**Erratum: Novel determination of density, temperature, and symmetry energy
for nuclear multifragmentation through primary fragment-yield reconstruction
[Phys. Rev. C **89**, 021601(R) (2014)]**

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(Received 10 January 2017; published 24 March 2017)

DOI: [10.1103/PhysRevC.95.039907](https://doi.org/10.1103/PhysRevC.95.039907)

In our original article, isotope yields from the reaction $^{64}\text{Zn} + ^{112}\text{Sn}$ at 40 MeV/nucleon were utilized to extract the density, temperature, and symmetry energy of the fragmenting system, based on the modified Fisher model (MFM). This is one of the series of similar analyses [1–5]. However, the Purdue’s MFM equation from the pioneering works of the Purdue group in Refs. [6–8] had a mistake which resulted in minor errors. We address the mistake in their equation and correct the errors in the extracted values in this Erratum. According to Refs. [6–8], the yield of the isotope with N neutrons and Z protons was expressed as

$$Y(I, A) = Y_0 A^{-\tau} \exp \left[\frac{W(I, A) + \mu_n N + \mu_p Z}{T} + N \ln \left(\frac{N}{A} \right) + Z \ln \left(\frac{Z}{A} \right) \right]. \quad (1)$$

Here $A = N + Z$, and $I = N - Z$. Following Refs. [6–8] and utilizing the generalized Weizsäcker-Bethe semiclassical mass formula, $W(I, A)$, is given as

$$W(I, A) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A} - a_p \frac{\delta_p}{A^{1/2}}, \quad (2)$$

$$\delta_p = -\frac{(-1)^Z + (-1)^N}{2}.$$

When the mixing entropy is defined following the standard positive definition as

$$S_{\text{mix}}(N, Z) = - \left[N \ln \left(\frac{N}{A} \right) + Z \ln \left(\frac{Z}{A} \right) \right], \quad (3)$$

the above equation is rewritten as

$$Y(I, A) = Y_0 A^{-\tau} \exp \left[\frac{W(I, A) + \mu_n N + \mu_p Z}{T} - S_{\text{mix}}(N, Z) \right]. \quad (4)$$

In this equation, the symmetry energy and the energy from the mixing entropy have the same sign. However the symmetry free energy is given as $F_{\text{sym}}(I, A) = E_{\text{sym}}(I, A) - T S_{\text{sym}}(I, A)$ where the energy from the symmetry entropy has a sign opposite to the symmetry energy. As addressed in the Appendix,

the symmetry energy and the energy from the mixing entropy should have signs opposite to each other in Eq. (4). The correct MFM formula is therefore,

$$Y(I, A) = Y_0 A^{-\tau} \exp \left[\frac{W(I, A) + \mu_n N + \mu_p Z}{T} + S_{\text{mix}}(N, Z) \right]. \quad (5)$$

When the mixing entropy is defined as in Eq. (3), the symmetry energy relative to the temperature a_{sym}/T is given as

$$a_{\text{sym}}/T = -\frac{A}{8} \{ \ln[R(3, 1, A)] - \ln[R(1, -1, A)] - \Delta(3, 1, A) + \Delta E_c \}, \quad (6)$$

which is also given as Eq. (1) in the original paper and $\Delta(3, 1, A)$ is defined as

$$\Delta(3, 1, A) = S_{\text{mix}}(3, A) - S_{\text{mix}}(1, A). \quad (7)$$

The original error in the sign affects only $\Delta(3, 1, A)$ in Eq. (6). Consequently, by using Eq. (3) in Eq. (7), the error is corrected, even though the numerical value of the mixing entropy has an opposite sign compared to the old value. All the formulas in Refs. [1–5] also are correct. However the signs of the mixing entropy values need to be reversed.

Fortunately, in our analysis and these references, isotope yield ratios for isobars have been utilized and the reversal of the signs tends to cancel each other. Therefore the correction in the extracted a_{sym}/T values becomes rather small, i.e., $\sim 20\%$ at most.

The corrected extracted values are $\rho/\rho_0 = 0.54 \pm 0.04$, $T = 5.9 \pm 0.2$ MeV, and $a_{\text{sym}} = 20.7 \pm 2.6$ MeV. These may be compared to the old values of $\rho/\rho_0 = 0.63 \pm 0.03$, $T = 4.9 \pm 0.2$ MeV, and $a_{\text{sym}} = 24.7 \pm 1.9$ MeV. The updated version of Table I shown here contains the new values.

In order to verify the validity of the qualitative discussions made in Refs. [1–5], the new and old a_{sym} values together with the available published data are plotted in Fig. 1. The new value still follows the same systematic curve as in the original paper, which is calculated by

$$a_{\text{sym}}(\rho/\rho_0) = 31.5 \times (\rho/\rho_0)^{0.69}. \quad (8)$$

Since the old and new values for the density and symmetry energy values agree within the error bars and the new a_{sym} value

TABLE I. Extracted parameters. The values from Ref. [9] are taken at $E^* = 7$ MeV, and they depend slightly on the excitation energy. The terms g0, g0AS, and g0ASS represent the Gogny interaction, the Gogny asymptotic stiff interaction, and the Gogny asymptotic superstiff interaction.

