Investigating $\Omega(2012)$ as a molecular state

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We conduct a dynamical calculation of pentaquark systems with quark contents $sss\bar{u}\bar{u}$ in the framework of two quark models: the chiral quark model (ChQM) and quark delocalization color screening model (QDCSM). The effective potentials between baryon and meson clusters are given, and the possible bound states are also investigated. Also, the study of the scattering process of the open channels is performed to look for any resonance state. The results show that the $\Omega(2012)$ is not suitable for the interpretation as a $\Xi^*\bar{K}$ molecular state in present quark models. Two resonance states: the $\Xi^*\bar{K}^*$ with $IJ^P = 0\frac{3}{2}^-$ ($M = 2328\text{--}2374\text{ MeV}$, $\Gamma = 57\text{--}65.5\text{ MeV}$) and $IJ^P = 1\frac{3}{2}^-$ ($M = 2341\text{--}2386\text{ MeV}$, $\Gamma = 31.5\text{--}100\text{ MeV}$) are obtained in both QDCSM and ChQM, which indicates that these two states are more likely to exist and worthy of being sought by future experiments.

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I. INTRODUCTION

The possible new resonance state is always an interesting topic in both experimental and theoretical physics, and it can help us to catch more details about the hadron-hadron and quark-quark interaction. The QCD theory has only predicted the existence of colorless hadronic states including two and three quarks. The hadrons which have more than three quarks lie outside the original quark model. These particles are the so-called exotic hadrons. The quantum numbers of these hadrons are unexplainable with the typical two and three quark bound states. Although the interaction fails in the description of many resonances, an example of the resonance better described by other approaches is the $\Lambda(1405)$ which can be interpreted as a meson baryon system [1–4] or the Roper resonance [5].

In 2018, the Belle Collaboration searched for excited Ω^- decaying into $\Xi_0 K^-$ and $\Xi^- K_s^0$ using $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ data samples with integrated luminosities of 5.7, 24.9, and 2.9 fb^{-1} , respectively [6]. A signal peak of around 2.012 GeV is seen with a signal significance of 8.3σ . This new resonance is denoted as $\Omega(2012)$, with a mass of $2012.4 \pm 0.7(\text{stat.}) \pm 0.6(\text{syst.})\text{ MeV}/c^2$ and a width of $6.4^{+2.5}_{-2.0}(\text{stat.}) \pm 1.6(\text{syst.})\text{ MeV}$. As we know that the ground Ω hyperon is a member of baryon decuplet in the quark model, which was unambiguously discovered in both production and decay at

BNL about half-century ago [7]. Three resonances: $\Omega(2250)$, $\Omega(2380)$, and $\Omega(2470)$ are also listed in the Particle Data Group (PDG) [8] with three- or two-star ratings, and their nature is still uncertain. So in experiments, there is only a little information of the excited Ω baryons. The discovery of the $\Omega(2012)$ state offers a great opportunity to study the hadron spectrum and understand the strong interactions.

Actually, before the Belle collaboration's report, there were some theoretical studies of the Ω excited states. References [9,10] investigated the Ω excited states by using the chiral unitary approach where the coupled channels interactions of the $\Xi^*\bar{K}$ and $\Omega\eta$ were taken into account. In Ref. [11], a dynamically generated Ω state with $J^P = \frac{3}{2}^-$ was obtained with the mass around 1800 MeV, which is in agreement with the predictions of the five-quark model. In addition, the Ω excited states were also investigated in classical quark models [12–15].

The Belle collaboration's observation of the $\Omega(2012)$ also prompted a lot of theoretical work on the issue, with pictures inspired by quark models and also molecular pictures based on the meson-baryon interaction. In various quark models, the masses of the first orbital excitations of Ω states were deemed to $\Omega(2012)$. In the chiral model, through the study of the strong decays of $\Omega(2012)$, it could be assigned to the spin-parity $J^P = \frac{3}{2}^-$ $1P$ wave state [16]. In Refs. [17,18], the mass and the two-body strong decays of the $\Omega(2012)$ state were studied by the QCD sum rule method, and the results have shown that the $\Omega(2012)$ could be interpreted as $1P$ orbital excitation of the ground state Ω baryon with the quantum number $J^P = \frac{3}{2}^-$. However, some work investigated $\Omega(2012)$ as the molecular state. Since its mass is very close to $\Xi^*\bar{K}$ threshold, some work considered it as a $J^P = \frac{3}{2}^-$ $\Xi^*\bar{K}$ molecular state [19–21]. In Ref. [22], the study of the $\Xi\bar{K}$ decay mode of the newly observed $\Omega(2012)$ indicated that the $\Omega(2012)$ was a dynamically generated state with spin-parity $J^P = \frac{3}{2}^-$ from the S -wave-coupled channel interactions of

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$\Xi\bar{K}$ and $\Omega\eta$. In Ref. [23], they performed the $SU(3)$ analysis of $\Omega(2012)$, discussed the implications of the $\bar{K}\Xi(1530)$ molecular picture of $\Omega(2012)$, and suggested crucial experimental tests to distinguish various pictures of $\Omega(2012)$.

