Polarizing fragmentation function and the Λ polarization in e^+e^- processes

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The surprising polarization of Λ 's and other hyperons measured in many unpolarized hadronic processes, $pN \rightarrow \Lambda^\dagger X$, has been a long-standing challenge for QCD phenomenology. One possible explanation was suggested, related to nonperturbative properties of the quark hadronization process, and encoded in the so-called polarizing fragmentation function (PFF). Recent Belle data have shown a nonzero Λ polarization also in unpolarized e^+e^- processes, $e^+e^- \rightarrow \Lambda X$ and $e^+e^- \rightarrow \Lambda h X$. We consider the single inclusive case and the role of the PFFs. Adopting a simplified kinematics, we show how they can originate a polarization $P_\Lambda \neq 0$ and give explicit expressions for it in terms of the PFFs. Although the Belle data do not allow yet, in our approach, an extraction of the PFFs, some clear predictions are given, suggesting crucial measurements, and estimates of P_Λ are computed, in qualitative agreement with the Belle data.

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I. INTRODUCTION

The polarization of Λ hyperons inclusively produced in the high energy interactions of unpolarized hadrons, $pN \rightarrow \Lambda^\dagger X$ [1,2], has been a major challenge for QCD theoretical interpretations for many years. A large amount of data is available [3], due to the fact that the weak decay of the Λ allows an easy measurement of its polarization P_Λ . No definite explanation of the origin of P_Λ , in a QCD framework, is convincingly available. In the usual application of perturbative QCD and collinear factorization, the elementary interactions among unpolarized partons cannot produce any significant final state quark polarization [4].

Nonperturbative QCD features have been invoked. In Refs. [5] and [6], transverse momentum dependent (TMD) effects in the fragmentation process were introduced, adopting a TMD factorization scheme, respectively, in proton-proton (pp) and lepton-proton (ℓp) interactions. In Ref. [7], collinear higher-twist quark-gluon-antiquark correlations in the nucleon were considered for ℓp and pp processes, while in Refs. [8] and [9], a complete twist-3 collinear fragmentation contribution to polarized hyperon production in unpolarized hadronic and e^+e^- collisions was presented.

The TMD effects in the quark fragmentation of Refs. [5,6] were encoded in the so-called polarizing fragmentation function (PFF), introduced and defined in Ref. [10].

Very recently, new data on the polarization of Λ hyperons produced in unpolarized e^+e^- annihilation processes, $e^+e^- \rightarrow \Lambda h X$ and $e^+e^- \rightarrow (\text{jet})\Lambda X$, were published by the Belle Collaboration [11], showing a nonzero value of P_Λ . Prompted by these data and the simplicity of the process which only depends on fragmentation functions, we consider in this paper the role of the transverse momentum dependent PFF in generating the Λ polarization. A general theoretical discussion of two-hadron inclusive production in e^+e^- interactions can be found in Ref. [12].

Our aim is that of understanding the physical mechanism through which the particular correlation between the momentum of the fragmenting quark, the momentum of the final Λ , and its polarization—described by the PFF—can build up the Λ polarization. To do so we will adopt a very simple kinematical configuration which allows analytical computations and a direct visualization of the process. It allows us to understand specific features of P_Λ , which are true beyond the approximate kinematics and provide genuine testable predictions. Although the actual data of the Belle Collaboration, as it will be explained in Sec. III, do not allow a precise direct comparison with our computation of P_Λ , our estimates result in good qualitative agreement with the data.

II. FORMALISM AND SIMPLE ANALYTICAL RESULTS FOR P_Λ

We consider the process $e^+e^- \rightarrow \Lambda(P)X$ in the c.m. reference frame defined in Fig. 1, with

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FIG. 2. Simple two-dimensional kinematical configuration. The $\hat{y}$ -axis is entering the xz plane.

