Diffractive excitation in pp and pA collisions at high energies

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(Received 6 May 2019; published 18 July 2019)

In this paper, we consider the Good-Walker approach for the diffractive excitation and present an update of the Miettinen-Pumplin (MP) model for $pp/\bar{p}p$ collisions considering the recent LHC data for the total and elastic pp cross sections. The behavior of the total, elastic, and diffractive cross sections is analyzed, and predictions for the energies of Run 3 of the LHC and those of the cosmic ray experiments are derived. Our results demonstrate that the MP model is able to describe the current data, and that it implies that the cross section for the diffraction excitation in pp collisions is almost constant in the energy range probed by the LHC and slowly decreases at higher energies. Our results also indicate that the Pumplin bound is not reached at the LHC and cosmic ray energies. Moreover, the implications of the diffractive excitation in pA collisions are discussed. In particular, the MP model, constrained by the pp data, is used to derive the main quantities present in the treatment of the diffractive excitation in pA collisions. Predictions for the total, elastic, and diffractive pA cross sections are presented considering different nuclei. We demonstrate that the effect of fluctuations decreases at larger energies and heavier nuclei. The energy dependence of the diffractive excitation cross section in pA collisions is estimated for different nuclei and compared with the predictions for the proton dissociation induced by photon interactions.

DOI: [10.1103/PhysRevD.100.014019](https://doi.org/10.1103/PhysRevD.100.014019)

I. INTRODUCTION

The recent LHC measurements for the total, elastic, and differential cross sections at high energies of 2.76, 7, 8, and 13 TeV [1–14] have motivated an intense debate about the treatment of the hadronic interactions at high energies (for a recent review see, e.g., Ref. [15]). As hadrons are composite objects, it is natural to expect that the description of these interactions could be performed in terms of elementary interactions between their parton constituents. During the last few decades, several models have been proposed, considering different assumptions and ingredients to treat the hadronic structure, the interactions between its constituents, and the unitarization effects (see, e.g., Refs. [16–35]). Some of them have been recently updated using the LHC data, and some new approaches have been proposed (see, e.g., Refs. [36–47]). Although the total and elastic cross sections are dominated by non-perturbative contributions, the energy rising is expected to

be driven by hard partonic subcollisions. Another important consequence of internal degrees of freedom (d.o.f.) is the diffractive excitation of the colliding particles, in which one (or both) particle(s) is (are) excited to a higher mass state with the same quantum numbers. In the case of single diffraction, one of the incident hadrons remains intact. As demonstrated many years ago by Good and Walker [16], the diffractive excitation arises from the fluctuating structure of the hadron, with its description being determined by the eigenstates of the scattering operator, which are the basis to express the physical states. Therefore, the diffractive excitation processes can be considered a direct probe of the proton wave function. Two other motivations to study the diffractive excitation processes are associated with the fact that its contribution to the hadronic cross sections is non-negligible and that its energy behavior is expected to be strongly sensitive to the approach of the black disk limit, which reduces the fluctuations in the eigenstates (see, e.g., Ref. [15]).

Our goal in this paper is to present predictions for the diffractive excitation in pp and pA collisions for the LHC and ultrahigh-energy cosmic ray ranges. Motivated by the recent LHC data for the total and elastic proton-proton cross sections at $\sqrt{s} = 8$ and 13 TeV, we update previous studies [36,37] of the diffractive excitation pp cross section (σ_{diff}) using the Miettinen-Pumplin (MP) model [24], which assumes that the scattering eigenstates correspond to parton showers, which interact via parton-parton scattering.

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The LHC data for σ_{tot} and σ_{el} are used to constrain the energy dependence of the main parameters of the MP model, and a parametrization for this dependence is derived. Parameter-free predictions for σ_{diff} are calculated, and the results are compared with the current experimental data. We demonstrate that the MP model is able to describe the data and that the contribution of the diffractive excitation slowly decreases at higher energies. Moreover, the dependence on the impact parameter (b) of the elastic and diffractive cross sections for different energies is estimated. In agreement with previous studies [24,37], our results demonstrate that the diffractive excitation is dominantly peripheral, with the maximum of the cross section occurring at larger impact parameters when the energy is increased. Finally, using our results for σ_{tot} , σ_{el} , and σ_{diff} , we compare our predictions for the ratio $R(s) \equiv (\sigma_{\text{el}} + \sigma_{\text{diff}})/\sigma_{\text{tot}}$ with the Pomplun bound [48], which establishes that this ratio should be smaller than $1/2$. We demonstrate that $R \approx 0.35$ at LHC energies, with the increase with the energy being very slow.