However, it was found that there is no significant signals for the $\Omega(2012) \rightarrow \Xi^*\bar{K} \rightarrow \bar{K}\pi\Xi$ in a very recent measurement of the Belle collaboration [24]. It is in sharp tension with the prediction of the S -wave $\Xi\bar{K}$ molecule assignment for $\Omega(2012)$, so this result strongly disfavors the molecular interpretation. Later on, based on the experiment results, a lot of work reinterprets the $\Omega(2012)$ state. In Ref. [25], the $\Omega(2012)$ was explained as the P -wave molecule assignments, and can be interpreted as the $\frac{1}{2}^+$ or $\frac{3}{2}^+$ $\Xi\bar{K}$ molecule state according to current experiment data. In Ref. [26], the $\Omega(2012)$ resonance was most likely to be the spin-parity $J^P = \frac{3}{2}^-$ $1P$ -wave state, and it had a large potential to be observed in the $\Omega(1672)\gamma$ channel within a nonrelativistic constituent quark potential model. In the hadronic molecular approach, the $\Omega(2012)$ state was considered to contain the mixed $\Xi^*\bar{K}$ and $\Omega\eta$ hadronic components [27]. A study of the interaction of the $\Xi^*\bar{K}$, $\Omega\eta$, and $\Xi\bar{K}$ (D -wave) channels within a coupled channel unitary approach showed that all data including the recent Belle experiment on $\tau_{\Omega^* \rightarrow \pi\bar{K}\Xi}/\tau_{\Omega^* \rightarrow \bar{K}\Xi}$, were compatible with the molecular picture stemming from meson baryon interaction of these channels [28,29]. While in Ref. [30], the production of $\Omega(2012)$ in the nonleptonic weak decays of $\Omega_c^0 \rightarrow \pi^+\bar{K}\Xi^*(1530)(\eta\Omega) \rightarrow \pi^+(\bar{K}\Xi)^-$ was investigated, where the final-state interactions of the $\bar{K}\Xi^*(1530)$ and $\eta\Omega$ pairs are taken into account.

Inspired by the experiment results of the $\Omega(2012)$ by the Belle collaboration and several theoretical explanations of this state, we take the chiral quark model (ChQM) and the quark delocalization color screening model (QDCSM) to investigate the pentaquark systems composed of $sssu\bar{u}$ to see whether the $\Omega(2012)$ be described as the molecular picture or not. By using the constituent quark models, we have studied the hidden-charm pentaquark systems [31,32] and predicated several resonance states, three of which were consistent with the reported $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ by the LHCb collaboration [33]. Extending to the study of the hidden-strange pentaquark systems, we also obtained several P_c -like resonance states [34]. Therefore, it is feasible to apply these models to the $sssu\bar{u}$ systems.

The structure of this paper is as follows. A brief introduction of two quark models is given in Sec. II. Section III devotes itself to the numerical results and discussions. The summary is shown in the last section.

II. TWO QUARK MODELS

A. Chiral quark model

The ChQM is based on the fact that nearly massless current light quark acquires a dynamical, momentum dependent mass, namely, the constituent quark mass due to its interaction with the gluon medium. So the chiral quark model contains Goldstone-boson exchange potentials, the perturbative one-gluon interaction, and a linear-screened confining potential. The model details can be found in Ref. [35]. Here only the Hamiltonian is given:

$$H = \sum_{i=1}^5 \left(m_i + \frac{p_i^2}{2m_i} \right) - T_{\text{CM}} + \sum_{j>i=1}^5 (V_{ij}^C + V_{ij}^G + V_{ij}^\chi + V_{ij}^{\sigma_a}), \quad (1)$$

$$V_{ij}^C = -a_c \lambda_i^c \cdot \lambda_j^c (r_{ij}^2 + v_0), \quad (2)$$

$$V_{ij}^G = \frac{1}{4} \alpha_s \lambda_i^c \cdot \lambda_j^c \left[\frac{1}{r_{ij}} - \frac{\pi}{2} \delta(\mathbf{r}_{ij}) \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{3m_i m_j} \right) - \frac{3}{4m_i m_j r_{ij}^3} S_{ij} \right], \quad (3)$$

$$V_{ij}^\chi = V_\pi(\mathbf{r}_{ij}) \sum_{a=1}^3 \lambda_i^a \lambda_j^a + V_K(\mathbf{r}_{ij}) \sum_{a=4}^7 \lambda_i^a \lambda_j^a + V_\eta(\mathbf{r}_{ij}) [(\lambda_i^8 \lambda_j^8) \cos \theta_P - (\lambda_i^0 \lambda_j^0) \sin \theta_P], \quad (4)$$

$$V_\chi(\mathbf{r}_{ij}) = \frac{g_{\text{ch}}^2}{4\pi} \frac{m_\chi^2}{12m_i m_j} \frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} m_\chi \left\{ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij}) \right] + \left[H(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} H(\Lambda_\chi r_{ij}) \right] S_{ij} \right\}, \quad \chi = \pi, K, \eta, \quad (5)$$

$$V_{ij}^{\sigma_a} = V_{a_0}(\mathbf{r}_{ij}) \sum_{a=1}^3 \lambda_i^a \lambda_j^a + V_\kappa(\mathbf{r}_{ij}) \sum_{a=4}^7 \lambda_i^a \lambda_j^a + V_{f_0}(\mathbf{r}_{ij}) \lambda_i^8 \lambda_j^8 + V_\sigma(\mathbf{r}_{ij}) \lambda_i^0 \lambda_j^0, \quad (6)$$

