Heavy quark spin symmetric molecular states from $\bar{D}^{(*)}\Sigma_c^{(*)}$ and other coupled channels in the light of the recent LHCb pentaquarks

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We consider the $\bar{D}^{(*)}\Sigma_c^{(*)}$ states, together with $J/\psi N$ and other coupled channels, and take an interaction consistent with heavy quark spin symmetry, with the dynamical input obtained from an extension of the local hidden gauge approach. By fitting only one parameter to the recent three pentaquark states reported by the LHCb Collaboration, we can reproduce the three of them in base to the mass and the width, providing for them the quantum numbers and approximate molecular structure as $1/2^ \bar{D}\Sigma_c$, $1/2^ \bar{D}^*\Sigma_c$, and $3/2^ \bar{D}^*\Sigma_c$, and the isospin I=1/2. We find another state around 4374 MeV, of the $3/2^ \bar{D}\Sigma_c^*$ structure, for which indications appear in the experimental spectrum. Two other near degenerate states of a $1/2^ \bar{D}^*\Sigma_c^*$ and $3/2^ \bar{D}^*\Sigma_c^*$ nature are also found around 4520 MeV, which although less clear, are not incompatible with the observed spectrum. In addition, a $5/2^ \bar{D}^*\Sigma_c^*$ state at the same energy appears, which however does not couple to $J/\psi p$ in an S wave, and hence, it is not expected to show up in the LHCb experiment.

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The discovery of some pentaquarks signals by the LHCb Collaboration in 2015 [1,2] generated a wave of enthusiasm in the hadron physics community. Two states were reported, one at 4380 MeV and width $\Gamma \sim 205$ MeV and another one at 4450 MeV and width 40 MeV. Actually, there had been several predictions for hidden charm molecular states in this region prior to the experimental discovery [3–10]. The hidden charm molecular states would have some resemblance with the $N^*(1535)$ resonance, which in the chiral unitary approach has large $K\Lambda$, $K\Sigma$ components [11–15]. Large $s\bar{s}$ components in that resonance have also been claimed in [16] from the study of the $pp \to pp\phi$ and $\pi^-p \to n\phi$ reactions.

A wave of theoretical papers with very different approaches, stimulated by the LHCb findings, were produced trying to match the masses and spin parity quantum numbers suggested in the experimental work, $(3/2^-, 5/2^+)$, $(3/2^+, 5/2^-)$, $(5/2^+, 3/2^-)$ for the two states, and other less likely combinations. We refer to review papers for references to all these works [17–25].

With the advent of run-2 data, the LHCb Collaboration updated the results of [1,2] reporting the observation of three clear narrow structures [26], branded as

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$$\begin{split} M_{P_{c1}} &= (4311.9 \pm 0.7^{+6.8}_{-0.6}) \text{ MeV}, \\ \Gamma_{P_{c1}} &= (9.8 \pm 2.7^{+3.7}_{-4.5}) \text{ MeV}, \\ M_{P_{c2}} &= (4440.3 \pm 1.3^{+4.1}_{-4.7}) \text{ MeV}, \\ \Gamma_{P_{c2}} &= (20.6 \pm 4.9^{+8.7}_{-10.1}) \text{ MeV}, \\ M_{P_{c3}} &= (4457.3 \pm 0.6^{+4.1}_{-1.7}) \text{ MeV}, \\ \Gamma_{P_{c3}} &= (6.4 \pm 2.0^{+5.7}_{-1.9}) \text{ MeV}. \end{split}$$

As one can see, the old peak at 4450 MeV is now split into two states at 4440 MeV and 4457 MeV, the last one very narrow, and a fluctuation observed in the old spectrum has given rise to a neat peak around 4312 MeV.

The new experimental findings have already had a reply from the theoretical community. In [27], sum rules are used that provide several scenarios to explain these states, the most favored ones being of $\Sigma_c^{(*)}D^{(*)}$ molecular nature. In [28], heavy quark spin symmetry (HQSS) is used with $\Sigma_c \bar{D}$, $\Sigma_c \bar{D}^*$, $\Sigma_c^* \bar{D}$, $\Sigma_c^* \bar{D}^*$ as single channels, and seven bound states are found, three of which can be associated with the experimental states. One should mention that in that line there is previous work, including other coupled channels, and which also predicts seven states with an isospin I = 1/2, and the widths of the states [8].

Another work [29] considers again the $\Sigma_c^{(*)}D^{(*)}$ coupled channels and, using meson exchange for the dynamics, generates three states that are associated to the new experimental resonances. There is also an interesting

suggestion to look into the isospin suppressed $\Lambda_b \rightarrow J/\psi \Delta K^-$ reaction, showing that the ratio of rates for $J/\psi \Delta$ to $J/\psi p$ production is largely enhanced due to the molecular nature of the states [30].