For the case of pA collisions, Gribov [49] has demonstrated that the diffractive excitation gives a significant contribution for the treatment of the multiple scatterings proposed by Glauber [50]. Different approaches to include these corrections in the Glauber formalism have been formulated during the last few decades. In this paper, we will consider the model proposed in Refs. [32,34] and discussed in detail in a series of papers (see, e.g., Refs. [38,51]). Such a model is based on the Good-Walker formalism and takes into account the cross-section fluctuations. One of the main inputs in this model is the broadness of the cross-section fluctuation around the average value, denoted as ω_σ in Refs. [32,34] and below, which can be determined by the forward diffractive and elastic pp cross sections. From our analysis of the pp scattering using the Miettinen-Pumplin model, such a quantity can be directly calculated, and its energy dependence is unambiguously determined. As a consequence, parameter-free predictions for the total, elastic, and diffractive excitation cross sections can be obtained. We will present our predictions for the energy dependence of these cross sections considering proton-lead ($p\text{Pb}$) and proton-nitrogen ($p\text{N}$) collisions. The impact of the fluctuations in the total and elastic cross sections is estimated, and a comparison with previous results is presented. Finally, in the case of the diffractive excitation, we also compare our results with the predictions for the proton dissociation by photon-induced interactions.

The paper is organized as follows: In Sec. II, we present a brief review of the Miettinen-Pumplin model and determine its main parameters using the recent LHC data. A parametrization for the energy dependence of these parameters is proposed, and the predictions for the diffractive excitation, compared with the experimental data, are presented. In Sec. III, we discuss the inclusion of the fluctuations in the pA cross section, as proposed in

Refs. [32,34], and the parameter ω_σ , in which its energy dependence is determined using the MP model. Predictions for the pA cross sections are presented, and the impact of the fluctuations is estimated. Finally, in Sec. IV, our main conclusions are summarized.

II. DIFFRACTIVE EXCITATION IN pp COLLISIONS

The model for diffractive excitation introduced by Miettinen and Pumplin (MP) [24] is based on the Good-Walker approach [16], which consists of expressing the physical state of the incoming (beam) hadron $|H\rangle$ as a superposition of eigenstates $\{|\psi_k\rangle\}$ of the scattering operator $\hat{T}$, which form a complete set of normalized states, such that $|H\rangle = \sum_k C_k |\psi_k\rangle$ and $\text{Im}\hat{T}|\psi_k\rangle = t_k |\psi_k\rangle$ with $0 \leq t_k \leq 1$ due to the unitarity. These eigenstates can only go through elastic scattering, and moreover, they interact with different intensities (eigenvalues) with the target. It is precisely these differences in the intensities that originate the diffractive excitation present in the final state of composed particles scattering. The MP model considers the eigenstates to be related to the partons inside the hadrons, and their d.o.f. are described in terms of their impact parameter $\mathbf{b}_i$ inside the hadron and their rapidities y_i , i.e., $|\psi_k\rangle \equiv |\mathbf{b}_1, \dots, \mathbf{b}_N; y_1, \dots, y_N\rangle$. Each eigenstate may contain N partons, and the state of the incoming hadron $|H\rangle$ is written as

$$|H\rangle = \sum_N \prod_{i=1}^N \int d^2\mathbf{b}_i dy_i C_N(\mathbf{b}_1, \dots, \mathbf{b}_N; y_1, \dots, y_N) \times |\mathbf{b}_1, \dots, \mathbf{b}_N; y_1, \dots, y_N\rangle. \quad (1)$$

One of the basic assumptions in the MP model is that the partons in the hadrons are uncorrelated, so that the probability to find such a state of N partons in the wave function is given by a Poisson distribution,

$$|C_N(\mathbf{b}_1, \dots, \mathbf{b}_N; y_1, \dots, y_N)|^2 = e^{-G^2} \left(\frac{G^2}{N!} \right) \prod_{i=1}^N |C_i(\mathbf{b}_i, y_i)|^2, \quad (2)$$

where G^2 is the average number of partons in the eigenstate. Moreover, the probability to find a parton is assumed to follow

$$|C_i(\mathbf{b}_i, y_i)|^2 = \frac{1}{2\pi\beta\lambda} \exp\left(-\frac{|y_i|}{\lambda} - \frac{|\mathbf{b}_i|^2}{\beta}\right), \quad (3)$$

with the widths in the rapidity and impact parameters, λ and β , respectively, to be determined. Assuming that the partons interact independently with the target, the total interaction can be written as

$$t(\mathbf{b}_1, \dots, \mathbf{b}_N; y_1, \dots, y_N; \mathbf{b}) = 1 - \prod_{i=1}^N (1 - \tau(\mathbf{b}_i - \mathbf{b}, y_i)), \quad (4)$$

where $\mathbf{b}$ is the impact parameter of the hadron-hadron collision and τ is the interaction probability for a single parton, considered to be given by

$$\tau(\mathbf{b}, y) = A \exp\left(-\frac{|y|}{\alpha} - \frac{|\mathbf{b}|^2}{\gamma}\right), \quad (5)$$

with, again, α , λ , and A parameters to be determined. In the Good-Walker approach, the cross sections in the $\mathbf{b}$ space can be expressed in terms of the mean value and dispersion of the eigenvalues t . With Eqs. (3)–(5), we obtain