As we said, the only polarization allowed by parity invariance is orthogonal to the xz production plane, that is, $\mathbf{P} = \hat{y}$. Notice that, in Eq. (7), according to our simple kinematical configuration of Fig. 2, we have

$$\hat{\mathbf{p}}_{q_1} \times \hat{\mathbf{p}}_{\perp}^{(1)} = +\hat{y} \quad \hat{\mathbf{p}}_{q_2} \times \hat{\mathbf{p}}_{\perp}^{(2)} = -\hat{y}. \quad (14)$$

The convolution in Eq. (7) then reads

$$\begin{aligned} & \frac{d\sigma^{\uparrow}}{dzd(\cos\theta)} - \frac{d\sigma^{\downarrow}}{dzd(\cos\theta)} \\ &= \sum_q e_q^2 \int p_{\perp} dp_{\perp} \left[\frac{d\hat{\sigma}^q}{d(\cos\theta_{q_1})} - \frac{d\hat{\sigma}^q}{d(\cos\theta_{q_2})} \right] \\ & \quad \times \Delta^N D_{\Lambda^{\uparrow/q}}(z, p_{\perp}), \end{aligned} \quad (15)$$

where the elementary $e^+e^- \rightarrow q\bar{q}$ cross section is given by

$$\frac{d\hat{\sigma}^q}{d\cos(\theta_q)} = \frac{3\pi\alpha^2}{2s} (1 + \cos^2\theta_q) \quad (16)$$

and the expressions of $\cos\theta_{q_{1,2}}$ are given in Eq. (12).

By using Eqs. (16) and (12) in Eq. (15), one has a very simple result:

$$\begin{aligned} & \frac{d\sigma^{\uparrow}}{dzd(\cos\theta)} - \frac{d\sigma^{\downarrow}}{dzd(\cos\theta)} \\ &= \frac{3\pi\alpha^2}{2s} \sum_q e_q^2 \int p_{\perp} dp_{\perp} (\cos^2\theta_{q_1} - \cos^2\theta_{q_2}) \\ & \quad \times \Delta^N D_{\Lambda^{\uparrow/q}}(z, p_{\perp}) \end{aligned} \quad (17)$$

$$\begin{aligned} &= \frac{3\pi\alpha^2}{2s} \sum_q e_q^2 \int p_{\perp} dp_{\perp} \frac{4zp_{\perp}\sqrt{s}}{z^2s + 4p_{\perp}^2} \sin(2\theta) \\ & \quad \times \Delta^N D_{\Lambda^{\uparrow/q}}(z, p_{\perp}). \end{aligned} \quad (18)$$

Some more comments on the physical interpretation of this expression and the approximations involved will be made in the next section.

We now have to compute the unpolarized cross section, appearing in the denominator of Eq. (8). As, differently from the polarized case, all components of $\mathbf{p}_q$ now contribute equally to the Λ production, and not only those in the xz plane, we write the convolution (6) as

$$\frac{d\sigma}{dzd(\cos\theta)} = \sum_q e_q^2 \int 2\pi p_{\perp} dp_{\perp} \frac{d\hat{\sigma}^q}{d(\cos\theta)} D_{\Lambda/q}(z, p_{\perp}) \quad (19)$$

$$= \sum_q e_q^2 \frac{d\hat{\sigma}^q}{d(\cos\theta)} D_{\Lambda/q}(z), \quad (20)$$

where, essentially, we have assumed $d\hat{\sigma}^q/d(\cos\theta_q) \simeq d\hat{\sigma}^q/d(\cos\theta)$ and used the relation $\int d^2\mathbf{p}_{\perp} D_{\Lambda/q}(z, p_{\perp}) = D_{\Lambda/q}(z)$. Equation (20) is the usual expression for the cross section in the collinear partonic configuration.

By collecting Eqs. (8) and (16)–(20) we have simple expressions for the Λ polarization along the $\hat{y}$ direction:

$$\begin{aligned} P_{\Lambda}(z, \cos\theta) &= \frac{\sum_q e_q^2 \int p_{\perp} dp_{\perp} \frac{4zp_{\perp}\sqrt{s}}{z^2s + 4p_{\perp}^2} \Delta^N D_{\Lambda^{\uparrow/q}}(z, p_{\perp})}{\sum_q e_q^2 D_{\Lambda/q}(z)} \\ & \quad \times \frac{\sin(2\theta)}{1 + \cos^2\theta} \end{aligned} \quad (21)$$

$$\begin{aligned} P_{\Lambda}(z, p_{\perp}) &= \frac{\sum_q e_q^2 \frac{4zp_{\perp}\sqrt{s}}{z^2s + 4p_{\perp}^2} \Delta^N D_{\Lambda^{\uparrow/q}}(z, p_{\perp})}{\sum_q e_q^2 2\pi D_{\Lambda/q}(z, p_{\perp})} \\ & \quad \times \frac{\int d(\cos\theta) \sin(2\theta)}{\int d(\cos\theta) (1 + \cos^2\theta)} \end{aligned} \quad (22)$$

$$P_{\Lambda}(z, p_{\perp}, \cos\theta) = \frac{\sum_q e_q^2 \frac{4zp_{\perp}\sqrt{s}}{z^2s + 4p_{\perp}^2} \Delta^N D_{\Lambda^{\uparrow/q}}(z, p_{\perp})}{\sum_q e_q^2 2\pi D_{\Lambda/q}(z, p_{\perp})} \frac{\sin(2\theta)}{1 + \cos^2\theta}. \quad (23)$$