$$V_k(\mathbf{r}_{ij}) = -\frac{g_{\text{ch}}^2}{4\pi} \frac{\Lambda_k^2}{\Lambda_k^2 - m_k^2} m_k \left[Y(m_k r_{ij}) - \frac{\Lambda_k}{m_k} Y(\Lambda_k r_{ij}) \right], \quad \chi = a_0, \kappa, f_0, \sigma, \quad (7)$$

$$S_{ij} = \left\{ 3 \frac{(\boldsymbol{\sigma}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\sigma}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right\}, \quad (8)$$

$$H(x) = (1 + 3/x + 3/x^2)Y(x), \quad Y(x) = e^{-x}/x. \quad (9)$$

Where α_s is the quark-gluon coupling constant. The coupling constant g_{ch} for chiral field is determined from the $NN\pi$ coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}. \quad (10)$$

The other symbols in the above expressions have their usual meanings.

B. Quark delocalization color screening model

The Hamiltonian of QDCSM is almost the same as that of ChQM but with two modifications [36,37]: First, there is no σ -meson exchange in QDCSM, and second, the screened color confinement is used between quark pairs resident in different clusters. That is

$$V_{ij}^C = \begin{cases} -a_c \lambda_i^c \cdot \lambda_j^c (r_{ij}^2 + v_0), & \text{if } i, j \text{ in the same baryon orbit,} \\ -a_c \lambda_i^c \cdot \lambda_j^c \left(\frac{1 - e^{-\mu_{ij} r_{ij}^2}}{\mu_{ij}} + v_0 \right), & \text{otherwise,} \end{cases} \quad (11)$$

where the color screening constant μ_{ij} is determined by fitting the deuteron properties, NN scattering phase shifts and $N\Lambda$, $N\Sigma$ scattering cross sections, with $\mu_{uu} = 0.45$, $\mu_{us} = 0.19$, and $\mu_{ss} = 0.08$, which satisfy the relation, $\mu_{us}^2 = \mu_{uu}\mu_{ss}$.

The single particle orbital wave functions in the ordinary quark cluster model are the left and right centered single Gaussian functions:

$$\begin{aligned} \phi_\alpha(S_i) &= \left(\frac{1}{\pi b^2} \right)^{\frac{3}{4}} e^{-\frac{(r - \frac{2}{3}S_i)^2}{2b^2}}, \\ \phi_\beta(-S_i) &= \left(\frac{1}{\pi b^2} \right)^{\frac{3}{4}} e^{-\frac{(r + \frac{2}{3}S_i)^2}{2b^2}}. \end{aligned} \quad (12)$$

The quark delocalization in QDCSM is realized by writing the single particle orbital wave function as a linear combination of the left and right Gaussians:

$$\begin{aligned} \psi_\alpha(S_i, \epsilon) &= (\phi_\alpha(S_i) + \epsilon \phi_\alpha(-S_i))/N(\epsilon), \\ \psi_\beta(-S_i, \epsilon) &= (\phi_\beta(-S_i) + \epsilon \phi_\beta(S_i))/N(\epsilon), \\ N(\epsilon) &= \sqrt{1 + \epsilon^2 + 2\epsilon e^{-S_i^2/4b^2}}, \end{aligned} \quad (13)$$

TABLE I. The parameters of two models: $m_\pi = 0.7 \text{ fm}^{-1}$, $m_K = 2.51 \text{ fm}^{-1}$, $m_\eta = 2.77 \text{ fm}^{-1}$, $m_\sigma = 3.42 \text{ fm}^{-1}$, $m_{a_0} = m_\kappa = m_{f_0} = 4.97 \text{ fm}^{-1}$, $\Lambda_\pi = 4.2 \text{ fm}^{-1}$, $\Lambda_K = 5.2 \text{ fm}^{-1}$, $\Lambda_\eta = 5.2 \text{ fm}^{-1}$, $\Lambda_\sigma = 4.2 \text{ fm}^{-1}$, $\Lambda_{a_0} = \Lambda_\kappa = \Lambda_{f_0} = 5.2 \text{ fm}^{-1}$, $g_{ch}^2/(4\pi) = 0.54$, $\theta_p = -15^\circ$.

	QDCSM	ChQM
b (fm)	0.518	0.518
m_u (MeV)	313	313
m_d (MeV)	313	313
m_s (MeV)	573	573
a_c (MeV)	58.03	48.59
v_0^{qq} (MeV)	-1.2883	-0.9610
$v_0^{q\bar{q}}$ (MeV)	-0.2012	-0.0602
α_{uu}	0.5652	0.8437
α_{us}	0.5239	0.7583
α_{ss}	0.4506	0.6197
$\alpha_{u\bar{u}}$	1.7930	1.7951
$\alpha_{u\bar{s}}$	1.7829	1.7839
$\alpha_{s\bar{s}}$	1.5114	1.4300

where $\epsilon(S_i)$ is the delocalization parameter determined by the dynamics of the quark system rather than adjusted parameters. In this way, the system can choose its most favorable configuration through its own dynamics in a larger Hilbert space.

The parameters of QDCSM are from our previous work [34] and the parameters of ChQM can be obtained by adjusting the baryons and mesons masses. We list all parameters in Table I. The calculated baryon masses and mesons masses in comparison with experimental values are shown in Table II.