$$\frac{d\sigma_{\text{tot}}}{d^2\mathbf{b}} = 2\langle t \rangle = 2 \left[1 - \exp\left(-\frac{G^2 A}{\beta \xi} \left(\frac{\alpha/\lambda}{1 + \alpha/\lambda}\right) e^{-b^2/(\gamma+\beta)}\right) \right], \quad (6)$$

$$\frac{d\sigma_{\text{el}}}{d^2\mathbf{b}} = \langle t^2 \rangle = \left[1 - \exp\left(-\frac{G^2 A}{\beta \xi} \left(\frac{\alpha/\lambda}{1 + \alpha/\lambda}\right) e^{-b^2/(\gamma+\beta)}\right) \right]^2, \quad (7)$$

$$\begin{aligned} \frac{d\sigma_{\text{diff}}}{d^2\mathbf{b}} &= \langle t^2 \rangle - \langle t \rangle^2 \\ &= \exp\left(-2\frac{G^2 A}{\beta \xi} \left(\frac{\alpha/\lambda}{1 + \alpha/\lambda}\right) e^{-b^2/(\gamma+\beta)}\right) \\ &\quad \times \left[\exp\left(\frac{G^2 A^2}{\beta \xi} \left(\frac{\alpha/\lambda}{2 + \alpha/\lambda}\right) e^{-2b^2/(\gamma+2\beta)}\right) - 1 \right], \end{aligned} \quad (8)$$

where the averages are taken over the probability distribution of the beam partons, $\xi = \beta^{-1} + \gamma^{-1}$ and $\zeta = \beta^{-1} + (\gamma/2)^{-1}$. By integrating these equations over $\mathbf{b}$, we are able to calculate the energy dependence of $\sigma_{\text{tot}}(s)$, $\sigma_{\text{el}}(s)$ and $\sigma_{\text{diff}}(s)$.

The MP model implies that the cross sections are completely specified once we know the parameters λ , α , A , G^2 , and β . As in the original paper [24], we will assume that $\alpha/\lambda = 2$ and $\gamma/\beta = 2$. Moreover, we will consider

TABLE I. Experimental data on σ_{tot} and σ_{el} from pp scattering considered in this analysis and the values of the parameters G^2 and β of the Miettinen-Pumplin model determined with Eqs. (6) and (7). References of experimental data are also shown.

Energy (GeV)	Ref.	σ_{tot} (mb)	σ_{el} (mb)	G^2	β (fm ²)
6.77	[53]	39.7 ± 1.5	8.3 ± 1.2	3.08 ± 0.80	0.208 ± 0.042
6.84	[54]	38.9 ± 0.1	8.3 ± 0.2	3.19 ± 0.14	0.198 ± 0.0063
6.91	[55]	39.3 ± 0.8	8.8 ± 0.3	3.48 ± 0.26	0.188 ± 0.012
9.78	[56]	38.14 ± 0.19	7.61 ± 0.29	2.85 ± 0.18	0.212 ± 0.010
11.54	[56]	38.24 ± 0.19	7.41 ± 0.31	2.71 ± 0.19	0.220 ± 0.012
13.76	[56]	38.39 ± 0.19	7.07 ± 0.35	2.50 ± 0.20	0.235 ± 0.014
16.66	[57]	38.47 ± 0.58	6.85 ± 0.24	2.37 ± 0.14	0.246 ± 0.013
16.83	[58]	38.62 ± 0.07	6.97 ± 0.11	2.423 ± 0.060	0.2427 ± 0.0047
18.17	[56]	38.76 ± 0.19	7.06 ± 0.28	2.46 ± 0.15	0.241 ± 0.012
19.66	[59]	39.00 ± 1.00	6.92 ± 0.44	2.36 ± 0.25	0.250 ± 0.024
23.5	[60]	39.65 ± 0.23	6.81 ± 0.33	2.25 ± 0.17	0.264 ± 0.016
23.5	[61]	39.13 ± 0.40	6.82 ± 0.08	2.297 ± 0.054	0.2564 ± 0.0066
23.5	[62]	38.90 ± 0.70	6.70 ± 0.30	2.26 ± 0.16	0.259 ± 0.017
23.76	[63]	40.68 ± 0.55	7.89 ± 0.52	2.72 ± 0.30	0.234 ± 0.020
23.88	[64]	39.00 ± 1.00	7.20 ± 0.40	2.51 ± 0.24	0.238 ± 0.021
30.6	[60]	40.11 ± 0.19	6.75 ± 0.32	2.18 ± 0.15	0.274 ± 0.016
30.6	[61]	39.91 ± 0.41	7.39 ± 0.08	2.522 ± 0.059	0.2431 ± 0.0064
30.6	[62]	40.20 ± 0.80	6.90 ± 0.40	2.24 ± 0.21	0.268 ± 0.022
44.9	[61]	41.89 ± 0.41	7.45 ± 0.08	2.368 ± 0.053	0.2679 ± 0.0067
52.8	[60]	42.38 ± 0.17	7.17 ± 0.30	2.20 ± 0.14	0.288 ± 0.015
52.8	[61]	42.85 ± 0.42	7.56 ± 0.08	2.340 ± 0.051	0.2767 ± 0.0069
62.3	[60]	43.55 ± 0.32	7.51 ± 0.36	2.26 ± 0.16	0.289 ± 0.017
62.5	[61]	44.00 ± 0.45	7.77 ± 0.10	2.343 ± 0.059	0.2838 ± 0.0077
2760	[8,10]	84.7 ± 3.3	21.8 ± 1.4	4.62 ± 0.77	0.335 ± 0.044
7000	[2]	98.0 ± 2.5	25.1 ± 1.1	4.57 ± 0.51	0.390 ± 0.035
8000	[4]	101.7 ± 2.9	27.1 ± 1.4	4.99 ± 0.69	0.382 ± 0.040
13000	[8]	110.6 ± 3.4	31.0 ± 1.7	5.66 ± 0.92	0.385 ± 0.045

