Erratum: Spontaneous growth of spinor fields in gravity **[Phys. Rev. D 98, 044011 (2018)]**

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The equation of motion for spontaneous spinorization is missing a term in the published version of Ref. [1]. Equation (27) should read

$$\begin{aligned}
 &(\zeta_\psi \hat{\gamma}^5 + \mathbb{I})\gamma^\mu \nabla_\mu \psi - [m - (\nabla_\mu \zeta_\psi) \hat{\gamma}^5 \gamma^\mu / 2]\psi = 0 \\
 &\Leftrightarrow \gamma^\mu \nabla_\mu \psi - \frac{\mathbb{I} - \zeta_\psi \hat{\gamma}^5}{1 - \zeta_\psi^2} [m - (\nabla_\mu \zeta_\psi) \hat{\gamma}^5 \gamma^\mu / 2]\psi = 0;
 \end{aligned}$$

i.e., $(\nabla_\mu \zeta_\psi) \hat{\gamma}^5 \gamma^\mu / 2$ was missing. This does not affect the conclusions of the paper, but it leads to other corrections we list below.

The first main correction is the discussion that leads to and follows Eq. (32). This equation is correct but has an alternative role when the missing terms are considered, as follows.

Equation (27) looks similar to Eq. (4) of a ghost-based (rather than tachyon-based) spontaneous scalarization at first sight, but there are differences in the behavior of the two differential equations. First, even though we grouped the $\nabla_\mu \zeta_\psi$ term with the mass in Eq. (27), it contains derivatives of ψ . ζ_ψ itself contains first derivatives of ψ through $\mathcal{L}_\psi^{5,K}$, so naively, $\nabla_\mu \zeta_\psi$ is a second order term in the partial differential equation for ψ . The spinor equation of motion (EOM) is first order for minimal coupling; hence, the nonminimal coupling seems to make a drastic change to the dynamics of spinors, beyond inciting an instability. One can see that this is not the case by using the fact that when ψ obeys Eq. (27),

$$\mathcal{L}_\psi^{5,K} = -m \frac{\zeta_\psi}{1 - \zeta_\psi^2} \bar{\psi} \psi,$$

which leads to

$$A_\psi^4 = \frac{\zeta_\psi}{4\pi\beta_\psi \tilde{T}} = \exp\left(-2m\beta_\psi \bar{\psi} \psi \frac{\zeta_\psi}{1 - \zeta_\psi^2}\right).$$

This implicitly defines ζ_ψ as a function of $\bar{\psi} \psi$ (and $\tilde{T}$), but not its derivatives. Hence, $\nabla_\mu \zeta_\psi$ preserves the first order nature of the spinor EOM.

Despite the complexity of Eq. (27), we can investigate the behavior of small perturbations around $\psi = 0$ (general relativity) relatively easily by keeping the leading terms in Eq. (27). Note that ζ_ψ has a quadratic dependence on ψ through $\bar{\psi} \psi$. One can also in general find regions where the matter changes slowly enough that $\nabla_\mu \tilde{T}$ also has a subleading effect. Overall, for a linearized analysis, we can ignore $\nabla_\mu \zeta_\psi$ and use $\zeta_0 \equiv \zeta_\psi(\bar{\psi} \psi = 0, \tilde{T})$ in the EOM. Keeping the leading terms in a nearly flat background and defining $\bar{m} = m/(1 - \zeta_0^2)$,

$$\begin{aligned}
 &(\gamma^\nu \nabla_\nu - \bar{m})\psi \approx -\bar{m} \zeta_0 \gamma^5 \psi \\
 \Rightarrow &(\gamma^\nu \nabla_\nu + \bar{m})(\gamma^\mu \nabla_\mu - \bar{m})\psi \approx -(\gamma^\nu \nabla_\nu + \bar{m})\bar{m} \zeta_0 \gamma^5 \psi \\
 &\approx \bar{m} \zeta_0 \gamma^5 (\gamma^\nu \nabla_\nu - \bar{m})\psi \\
 &= -\bar{m}^2 \zeta_0^2 \psi.
 \end{aligned}$$

We can see the instability using an analysis similar to that of Eq. (14), i.e., using $\psi = u(\vec{k})e^{-ik_\mu x^\mu}$,

$$(-\eta^{\mu\nu}k_\mu k_\nu - \bar{m}^2)\psi = -\bar{m}^2\zeta_0^2\psi$$

$$\Rightarrow E^2 = \vec{k} \cdot \vec{k} - \frac{m^2}{\zeta_0^2 - 1}.$$

The mass-square term has the tachyonic sign for $|\zeta_0| > 1$.

The second main correction is the effect of the changes in the equation of motion to the specific case of spherically symmetry solutions discussed in Sec. IV, Eq. (39), and its repetition in Eq. (43) are corrected as

$$\sqrt{1-\bar{\mu}}\Phi^{\pm'} = \left[\Xi_\Psi \pm \frac{1}{r} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{m}{1-\zeta_\Psi^2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right] \Phi^\pm$$

$$\pm \left[\frac{m\zeta_\Psi}{1-\zeta_\Psi^2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{\Xi_\Psi}{\zeta_\Psi} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right] \Phi^\mp$$

and

$$\Xi_\Psi \equiv \sqrt{1-\bar{\mu}} \frac{\zeta_\Psi \partial_r \zeta_\Psi}{2(1-\zeta_\Psi^2)}.$$

This is an implicit equation since $\partial_r \zeta_\Psi$ also contains $\Phi^{\pm'}$. These corrections do not affect any of the stress-energy terms T^Ψ or $\mathcal{T}^{\Psi,5}$ in Eqs. (41) and (42).

A series expansion of the spinor components also contain an unrelated minor mistake, and Eq. (44) should be corrected as

$$\Phi_1^+ = C^+ r + \mathcal{O}(r^3)$$

$$\Phi_2^+ = -\frac{m(C^+ + \zeta_{\Psi,0}C^-)}{3(1-\zeta_{\Psi,0}^2)} r^2 + \mathcal{O}(r^3)$$

$$\Phi_1^- = -\frac{m(C^- + \zeta_{\Psi,0}C^+)}{3(1-\zeta_{\Psi,0}^2)} r^2 + \mathcal{O}(r^3)$$

$$\Phi_2^- = C^- r + \mathcal{O}(r^3)$$

Ξ_Ψ does not appear in this equation since it vanishes at $r = 0$.

[1] F. M. Ramazanoğlu, Spontaneous growth of spinor fields in gravity, [Phys. Rev. D **98**, 044011 \(2018\)](#).