III. RESULTS AND DISCUSSIONS

In this work, we systematically investigate the pentaquark systems composed of $sssu\bar{u}$ with the quantum numbers $I = 0, 1$ and $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$. Two kinds of quark models are used here, which are QDCSM and ChQM. Our purpose is to see whether the $\Omega(2012)$ can be explained as the molecular pentaquarks. Moreover, we also attempt to explore if there is any other pentaquark state. For the first step, the effective potential of all channels is studied because the attractive potential is necessary for forming a bound state or a resonance. Then, in order to check whether there is any bound state a dynamic calculation including both the single-channel and channel-coupling is carried out. Finally, the scattering process of the

TABLE II. The masses (in MeV) of the ground baryons and mesons.

	N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*	Ω
QDCSM	939	1232	1124	1238	1360	1374	1496	1642
ChQM	939	1232	1122	1266	1343	1396	1473	1622
Expt.	939	1232	1116	1193	1385	1318	1533	1672
	ϕ	$\bar{K}^*$	$\bar{K}$	π	ρ	ω	η	
QDCSM	1020	892	495	139	890	842	283	
ChQM	1020	892	495	139	891	788	227	
Expt.	1020	892	495	139	770	782	547

FIG. 1. The effective potentials of the $sss\bar{u}\bar{u}$ system in QDCSM, and (a) $IJ^P = 0_2^{1-}$, (b) $IJ^P = 0_2^{3-}$, (c) $IJ^P = 0_2^{5-}$, (d) $IJ^P = 1_2^{1-}$, (e) $IJ^P = 1_2^{3-}$, (f) $IJ^P = 1_2^{5-}$.

corresponding open channels is observed to search for any resonance states.

A. Effective potentials

The effective potential between two clusters is defined as, $V(S) = E(S) - E(\infty)$, where $E(S)$ is the diagonal matrix elements of the Hamiltonian of the system in the generating coordinate with the distance S between two clusters. All the effective potentials in two quark models are shown in Figs. 1 and 2, respectively.

Figure 1 shows the results in the QDCSM. For the $IJ^P = 0_2^{1-}$ system, we can see that the potentials are attractive for the $\Omega\omega$, $\Xi^*\bar{K}^*$, and $\Xi\bar{K}^*$ channels, while the one for the $\Xi\bar{K}$ channel is repulsive. It is obvious that the attraction of the $\Omega\omega$ is the largest one, followed by the attractions of $\Xi\bar{K}^*$ and $\Xi^*\bar{K}^*$ channels. So it is more possible for the $\Omega\omega$ channel to form a bound state. For the $IJ^P = 0_2^{3-}$ system, there are five physical channels, and there is only two channels $\Xi^*\bar{K}^*$ and $\Omega\omega$, which are attractive. For the $IJ^P = 0_2^{5-}$, the system have

two physical channels: $\Omega\omega$ and $\Xi^*\bar{K}^*$, but the interactions of both two channels are repulsive. So it is difficult for these two channels to form any bound state. For the $IJ^P = 1_2^{1-}$, the potential of the $\Omega\rho$, $\Xi^*\bar{K}^*$, and $\Xi\bar{K}^*$ channels show an attractive property, while the potential of the $\Xi\bar{K}$ channel is repulsive. For the $IJ^P = 1_2^{3-}$ and $IJ^P = 1_2^{5-}$, similar results to that of the $IJ^P = 0_2^{3-}$ and $IJ^P = 0_2^{5-}$ are obtained. The potential is attractive for the channel $\Xi^*\bar{K}^*$ and $\Omega\rho$ with $IJ^P = 1_2^{3-}$, while the interaction of the rest of channels is all repulsive.

Figure 2 lists the potentials of all channels in the ChQM. For the $IJ^P = 0_2^{1-}$ and $IJ^P = 1_2^{1-}$ systems, the behavior of the effective potentials is similar to that of QDCSM. Most channels are attractive, except for the $\Xi\bar{K}$ channel. For the $J^P = \frac{3}{2}^-$ systems, the behavior of the effective potentials of both $I = 0$ and $I = 1$ is similar to each other. There are four channels with weak attractions, which are $\Omega\omega$ and $\Omega\eta$ with $IJ^P = 0_2^{3-}$, and $\Omega\pi$ and $\Omega\rho$ with $IJ^P = 1_2^{3-}$. Besides, there are two channels with deep attractions, which are the $\Xi^*\bar{K}^*$

FIG. 2. The effective potentials of the $sss\bar{u}\bar{u}$ system in ChQM, and (a) $IJ^P = 0_2^{1-}$, (b) $IJ^P = 0_2^{3-}$, (c) $IJ^P = 0_2^{5-}$, (d) $IJ^P = 1_2^{1-}$, (e) $IJ^P = 1_2^{3-}$, (f) $IJ^P = 1_2^{5-}$.

TABLE III. The masses and binding energies (in MeV) of the $ssu\bar{u}$ systems in QDCSM.

