Exclusive production ratio of neutral to charged kaon pair in e^+e^- annihilation continuum via a relativistic quark model

Yi Jin^{1,*}, Shi-Yuan Li^{2,†}, Yan-Rui Liu², Zhao-Xia Meng¹, Zong-Guo Si², and Tao Yao²

¹*School of Physics and Technology, University of Jinan, Jinan 250022, People's Republic of China*

²*School of Physics, Shandong University, Jinan 250100, People's Republic of China*

(Received 20 August 2019; revised 18 February 2020; accepted 6 April 2020; published 2 July 2020)

A completely relativistic quark model in the Bethe-Salpeter framework is employed to calculate the exclusive production ratio of the neutral to charged kaon pair in e^+e^- annihilation continuum region below the J/Ψ mass. The valence quark charge plays the key role. The cancellation of the diagrams for the same charge case ($K_S + K_L$) and the noncancellation of the diagrams for the different charge case ($K^- + K^+$) lead to the ratio as $(m_s - m_d)^2/M_{kaon}^2 \approx 1/10$.

DOI: [10.1103/PhysRevC.102.015201](https://doi.org/10.1103/PhysRevC.102.015201)

In the low energy region below the J/Ψ mass, it is generally difficult to employ a quark degree of freedom to calculate for the hadron production, since the perturbative QCD calculation is not valid any more. However, in some simple exclusive processes, we can employ the quarks in the Bethe-Salpeter (BS) framework to investigate. The information of strong interaction between quarks can be absorbed into the BS wave function (or BS vertex) to some approximation level. As a matter of fact, this idea emerged just at the time of the birth of quark model before the birth of QCD. One of the most famous examples is the Chinese ‘Straton Model’ from the mid-1960s, namely, ‘Relativistic Structure Theory Of Mesons And Baryons’ [1,2]. Almost all these kind of works deal with decay processes rather than production ones, while the latter are more complex.

The aim of this paper is to investigate the exclusive production ratio of the neutral to charged kaon pair in e^+e^- annihilation continuum below the J/Ψ mass under the spirit of the Straton model. More concretely, we employ a completely relativistic BS framework to describe the coupling of the virtual photon to the kaons via the triangle quark loop as shown in Fig. 1. The photon-quark-quark vertex is exactly that in the standard model. The vertices between the quarks and the corresponding kaon can be given by the BS wave function of the kaon in term of ‘valence’ quark field. When a resonance is produced in e^+e^- annihilation process, the production ratios of its decay channels at the relevant $\sqrt{s}$ are mainly determined by the properties of the resonance. On

the other hand, in the continuum region, the mechanism is different. For this case, the electromagnetic interaction and nonperturbative QCD interaction are separately assigned in the diagrams of Fig. 1.

Because of the scalar function of the momentum in the quark-quark-hadron BS vertex, the loop integral is finite. The ratio can be calculated straightforwardly. The calculation demonstrates how the naive suppression on the production rate of the neutral kaon pair with respect to that of the charged pair is obtained. This is consistent with the fact that the coupling of neutral particles with the photon is suppressed with respect to the case of the charged ones [3]. The experiment [3] shows that the suppression can be of one order of magnitude. Our calculation obtains the suppression factor as $(\Delta m/M)^2$, where Δm is the difference between the down quark mass and the strange quark mass, while M is the kaon mass.

In quantum field theory, the BS wave function χ is written as

$$\begin{aligned}\chi(P, q) &= \int \frac{d^4x}{(2\pi)^4} e^{-iqx} \frac{1}{\sqrt{3}} \delta_{ij} \langle 0 | T \psi^i\left(\frac{x}{2}\right) \bar{\psi}^j\left(-\frac{x}{2}\right) | B(P) \rangle \\ &=: S_F(p_1) \Gamma(P, q) S_F(-p_2),\end{aligned}\quad (1)$$

and hence the BS vertex $\Gamma(P, q)$ is defined. Here, P is the four-momentum of the bound state $|B\rangle$, while q is the four-momentum to describe the inner movement of the constituent particles. They can be expressed as the linear combination by two four-momenta p_1 and p_2 of the propagators, $S_F(p_1)$ and $S_F(p_2)$, and vice versa. The only restriction of these momenta is $P^2 = M_B^2$ with M_B the mass of the bound state. We emphasize that the constituent particles can be off mass shell in the BS framework, and are treated completely as propagators.