TABLE II. Experimental data on σ_{tot} and σ_{el} from $\bar{p}p$ scattering considered in this analysis and the values of the parameters G^2 and β of the Miettinen-Pumplin model determined with Eqs. (6) and (7). References of experimental data are also shown.

Energy (GeV)	Ref.	σ_{tot} (mb)	σ_{el} (mb)	G^2	β (fm ²)
9.78	[56]	43.86 ± 0.22	8.2 ± 0.4	2.56 ± 0.20	0.264 ± 0.016
11.54	[56]	43.00 ± 0.22	7.30 ± 0.47	2.21 ± 0.21	0.291 ± 0.022
13.76	[56]	42.04 ± 0.21	7.8 ± 0.6	2.53 ± 0.31	0.255 ± 0.024
16.26	[56]	41.80 ± 0.21	7.52 ± 0.6	2.41 ± 0.30	0.264 ± 0.026
18.17	[56]	41.60 ± 0.21	7.12 ± 0.52	2.23 ± 0.25	0.279 ± 0.024
30.4	[60]	42.13 ± 0.58	7.16 ± 0.44	2.21 ± 0.21	0.284 ± 0.023
52.6	[60]	43.32 ± 0.34	7.44 ± 0.44	2.25 ± 0.20	0.289 ± 0.021
62.3	[60]	44.12 ± 0.40	7.46 ± 0.44	2.19 ± 0.19	0.300 ± 0.022
546	[65,66]	61.26 ± 0.93	12.87 ± 0.30	3.11 ± 0.15	0.319 ± 0.015
547	[67]	61.90 ± 1.6	13.30 ± 0.61	3.23 ± 0.31	0.313 ± 0.027
1800	[68]	71.71 ± 2.02	15.79 ± 0.87	3.38 ± 0.38	0.351 ± 0.034
1800	[65,66]	80.03 ± 2.24	19.70 ± 0.85	4.20 ± 0.45	0.337 ± 0.030
1800	[69]	72.10 ± 3.3	16.6 ± 1.6	3.67 ± 0.75	0.333 ± 0.056

$A = 1$, in order to account for the maximal probability allowed. Therefore, we are left with only two free parameters to be determined: G^2 and β . Our strategy, similar to that used in Refs. [24,37], is to determine these parameters from the experimental values of σ_{tot} and σ_{el} by means of the integrals of Eqs. (6) and (7). Specifically, we considerably expand the analysis presented in Ref. [37] by including data from both pp and $\bar{p}p$ scattering in the largest energy range considered so far. In particular, we will include the most recent experimental information obtained in the LHC (up to 13 TeV), and consider data at low energies ($\gtrsim 10$ GeV). Since G^2 and β are determined simultaneously by σ_{tot} and σ_{el} , we selected for each energy those data that have been obtained by the same experimental group, in order to minimize the effects of systematics of different experiments. For this, we used the dataset provided by the Particle Data Group (PDG) [52] and Refs. [2,4,8,10]. All selected data for pp [2,4,8,10,53–64] and $\bar{p}p$ [56,60,65–69] scattering are displayed in Tables I and II, respectively, together

with our results for G^2 and β . The errors in the experimental data correspond to statistical and systematic uncertainties added in quadrature, and the errors in the parameters of the model were obtained from error propagation from the data uncertainties. Once we have determined G^2 and β from the experimental information, we can find analytical expressions that describe their energy dependence by fitting the G^2 and β values. Specifically, we note that G^2 presents an energy dependence very close to that shown by σ_{tot} data for pp and $\bar{p}p$ scattering. For this reason, we consider an analytical expression inspired by Regge phenomenology:

$$G^2(s) = a \left(\frac{s}{s_0} \right)^{-b} + c \left(\frac{s}{s_0} \right)^d, \quad (9)$$

where a , b , c , and d are free parameters and $s_0 = 1 \text{ GeV}^2$ is fixed. On the other hand, β has a linear dependence on the $\ln s$ variable. Therefore, we consider

FIG. 1. Total (left), elastic (central), and single diffractive dissociation (right) cross sections as a function of energy, calculated from Eqs. (6)–(11), and comparison with the available experimental data.

