated by sliding μ_F from 1 GeV to twice the meson mass.

	Γ^{LO} (GeV)	Γ^{NLO} (GeV)	Br^{NLO}
$W^+ \rightarrow B^+ \gamma$	$(0.75 \sim 1.9) \times 10^{-11}$	$(3.1 \sim 7.7) \times 10^{-12}$	$(1.5 \sim 3.7) \times 10^{-12}$
$W^+ \rightarrow D_s^+ \gamma$	$(0.72 \sim 1.3) \times 10^{-7}$	$(4.9 \sim 8.4) \times 10^{-8}$	$(2.3 \sim 4.0) \times 10^{-8}$

FIG. 8. Branching fractions of $W^+ \rightarrow B^+ \gamma$ and $W^+ \rightarrow D_s^+ \gamma$ as a function of μ_F , which ranges from 1 GeV to twice the meson mass. Our predictions are juxtaposed with the existing ones obtained from the collinear factorization [11], which are represented by the green bands.

Very recently, we have proposed a novel factorization theorem, which attempts to refactorize the B meson LCDA in full QCD into the LCDA defined in HQET, convoluted with a perturbative calculable Z function [32]. The underlying motivation is to separate the short-distance effect of order m_b out of the QCD LCDA, which cannot be a genuinely perturbative object. Starting from the standard collinear factorization approach, armed with this factorization formula, we may readily make an optimized prediction which combines the virtues of both collinear and HQET factorization approaches. This improved approach will greatly facilitate resummation of both types of logarithms, $\ln m_b/\Lambda_{\text{QCD}}$ and $\ln m_W^2/m_b^2$. We hope to present the optimized prediction for this process in future publication.

We employ (2) to compute the partial decay widths. For the NLO prediction, we take the absolute square of the form factors, including their imaginary part in $F_{V/A}^{(1)}$, without truncating the partial width strictly at $\mathcal{O}(\alpha_s)$. We present the LO and NLO predictions for the partial widths and branching ratios of $W^+ \rightarrow B^+ + \gamma$ and $W^+ \rightarrow D_s^+ + \gamma$ in Table I and Fig. 8, respectively. Several orders of magnitude difference between $W^+ \rightarrow B^+ + \gamma$ and $W^+ \rightarrow D_s^+ + \gamma$ is primarily due to $|V_{cs}| \gg |V_{ub}|$.

The NLO corrections turns out to be sizable, shift the LO prediction by $-83\% \sim 2\%$ for $W^+ \rightarrow B^+ \gamma$, and $-62\% \sim 18\%$ for $W^+ \rightarrow D_s^+ \gamma$, respectively. Recall for a similar decay process $W \rightarrow B_c + \gamma$, the $\mathcal{O}(\alpha_s)$ correction has also been found to considerably reduce the LO result [30].

Our state-of-the-art predictions for the branching fraction for $W^+ \rightarrow B^+ \gamma$ lies between $(1.5 \sim 3.7) \times 10^{-12}$. It is difficult to observe such an extremely rare decay channel in the foreseeable future, even after the integrated luminosity of LHC reaches 3000 fb^{-1} . On the other hand, the branching fraction for $W^+ \rightarrow D_s^+ \gamma$ is predicted to be within $(2.3 \sim 4.0) \times 10^{-8}$, which may have bright observation prospect in the future LHC experiments. Lastly, it is also interesting to remark that our NLO predictions in HQET factorization are somewhat greater than what were

obtained from the standard collinear factorization approach for $W^+ \rightarrow B^+ + \gamma$, but compatible with theirs for $W^+ \rightarrow D_s^+ + \gamma$, albeit within large errors [11].

VII. SUMMARY AND OUTLOOK

In this work, inspired by the NRQCD factorization for hard exclusive heavy quarkonium production, we have formulated the HQET factorization approach, tailored for describing the hard exclusive production of heavy-flavor meson. This approach in spirit is quite different from the standard collinear factorization for hard exclusive production. We have taken the $W^+ \rightarrow B^+(D_s^+) + \gamma$ as the prototype processes to illustrate our theoretical framework, especially including the complete NLO perturbative correction. By examining that the NLO hard-scattering kernel is IR finite, we have explicitly verified that the HQET factorization formula in (18) indeed holds at first nontrivial order in α_s yet at lowest order in $1/m_b$. It is conceivable that this factorization formula may hold to all orders in α_s .

Interestingly, both vector/axial-vector form factors $F_{V/A}$ remain identical through NLO in α_s , which may be attributed to the heavy-quark spin symmetry. The NLO perturbative corrections turn to be substantial and negative. Our predictions for $W^+ \rightarrow B^+(D_s^+) + \gamma$ are compared with the existing ones using the collinear factorization approach. While the $W^+ \rightarrow B^+ + \gamma$ process is perhaps too suppressed to experimentally tag, the process $W^+ \rightarrow D_s^+ + \gamma$ may have a positive chance to be observed in future LHC experiment.

A nuisance of the fixed-order calculation in HQET factorization approach is that the hard-scattering kernel is inevitably plagued with large collinear logarithms of m_W/m_b , which may potentially ruin the convergence of perturbative expansion. This reflects two hard scales m_W and m_b are not yet disentangled in this approach. Recently a new factorization theorem that links the B meson LCDAs defined between QCD and HQET has been established [41]. This factorization formula opens the gate for

effectively merging both HQET factorization and collinear factorization to make the optimized predictions. Within this new scheme, both large logarithms of type $\ln m_b/\Lambda_{\text{QCD}}$ and $\ln m_W/m_b$ can be efficiently resummed with the aid of LN equation and ERBL equation. We hope to illustrate this improved theoretical framework in a future publication.

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