The BS framework has been applied to describe light and heavy hadrons in term of constituent quarks, as well as hadron molecule (e.g., [4–6]). The BS wave function can be obtained by fitting corresponding data for each kind of meson, in term of the valence quark-antiquark field. To go further, one can set up some certain model for the structure of each kind of multistates of hadrons, and get the BS wave function by

*Corresponding author: ss_jiny@ujn.edu.cn

†Corresponding author: lishy@sdu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

FIG. 1. Triangle diagrams for the exclusive productions of $e^+e^- \rightarrow K^0\bar{K}^0$ and $e^+e^- \rightarrow K^+K^-$. The four-momenta of the corresponding kaons and quark propagator are shown in the bracket.

solving the BS equation. For this purpose some of the model parameters may need to be fixed by data. Here, since we only calculate the ratio, we will not go into the details of the functional of the scalar function $\phi_P(q^2)$ [hereafter we will write as $\phi(q^2)$ for simplicity] in the BS vertex,

$$\Gamma(P, q) = \Omega^P \cdot \phi(q^2). \quad (2)$$

Here, the Dirac structure Ω^P [7,8] is arranged as

$$\Omega^P = \gamma_5 - i\gamma_5 \not{P} \frac{B_1}{M} - i\gamma_5 \not{q} \frac{B_2}{M} - \gamma_5 (\not{P} \not{q} - \not{q} \not{P}) \frac{B_3}{M^2}. \quad (3)$$

In the above equation, B_i ($i = 1, 2, 3$) are the dimensionless coefficients. From this explicit form, a power counting rule according to powers of q/M [9–12] can be directly read out and the first two terms are of the leading order, because they both belong to the $(q/M)^0$ order.

Since we have separated out the factor $\not{q}$ from the vertex, the scalar wave function ϕ can be taken as a function of q^2 . The normalization of the BS wave function is absorbed into $\phi(q^2)$.

As in Ref. [13], the amplitude for Fig. 1(a) is written as

$$\begin{aligned} \mathcal{M}_1 = & \frac{-1}{3} \frac{ie^2}{K^2} \bar{v}_{e^+} \gamma_\mu u_{e^-} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{[(q+k_1)^2 - m_s^2][(q-k_2)^2 - m_s^2]} \\ & \times \text{Tr}[\gamma^\mu (\not{q} - \not{k}_2 + m_s) \Gamma_2(\not{q} + m_d) \Gamma_1(\not{q} + \not{k}_1 + m_s)] \phi_{k_1}(q^2) \phi_{k_2}(q^2), \end{aligned} \quad (4)$$

where we only consider the leading Dirac structures, i.e., $\Gamma_2 = \gamma^5(1 + B_1 \not{k}_2/M)$ and $\Gamma_1 = \gamma^5(1 + B_1 \not{k}_1/M)$ with $K = k_1 + k_2$ [10,12]. The other three amplitudes for Fig. 1(b)–1(d) are, respectively,

$$\begin{aligned} \mathcal{M}_2 = & \frac{-1}{3} \frac{ie^2}{K^2} \bar{v}_{e^+} \gamma_\mu u_{e^-} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{[(q-k_1)^2 - m_d^2](q^2 - m_s^2)[(q+k_2)^2 - m_d^2]} \\ & \times \text{Tr}[\gamma^\mu (\not{q} - \not{k}_1 + m_d) \Gamma_1(\not{q} + m_s) \Gamma_2(\not{q} + \not{k}_2 + m_d)] \phi_{k_1}(q^2) \phi_{k_2}(q^2), \\ \mathcal{M}'_1 = & \frac{-1}{3} \frac{ie^2}{K^2} \bar{v}_{e^+} \gamma_\mu u_{e^-} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{[(q+k_1)^2 - m_s^2](q^2 - m_u^2)[(q-k_2)^2 - m_s^2]} \end{aligned} \quad (5)$$