Channel	$IJ^P = 0\frac{1}{2}^-$	$IJ^P = 0\frac{3}{2}^-$	$IJ^P = 0\frac{5}{2}^-$	$IJ^P = 1\frac{1}{2}^-$	$IJ^P = 1\frac{3}{2}^-$	$IJ^P = 1\frac{5}{2}^-$	E_{th}
$\Xi\bar{K}$	1878.6	...	...	1878.5	...	...	1869
$\Xi\bar{K}^*$	2269.3	2276.1	...	2271.2	2275.5	...	2266
$\Xi^*\bar{K}^*$	2394.8	2365.6/−23.4	2398.5	2391.6	2372.3/−16.7	2398.3	2389
$\Xi^*\bar{K}$	...	2000.7	...	...	2000.4	...	1991
$\Omega\omega$	2479.8/−4.2	2479.7/−4.3	2539.3	...	...	...	2484
$\Omega\rho$	...	...	...	2517.9/−14.1	2524.6/−7.4	2539.3	2532
$\Omega\eta$	...	1929.8	...	...	...	...	1924
$\Omega\pi$	...	...	...	...	1786.8	...	1780
$E_{\text{c.c.}}$	1878.6	1864.5/−59.5	2398.1	1878.1	1741.8/−38.2	2398.1	

with $IJ^P = 0\frac{3}{2}^-$ and $IJ^P = 1\frac{3}{2}^-$. For the $J^P = \frac{5}{2}^-$ systems, the potentials of the channels $\Omega\omega$ with $I = 0$ and $\Omega\rho$ with $I = 1$ are weakly attractive, while the other two channels are repulsive.

According to the results of these two models, we find that the potential of both the $IJ^P = 0\frac{3}{2}^-$ and $IJ^P = 1\frac{3}{2}^-$ $\Xi^*\bar{K}^*$ channels are attractive in both two quark models, and the attraction is larger than the one of other channels. So, among all the channels of the $ssu\bar{u}$ system, the $\Xi^*\bar{K}^*$ channel with $IJ^P = 0\frac{3}{2}^-$ and $IJ^P = 1\frac{3}{2}^-$ $\Xi^*\bar{K}^*$ is the most possible channel to form a bound state.

B. Bound state calculation

To check whether the possible bound state can be realized, a dynamic calculation is a need. Here, the resonating group method (RGM) [38] is employed. Expanding the relative motion wave function between two clusters in the RGM equation by Gaussians, the integrodifferential equation of RGM can be reduced to an algebraic equation, the generalized eigenequation. The energy of the system can be obtained by solving the eigenequation. In the calculation, the baryon-meson separation ($|s_n|$) is taken to be less than 6fm (to keep the matrix dimension manageably small). The masses and binding energies of every single channel and those with channel coupling are listed in Tables III and IV, in which $E_{\text{c.c.}}$ is the results of the channel coupling, and “...” means that there is no this channel.

The results of all the systems in the QDCSM are listed in Table III. For the $IJ^P = 0\frac{1}{2}^-$ system, the single-channel

calculation shows that $\Xi\bar{K}$ is unbound, which agrees with the repulsive nature of the interaction of this channel. For the $\Xi\bar{K}^*$ and $\Xi^*\bar{K}^*$ channels, the attractions are too weak to tie the two particles together, so neither of them is a bound state. However, due to the stronger attraction, the obtained lowest energy of $\Omega\omega$ is below its threshold, with the binding energy of −4.2 MeV. We also consider the effect of the multichannel coupling. The lowest energy by the channel-coupling is 1878.6 MeV, which is above the threshold of the lowest channel $\Xi\bar{K}$. So there is no bound state for the $IJ^P = 0\frac{1}{2}^-$ system. Nevertheless, we should check if the $\Omega\omega$ is a resonance state in the scattering calculation, which will be presented in the next subsection.

For the $IJ^P = 0\frac{3}{2}^-$, there are two bound states ($\Xi^*\bar{K}^*$ and $\Omega\omega$) by the single calculation. After the multichannel coupling calculation, the lowest energy of this system is 1864.5 MeV, −59.5 MeV lower than the threshold of the lowest channel $\Omega\eta$, which implies that the $IJ^P = 0\frac{3}{2}^-$ system is bound by the channel coupling. Besides, we also pay attention to the states $\Xi^*\bar{K}^*$ and $\Omega\omega$, which is possible to be resonance states by the channel coupling. To confirm whether the states $\Xi^*\bar{K}^*$ and $\Omega\omega$ can survive as resonance states after the full channels coupling, the study of the scattering processes of the open channels is carried out, and the results are shown in the next section. For the $IJ^P = 0\frac{5}{2}^-$, the interaction of two channels is repulsive, so there is no any bound state. The channel coupling calculation cannot help much.

For the $I = 1$ system, the behavior is similar to that of the $I = 0$ system. There is only one bound state in $IJ^P = 1\frac{1}{2}^-$ system, which is $\Omega\rho$, with the binding energy of −14.1 MeV.

 TABLE IV. The masses and binding energies (in MeV) of the $ssu\bar{u}$ systems in ChQM.