Ratio	ρ/ρ_0	a_{sym} (MeV)	a_{sym}/T	T (MeV)
g0		23.7 ± 0.9	4.12 ± 0.11	5.74 ± 0.3
g0AS	1.19 ± 0.03	19.1 ± 1.0	3.17 ± 0.09	6.1 ± 0.4
g0ASS	1.44 ± 0.05	14.9 ± 1.2	2.36 ± 0.08	6.3 ± 0.5
Expt.	0.54 ± 0.04	20.7 ± 2.6	3.5 ± 0.4	5.9 ± 0.2
Ref. [9]	0.45 ± 0.12	17 ± 2		6.5 ± 0.5

follows the same curve as that in the original paper, these facts verify that our conclusions in the paper are still valid.

APPENDIX: MFM

In the Fisher model [10], for a single constituent system, a spherical droplet from A particles is formed in the gas phase with B particles through a phase transition, and both phases are derived from a parent source with mass $A + B$ particles. The free energy of the system in the initial and final phases, respectively, can be written as

$$F_{\text{initial}} = \mu_g(A + B) - TS, \quad (\text{A1})$$

$$F_{\text{final}} = \mu_l A + \mu_g B + 4\pi R^2 \sigma A^{2/3} - T(S - \tau \ln A). \quad (\text{A2})$$

FIG. 1. Comparisons the new and old results together with the available published results. The line is from Eq. (8). Data are taken from Khoa and Than [11], Kowalski *et al.* [12], Wada *et al.* [13], Roca-Maza *et al.* [14], Shetty *et al.* [15], Shetty *et al.* [9], Trippa *et al.* [16], and Tsang *et al.* [17].

In Eqs. (A1) and (A2), μ_l and μ_g are, respectively, the chemical potentials of the liquid and gas phases, and S is the total entropy of the initial phases. The third term in Eq. (A2) is the surface contribution for the spherical droplet with radius R and surface tension parameter σ . Near the critical point σ can be expressed as a function of temperature as

$$\sigma(T, T_c) = \begin{cases} \sigma_0(1 + \frac{3T}{2T_c})(1 - \frac{T}{T_c})^{3/2} & (T < T_c), \\ 0 & (T \geq T_c), \end{cases} \quad (\text{A3})$$

where T_c is the temperature at the critical point. The last term in Eq. (A2) originates from the entropy change of the system when the droplet is formed. The term $\tau \ln A$ is the entropy change caused by the clusterization, introduced by Fisher in his model [10]. $\tau \ln A$ is subtracted from the total entropy because, when the clusterization occurs, the entropy decreases.

The free energy of the droplet can be given as the difference between Eqs. (A1) and (A2) as

$$F_{\text{droplet}} = F_{\text{final}} - F_{\text{initial}} = (\mu_l - \mu_g)A + 4\pi R^2 \sigma A^{2/3} + T\tau \ln A. \quad (\text{A4})$$

In a canonical ensemble, the free energy can be deduced as

$$-F_{\text{droplet}} = T \ln(\mathbf{Z}), \quad (\text{A5})$$

where $\mathbf{Z}$ is the partition function and it is proportional to the yield $Y(A)$ of a given type of droplet with A particles,

$$Y(A) \propto \mathbf{Z} = \exp\left(-\frac{F_{\text{droplet}}}{T}\right). \quad (\text{A6})$$

Equation (A6) is the mathematical expression of the Fisher model.

In order to apply the Fisher model to a nuclear multifragmentation process, two constituents (neutrons and protons) and the characteristics of nuclear force have to be taken into account. In the framework of MFM, from the analogy to Eq. (A4), the free energy of a fragment with mass number A and $I = N - Z$ (N neutrons and Z protons) is expressed as

$$F(I, A) = [-W(I, A) - \mu_n N - \mu_p Z] + T(\tau \ln A - \Delta S). \quad (\text{A7})$$

ΔS is the entropy change from a single-constituent system to a two-constituent system.

The entropy change ΔS in Eq. (A7) is called mixing entropy. For classical particles, the total number of the

microstates of Maxwell-Boltzmann (M.B.) distribution, $\Omega_{\text{M.B.}}$ can be expressed as

$$\Omega_{\text{M.B.}} = \frac{N!}{\prod_l a_l} \prod_l \omega_l^{a_l}, \quad (\text{A8})$$

where N is the particle number and a_l is the particle number in the l state and that $N = \sum a_l$. ω_l is the degeneracy of the l state. Going to a nuclear system and ignoring the spin, nucleons only have two states, the proton and the neutron, defined as the “ n ” state and the “ p ” state here. The degeneracies of the two states are both 1.

Therefore for a nucleus with Z protons ($a_p = Z$) and N neutrons ($a_n = N$), the total number of the microstates becomes

$$\Omega_{\text{M.B.}}(N, Z) = \frac{A!}{N!Z!}. \quad (\text{A9})$$

Thus $S_{\text{mix}}(N, Z)$ is calculated simply as

$$S_{\text{mix}}(N, Z) = -[\ln(N!Z!) - \ln(A!)] \\ \approx -\left[N \ln\left(\frac{N}{A}\right) + Z \ln\left(\frac{Z}{A}\right) \right]. \quad (\text{A10})$$

Since the mixing entropy increases when the system changes from a single constituent to two constituents, ΔS in Eq. (A7) is expressed as

$$\Delta S = S_{\text{mix}}(N, Z). \quad (\text{A11})$$

Putting Eq. (A11) into Eq. (A7), one can get Eq. (5) in the text.

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