In the single inclusive Λ production process that we are considering, the only observables are z and $\cos\theta$, while the values of $p_{\perp}$ are integrated. We have also given Eqs. (22) and (23) as they might allow some comparison with the data of Ref. [11].

III. COMMENTS, SUGGESTED MEASUREMENTS, AND SOME PREDICTIONS

Before trying to give some estimates of the polarization, a few comments are in order, which illustrate the meaning and validity of the results obtained in the previous section:

- (i) Our simple two-dimensional kinematical configuration shows clearly the mechanism which, thanks to the $\mathbf{P} \cdot \mathbf{p}_q \times \mathbf{p}_{\perp}$ correlation of the PFF, builds up the

Λ polarization. As illustrated in Fig. 2, at any fixed values of z and $p_\perp = |\mathbf{p}_\perp|$ there are two possible vectors $\mathbf{p}_q$ leading to the same $\mathbf{p}_\Lambda$. These two vectors lead to opposite vectors $\mathbf{p}_q \times \mathbf{p}_\perp = \mathbf{p}_q \times \mathbf{p}_\Lambda$ and then to opposite values of the polarization along the $\mathbf{P} = \hat{\mathbf{y}}$ direction. However, in the convolution of Eqs. (7) and (15), to each of them there corresponds a different value of the scattering angle θ and then of the elementary cross section (16). This leads to a clear physical interpretation of Eqs. (17) and (18).

- (ii) Notice also that the annihilation cross section is symmetric around $\theta = \pi/2$, where $\cos^2 \theta_{q_1} = \cos^2 \theta_{q_2}$, Eq. (12); thus, the polarization vanishes at $\theta = \pi/2$, reflecting the nature of the $e^+e^- \rightarrow q\bar{q}$ partonic interaction at leading order. In addition, as we already remarked, the polarization must vanish at $\theta = 0$ and $\theta = \pi$ for parity invariance: then one understands the $\sin(2\theta)$ behavior of P_Λ .
- (iii) The true kinematical implementation of our mechanism should take into account a three-dimensional configuration of $\mathbf{p}_q$ around $\mathbf{p}_\Lambda$. Then, always at fixed values of z and $p_\perp$, we would have contributions from other pairs of vectors $\mathbf{p}_{q_1}$ and $\mathbf{p}_{q_2}$ out of the xz plane. Such vectors contribute (oppositely) to P_Λ only with their components in the xz plane, which is smaller than $p_\perp$. In addition, for them the difference between θ_{q_1} and θ_{q_2} is also smaller. We estimate that their total contribution to P_Λ would not change the value obtained in our limited two-dimensional kinematical model by more than a factor 2. Our numerical computations will actually underestimate the value of the polarization.
- (iv) The conclusions of the first two paragraphs of this section are valid also in full three-dimensional kinematics; the first one simply illustrates the PFF mechanism, while the second one is based on the symmetry properties, at leading order, of the elementary cross section, proportional to $(1 + \cos^2 \theta)$. Then, our result

$$P_\Lambda \sim \sin(2\theta) \quad (24)$$

is a genuine prediction of the way in which a PFF would build up the Λ polarization, a prediction which could and should be easily tested experimentally.

- (v) We have given explicit expressions of $P_\Lambda(z, p_\perp, \cos \theta)$, and some of their integrated forms, in terms of the unknown polarizing fragmentation function $\Delta^N D_{\Lambda^\dagger/q}(z, p_\perp)$. The θ dependence, as we stressed in the previous paragraph, is instead well defined. Notice that we also find

$$P_\Lambda = \mathcal{O}(p_\perp/\sqrt{s}), \quad (25)$$

meaning that the polarization in the single inclusive process $e^+e^- \rightarrow \Lambda X$ is a higher twist effect. It agrees

with the observation of a vanishing Λ transverse polarization in e^+e^- at the Z^0 pole by the OPAL Collaboration [14].

