Erratum: Ultrastrong coupling in the near field of complementary split-ring resonators **[Phys. Rev. B *90*, 205309 (2014)]**

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The equations reported in Sec. II C in the original paper contain multiple errors. First, the relative material permittivity ϵ has been accounted for twice: once directly in the definition of the electromagnetic vector potential [Eq. (2) in the original paper] and a second time via the definition of the effective volume [Eq. (3)]. Since ϵ is varying for the present case, it is convenient to account for its contribution via the effective volume. In this convention, the electromagnetic vector potential is

$$\hat{\mathbf{A}}_{\text{LC}}(\mathbf{r}, t) = \sqrt{\frac{\hbar}{2\epsilon_0\omega V}} [\hat{a}\mathbf{u}(\mathbf{r})e^{-i\omega t} + \hat{a}^\dagger\mathbf{u}^*(\mathbf{r})e^{i\omega t}]. \quad (1)$$

Second, in Eq. (4), the weight factor $(u_x + iu_y)/\sqrt{2}$ was omitted in the sum, and a factor of $\sqrt{2}$, accounting for the circular polarization selection rule of the cyclotron transition, was omitted in the normalization factor. The correct expression for the bright mode creation operator is

$$\hat{b}^\dagger = \frac{1}{\sqrt{N_e}} \sum_k \frac{u_x + iu_y}{\sqrt{2}} \hat{c}_{v,k}^\dagger \hat{c}_{v-1,k}. \quad (2)$$

with

$$N_e = \frac{\rho_{\text{2DEG}}}{v} \int_{\text{2DEG}} \frac{|u_x(\mathbf{r})|^2 + |u_y(\mathbf{r})|^2}{2} dx dy. \quad (3)$$

These two errors lead to a factor of $\sqrt{\epsilon v}$ in Eq. (6) which becomes

$$\Omega_R = \sqrt{\frac{\omega_c e^2 N_e v}{4\epsilon_0 \omega_{\text{LC}} V m^*}}. \quad (4)$$

Moreover, in Eq. (7) a factor of $\sqrt{\epsilon/8\pi}$ is missing

$$\frac{\Omega_R}{\omega_c} = \sqrt{\frac{\lambda_{\text{LC}}}{V}} \sqrt{\int_{\text{2DEG}} \frac{|u_x|^2 + |u_y|^2}{2} dx dy} \sqrt{\frac{\alpha v}{4\pi}}. \quad (5)$$

Values for the mode volume and effective number of coupled electrons calculated with the corrected formulas are $V = 1.3 \times 10^{-15} \text{ m}^3$ and $N_e = 5200$. For this calculation, the electric-field distribution $\mathbf{u}$ (calculated with the eigenfrequency solver of the COMSOL rf module) was normalized according to $\max_{\text{2DEG}} \left(\frac{|u_x + iu_y|^2}{2} \right) = 1$. The resulting coupling rate of $(\Omega_R/\omega_c)_{\text{BEM}} = 0.22$ is independent of the chosen normalization condition, whereas both V and N_e are inversely proportional to the normalization value.

The remainder of the paper discusses relative changes in the normalized coupling rate $\frac{\Omega_R}{\omega_c}$. The erroneous factors cancel out for these quantities. Thus, the results and conclusions reported in the paper remain valid.