The blind predictions for the molecular hidden charm states have necessarily uncertainties, which are tied to the cutoff or subtraction constants needed to regularize the loops involved in the calculations. The differences in the results found among different approaches are mostly due to this point (see Refs. [3] and [7], for instance). In this sense, differences of masses between the $3/2^-$ and $1/2^-$ states are more reliable. Thus, in [3], one finds that this difference is 149 MeV, and in [7], it is 141 MeV. Actually, these numbers are very close to the differences between the masses of the $P_c(4457)$ and $P_c(4312)$, which is 145 MeV. In [8], this difference is 155 MeV.

In the works of Refs. [3,7], $\bar{D}\Sigma_c$ and $\bar{D}^*\Sigma_c$, among other coupled channels, were used, but not $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c^*$. HQSS [31–33] relates the strength of the interaction of these channels, and they were considered in [8]. The advent of the LHCb data offers an opportunity to tune the regulator of

the loops to adjust to some experimental data. This is the purpose of the present work. It is similar to the study of Ref. [28], but includes more channels than the $\Sigma_c^{(*)}D^{(*)}$ used in [28], and in addition, we work with coupled channels rather than using single channels, which allows us to obtain also the widths.

In [8], the Bethe-Salpeter equation (BSE) is used with the coupled channels in $I=1/2,\,\eta_c N,\,J/\psi N,\,\bar{D}\Lambda_c,\,\bar{D}\Sigma_c,\,\bar{D}^*\Lambda_c,\,\bar{D}^*\Sigma_c,\,\bar{D}^*\Sigma_c^*$ for spin parity $J^P=1/2^-$ and $J/\psi N,\,\bar{D}^*\Lambda_c,\,\bar{D}^*\Sigma_c,\,\bar{D}\Sigma_c^*,\,\bar{D}\Sigma_c^*$ for $J^P=3/2^-$. In addition, a single channel for $\bar{D}^*\Sigma_c^*$ in the $J^P=5/2^-$ sector is also studied. The BSE in matrix form for the scattering matrix reads

$$T = [1 - VG]^{-1}V, (2)$$

where G is the loop function of the meson-baryon intermediate states and the potential V, respecting leading order (LO) HQSS constraints, is given in Eqs. (3)–(5) (taken from Ref. [8]).

(i)
$$J = 1/2$$
, $I = 1/2$

(ii)
$$J = 3/2$$
, $I = 1/2$

$$J/\psi N \quad \bar{D}^* \Lambda_c \quad \bar{D}^* \Sigma_c \qquad \bar{D} \Sigma_c^* \qquad \bar{D}^* \Sigma_c^*$$

$$\begin{pmatrix} \mu_1 & \mu_{12} & \frac{\mu_{13}}{3} & -\frac{\mu_{13}}{\sqrt{3}} & \frac{\sqrt{5}\mu_{13}}{3} \\ \mu_{12} & \mu_2 & \frac{\mu_{23}}{3} & -\frac{\mu_{23}}{\sqrt{3}} & \frac{\sqrt{5}\mu_{23}}{3} \\ \frac{\mu_{13}}{3} & \frac{\mu_{23}}{3} & \frac{1}{9}(8\lambda_2 + \mu_3) & \frac{\lambda_2 - \mu_3}{3\sqrt{3}} & \frac{1}{9}\sqrt{5}(\mu_3 - \lambda_2) \\ -\frac{\mu_{13}}{\sqrt{3}} & -\frac{\mu_{23}}{\sqrt{3}} & \frac{\lambda_2 - \mu_3}{3\sqrt{3}} & \frac{1}{3}(2\lambda_2 + \mu_3) & \frac{1}{3}\sqrt{\frac{5}{3}}(\lambda_2 - \mu_3) \\ \frac{\sqrt{5}\mu_{13}}{3} & \frac{\sqrt{5}\mu_{23}}{3} & \frac{1}{9}\sqrt{5}(\mu_3 - \lambda_2) & \frac{1}{3}\sqrt{\frac{5}{3}}(\lambda_2 - \mu_3) & \frac{1}{9}(4\lambda_2 + 5\mu_3) \end{pmatrix}_{I=1/2}$$

(iii)
$$J = 5/2$$
, $I = 1/2$
$$\bar{D}^* \Sigma_c^* \colon (\lambda_2)_{I=1/2}$$
 (5)

LO HQSS interactions for I = 3/2 can be also found in Ref. [8].

