Erratum: Weyl superconductors [Phys. Rev. B **86, 054504 (2012)]**

Tobias Meng and Leon Balents

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The main conclusion of our paper is that proximity-induced BCS s -wave superconductivity does not gap an inversion-symmetric, time-reversal symmetry breaking Weyl semimetal, but that it can split each Weyl node into a pair of Bogoliubov-Weyl nodes. While this general statement is correct, as illustrated by the argument given in Appendix A of the paper, the diagonalization of the Hamiltonian detailed in Eq. (1) of the main text is incorrect, and the authors are grateful to V. Kornich for pointing out this mistake. The Hamiltonian in Eq. (1) does not, in fact, give rise to a splitting of the Weyl nodes, which in turn invalidates the applicability of our conclusions to this Hamiltonian. As we show below, a splitting of the Weyl nodes into Bogoliubov-Weyl nodes still occurs for more general Hamiltonians. The discussion in the main text in fact applies to a multilayer structure in which the proximity-induced superconductivity has a relative phase of π between the top and bottom surfaces of the topological insulator layers.

In the following, we discuss a generalized version of the Hamiltonian analyzed in the main text to illustrate these statements. The normal state Hamiltonian remains identical to the main text,

$$H_0 = \sum_{\vec{k}} \Psi_{\vec{k}}^\dagger \mathcal{H}_0(\vec{k}) \Psi_{\vec{k}}, \quad (1)$$

$$\mathcal{H}_0(\vec{k}) = v_F(\sigma_x k_y - \sigma_y k_x) \tau_z + m \sigma_z \mathbb{1}_\tau + (t_s + t_d) \cos\left(\frac{k_z d}{2}\right) \mathbb{1}_\sigma \tau_x + (t_s - t_d) \sin\left(\frac{k_z d}{2}\right) \mathbb{1}_\sigma \tau_y, \quad (2)$$

$$\Psi_{\vec{k}}^T = (c_{\vec{k},\uparrow}^{\text{top}}, c_{\vec{k},\downarrow}^{\text{top}}, e^{ik_z d/2} c_{\vec{k},\uparrow}^{\text{bot}}, e^{ik_z d/2} c_{\vec{k},\downarrow}^{\text{bot}}). \quad (3)$$

In the superconducting proximity effect, however, we now allow for asymmetries between the top and bottom layers,

$$H_{\text{SC}} = H_{\text{SC}}^{\text{top}} + H_{\text{SC}}^{\text{bot}}, \quad (4)$$

$$H_{\text{SC}}^{\text{top}} = (\Delta + \delta\Delta) e^{i\varphi/2} c_{\vec{k},\uparrow}^{\text{top} \dagger} c_{-\vec{k},\downarrow}^{\text{top} \dagger} + \text{H.c.}, \quad (5)$$

$$H_{\text{SC}}^{\text{bot}} = (\Delta - \delta\Delta) e^{-i\varphi/2} c_{\vec{k},\uparrow}^{\text{bot} \dagger} c_{-\vec{k},\downarrow}^{\text{bot} \dagger} + \text{H.c.} \quad (6)$$

Here, φ is the phase difference between the top and bottom layer, Δ is the average pairing amplitude, and $2\delta\Delta$ the difference in the pairing amplitudes of top and bottom layers. To realize this Hamiltonian experimentally, the heterostructure needs to be modified as compared to the originally proposed one (see Fig. 1 in the original paper). Different amplitudes of the proximity-induced order parameters without a phase difference between the top and bottom layers require interfaces of different transparency. This situation can be realized by controlling the interface roughnesses, or may arise due to additional normal insulating spacer layers between one of the surfaces of the topological insulator layers and the superconductors [see Fig. 1 (left)]. Finite phase differences between the top and bottom layers can be created by Josephson junctions in which the phase difference is controlled by magnetic fields [see Fig. 1 (right)]. As we will show now, *either* a finite phase difference, *or* a different pairing amplitude on the top and bottom surfaces is already enough to split the Weyl nodes into Bogoliubov-Weyl nodes.