TABLE III. Predictions for σ_{tot} , σ_{el} , and σ_{diff} at energies of the LHC and of cosmic ray experiments obtained from Miettinen-Pumpkin model with the energy dependence of parameters G^2 and β given by Eqs. (9)–(11).

Energy	8 TeV	14 TeV	57 TeV	95 TeV
σ_{tot} (mb)	103.1	114.1	144.8	157.1
σ_{el} (mb)	27.5	31.8	44.5	49.8
σ_{diff} (mb)	9.7	9.6	9.0	8.8

$$\beta(s) = \beta_0 + \beta_1 \ln\left(\frac{s}{s_0}\right), \quad (10)$$

with β_0 and β_1 to be determined from the fit. For energies above 10 GeV, the values of G^2 and β for pp and $\bar{p}p$ are consistent within uncertainties; therefore, we consider all data as being of the same reaction. The results of the fits for $\sqrt{s_{\text{min}}} = 10$ GeV are

$$\begin{aligned} a &= 4.1 \pm 1.7 \\ b &= 0.25 \pm 0.13 \\ c &= 0.80 \pm 0.31 \\ d &= 0.101 \pm 0.023 \end{aligned} \quad \left| \begin{aligned} \beta_0 &= 0.1809 \pm 0.0078 \text{ fm}^2 \\ \beta_1 &= 0.0115 \pm 0.0011 \text{ fm}^2. \end{aligned} \right. \quad (11)$$

The fit to G^2 points presents a reduced χ^2 of 0.836 for 31 d.o.f., while for the fit to β we have 0.875 for 33 d.o.f., indicating that our choices for the parametrizations describe successfully the energy dependence of the parameters of the MP model.

Using our parametrizations for G^2 and β , we are able to calculate the energy dependence of pp and $\bar{p}p$ cross sections. The results are presented in Fig. 1, where we compare our predictions with the available data for σ_{tot} , σ_{el} [3–6,8,10,52], and for single dissociation (SD) cross sections [69–78]. We have that the MP model is able to describe the current experimental data. In particular, the data for the diffractive cross section, which were not

considered in the fitting of the parameters, are quite well described. For the diffractive cross section, we predict that the cross section for $\sqrt{s} = 14$ TeV will be almost identical to that measured in Run 1 of the LHC. Finally, it is also interesting to note that the total cross section for pp scattering in the MP model behaves asymptotically as $\sigma_{\text{tot}} \sim \beta \ln G^2$. Therefore, with our choices of G^2 and β , we have $\sigma_{\text{tot}} \sim \ln^2 s$, in accordance with the Froissart-Martin bound [79,80].

In Table III, we summarize the predictions for these cross sections at the energies of the LHC and cosmic ray experiments. The values of σ_{tot} and σ_{el} derived using the MP model are in agreement with predictions of other approaches. For instance, the σ_{tot} values from the analysis performed in Ref. [81] are 112.1 ± 1.3 mb at 14 TeV, 143.4 ± 2.8 mb at 57 TeV, and 156.2 ± 3.6 mb at 95 TeV, while for σ_{el} (considering a combination of these predictions with those obtained in Ref. [82]) we have 31.05 ± 0.67 mb, 45.0 ± 1.3 mb, and 51.2 ± 1.6 mb at 14, 57, and 95 TeV, respectively. Moreover, our predictions for σ_{tot} , σ_{el} , and σ_{diff} at LHC energies are similar to those derived using the DIPSY dipole model in Refs. [40] and [83], denoted in Fig. 1 by DIPSY(2007) and DIPSY(2015), respectively. In this model, similarly to the approach used in this paper, the cross sections are estimated considering the Good-Walker formalism for diffractive excitation. However, in the DIPSY dipole model, the scattering eigenstates are given by the dipole states, with the fluctuations in the collision process originating from the initial wave functions, from the dipole cascades, and from the dipole-dipole scattering probability. In addition, we also have verified that our results at LHC energies are similar to those derived using the modified ABMST model proposed in Ref. [84], and made available in the latest PYTHIA 8 release.

One of the important aspects of the MP model is that it also allows us to study the energy evolution of the cross sections in the impact parameter space. The elastic and

FIG. 2. Energy evolution of the elastic and diffractive dissociation cross sections in the impact parameter space (left and central panels, respectively) and of the ratio $R = (\sigma_{\text{el}} + \sigma_{\text{diff}})/\sigma_{\text{tot}}$ (right panel).