Note, that the single channel interactions used in [28] are recovered from Eqs. (3)–(5), identifying the terms C_a and C_b introduced in that reference to $(2\lambda_2/3 + \mu_3/3)_{I=1/2}$ and $(\lambda_2/3 - \mu_3/3)_{I=1/2}$, respectively.

There are seven parameters relying upon HQSS only, but when one imposes a particular dynamics, restrictions among them appear, as shown in [34]. In the present work, we shall consider the same constraints as in [8], which stem from the use of an extension of the local hidden gauge approach, where the source of interaction is the exchange of vector mesons [35–37]. Detailed discussions justifying this extension to the charm, or bottom sector, are given in [38,39]. These constraints are for I = 1/2,

$$\mu_{1} = 0, \quad \mu_{23} = 0, \quad \lambda_{2} = \mu_{3}, \quad \mu_{13} = -\mu_{12},$$

$$\mu_{2} = \frac{1}{4f^{2}}(k^{0} + k'^{0}), \quad \mu_{3} = -\frac{1}{4f^{2}}(k^{0} + k'^{0}),$$

$$\mu_{12} = -\sqrt{6}\frac{m_{\rho}^{2}}{p_{D^{*}}^{2} - m_{D^{*}}^{2}} \frac{1}{4f^{2}}(k^{0} + k'^{0}), \tag{6}$$

with $f_{\pi}=93$ MeV, and k^0 , k'^0 the center of mass energies of the mesons in the $MB\to M'B'$ transition. In addition, $p_{D^*}^2$ applies to the *t*-channel exchanged D^* in the tree level of some suppressed transitions ($\eta_c N \to \bar{D} \Lambda_c$, for instance). We should note that there are corrections to the strict heavy quark limit responsible for the different mass of the D and D^* , but these breaking terms are expected to be smaller for the irreducible interaction terms used as the kernel of the BSE. This is similar to the case of LO chiral Lagrangians, which are SU(3) invariant, though the mass of the pion and

kaon are quite different, as is also the case for the baryons within the same multiplet. The idea is that a significant part of the breaking terms give rise to the different masses of the particles, but to calculate their interaction, these LO Lagrangians are rather good, once the values of the physical masses are used in the calculations. This is because analytical properties related to thresholds and unitarity require the use of exact masses. A clear example of this is the study of the two pole pattern exhibited by the $\Lambda(1405)$ using unitarized LO chiral amplitudes and physical hadron masses [40,41].

This is similar to the case of chiral Lagrangians, which are SU(3) symmetric for the meson interaction but however lead to SU(3) breaking in the mass of the ground stable particles of the SU(3) multiplets [42].

The novelty with respect to Ref. [8] is a different choice of the subtraction constant to renormalize the mesonbaryon loops (G) in dimensional regularization, since the position of the poles is tied to its value. A subtraction constant $a(\mu) = -2.3$ with $\mu = 1$ GeV was used in [8]. This value was justified since it falls in the range of "natural values" discussed in [43] and was also used in [3]. The scheme produces seven states, three of which can be clearly associated to the recently found experimental resonances. The new information allows us to take a new value of $a(\mu = 1 \text{ GeV}) = -2.09$, such that the sum of the masses of the three theoretical states matches the experimental results. With this constraint, we fix the only free parameter of the model of Ref. [8]. The energies of the states are found by looking at the poles of the scattering matrix, Eq. (2), in the second Riemann sheet of the complex energy plane. The results are reported in Tables I and II for $J^P = 1/2^-$ and 3/2-, respectively. In addition, we get a mass of 4519.23 MeV and a zero width for the single channel $\bar{D}^*\Sigma_c^*$ with $J=5/2^-$. This channel obviously does not couple to $J/\psi N$ so we should not see it in the $\Lambda_b \rightarrow$ $J/\psi pK^-$ experiment. The states in Tables I and II all couple to $J/\psi N$, and in principle, they could be seen in the

TABLE I. Dimensionless coupling constants of the $(I = 1/2, J^P = 1/2^-)$ poles found in this work to the different channels. The imaginary part of the energies corresponds to $\Gamma/2$.