$$\times \text{Tr}[\gamma^\mu (\not{q} - \not{k}_2 + m_s) \Gamma_2 (\not{q} + m_u) \Gamma_1 (\not{q} + \not{k}_1 + m_s)] \phi_{k_1}(q^2) \phi_{k_2}(q^2), \quad (6)$$

$$\begin{aligned} \mathcal{M}'_2 = & \frac{2ie^2}{3K^2} \bar{v}_{e^+} \gamma_\mu u_{e^-} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{[(q - k_1)^2 - m_u^2] (q^2 - m_s^2) [(q + k_2)^2 - m_u^2]} \\ & \times \text{Tr}[\gamma^\mu (\not{q} - \not{k}_1 + m_u) \Gamma_1 (\not{q} + m_s) \Gamma_2 (\not{q} + \not{k}_2 + m_u)] \phi_{k_1}(q^2) \phi_{k_2}(q^2). \end{aligned} \quad (7)$$

Here, (ϕ_{k_1}, ϕ_{k_2}) , respectively, refers to the scalar wave function in the vertex for $(\bar{K}^0, K^0)$ or (K^-, K^+) , and m_u, m_d, m_s are the masses of the quarks. In this work, we adopt the SU(2) symmetry and thus the scalar wave functions of these four kaons are all the same in term of the inner momentum properly set. We use the loop variable q as the argument. It is not the original argument of ϕ as defined in Eq. (2), which is the relative momentum of the two constituents. This specific choice makes the functional forms of the $\phi_{k_1}(q^2)$ and $\phi_{k_2}(q^2)$ could be different.

If one only takes the γ^5 term in the BS vertex, i.e., $B_1 = 0$, one can find the exact relation $\mathcal{M}_1 = -\mathcal{M}_2$ with a straightforward calculation. Therefore, these two diagrams for the $K^0 \bar{K}^0$ case [Fig. 1(a) and (b)] completely cancel. On the contrary, we get $\mathcal{M}'_1 + \mathcal{M}'_2 = 3\mathcal{M}'_1$ for the $K^+ K^-$ case [Fig. 1(c) and (d)]. Conventionally, before the power counting rule [9–12] was suggested, it is considered that the leading Dirac structure is only γ^5 . If one adopts this, the process $e^+ e^- \rightarrow K_L + K_S$ is vanishing with respect to the process $e^+ e^- \rightarrow K^- + K^+$ (K_S and K_L are just the perpendicular states constructed as the linear combination from the other two states K^0 and $\bar{K}^0$).

Here, we address this simplified result, just to call the attention on the fact that, when the coupling to the photon via the valence quark is the same for the corresponding two diagrams, they completely cancel for the vertex taken as γ^5 . This is the case for the down quark and strange quark having the same charge [Fig. 1 (a) and (b)]. Because the charge of the up quark and strange quark are different, especially with opposite sign, the corresponding two diagrams do not cancel, rather are enhanced [Fig. 1 (c) and (d)]. However, as pointed out above, the power counting rule [9–12] suggests that the leading Dirac structure includes two terms rather than one. This has significantly improved the description on the decay constant [9–12]. Especially, the pion decay constant data can be naturally included and well described without any special prioritization on the wave function in our previous papers [9–12].

Adopting the vertex as $\gamma^5(1 + B_1 \not{P}/M)\phi(q^2)$ [the normalization factor in $\phi(q^2)$ will be changed], and by straightforward calculations, one can conclude that the neutral to charged production ratio is

$$\frac{\sigma(e^+ e^- \rightarrow K_S K_L)}{\sigma(e^+ e^- \rightarrow K^+ K^-)} \cong \left(\frac{\Delta m}{M}\right)^2 \quad (8)$$

with $\Delta m = m_s - m_d$. The significant contribution by the loop integral on q is tamed by the scalar wave function for $\sqrt{s}$ smaller than the J/Ψ mass, leading to its contribution of the order of kaon mass. In this improved result with the full leading Dirac structure of the vertex, one finds that the

cancellation between (a) and (b) in Fig. 1 still works, but not completely, rather, leading to the Δm factor.