FIG. 3. Differential elastic cross sections as a function of the squared momentum transfer for pp collisions at $\sqrt{s} = 52.8$ GeV and 13 TeV. The data and prediction for 13 TeV have been rescaled by a factor 10^{-2} .

diffractive differential cross sections as a function of b are presented in the left and central panels of Fig. 2, respectively, from Tevatron to cosmic ray energies. We observe that the elastic scattering is mainly central and its magnitude increases with the energy, approaching the black disk limit, although it is not yet saturated at 57 TeV. In contrast, the diffractive dissociation becomes more peripheral, with its maximum moving to larger impact parameters as the energy increases. We also note the decrease of the magnitude of this cross section from lower to higher energies. Finally, in the right panel of Fig. 2, we present the evolution with the energy of the ratio $R(s) = (\sigma_{\text{el}} + \sigma_{\text{diff}})/\sigma_{\text{tot}}$, where the cross sections are those of Eqs. (6)–(8) integrated in $\mathbf{b}$ space. According to the Pomplin bound [48], we should have $R \leq 1/2$. Our results indicate that this bound is not saturated and that R approaches the upper limit at a slow rate.

Before presenting our generalization of the MP model for pA collisions in the next section, it is important to emphasize one of the shortcomings of the approach. As demonstrated in Fig. 3, the MP model is not able to describe the experimental data for the differential elastic cross section at large values of the squared momentum transfer q^2 . Similar conclusions have also been derived in Ref. [85] using the DIPSY dipole model. These results indicate that the simplistic approach for the description of the spatial distribution of the partons inside the hadron, present in the MP model, should be improved. Such study is a work in progress.

III. DIFFRACTIVE EXCITATION IN pA COLLISIONS

Our goal in this section is to extend our previous analysis for proton-nucleus collisions. The extrapolation of results

from pp to pA collisions is generally performed using the Glauber formalism [50], which assumes that the projectile nucleon travels along straight lines, undergoing multiple elastic collisions with the nucleons in the target. As pointed out by Gribov [49], such formalism disregards the diffractive excitation of the intermediate nucleons, which gives a significant contribution for the total and elastic pA cross sections. We will include the contribution of the diffractive excitation using the formalism formulated in Refs. [32,34], which is based on the Good-Walker approach (for a review, see Ref. [51]). As in the GW approach, the incoming hadron state is expressed as a superposition of the eigenstates of the scattering operator. The basic assumption in Refs. [32,34] is that each of these eigenstates interacts with the target with its own cross section σ , with the probability of interaction given by $P(\sigma, s)$. As a consequence, the total, elastic, and diffractive pA cross sections can be expressed as follows:

$$\sigma_{\text{tot}}^{pA}(s) = 2 \int d\sigma P(\sigma, s) \int d^2\mathbf{b} \text{Re} \Gamma_A(\mathbf{b}, \sigma), \quad (12)$$

$$\sigma_{\text{el}}^{pA}(s) = \int d\sigma \left| \int d^2\mathbf{b} P(\sigma, s) \Gamma_A(\mathbf{b}, \sigma) \right|^2, \quad (13)$$

$$\sigma_{\text{diff}}^{pA}(s) = \int d^2\mathbf{b} \left[\int d\sigma P(\sigma, s) |\Gamma_A(\mathbf{b}, \sigma)|^2 - \left| \int d\sigma P(\sigma, s) \Gamma_A(\mathbf{b}, \sigma) \right|^2 \right], \quad (14)$$

where Γ_A is the profile function for the pA scattering, given by

$$\Gamma_A(\mathbf{b}, \sigma) = 1 - \exp \left(-\frac{A}{2} \sigma (1 - i\eta) T(\mathbf{b}) \right). \quad (15)$$

Here η is the ratio of the real to the imaginary part of the forward pp elastic scattering amplitude (which, given that it is small at high energies, we shall consider $\eta \approx 0$), and $T(b)$ is the thickness function of the nucleus,

$$T(\mathbf{b}) = \frac{1}{2\pi B(s)} \int dz d^2\mathbf{s} e^{-(\mathbf{b}-\mathbf{s})^2/(2B(s))} \rho_A(\sqrt{|\mathbf{s}|^2 + z^2}), \quad (16)$$

with $\mathbf{b}$ being the impact parameter of the pA scattering, $\mathbf{s}$ a vector in the $\mathbf{b}$ plane, $B(s)$ the forward slope of the pp differential elastic cross section, and ρ_A the single nucleon distribution. For heavy nuclei, we will consider the Wood-Saxon distribution

$$\rho_A(r) = \frac{\rho_0}{1 + \exp((r - R_0)/a)}, \quad (17)$$

where ρ_0 is a normalization constant (calculated to have ρ normalized to 1) and the parameters R_0 and a are available in Ref. [86].