Defining

$$\tilde{\Psi}_{\vec{k}}^T = (c_{\vec{k},\uparrow}^{\text{top}}, c_{\vec{k},\downarrow}^{\text{top}}, e^{ik_z d/2} c_{\vec{k},\uparrow}^{\text{bot}}, e^{ik_z d/2} c_{\vec{k},\downarrow}^{\text{bot}}, c_{-\vec{k},\uparrow}^{\text{top} \dagger}, c_{-\vec{k},\downarrow}^{\text{top} \dagger}, e^{ik_z d/2} c_{-\vec{k},\uparrow}^{\text{bot} \dagger}, e^{ik_z d/2} c_{-\vec{k},\downarrow}^{\text{bot} \dagger}), \quad (7)$$

we can write the Hamiltonian as

$$H = H_0 + H_{\text{SC}} = \sum_{\vec{k}, k_y > 0} \tilde{\Psi}_{\vec{k}}^\dagger \tilde{\mathcal{H}}(\vec{k}) \tilde{\Psi}_{\vec{k}} + H_{k_y=0}, \quad (8)$$

$$\tilde{\mathcal{H}}(\vec{k}) = \begin{pmatrix} \mathcal{H}_0(\vec{k}) & \tilde{\Delta}(\vec{k}) \\ \tilde{\Delta}(\vec{k})^\dagger & -\mathcal{H}_0^*(-\vec{k}) \end{pmatrix}, \quad (9)$$

$$\tilde{\Delta}(\vec{k}) = \begin{pmatrix} 0 & (\Delta + \delta\Delta) e^{i\varphi/2} & 0 & 0 \\ -(\Delta + \delta\Delta) e^{i\varphi/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & (\Delta - \delta\Delta) e^{-i\varphi/2} \\ 0 & 0 & -(\Delta - \delta\Delta) e^{-i\varphi/2} & 0 \end{pmatrix}. \quad (10)$$

FIG. 1. Generalized heterostructures realizing different order parameter amplitudes (left) and phase differences (right) for the top and bottom surfaces of the topological insulator layer (TI). Different superconducting order parameter amplitudes can, for example, be realized by introducing normal insulating spacer layers (NI) at one of the interfaces with the superconducting layers (SC), while controllable phase differences may be achieved using Josephson junctions threaded by magnetic fluxes Φ . The period of these superlattices is denoted by d .

Here, $H_{k_y=0}$ is the Hamiltonian restricted to $k_y = 0$. Defining

$$\Phi_{\vec{k}}^T = (c_{\vec{k},\uparrow}^{\text{top}}, c_{\vec{k},\downarrow}^{\text{top}}, e^{ik_z d/2} c_{\vec{k},\uparrow}^{\text{bot}}, e^{ik_z d/2} c_{\vec{k},\downarrow}^{\text{bot}}, c_{-\vec{k},\downarrow}^{\text{top}\dagger}, c_{-\vec{k},\uparrow}^{\text{top}\dagger}, e^{ik_z d/2} c_{-\vec{k},\downarrow}^{\text{bot}\dagger}, e^{ik_z d/2} c_{-\vec{k},\uparrow}^{\text{bot}\dagger}) \quad (11)$$

allows us to bring the Hamiltonian to the form

$$H = H_0 + H_{\text{SC}} = \sum_{\vec{k}, k_y > 0} \Phi_{\vec{k}}^\dagger \mathcal{H}(\vec{k}) \Phi_{\vec{k}} + H_{k_y=0}, \quad (12)$$

$$\mathcal{H}(\vec{k}) = \begin{pmatrix} \mathcal{H}_0(\vec{k}) & \Delta(\vec{k}) \\ \Delta(\vec{k})^\dagger & \bar{\mathcal{H}}_0(\vec{k}) \end{pmatrix}, \quad (13)$$

$$\Delta(\vec{k}) = \begin{pmatrix} (\Delta + \delta\Delta)e^{i\varphi/2} & 0 & 0 & 0 \\ 0 & -(\Delta + \delta\Delta)e^{i\varphi/2} & 0 & 0 \\ 0 & 0 & (\Delta - \delta\Delta)e^{-i\varphi/2} & 0 \\ 0 & 0 & 0 & -(\Delta - \delta\Delta)e^{-i\varphi/2} \end{pmatrix}, \quad (14)$$

where $\bar{\mathcal{H}}_0(\vec{k}) = \mathcal{H}_0(\vec{k})|_{t_s, d \rightarrow -t_s, d}$. Introducing a pseudospin $\vec{\kappa}$ in particle-hole space finally yields