Channel	$IJ^P = 0\frac{1}{2}^-$	$IJ^P = 0\frac{3}{2}^-$	$IJ^P = 0\frac{5}{2}^-$	$IJ^P = 1\frac{1}{2}^-$	$IJ^P = 1\frac{3}{2}^-$	$IJ^P = 1\frac{5}{2}^-$	E_{th}
$\Xi\bar{K}$	1899.5	...	...	1899.7	...	...	1891
$\Xi\bar{K}^*$	2279.9/−8.1	2296.6	...	2292.3	2297.3	...	2288
$\Xi^*\bar{K}^*$	2368.1	2306.9/−58.1	2374.3	2368.1	2351.4/−13.6	2371.3	2365
$\Xi^*\bar{K}$	...	1975.4	...	...	1976.1	...	1968
$\Omega\omega$	2417.2	2417.5	2417.8	...	...	...	2410
$\Omega\rho$	...	...	...	2519.6	2519.9	2520.2	2513
$\Omega\eta$	...	1856.4	...	...	...	...	1849
$\Omega\pi$	...	...	...	...	1768.6	...	1761
$E_{\text{c.c.}}$	1899.4	1856.1	2374.1	1899.7	1768.3	2371.2	

After the channel coupling calculation, the lowest energy of this system is still above the threshold of the $\Xi\bar{K}$ channel, which indicates that this system is unbound by the channel coupling. However, the $\Omega\rho$ may turn out to be a resonance state by coupling to open channels, which should be investigated in the scattering process of the open channels. For the $IJ^P = 1\frac{3}{2}^-$ system, similar results to the case of $IJ^P = 0\frac{3}{2}^-$ system is obtained. The single-channel calculation shows that both the $\Xi^*\bar{K}^*$ and $\Omega\rho$ are bound to state, but both of them can be coupled to the open channels. The scattering process of the open channels will be shown in the next section. Besides, stable energy is obtained by the channel coupling calculation, the mass of which is -38.2 MeV lower than the threshold of $\Omega\pi$. So $IJ^P = 1\frac{3}{2}^-$ system is possible to a bound state after the channel coupling. For the $IJ^P = 1\frac{5}{2}^-$ system, there is no bound state no matter the single-channel calculation or the channel coupling calculation.

In the ChQM, for the system with $IJ^P = 0\frac{1}{2}^-$, although there are three channels ($\Xi\bar{K}^*$, $\Xi^*\bar{K}^*$, and $\Omega\omega$), who have attractive potentials, the $\Xi^*\bar{K}^*$ and $\Omega\omega$ are unbound. It is reasonable. As shown in Fig. 2, the effective potential of $\Xi^*\bar{K}^*$ and $\Omega\omega$ is weakly attractive, so neither $\Xi^*\bar{K}^*$ nor $\Omega\omega$ is bound here. However, the attraction between Ξ and $\bar{K}^*$ is strong enough to bind Ξ and $\bar{K}^*$, so the $\Xi\bar{K}^*$ is a bound state with the binding energy of -8.1 MeV in the single calculation. Considering the multichannel coupling, the results show that no bound state exists because the channel coupling cannot push the lowest energy below the threshold of the $\Xi\bar{K}$. However, we should check if $\Xi\bar{K}^*$ is a resonance state by coupling the open channel in the following work. For the $IJ^P = 0\frac{3}{2}^-$ system, since the attraction of the $\Xi^*\bar{K}^*$ state is the largest one, a bound state with the binding energy of -58.1 MeV is obtained in the single-channel calculation. However, the either $\Omega\omega$ or $\Omega\eta$ is bound due to the weak attraction in these two channels. For the $IJ^P = 0\frac{5}{2}^-$ system, the attraction potential between Ω and ω is not enough to make a bound state. For the $I = 1$ systems, there is only one channel $\Xi^*\bar{K}^*$ with $J^P = \frac{3}{2}^-$, which can form a bound state in a single-channel calculation, because of the strong attraction between Ξ^* and $\bar{K}^*$ as shown in Fig. 2. Other channels are all unbound, even with the help of multichannel coupling calculation. However, we still have to investigate the scattering process of the corresponding open channels to confirm if this $\Xi^*\bar{K}^*$ is a resonance state or not.

Comparing the results of two quark models, although different models obtain several different bound states, some similar conclusions are reached. We find that the interaction between Ξ^* and $\bar{K}$ is repulsive, so it is difficult to form a $\Xi^*\bar{K}$ bound state. Therefore, the $\Omega(2012)$ cannot be explained as a $J^P = \frac{3}{2}^-$ $\Xi^*\bar{K}$ molecular state in present work. However, the interaction between Ξ^* and $\bar{K}^*$ is strong enough to form a bound state in both QDCSM and ChQM with $I = 0$ or 1 and $J^P = \frac{3}{2}^-$, so it is more possible to be a resonance state in the scattering process of the open channels. To check whether or not the $\Xi^*\bar{K}^*$, as well as other single-channel bound states can survive as resonance states after coupling to the open channels, the study of the scattering process of the open channels is performed in the following section.

FIG. 3. The phase shifts of the open channels in QDCSM, and (a) $IJ^P = 0\frac{1}{2}^-$, (b) $IJ^P = 0\frac{3}{2}^-$, (c) $IJ^P = 1\frac{1}{2}^-$, (d) $IJ^P = 1\frac{3}{2}^-$.

C. Resonance states

In this section, we calculate the scattering phase shifts of all the open channels in both QDCSM and ChQM, the results of which are shown in Figs. 3 and 4, respectively. The energy and decay width of the resonance states can be obtained from the phase shifts, and they are listed in Tables V–VII, where M_r is the resonance mass, Γ_i is the partial decay width of the resonance state, and Γ_{total} is the total decay width. We should mention that all of the states that we study here are in the S -wave. The S -wave bound states that decay to D -wave open channels through tensor interactions are neglected here due to their small decay widths, so the total decay widths of the states given below are the lower limits. The general features of the calculated results are as follows.