The main input in Eqs. (12), (13), and (14) is the probability distribution $P(\sigma, s)$ parametrized by [32,34]

$$P(\sigma, s) = N(s) \frac{\sigma}{\sigma + \sigma_0(s)} \exp\left(-\frac{(\sigma/\sigma_0(s) - 1)^2}{\Omega^2(s)}\right). \quad (18)$$

The parameters N , σ_0 , and Ω are energy dependent and are constrained by the moments of the distribution, which are assumed to satisfy the following properties:

$$\int P(\sigma, s) d\sigma = 1, \quad (19)$$

$$\int \sigma P(\sigma, s) d\sigma = \sigma_{\text{tot}}(s), \quad (20)$$

$$\int \sigma^2 P(\sigma, s) d\sigma = \sigma_{\text{tot}}^2(1 + \omega_\sigma(s)), \quad (21)$$

where σ_{tot} is the total $pp/\bar{p}p$ cross section and ω_σ , which is related to the broadness of the cross section fluctuations, is expressed in terms of the diffractive and elastic $pp/\bar{p}p$ cross sections [34]

$$\omega_\sigma = \frac{d\sigma_{\text{diff}}/dq^2|_{q^2=0}}{d\sigma_{\text{el}}/dq^2|_{q^2=0}}. \quad (22)$$

In what follows, we will use the results obtained with the Miettinen-Pumplin model and discussed in the previous section to constrain the parameters present in $P(\sigma, s)$ and to calculate ω_σ . Moreover, the intrinsic predictive power of the MP model that follows from our parametrizations for $G^2(s)$ and $\beta(s)$, Eqs. (9)–(11), allows us to calculate ω_σ for any desirable energy, and consequently, to estimate the energy dependence of the total, elastic, and diffractive pA cross sections. In fact, from the MP model, we have

$$\frac{d\sigma_{\text{el}}}{dq^2}\bigg|_{q^2=0} = \frac{1}{4\pi} \left[\int d^2\mathbf{b} (1 - e^{-G^2(\tau(\mathbf{b}))}) \right]^2, \quad (23)$$

$$\frac{d\sigma_{\text{diff}}}{dq^2}\bigg|_{q^2=0} = \frac{1}{4\pi} \int d^2\mathbf{b} d^2\mathbf{b}' \exp\{-G^2[\langle\tau(\mathbf{b})\rangle + \langle\tau(\mathbf{b}')\rangle]\} \{\exp[G^2\langle\tau(\mathbf{b})\tau(\mathbf{b}')\rangle] - 1\}, \quad (24)$$

where

$$\langle\tau(\mathbf{b})\rangle = \frac{A}{\beta\xi} \left(\frac{\alpha/\lambda}{1 + \alpha/\lambda} \right) e^{-b^2/(\gamma+\beta)}, \quad (25)$$

$$\langle\tau(\mathbf{b})\tau(\mathbf{b}')\rangle = \frac{A\gamma}{\gamma + 2\beta} \left(\frac{\alpha/\lambda}{2 + \alpha/\lambda} \right) \exp\{-\eta(b^2 + b'^2) + \mu b b' \cos(\phi - \phi')\}, \quad (26)$$

where $\eta = (\gamma + \beta)/(\gamma^2 + 2\gamma\beta)$, $\mu = 2\beta/(\gamma^2 + 2\gamma\beta)$, $b = |\mathbf{b}|$, and ϕ is the polar angle of $\mathbf{b}$ (analogous for $\mathbf{b}'$)

FIG. 4. Energy evolution of ω_σ from the Miettinen-Pumplin model (red solid curve) compared with the values presented in Ref. [38] (black dots).

and ϕ'). Using the parameters constrained in the previous section, the resulting energy dependence of ω_σ is presented in Fig. 4 together with some values for this quantity estimated by Guzey and Strikman (GS) in Ref. [38]. We have that our predictions are, on average, similar to those presented in Ref. [38], as well as the behavior predicted for the energy dependence. At low energies, ω_σ rises with energy, as expected from Regge phenomenology [34], and at high energies, it decreases and tends to zero. Therefore, asymptotically the cross section fluctuations cease, which is expected to occur in the black disk limit.

In order to estimate the pA cross sections, we will consider the predictions for the total $pp/\bar{p}p$ cross sections derived using the MP model. Moreover, we will assume that the slope $B(s)$ of the pp elastic scattering amplitude is given by

$$B(s) = 11.21 - 0.176 \ln(s/s_0) + 0.0372 \ln^2(s/s_0), \quad (27)$$

where the parameters are given in GeV^{-2} and have been constrained using the recent LHC data with $s_0 = 1 \text{ GeV}^2$. In Fig. 5, we present our predictions for the energy dependence of the total and elastic cross sections considering proton-lead ($p\text{Pb}$) and proton-nitrogen ($p\text{N}$) collisions.¹ For comparison, we also present the results obtained without cross section fluctuations, i.e., using the standard Glauber formalism. In addition, for $p\text{Pb}$ collisions, we show the results reported in Ref. [38]. We see that the effects of the fluctuations are stronger at lower energies and heavier nuclei. As already expected from the results for ω_σ , the impact of the fluctuations becomes less important at

¹For the nitrogen nucleus, we considered a Gaussian distribution for $\int \rho_A(\mathbf{s}, z) dz$.

