Erratum: Subleading contributions to the three-nucleon contact interaction **[Phys. Rev. C 84, 014001 (2011)]**

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As a result of a careful reanalysis of the Fierz relations, we find additional contributions to the short-range three-nucleon potential as compared to the ones already identified. In total, it depends on 13 low-energy constants instead of 10. We found a mistake in the code generating the Fierz relations listed in the Appendix, which affects Eqs. (A2), (A6), (A12), (A61), (A62), (A84), (A88), (A102), and (A103) of the original paper. They should be replaced by the following ones:¹

$$2o_{26} = o_{80} - o_{82} + 2o_{85} - 2o_{86} + o_{120} - o_{122} + 2o_{125} - 2o_{126}, \quad (\text{A2})$$

$$2o_{58} = o_{82} - o_{80} + o_{122} - o_{120}, \quad (\text{A6})$$

$$4o_{83} = o_{79} + o_{81} + o_{83} + o_{84} + o_{119} + o_{121} + o_{123} + o_{124}, \quad (\text{A12})$$

$$4o_{79} = -o_{79} - o_{81} - o_{119} - o_{121}, \quad (\text{A61})$$

$$4o_{80} = 2o_{80} + 2o_{82} - o_{58} - o_{64}, \quad (\text{A62})$$

$$2o_{25} = o_{76} - o_{77} + o_{116} - o_{117}, \quad (\text{A84})$$

$$2o_{57} = o_{82} - o_{80} + o_{122} - o_{120}, \quad (\text{A88})$$

$$4o_{75} = -o_{75} - o_{78} - o_{115} - o_{118}, \quad (\text{A102})$$

$$4o_{76} = o_{25} + o_{31} - o_{76} - o_{77} - o_{116} - o_{117}. \quad (\text{A103})$$

By combining them with the remaining relations, it turns out that only 128 are linearly independent, instead of 132 as stated in the original paper. Correspondingly, the independent operators are 18 instead of 14. They can be chosen to be

$$o_1, \quad o_2, \quad o_4, \quad o_5, \quad o_7, \quad o_{33}, \quad o_{34}, \quad o_{35}, \quad o_{36}, \quad o_{37}, \quad o_{38}, \quad o_{39}, \quad o_{40}, \quad o_{42}, \quad o_{43}, \quad o_{45}, \quad o_{58}, \quad o_{64}. \quad (8)$$

The operators o_1 , o_2 , o_4 , o_5 , and o_7 are eliminated by the constraints of relativity. All the remaining 13 independent operators enter the minimal two-derivative three-nucleon contact Lagrangian in Eq. (14), which should read

$$\begin{aligned} \mathcal{L}_{3N}^{(2)} = & E'_1 \vec{\nabla} (N^\dagger N) \cdot \vec{\nabla} (N^\dagger N) N^\dagger N + E'_2 \vec{\nabla} (N^\dagger \tau^a N) \cdot \vec{\nabla} (N^\dagger \tau^a N) N^\dagger N \\ & + E'_3 \vec{\nabla} (N^\dagger \tau^a N) \cdot \vec{\nabla} (N^\dagger N) N^\dagger \tau^a N + E'_4 \vec{\nabla} \cdot (N^\dagger \vec{\sigma} N) \vec{\nabla} \cdot (N^\dagger \vec{\sigma} N) N^\dagger N \\ & + E'_5 \vec{\nabla} \cdot (N^\dagger \vec{\sigma} \tau^a N) \vec{\nabla} \cdot (N^\dagger \vec{\sigma} \tau^a N) N^\dagger N + E'_6 \vec{\nabla} \cdot (N^\dagger \vec{\sigma} \tau^a N) \vec{\nabla} \cdot (N^\dagger \vec{\sigma} N) N^\dagger \tau^a N \\ & + E'_7 \vec{\nabla}_i (N^\dagger \sigma^j N) \vec{\nabla}_j (N^\dagger \sigma^i N) N^\dagger N + E'_8 \vec{\nabla}_i (N^\dagger \sigma^j \tau^a N) \vec{\nabla}_j (N^\dagger \sigma^i \tau^a N) N^\dagger N \\ & + E'_9 \vec{\nabla}_i (N^\dagger \sigma^j N) \vec{\nabla}_i (N^\dagger \sigma^j N) N^\dagger N + E'_{10} \vec{\nabla}_i (N^\dagger \sigma^j \tau^a N) \vec{\nabla}_i (N^\dagger \sigma^j \tau^a N) N^\dagger N \\ & + E'_{11} \vec{\nabla} \cdot (N^\dagger \vec{\sigma} N) \vec{\nabla} (N^\dagger N) \cdot N^\dagger \vec{\sigma} N + E'_{12} \epsilon^{abc} \vec{\nabla} (N^\dagger \tau^a N) \times \vec{\nabla} (N^\dagger \tau^b N) \cdot N^\dagger \vec{\sigma} \tau^c N \\ & + E'_{13} \epsilon^{abc} \vec{\nabla} (N^\dagger \sigma^i \tau^a N) \times \vec{\nabla} (N^\dagger \sigma^i \tau^b N) \cdot N^\dagger \vec{\sigma} \tau^c N. \end{aligned} \quad (14)$$

¹We number the equations as the corresponding ones in the original paper.