In the single inclusive process that we are considering, one only observes the final Λ and its polarization, given by Eq. (21). Such a process allows us to exploit the fact that one knows the direction of $\mathbf{P}_\Lambda$, perpendicular to the production plane. In addition, we have assumed that also the elementary dynamics predominantly takes place in the same plane. While the $\cos \theta$ dependence of the polarization is definitely fixed by the PFF mechanism, independently of the simplified kinematics, its magnitude, $P_\Lambda(z, \cos \theta)$, depends on our assumption and on the functional form of the PFF $\Delta^N D_{\Lambda^\dagger/q}(z, p_\perp)$. If abundant and precise data were available for P_Λ in the $e^+e^- \rightarrow \Lambda X$ process, one could try to parametrize the PFF and, performing a best fit of the experimental points, extract information on the PFF.

Unfortunately, the available experimental information is not exactly what we would like to have in order to perform such a procedure. In Ref. [11] the inclusive Λ polarization is measured along the direction $\hat{\mathbf{T}} \times \hat{\mathbf{p}}_\Lambda$, where $\hat{\mathbf{T}}$ is the thrust axis, which, apart from unavoidable experimental inaccuracies, coincides with $\hat{\mathbf{p}}_q$. What actually the Belle Collaboration measures is the polarization in the process $e^+e^- \rightarrow \text{jet} \Lambda X$, which, in general, need not be perpendicular to the Λ production plane. Their polarization is presented as a function of $p_\perp$ and z . Presumably, data have been collected over a wide range of the angle θ , although this information cannot be found in the paper.

However, we can attempt a very qualitative comparison with the Belle data, by exploiting Eq. (23), which also gives, with some approximations, the polarization along $\mathbf{p}_q \times \mathbf{p}_\Lambda$ as a function of $p_\perp$, z and $\cos \theta$. Similarly, if we knew the range of θ covered by the experiment, we could use Eq. (22).

In order to obtain simple numerical estimates of P_Λ , we assume the PFFs, $\Delta^N D_{\Lambda^\dagger/q}(z, p_\perp)$, to be 20% of the corresponding unpolarized TMD-FFs, $D_{\Lambda/q}(z, p_\perp)$. In Ref. [6] the PFFs have been parametrized in terms of the unpolarized FFs and have been used to compute the Λ polarization in $pp \rightarrow \Lambda X$ processes. From the best fit parameters it is found that the overall normalization factor for the u and d quark PFFs has to be negative, whereas for s quarks it has to be positive. Motivated by this, we choose

$$\begin{aligned} \Delta^N D_{\Lambda^\dagger/u}(z, p_\perp) &= -0.2 \times D_{\Lambda/u}(z, p_\perp) \\ \Delta^N D_{\Lambda^\dagger/d}(z, p_\perp) &= -0.2 \times D_{\Lambda/d}(z, p_\perp) \\ \Delta^N D_{\Lambda^\dagger/s}(z, p_\perp) &= 0.2 \times D_{\Lambda/s}(z, p_\perp). \end{aligned} \quad (26)$$

As usual, for the unpolarized TMD-FFs, we consider factorized z and $p_\perp$ dependences, with a Gaussian parametrization for the transverse momentum dependent part:

$$D_{\Lambda/q}(z, p_{\perp}) = D_{\Lambda/q}(z) \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-\frac{p_{\perp}^2}{\langle p_{\perp}^2 \rangle}}, \quad (27)$$

where $q = u, d, s$ and $D_{\Lambda/q}(z)$ is the collinear FF. The width of the Gaussian, $\langle p_{\perp}^2 \rangle$, is a free parameter which could be extracted by fitting data from experiments.

In order to further simplify our order of magnitude estimates, we consider the LO collinear FF from Ref. [15], where it has been assumed, following simple $SU_f(3)$ symmetry arguments, that all the light flavor quarks fragment to Λ with equal probability, i.e.,

$$D_{\Lambda/u} = D_{\Lambda/d} = D_{\Lambda/s} \equiv D_{\Lambda/q}, \quad (28)$$

while the antiquark fragmentation into a Λ is taken to be negligible, $D_{\Lambda/\bar{q}} = 0$.