In the BS framework, the light quark masses are parameters. Generally the constituent quark mass values are adopted and can work well [9–12], hence the value of Δm is around 160 MeV. With the kaon mass $M \approx 500$ MeV, we get a rough estimation $(\Delta m/M)^2 \approx 1/10$, i.e., one order of magnitude as the experiment [3] indicated. The above calculations also show that, to get the quantitative result for the ratio between these two exclusive processes, especially the dependence on the difference of the value of the quark masses, the full leading Dirac structures that the power counting rule [9] dictates, i.e., the first two terms of Eq. (3), are necessary.

Here, we calculate the ratio $\frac{\sigma(e^+ e^- \rightarrow K^+ K^-)}{\sigma(e^+ e^- \rightarrow K_S K_L)}$ from data of Ref. [3] (Fig. 2) and compare with our calculation. For easy to compare with further data we take the inverse of Eq. (8), which is 9.8 with $\Delta m = 160$ MeV. The fitting on the data gives a ratio of 8.3, which corresponds to $\Delta m = 170$ MeV. We would like to address that the data include the contribution of resonances which can not be singled out by a GLOBAL fitting. The contribution of resonances will make the production rates of the neutral kaon pair and of the charged pair more near to each other. In Fig. 2, we draw the line 9.8

FIG. 2. The ratio $\frac{\sigma(e^+ e^- \rightarrow K^+ K^-)}{\sigma(e^+ e^- \rightarrow K_S K_L)}$ from data of Ref. [3] comparing with our calculation, which is 9.8 with $\Delta m = 160$ MeV (solid line). The global fitting on the data gives a ratio of 8.3, which corresponds to $\Delta m = 170$ MeV (dotted line).

with $\Delta m = 160$ MeV up to 3 GeV as the prediction for the forthcoming BESIII continuum region data.

Besides the ground state kaons, the excited states such as K^* , etc., can also be studied. Without referring to the full calculation, but just investigating the relative sign between two contributing diagrams based on different Dirac structures in the quark-hadron coupling vertex, one can predict the interesting behaviors: For the case of double vectors ($K^* + K^*$), the ratio of neutral to charged is smaller than one, similar as this ground state case. For the case of one pseudo-scalar with one vector ($K + K^*$), the ratio will turn over to be larger than one. This can be checked by future experiments.

BESIII can give better experimental measurements in energy region above 2.00 GeV on the charged and neutral kaon pair exclusive production processes. For example, production cross sections of K^+K^- have been measured at $\sqrt{s}$ from 2.00 to 3.08 GeV with largely reduced uncertainty comparing to previous experiments [14]. The ratio $\frac{\sigma(e^+e^- \rightarrow K_S K_L)}{\sigma(e^+e^- \rightarrow K^+ K^-)}$ in our study can be better checked once the process of $K_S K_L$ exclusive production were soon to be obtained at BESIII, and the scalar wave function fitting both exclusive processes can also be achieved. A concrete model for the BS wave function to describe the strong interaction has been introduced [9–12].

This model can be used to calculate the cross section and be checked by the data.