In QDCSM, for the $IJ^P = 0\frac{1}{2}^-$ system, the bound state $\Omega\omega$ can decay to three open channels: the S -wave $\Xi\bar{K}$, $\Xi\bar{K}^*$, and $\Xi^*\bar{K}^*$. The scattering phase shifts of these three open channels

FIG. 4. The phase shifts of the open channels in ChQM, and (a) $IJ^P = 0\frac{3}{2}^-$, (b) $IJ^P = 1\frac{3}{2}^-$.

TABLE V. The resonance mass and decay width (in MeV) of the molecular pentaquarks with $J^P = \frac{1}{2}^-$ in QDCSM.

Open channels	$\Omega\omega(I=0)$		$\Omega\rho(I=1)$	
	M_r	Γ_i	M_r	Γ_i
$\Xi\bar{K}^*$	2446	80	2500	30
$\Xi^*\bar{K}^*$	2464	7	2511	39
$\Xi\bar{K}$	—	—	—	—
Γ_{total}	87		69	

are shown in Fig. 3(a), from which we can see that there is no resonance state in the $\Xi\bar{K}$ channel, while the $\Omega\omega$ appears as a resonance state in both $\Xi\bar{K}^*$ and $\Xi^*\bar{K}^*$ channels, with the resonance energy of 2446 MeV and 2464 MeV, and the total decay width of 87 MeV. This indicates that the $\Omega\omega$ with $IJ^P = 0\frac{1}{2}^-$ is possible to be a broad resonance state in the QDCSM.

For the $IJ^P = 0\frac{3}{2}^-$ system, Fig. 3(b) shows the phase shifts of the open channels $\Xi\bar{K}^*$, $\Xi^*\bar{K}$ and $\Omega\eta$. There are two resonance states in both the $\Xi\bar{K}^*$ and $\Xi^*\bar{K}$ scattering phase shifts corresponding to $\Xi^*\bar{K}^*$ and $\Omega\omega$ respectively, while in the $\Omega\eta$ scattering channel there is only one resonance state corresponding to $\Xi^*\bar{K}^*$. The reason is that the channel coupling pushes the higher state $\Omega\omega$ above the threshold. The resonance masses and decay widths of resonance states are listed in Table VI, which shows that the $IJ^P = 0\frac{3}{2}^-$ $\Xi^*\bar{K}^*$ is a resonance state with energy of 2328–2374 MeV and decay width of 57 MeV, and $IJ^P = 0\frac{3}{2}^-$ $\Omega\omega$ is a resonance state with energy of 2477–2479 MeV and decay width of 60 MeV.

The results of the $IJ^P = 1\frac{1}{2}^-$ system is similar to that of the $IJ^P = 0\frac{1}{2}^-$ system. Figure 3(c) shows that there is no resonance state in the $\Xi\bar{K}$ channel, while the resonance state $\Omega\rho$ appears clearly in both the $\Xi\bar{K}^*$ and $\Xi^*\bar{K}^*$ channels. The resonance energy and decay width are listed in Table V, which are 2500–2511 MeV and 69 MeV, respectively.

 TABLE VI. The resonance mass and decay width (in MeV) of the molecular pentaquarks with $J^P = \frac{3}{2}^-$ in QDCSM.

Open channels	$\Omega\omega(I=0)$		$\Xi^*\bar{K}^*(I=0)$	
	M_r	Γ_i	M_r	Γ_i
$\Xi\bar{K}^*$	2479	39	2333	15
$\Xi^*\bar{K}$	2477	21	2374	0.001
$\Omega\eta$	—	—	2328	42
Γ_{total}	60		57	
Open channels	$\Omega\rho(I=1)$		$\Xi^*\bar{K}^*(I=1)$	
	M_r	Γ_i	M_r	Γ_i
$\Xi\bar{K}^*$	2522	27	2342	20
$\Xi^*\bar{K}$	2528	72	2386	50
$\Omega\pi$	—	—	2362	30
Γ_{total}	99		100	

 TABLE VII. The resonance mass and decay width (in MeV) of the molecular pentaquarks with $J^P = \frac{3}{2}^-$ in ChQM.

Open channels	$\Xi^*\bar{K}^*(I=0)$		$\Xi^*\bar{K}^*(I=1)$	
	M_r	Γ_i	M_r	Γ_i
$\Xi^*\bar{K}$	2352	0.5	2350	1.5
$\Omega\eta$	2337	65	—	—
$\Omega\pi$	—	—	2341	30
Γ_{total}	65.5		31.5	

For the $IJ^P = 1\frac{3}{2}^-$ system, there are two singly-bound states $\Omega\rho$ and $\Xi^*\bar{K}^*$, which can decay to three open channels: the S -wave $\Xi\bar{K}^*$, $\Xi^*\bar{K}$, and $\Omega\pi$. The phase shifts of these open channels are shown in Fig. 3(d), from which we can see that both $\Omega\rho$ and $\Xi^*\bar{K}^*$ survive as resonance states in the $\Xi\bar{K}^*$ and $\Xi^*\bar{K}$ scattering process. However, in the $\Omega\pi$ scattering channel, there is only one resonance state, which is $\Xi^*\bar{K}^*$ state, since the channel coupling pushes the higher state $\Omega\rho$ above its threshold. From Table VI, we can see that the resonance energy of $IJ^P = 1\frac{3}{2}^-$ $\Xi^*\bar{K}^*$ is 2342–2386 MeV and decay width is 100 MeV, and the resonance energy of $IJ^P = 1\frac{3}{2}^-$ $\Omega\rho$ is 2522–2528 MeV and decay width is 99 MeV.