In the plots given in Fig. 3 we show P_{Λ} as a function of $p_{\perp}$, computed according to Eq. (23), for different bins of z , similar to those of the Belle data [11], and fixing $\theta = \pi/3$. For each plot we have integrated the numerator and denominator of Eq. (23) over z according to the corresponding bin range. The LO collinear FFs $D_{\Lambda/q}(z)$ for light quarks have been taken from Ref. [15]. Notice that, in this case, as a consequence of Eqs. (27) and (26), the flavor independent Gaussian widths $\langle p_{\perp}^2 \rangle$ do not affect the value of $P_{\Lambda}(p_{\perp}, \theta)$.

As we commented before, the actual comparison between our computation of P_{Λ} and the Belle data can only be considered at a qualitative level. Our definition of z , Eq. (2), differs from the value of $z_{\Lambda} = 2E_{\Lambda}/\sqrt{s}$, used in

FIG. 3. Plots of $P_{\Lambda}(p_{\perp}, \theta = \pi/3)$ vs $p_{\perp}$, computed according to Eq. (23). For each bin of z values the numerator and denominator of Eq. (23) have been accordingly integrated. The PFFs are given in Eq. (26) and the FFs $D_{\Lambda/q}(z)$ are from Ref. [15].

FIG. 4. Plots of $P_{\Lambda}(z, \theta = \pi/3)$ vs z , computed according to Eq. (21), for different values of $\langle p_{\perp}^2 \rangle$. We have integrated the numerator and denominator of Eq. (21) in the range $0 \leq p_{\perp} \leq 3$ GeV.

Ref. [11], by terms of the order $p_{\perp}^2/(sz^2)$. Moreover, our estimates of P_{Λ} undervalue it. The signs of the PFFs have been assumed to be the same as those obtained in fitting the Λ polarization in $pp \rightarrow \Lambda X$ processes [6]. Despite all this, when comparing with Fig. 1 of Ref. [11], the qualitative agreement is remarkable, with negative values of $P_{\Lambda}(p_{\perp})$ of the order of a few percents or less. Notice, however, that the sign of P_{Λ} depends on the value of the production angle θ and changes at $\theta = \pi/2$.

In Fig. 4 we plot P_{Λ} as a function of z according to Eq. (21), for different values of $\langle p_{\perp}^2 \rangle$, fixing $\theta = \pi/3$ and integrating the numerator and denominator over $p_{\perp}$ between 0 and 3 GeV. Increasing the upper integration limit has negligible effects. Notice that, in this case, again as a consequence of Eqs. (27) and (26), the collinear FFs $D_{\Lambda/q}(z)$ cancel out in the ratio giving $P_{\Lambda}(z, \theta)$.

Again, with our choice of the PFFs, Eq. (26), we obtain, at the chosen θ angle, negative values of $P_{\Lambda}(z)$, of the order of 1%, in qualitative agreement with the Belle data [11].

IV. CONCLUSIONS

Motivated by new data on the polarization of Λ hyperons, measured by the Belle Collaboration in unpolarized e^+e^- annihilation processes [11], we have investigated the role of the polarizing fragmentation functions [5,6,10] in generating the polarization. These new data are particularly interesting as they refer to a process in which P_{Λ} can only depend on the properties of the quark fragmentation, without the complication of convolutions with partonic distributions, such as in the case of $pN \rightarrow \Lambda^{\uparrow} X$ and $\ell N \rightarrow \Lambda^{\uparrow} X$ processes.

We have adopted a simplified two-dimensional kinematics, which might lead to an underestimate of P_Λ , but has the merit of visualizing the partonic process and the role of the PFFs. In addition, in our LO TMD factorization scheme, we obtain a general clear prediction for the θ dependence of P_Λ , Eq. (24), which could be easily tested.

A precise comparison of the results of our approach with the few Belle data is not possible yet, because of the nature of the Belle data, which do not refer to a fully inclusive process, and to the uncertainty on the Λ production angle θ , not discussed in Ref. [11]. However, by making simple assumptions on the PFFs and the collinear FF, and following indications on the sign of the PFFs obtained

by fitting the Λ polarization in $pp \rightarrow \Lambda^\dagger X$ processes [5], we have given some estimates of the expected values of $P_\Lambda(p_\perp)$ and $P_\Lambda(z)$ in the same $p_\perp$ and z kinematical regions covered by the Belle measurements, fixing a particular value of θ . Such estimates show a negative and small Λ polarization, in agreement with the Belle data.

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