Denoting $\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}'_i$ and $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}'_i$, $\mathbf{p}_i$ and $\mathbf{p}'_i$ being the initial and final momenta of nucleon i , the resulting momentum-space potential reads

$$\begin{aligned}
 V = \sum_{i \neq j \neq k} & \left[-E_1 \mathbf{k}_i^2 - E_2 \mathbf{k}_i^2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - E_3 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - E_4 \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - E_5 (3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j - \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \right. \\
 & - E_6 (3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j - \mathbf{k}_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - \frac{i}{4} E_7 \mathbf{k}_i \times (\mathbf{Q}_i - \mathbf{Q}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \\
 & - \frac{i}{4} E_8 \mathbf{k}_i \times (\mathbf{Q}_i - \mathbf{Q}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k - E_9 \mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j - E_{10} \mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\
 & \left. - E_{11} \mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i - E_{12} \mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j - E_{13} \mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k \right], \tag{15}
 \end{aligned}$$

with

$$\begin{aligned}
 E_1 &= \frac{1}{2} E'_1 + \frac{3}{2} E'_2 - \frac{3}{2} E'_3 - \frac{1}{2} E'_9 + \frac{3}{2} E'_{10}, \\
 E_2 &= E'_2 + E'_6 + 2E'_{12} - 8E'_{13}, \\
 E_3 &= E'_2 - E'_3 - E'_6 + \frac{1}{3} E'_{11} - 2E'_{12} + 8E'_{13}, \\
 E_4 &= \frac{1}{3} E'_2 - \frac{1}{3} E'_3 - \frac{1}{3} E'_6 - \frac{1}{3} E'_9 + E'_{10} - \frac{2}{3} E'_{12} + \frac{8}{3} E'_{13}, \\
 E_5 &= E'_2 - E'_3 - E'_6 - E'_9 + \frac{1}{3} E'_{11} - 2E'_{12} + 8E'_{13}, \\
 E_6 &= \frac{1}{3} E'_2 - \frac{1}{3} E'_3 - \frac{1}{3} E'_6 - \frac{1}{3} E'_9 - \frac{2}{3} E'_{12} + \frac{8}{3} E'_{13}, \\
 E_7 &= -8E'_{12} + 40E'_{13}, \\
 E_8 &= -4E'_{12} + 12E'_{13}, \\
 E_9 &= \frac{3}{2} E'_2 - \frac{3}{2} E'_3 - E'_4 - \frac{3}{2} E'_9 + E'_{11} - 3E'_{12} + 12E'_{13}, \\
 E_{10} &= \frac{1}{2} E'_2 - \frac{1}{2} E'_3 - E'_5 - \frac{1}{2} E'_9 - E'_{12} + 6E'_{13}, \\
 E_{11} &= \frac{3}{2} E'_2 - \frac{3}{2} E'_3 - E'_7 - \frac{3}{2} E'_9 + 3E'_{12} - 12E'_{13}, \\
 E_{12} &= \frac{1}{2} E'_2 - \frac{1}{2} E'_3 - E'_8 - \frac{1}{2} E'_9 + E'_{12} - 6E'_{13}, \\
 E_{13} &= E'_6 + 4E'_{12} - 16E'_{13}. \tag{16}
 \end{aligned}$$

Its coordinate space expression, using the same momentum cutoff as in the original paper,

$$Z_0(r; \Lambda) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p} \cdot \mathbf{r}} F(\mathbf{p}^2; \Lambda), \tag{17}$$

is

$$\begin{aligned}
 V = \sum_{i \neq j \neq k} & (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2 \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
 & + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k) (\mathbf{L} \cdot \mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
 & + [(E_9 + E_{10} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} + (E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ik}] Z_0'(r_{ij}) Z_0'(r_{ik}) \tag{18}
 \end{aligned}$$

where S_{ij} and $(\mathbf{L} \cdot \mathbf{S})_{ij}$, respectively, are the tensor and spin-orbit operators for particles i and j . Therefore, the subleading three-nucleon contact potential depends on the three additional low-energy constants E_{11} – E_{13} besides the already identified E_1 – E_{10} .