Erratum: Berry curvature and orbital angular momentum of electrons in angle-resolved photoemission spectroscopy [Phys. Rev. B **91, 245133 (2015)]**

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There are mistakes in our proposal for experimental setup in Sec. IV of our original paper; the electric field is treated as imaginary by error in Eq. (23). In the right-hand side of the first equation in Eq. (23), the operator between $|\mathbf{k}\rangle$ and $|\mathbf{k}'\rangle$ is the time-evolution operator in the interaction representation. Then, by the relation $\langle \mathbf{k}|X|\mathbf{k}'\rangle \sim -i\langle \mathbf{k}|\partial_{k_x}|\mathbf{k}'\rangle$, the second term in the second equation in Eq. (23) should be changed to $b \rightarrow ib$, and consequently, the observation scheme in Sec. IV breaks down.

We give another scheme for observing the Hall current of the electron with orbital angular momenta (OAM) and claim that the observation of the OAM is experimentally feasible. We consider a situation where the photoelectron passes through a plate which absorbs injected electrons. We put a plate parallel to the surface (the xy plane) of the material where photoelectron comes from. The plate is not magnetically ordered. When we focus only on the electrons passing through the plate, the Hamiltonian is effectively given as $H = H_0 - i\frac{\gamma p_z}{2m_e}$, where H_0 is the Hamiltonian of free electrons, m_e is the electron mass, and γ^{-1} is the attenuation length determined by kind of chemical compounds of the plate. At $t = 0$ the electron enters the region of the plate with finite thickness, and then the wave function is written as

$$\Psi'(\mathbf{R}, \mathbf{k}_c, t) = e^{-itH_0 - \frac{\gamma p_z}{2m_e}t} \Psi(\mathbf{R}, \mathbf{k}_c).$$

Here, we assume that the attenuation length has the form $\gamma = \gamma_0[1 + \epsilon_s(X - X_g)]$ where ϵ_s is the modulation of the attenuation length and X_g is the center of the modulation. Then, when the modulation and transit time (t) are small, the wavefunction has the form

$$\Psi'(\mathbf{R}, \mathbf{k}_c, t) = e^{-itH_0 - \frac{\gamma_0 p_z}{2m_e}t} \left[1 - \frac{\gamma_0 \epsilon_s p_z}{2m_e} t (X - X_g) \right] \Psi(\mathbf{R}, \mathbf{k}_c),$$

and we have the expectation value

$$\begin{aligned} \langle P'_y \rangle &= \frac{\langle \Psi'(\mathbf{R}, \mathbf{k}_c, t) | k_y | \Psi'(\mathbf{R}, \mathbf{k}_c, t) \rangle}{\langle \Psi'(\mathbf{R}, \mathbf{k}_c, t) | \Psi'(\mathbf{R}, \mathbf{k}_c, t) \rangle} = \frac{\langle k_y \rangle - \frac{\gamma_0 \epsilon_s p_z}{m_e} t \langle k_y (X - X_g) \rangle}{1 - \frac{\gamma_0 \epsilon_s p_z}{m_e} t \langle (X - X_g) \rangle} \\ &= \langle k_y \rangle - \frac{\gamma_0 \epsilon_s p_z}{m_e} t [\langle k_y (X - X_g) \rangle - \langle k_y \rangle \langle (X - X_g) \rangle], \end{aligned}$$

where $\langle \mathcal{O} \rangle = \langle \Psi(\mathbf{R}, \mathbf{k}_c) | \mathcal{O} | \Psi(\mathbf{R}, \mathbf{k}_c) \rangle$ and $\langle \mathcal{O}' \rangle = \frac{\langle \Psi'(\mathbf{R}, \mathbf{k}_c, t) | \mathcal{O} | \Psi'(\mathbf{R}, \mathbf{k}_c, t) \rangle}{\langle \Psi'(\mathbf{R}, \mathbf{k}_c, t) | \Psi'(\mathbf{R}, \mathbf{k}_c, t) \rangle}$, assuming the condition $\frac{\gamma_0 \epsilon_s p_z}{m_e} t \langle X \rangle \sim -\frac{\gamma_0 \epsilon_s p_z}{m_e \sigma} t \ll 1$ for $X_g = 0$. σ corresponds to the size of the wave packet in momentum space (see original paper). The momentum shift becomes

$$\langle P'_y \rangle = \langle k_y \rangle - \frac{\gamma_0 \epsilon_s p_z}{m_e} t (\langle k_y X \rangle - \langle k_y \rangle \langle X \rangle).$$

In the same manner we obtain the change of the momentum $\langle P'_x \rangle$ by the anisotropy of γ_0 along the y axis, $\gamma = \gamma_0(1 + \epsilon_s Y)$, and we have

$$\langle L_z \rangle = \frac{m_e}{\gamma_0 \epsilon_s p_z t} (\langle \delta P_x \rangle - \langle \delta P_y \rangle),$$

where $\langle \delta P_i \rangle = \langle P'_i \rangle - \langle P_i \rangle$. Here, we choose $\frac{\gamma_0 p_z}{m_e} t \sim 1$, and then the condition $\epsilon_s < \sigma$ is obtained. The thickness of the plate is given by γ^{-1} ; typically γ^{-1} is given by the order of the inelastic scattering length ~ 0.1 – 10 nm [1]. The momentum shift Q is estimated as

$$\frac{Q}{\hbar} \sim \frac{\gamma_0 \epsilon_s p_z}{\hbar m_e} t \langle L_z \rangle \sim \epsilon_s \frac{1}{\hbar} \langle L_z \rangle \sim \epsilon_s \frac{\hbar^2 v^2 \sigma^2}{\Delta^2} < k_0,$$

with $k_0 \equiv \frac{\hbar^2 v^2 \sigma^2}{\Delta^2} = 7.07 \times 10^5 \text{ m}^{-1}$, where the same values in the previous paper are used for v , σ , and Δ . Experimentally, ϵ_s can be obtained by methods such as strain or junction. Here, we consider a junction of materials by connecting two materials labeled by A and B, which have the attenuation lengths γ_A^{-1} and γ_B^{-1} , respectively. The middle of the junction region is composed of the alloy of A and B, and γ can be assumed to be proportional to the density of the two materials. According to scattering theory, the intensity after passing through the target is given as $I \propto e^{-n\ell\Sigma}$, where n is the density of the target, ℓ is the length of the target region, and Σ is the total scattering cross section determined by component of the target; in the present case γ corresponds to $n\Sigma$.

Then, for example $\gamma_A \ll \gamma_B$, we have $\epsilon_s = \frac{2}{L} \frac{\gamma_B - \gamma_A}{\gamma_A + \gamma_B} \sim 2L^{-1}$, where L is the width of the junction region, and when $L \sim \sigma^{-1}$, we have the largest value of the Hall velocity $\sim \frac{\hbar k_0}{m_e}$ obtained by the above relation.

- [1] I. Lindau and W. E. Spicer, *J. Electron Spectrosc. Relat. Phenom.* **3**, 409 (1974).