(4306.38 + i7.62) MeV									
	$\eta_c N$	$J/\psi N$	$ar{D}\Lambda_c$	$ar{D}\Sigma_c$	$ar{D}^*\Lambda_c$	$ar{D}^*\Sigma_c$	$ar{D}^*\Sigma_c^*$		
g_i	0.67 + i0.01	0.46 - i0.03	0.01 - i0.01	2.07 - i0.28	0.03 + i0.25	0.06 - i0.31	0.04 - i0.15		
$ g_i $	0.67	0.46	0.01	2.09	0.25	0.31	0.16		
(4452.96 + i11.72) MeV									
	$\eta_c N$	$J/\psi N$	$ar{D}\Lambda_c$	$ar{D}\Sigma_c$	$ar{D}^*\Lambda_c$	$ar{D}^*\Sigma_c$	$ar{D}^*\Sigma_c^*$		
g_i	0.24 + i0.03	0.88 - 0.11	0.09 - i0.06	0.12 - i0.02	0.11 - i0.09	1.97 - i0.52	0.02 + i0.19		
$ g_i $	0.25	0.89	0.11	0.13	0.14	2.03	0.19		
(4520.45 + i11.12) MeV									
	$\eta_c N$	$J/\psi N$	$ar{D}\Lambda_c$	$ar{D}\Sigma_c$	$ar{D}^*\Lambda_c$	$ar{D}^*\Sigma_c$	$ar{D}^*\Sigma_c^*$		
g_i	0.72 - i0.10	0.45 - i0.04	0.11 - i0.06	0.06 - i0.02	0.06 - i0.05	0.07 - i0.02	1.84 - i0.56		
$ g_i $	0.73	0.45	0.13	0.06	0.08	0.08	1.92		

TABLE II. Same as Table I for $J^P = 3/2^-$.

(4374.33 + i6.87) MeV	$J/\psi N$	$ar{D}^*\Lambda_c$	$\bar{D}^*\Sigma_c$	$ar{D}\Sigma_c^*$	$\bar{D}^*\Sigma_c^*$
$g_i \ g_i $	0.73 - i0.06 0.73	0.11 - i0.13 0.18	0.02 - i0.19 0.19	1.91 – i0.31 1.94	$0.03 - i0.30 \\ 0.30$
(4452.48 + i1.49) MeV $g_i g_i $	$J/\psi N$ $0.30 - i0.01$ 0.30	$ar{D}^* \Lambda_c \\ 0.05 - i 0.04 \\ 0.07$	$ar{D}^*\Sigma_c$ 1.82 - i0.08 1.82	$ar{D}\Sigma_c^* \ 0.08 - i0.02 \ 0.08$	
(4519.01 + i6.86) MeV $g_i g_i $	$J/\psi N$ $0.66 - i0.01$ 0.66	$\bar{D}^*\Lambda_c$ $0.11 - i0.07$ 0.13	$\bar{D}^*\Sigma_c$ $0.10 - i0.3$ 0.10	$\bar{D}\Sigma_c^*$ $0.13 - i0.02$ 0.13	$ar{D}^*\Sigma_c^*$ 1.79 – i0.36 1.82

experiment, although we cannot predict their strength in the spectrum. In Table III, we show the results for the three resonances that we identify with the experimental states. The main channel is taken from the largest coupling. We find the last two states nearly degenerate, yet, the widths of the states force us to identify the $3/2^-$ state with the $P_c(4457)$. Note that the masses divert only in a few MeV from the experimental ones, and the three widths obtained are compatible with the experiment. The results of Table III are similar to those of [28], where the input has been adjusted to reproduce the $P_c(4440)$ and $P_c(4457)$ states. There is only a small difference since in [28] the J^P assignments to the $P_c(4440)$ and $P_c(4457)$ are opposite to ours in their option A, but the same in their option B. Our approach, providing the width, gives us one additional reason to support our assignment. As to the molecular nature of the states, the single channel calculation of [28] gives the same state as those written in Table III as our main channel. Option B of [28] is further studied in [44] where the widths obtained are consistent with experiments.

We should note that the reason why $\mu_{23} = 0$ in Eq. (6) is the neglect of pion exchange which was found small, although not negligible in [8]. Its consideration would break the near degeneracy that we have in the two higher states of Table III, as was found in [9], where, however, the effect of pion exchange was found more important as a consequence of the choice of large cutoffs that made the binding much larger.

It looks strange that the widths obtained here are smaller than those reported in [8] in spite that the masses of the states are bigger, and hence, there is more phase space for decay. The answer has to be found in the fact that the

TABLE III. Identification of some of the I = 1/2 resonances found in this work with experimental states.