$$\begin{aligned} \mathcal{H}(\vec{k}) = & v_F(\sigma_x k_y - \sigma_y k_x) \tau_z \mathbb{1}_\kappa + m \sigma_z \mathbb{1}_\tau \mathbb{1}_\kappa + (t_s + t_d) \cos\left(\frac{k_z d}{2}\right) \mathbb{1}_\sigma \tau_x \kappa_z + (t_s - t_d) \sin\left(\frac{k_z d}{2}\right) \mathbb{1}_\sigma \tau_y \kappa_z \\ & + \Delta \cos(\varphi/2) \sigma_z \mathbb{1}_\tau \kappa_x + \delta\Delta \cos(\varphi/2) \sigma_z \tau_z \kappa_x - \Delta \sin(\varphi/2) \sigma_z \tau_z \kappa_y - \delta\Delta \sin(\varphi/2) \sigma_z \mathbb{1}_\tau \kappa_y. \end{aligned} \quad (15)$$

This Hamiltonian can be diagonalized with the canonical transformations

$$\sigma_x \rightarrow \sigma_x \tau_z, \quad \sigma_y \rightarrow \sigma_y \tau_z, \quad \sigma_z \rightarrow \sigma_z, \quad \tau_x \rightarrow \tau_x \sigma_z, \quad \tau_y \rightarrow \tau_y \sigma_z, \quad \tau_z \rightarrow \tau_z, \quad (16)$$

followed by

$$\tau_x \rightarrow \tau_x \kappa_z, \quad \tau_y \rightarrow \tau_y \kappa_z, \quad \tau_z \rightarrow \tau_z, \quad \kappa_x \rightarrow \kappa_x \tau_z, \quad \kappa_y \rightarrow \kappa_y \tau_z, \quad \kappa_z \rightarrow \kappa_z, \quad (17)$$

which yields

$$\begin{aligned} \mathcal{H}(\vec{k}) = & v_F(\sigma_x k_y - \sigma_y k_x) \mathbb{1}_\tau \mathbb{1}_\kappa + m \sigma_z \mathbb{1}_\tau \mathbb{1}_\kappa + \left[(t_s + t_d) \cos\left(\frac{k_z d}{2}\right) \tau_x + (t_s - t_d) \sin\left(\frac{k_z d}{2}\right) \tau_y \right] \sigma_z \mathbb{1}_\kappa \\ & + \sigma_z [\Delta \cos(\varphi/2) \tau_z \kappa_x + \delta\Delta \cos(\varphi/2) \mathbb{1}_\tau \kappa_x - \Delta \sin(\varphi/2) \mathbb{1}_\tau \kappa_y - \delta\Delta \sin(\varphi/2) \tau_z \kappa_y]. \end{aligned} \quad (18)$$

For the case $\delta\Delta = 0 = \varphi$ considered in the main text, this Hamiltonian has a twofold degenerate spectrum, and the Weyl nodes are not split into pairs of Bogoliubov-Weyl nodes. For $\delta\Delta = 0$ and $\varphi = \pi$, on the other hand, we obtain the spectrum discussed in the main text. For general finite values of $\delta\Delta$ and/or φ , corresponding to a generic heterostructure without perfect symmetry between the top and bottom surfaces of the topological insulator layers, the spectrum is also nondegenerate, and each Weyl node is split into a pair of Bogoliubov-Weyl nodes. Therefore, a multilayer structure of topological insulators and BCS s -wave superconductors will generally exhibit split Bogoliubov-Weyl nodes, in agreement with our initial finding. Similar results have also been obtained for intrinsic superconductivity in Weyl semimetals (see, for instance, Refs. [1–4]), which also demonstrate the existence of Bogoliubov-Weyl modes in systems that might experimentally be more accessible than topological insulator heterostructures.

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- [1] M. Gong, S. Tewari, and C. Zhang, [Phys. Rev. Lett. **107**, 195303 \(2011\)](#).
- [2] J. D. Sau and S. Tewari, [Phys. Rev. B **86**, 104509 \(2012\)](#).
- [3] G. Y. Cho, J. H. Bardarson, Y.-M. Lu, and J. E. Moore, [Phys. Rev. B **86**, 214514 \(2012\)](#).
- [4] G. Bednik, A. A. Zyuzin, and A. A. Burkov, [Phys. Rev. B **92**, 035153 \(2015\)](#).