FIG. 5. Total (left) and elastic (right) cross section for proton-lead (top) and proton-nitrogen (bottom) scattering, with and without cross-section fluctuations as a function of the energy in the center-of-mass system. In the $p\text{Pb}$ case, we also present the results by Guzey and Strikman (GS) [38].

high energies. Our predictions for $p\text{Pb}$ collisions are similar to those derived in Ref. [38], mainly at smaller energies. The difference between the predictions at higher energies arises from the different inputs used to constrain the probability parameters. Here we have considered σ_{tot} from the MP model and $B(s) \sim \ln^2 s$, while in Ref. [38] the authors have used σ_{tot} from a fit by Donnachie and Landshoff [87] and assumed $B(s) \sim \ln s$. Using our results for the total and elastic $p\text{Pb}$ cross sections, we can estimate the inelastic cross section and compare it with the experimental data measured by the CMS [88] and the LHCb [89] Collaborations, and also with data from Ref. [90]. In Fig. 6, we present our predictions for the energy dependence of the inelastic cross sections derived using the standard Glauber approach (without fluctuations) and using the fluctuation approach. In this case, we have only considered the nondiffractive contribution. We have that our approach overestimates the current data at high energies. In contrast, the DIPSY model generalized for pA collisions in Ref. [83]

FIG. 6. Inelastic cross sections for proton-lead scattering as a function of energy and comparison with the available experimental data.

FIG. 7. Diffractive dissociation cross section for proton-lead (left), proton-argon (central), and proton-nitrogen (right) scattering as a function of the energy in the center-of-mass system. The cross sections due to hadronic cross section fluctuations are represented by red solid curves, and those from photon-induced interactions are represented by blue dashed curves. In the pPb case, the points represent the predictions for the diffractive excitation derived in Ref. [38].

describes these data. Such differences can be associated to the fact that in our approach for the diffractive excitation, we are disregarding possible fluctuations in the target, which reduces the inelastic cross section. The inclusion of target fluctuations in the MP approach is currently under study.

Finally, we can estimate the magnitude of the diffractive excitation in pA collisions at high energies. Our predictions are presented in Fig. 7 for proton-lead, proton-argon (pAr), and proton-nitrogen scattering and are represented by the red solid lines. For comparison, we also present in the pPb case the predictions derived in Ref. [38]. We have that the diffractive excitation cross section increases with the atomic nuclei and decreases with the energy, in agreement with the results derived in Ref. [38]. In Ref. [38], the authors have pointed out that the electromagnetic contribution for the diffractive excitation in pA collisions becomes important. The basic idea is that in ultraperipheral collisions, the nucleus acts as a source of photons which interact with the proton [91]. This contribution can be estimated in terms of the nuclear photon flux (n_A) and the photon-proton cross section ($\sigma_{\gamma p \rightarrow X}$) as follows:

$$\sigma_{e.m.}^{pA} = \int \frac{d\omega}{\omega} n_A(\omega) \sigma_{\gamma p \rightarrow X}(\omega), \quad (28)$$

where ω is the photon energy. In our calculations we consider the same inputs used in Ref. [38], and the resulting predictions are represented in Fig. 7 by blue dashed curves. We have that the electromagnetic contribution increases with the energy and the atomic number, being dominant for pPb collisions. For pAr collisions, both contributions are similar at the LHC energies. Finally, it is important to emphasize that the electromagnetic

contribution becomes dominant in pN collisions at ultra-high cosmic ray energies.

IV. SUMMARY

The contribution of the diffractive excitation for the hadronic processes is directly related to the treatment of the internal d.o.f. of the hadrons. During the last few decades, several approaches have been proposed. In this paper, we have considered the Good-Walker approach and updated the Miettinen-Pumplin (MP) model, considering the recent LHC data for the total and elastic pp cross sections and older experimental data as well. We have demonstrated that this model is able to successfully describe the current data, and a parametrization for the energy dependence of the main parameters of the MP model was proposed. The behavior of the cross sections for higher energies has been analyzed, and predictions for the energies of Run 3 of the LHC and those of the cosmic ray experiments have been derived. Our results demonstrated that the cross section for the diffraction excitation in pp collisions is almost constant in the energy range probed by the LHC and slowly decreases at higher energies. Moreover, our results indicated that the Pumplin bound is not reached at the LHC and cosmic ray energies. We have also presented our results for the diffractive excitation in pA collisions, as well as its impact on the predictions of the total and elastic cross sections considering different nuclei. The MP model, constrained by the pp data, has been used to derive the main quantities present in the treatment of the diffractive excitation in pA collisions. We demonstrated that the effect of fluctuations decreases at larger energies and heavier nuclei. Moreover, our results indicated that the proton dissociation induced by photon interactions becomes dominant with the increase of the energy and the atomic number.

ACKNOWLEDGMENTS

This work was partially financed by the Brazilian funding agencies CNPq, CAPES, FAPERGS, and Instituto Nacional de Ciência e Tecnologia–Física Nuclear e Aplicações (INCT-FNA) (Process No. 464898/2014-5).

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