Mass [MeV]	Width [MeV]	Main channel	J^P	Experimental state
4306.4	15.2	$ar{D}\Sigma_c$	1/2-	$P_c(4312)$
4453.0	23.4	$ar{D}^*\Sigma_c$	1/2-	$P_{c}(4440)$
4452.5	3.0	$ar{D}^*\Sigma_c^c$	3/2-	$P_c(4457)$

couplings have also become smaller. This is not an accident but the consequence of one important property. Indeed, it is well known that in the case of a one channel bound state, the coupling square, g^2 , goes as the square root of the binding energy as a consequence of the most celebrated Weinberg's compositeness condition [45,46]. It is, however, less known that in the case of coupled channels, all couplings go to zero close to the threshold of one channel [47,48]. In the present case, the $P_c(4312)$ is close to the $\Sigma_c \bar{D}$ threshold, and the $P_c(4440)$ and $P_c(4457)$ are very close to the $\Sigma_c \bar{D}^*$ threshold. We have made a study of the uncertainties in our approach to account for a small breaking of the HQSS and the lack of pion exchange. We have taken the μ_2 , μ_3 , μ_{12} terms in Eq. (6) and have allowed them to vary randomly with $\pm 15\%$. By means of this, we find an uncertainty in the mass of the states of about 3 MeV and about 15% uncertainty in the width. The couplings of the main channels are more stable and modified at the level of 5%. These variations are further reduced if after any random choice of $\mu_{2,3,12}$ we tune the subtraction constant to have the same average mass than the experiment for the three corresponding states.

The pole positions have been obtained without considering the width of the $\Sigma_c^*(\Gamma \simeq 15 \text{ MeV})$. We have redone the calculations considering it by convoluting the $\bar{D}^{(*)}\Sigma_c^* - G$ functions with the spectral function of the Σ_c^* , as done in [49]. There are only minor changes in the last state of Tables I and II; the change in the mass leads to a reduction of about 1 MeV, the couplings to $\bar{D}^*\Sigma_c^*$ are affected at the level of 3%, and the widths are reduced by about 20%, mostly due to the reduction of the mass.

The association that we have done of the states found in this work with the experimental ones agrees with the one proposed in [29] where, however, the widths are not evaluated. One should also note that in [28] and here we find seven states, while only three states are reported in [29]. Actually, it is worth noting that in [28] a $3/2^ \bar{D}\Sigma_c^*$ state is reported at 4371 MeV, while we find a state in Table II, coupling mostly to $\bar{D}\Sigma_c^*$, at 4374 MeV with a width of about 14 MeV. It is interesting to call attention to the fact that the $J/\psi p$ spectrum of Ref. [26] shows high bins around 4370 MeV, but with the present statistics, one

cannot make any assertion about this corresponding to a new state. We find two more states that can decay to $J/\psi p$ in Tables I and II, a state of $1/2^-$ at 4520 MeV and a $3/2^-$ state at 4519 MeV, which couple mostly to $\bar{D}^*\Sigma_c^*$. The single channel results reported in [28] also find these two states at 4523 MeV and 4517 MeV, respectively, in their option A. With the risk of stretching too much the imagination, there is indeed a peak in the $J/\psi p$ spectrum of [26] in that region that, however, could as well be a statistical fluctuation. Note that we also obtain a near degenerate state with this nature for $5/2^-$. This state appears at 4500 MeV in option A and at 4523 in option B of [28].

In summary, the molecular picture in the coupled channels to $J/\psi p$ in the S wave, using the constraints of HQSS and dynamics from the extension of the local hidden gauge approach, basically an extension of the chiral unitary approach to the charm sector, renders six states that couple to $J/\psi p$. We have estimated, by means of a Monte-Carlo simulation, the uncertainties due to the breaking of the HQSS and found changes in the masses of the states of about 3 MeV and about 15% in the widths. The couplings of the main channels are more stable and modified only at the level of 5%. Three of these resonances can be identified with the three states reported in [26] in base to their masses

and widths. In addition, we provide a prediction of their J^P quantum numbers and of the nature of these states as basically $1/2^- \bar{D}\Sigma_c$, $1/2^- \bar{D}^*\Sigma_c$, and $3/2^- \bar{D}^*\Sigma_c$. We find a fourth state, which couples mostly to $\bar{D}\Sigma_c^*$ with $3/2^-$, in a region where there is a small enhancement in the $J/\psi p$ spectrum of [26]. The other two states, of $\bar{D}^*\Sigma_c^*$ nature, are around 4520 MeV (close to the threshold of this mesonbaryon pair), and although there are small peaks in that region in [26], one can only speculate at the present stage. They are also near degenerate with a $5/2^-$ state of the same nature, which however is not expected to show up in the LHCb experiment. This degeneracy is obvious from the diagonal $\bar{D}^*\Sigma_c^*$ interactions given in Eqs. (3)–(5), taking into account that the hidden gauge model used here leads to $\lambda_2 = \mu_3$ for I = 1/2.

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