With regards to the ChQM, for the $IJ^P = 0\frac{1}{2}^-$ system, we study the scattering process of the open channel $\Xi\bar{K}$, and we do not find any resonance state. The reason is that the channel coupling to the $\Xi\bar{K}$ pushes the energy of $\Xi\bar{K}^*$ above its threshold. For the $IJ^P = 0\frac{3}{2}^-$ system, the singly-bound state $\Xi^*\bar{K}^*$ can be coupled to three open channels: $\Xi\bar{K}^*$, $\Xi^*\bar{K}$, and $\Omega\eta$, the scattering phase shifts of which are shown in Fig. 4(a). It is obvious in Fig. 4(a) that the $\Xi^*\bar{K}^*$ appears as a resonance state in both $\Xi^*\bar{K}$ and $\Omega\eta$ scattering process, while in the $\Xi\bar{K}^*$ channel, there are no resonance states because the channel coupling pushes the higher state $\Xi^*\bar{K}^*$ above its threshold.

The results of the $IJ^P = 1\frac{3}{2}^-$ system is similar to that of the $IJ^P = 0\frac{3}{2}^-$ system. The phase shifts of three open channels are shown in Fig. 4(b), from which we can see that the singly-bound state $\Xi^*\bar{K}^*$ appears as a resonance state in both $\Xi^*\bar{K}$ and $\Omega\pi$ phase shifts, while in the $\Xi\bar{K}^*$ channel, there is no resonance states.

The resonance energy and decay width of the $\Xi^*\bar{K}^*$ with $I=0$ and 1, and $J^P = \frac{3}{2}^-$ are listed in Table VII, from which we can see that the resonance energy of the $IJ^P = 0\frac{3}{2}^-$ $\Xi^*\bar{K}^*$ is 2337–2352 MeV and decay width is 65.5 MeV, and those of the $IJ^P = 1\frac{3}{2}^-$ $\Xi^*\bar{K}^*$ is 2341–2350 MeV and decay width is 31.5 MeV.

IV. SUMMARY

In this work, we systematically investigate the pentaquark systems composed of $sssu\bar{u}$ in the framework of two kinds of quark models, which are QDCSM and ChQM. Our purpose is to see whether the $\Omega(2012)$ can be explained as the molecular pentaquarks. Moreover, we also attempt to explore if there is any other pentaquark state. Our results show that:

- (1) Although some work considered the $\Omega(212)$ as a $J^P = \frac{3}{2}^- \Xi^* \bar{K}$ molecular state [19–21] because its mass is very close to the $\Xi^* \bar{K}$ threshold, we find that the interaction between Ξ^* and $\bar{K}$ is repulsive, which leads to the difficulty to form a $\Xi^* \bar{K}$ bound state. Therefore, the $\Omega(212)$ cannot be explained as a $J^P = \frac{3}{2}^- \Xi^* K$ molecular state in present work. Ref. [39] investigated the $1S$, $2S$, $1P$, and $2P$ states of light baryons within the chiral quark model, and found that the baryons could not be used to explain the $\Omega(212)$ well. Therefore, the $\Omega(212)$ is more possible to be a mixture state of the three-quark system and five-quark system. An unquenched quark model can be used to study this state, which is our next work.
- (2) We also find several bound states and resonance states in present work. The $sssu\bar{u}$ systems with both $IJ^P = 0\frac{3}{2}^-$ and $IJ^P = 1\frac{3}{2}^-$ are bound by the multichannel coupling in QDCSM. We also obtain four resonance states in QDCSM, which are $\Omega\omega$ with $IJ^P = 0\frac{1}{2}^-$ ($M = 2446$ – 2464 MeV, $\Gamma =$

87 MeV) and $IJ^P = 0\frac{3}{2}^-$ ($M = 2477$ – 2479 MeV, $\Gamma = 60$ MeV), and $\Omega\rho$ with $IJ^P = 1\frac{1}{2}^-$ ($M = 2500$ – 2511 MeV, $\Gamma = 69$ MeV) and $IJ^P = 1\frac{3}{2}^-$ ($M = 2522$ – 2528 MeV, $\Gamma = 99$ MeV). However, all these states cannot be obtained in ChQM. So more evidence should be explored to confirm the existence of these states. Nevertheless, the $\Xi^* \bar{K}^*$ with $IJ^P = 0\frac{3}{2}^-$ ($M = 2328$ – 2374 MeV, $\Gamma = 57$ – 65.5 MeV) and $IJ^P = 1\frac{3}{2}^-$ ($M = 2341$ – 2386 MeV, $\Gamma = 31.5$ – 100 MeV) are verified to be resonance states in both QDCSM and ChQM. So these two resonance states are worthy of being searched by future experiments.

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