In the above analysis, there is still an interesting point worthy to be discussed. The photon and quark interaction is calculated by perturbative QED, which is the expansion arranged by the small electromagnetic coupling constant e . The hadron and quark coupling is described by the BS vertex, and its Dirac structures are arranged according to the powers of q/M . Our calculation is done at the leading order for both parameters. Since there is no expectation to get result with positive powers of M to cancel the arrangement of the Dirac structures according to powers of q/M , we can safely rely on these two parameters as the only independent expansion parameters. In the most general case, the photon-quark-quark vertex may have many Dirac structures. However, the $e\gamma^\mu$ is the leading contribution for photon quark interaction. The reason is as following. Assuming that there exists descriptions for this process by a consistent effective field theory, all the higher order effects can be absorbed into the vertices

and the effective quark propagators for this triangle diagram. For the propagators, the quark mass is only a parameter. For the vertex, first we consider the simplest toy model: the interaction is only QED. The effects of high order in e can lead to Dirac structures like $\sigma^{\mu\nu}$ in the vertex, but they are all suppressed by the small electromagnetic coupling constant e . Same argument is also valid when considering standard model (including perturbative QCD) and/or other perturbatively calculable effective theory. Other Dirac structures are suppressed by a small coupling constant. The difficulty arises when perturbative calculation is not valid, e.g., that soft strong interactions cannot be factorized from this photon-quark-quark vertex. However, in either the continuum or the resonance (heavy photon) case, the current constructed with the vertex coupling to the vector (heavy) photon field must be a vector and has the same quantum number as the (heavy) photon. The conservation of the discrete quantum numbers, e.g., charge conjugation, of strong interaction requires the asymmetric vector current, so that the leading vertex is γ^μ . Other vector-like vertex can be, e.g., $\sigma^{\mu\nu}q_\nu/M$, where q is the small relative momentum between constituent quarks¹ and can give a minus sign under charge conjugation. The small momentum leads to the suppression of order q/M , just the same reason as the power counting rule [9–12]. Therefore, the Dirac structures except γ^μ can contribute, but they are all next to leading order corrections on e or on powers of q/M . The expansion on e and on powers of q/M do not mix at the leading order. The $\Delta m/M$ in our result is not an independent parameter in the above calculations. Taking into account next to leading order corrections on e or on powers of q/M will lead to suppressed correction to this result.

This work is supported by National Natural Science Foundation of China (Grants No. 11635009, No. 11775130, No. 11775132, No. 11605074, and No. U1732263) and the Natural Science Foundation of Shandong Province (Grants No. ZR2018MA047, No. ZR2017MA002, and No. ZR2016AM16).

¹If q is large, it is a perturbative scale.

- [1] PRINT-67-903 (by Research Group of the Theory of Elementary Particles, Peking).
- [2] Hung-yuan Tzu, *Proceedings of the 1980 Guangzhou Conference on Theoretical Physics* (1980), pp. 4–31.
- [3] J. P. Lees *et al.* (BaBar Collaboration), *Phys. Rev. D* **89**, 092002 (2014).
- [4] P. Maris and P. C. Tandy, *Phys. Rev. C* **60**, 055214 (1999).
- [5] W. Lucha and F. F. Schöberl, *Phys. Rev. D* **93**, 056006 (2016).
- [6] Q.-X. Yu and X.-H. Guo, *Nucl. Phys. B* **947**, 114727 (2019).
- [7] Division of Elementary Particles, Laboratory of Theoretical Physics, Peking University, and Laboratory of Theoretical Physics, Institute of Mathematics, Academia Sinica,

- Acta scientiarum naturalium Universitatis Pekinensi*, 1966, 2: 209.
- [8] C. H. Llewellyn-Smith, *Ann. Phys. (NY)* **53**, 521 (1969).
- [9] S. Bhatnagar and S.-Y. Li, *J. Phys.* **32**, 949 (2006).
- [10] S. Bhatnagar and S.-Y. Li, *arXiv:hep-ph/0612084*.
- [11] S. Bhatnagar, S.-Y. Li, and J. Mahecha, *Int. J. Mod. Phys. E* **20**, 1437 (2011).
- [12] S. Bhatnagar, S.-Y. Li, and J. Mahecha, *DAE Symp. Nucl. Phys.* **54**, 516 (2009).
- [13] S.-Y. Li, *Proceedings of the 47th Recontres de Moriond (QCD)* (2012), p. 149.
- [14] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **99**, 032001 (2019).