Erratum: Diffusive quantum criticality in three-dimensional disordered Dirac semimetals **[Phys. Rev. B **90**, 241112(R) (2014)]**

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We correct erroneous conclusions from our previous article [Phys. Rev. B **90**, 241112(R) (2014)] regarding the values of various critical exponents, calculated to two-loop order. Furthermore, from a three-loop renormalization group flow equation, we argue, that ϵ -expansion near two spatial dimensions, with $\epsilon = d - 2$, may not be reliable to address the critical properties of the disorder-driven Dirac semimetal-metal quantum phase transition in $d = 3$.

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We clarify some errors in our previous two-loop calculation for a potential disorder-driven Dirac semimetal (DSM) - compressible diffusive metal (CDM) quantum phase transition (QPT) in three spatial dimensions ($d = 3$). We performed a two-loop analysis within the framework of an ϵ -expansion near $d = 2$ (the lower critical dimension for the problem), and we reported that the correlation length exponent (CLE) (ν) is given by $\nu^{-1} = \epsilon - \frac{\epsilon^2}{8} + O(\epsilon^3)$ and the dynamic scaling exponent (DSE) $z = 1 + \frac{\epsilon}{2} - \frac{3}{16}\epsilon^2 + O(\epsilon^3)$, where $\epsilon = d - 2$, near the DSM-CDM quantum critical point (QCP). Hence, for $d = 3$, we found $\nu \approx 1.14$ and $z \approx 1.31$. We realized that the quoted values of these two exponents are incorrect because we missed some relevant two-loop diagrams, which we correct here. But, more importantly, we question here the validity of such an ϵ -expansion near $d = 2$, and we argue that the ϵ -expansion may not be reliable for this problem.

We realized that instead of reevaluating the diagrams shown in our paper, one can derive the flow equation for disorder coupling (Δ) by analyzing the Gross-Neveu (GN) model for a *discrete* chiral symmetry-breaking mass condensation for which the flow equation of GN coupling (g) is known to three-loop order [1–4]. The Euclidean actions for these two theories are, respectively,

$$\bar{S}_D = \int d^d \vec{x} d\tau (\bar{\Psi}_a [-i\partial_\tau \gamma_0 - i v \gamma_j \partial_j] \Psi_a)_\tau - \frac{\Delta}{2} \int d^d \vec{x} d\tau d\tau' (\bar{\Psi}_a \gamma_0 \Psi_a)_\tau (\bar{\Psi}_b \gamma_0 \Psi_b)_{\tau'}, \quad (1)$$

$$S_{GN} = \int d^D \vec{x} [\bar{\Psi} (-i\gamma_\mu \partial_\mu) \Psi - g(\bar{\Psi} \Psi)^2], \quad (2)$$

where $\bar{\Psi} = \Psi^\dagger \gamma_0$, and a, b are replica indices. Due to fermion doubling in a lattice [5], we work with four-dimensional γ matrices, satisfying the anticommutation relation $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. The critical behavior of S_{GN} and $\bar{S}_D$ can be studied by performing ϵ -expansions near two space-time and spatial dimensions, respectively. The infrared flow equation for g to three-loop order reads [1–4]

$$\beta_g = -\epsilon g + (n-2)g^2 - (n-2)g^3 - (n-2)(n-7)\frac{g^4}{4}, \quad (3)$$

where $\epsilon = D - 2$, with D as the *space-time* dimensions, and n is the number of spinor components.

To derive the flow equation of (Δ), one can set the external frequency to zero, for which we establish a correspondence between diagrammatic contributions for S_{GN} and $\bar{S}_D$. (i) Contributions from relevant one-loop [Fig. 1, (i)–(iv) of our paper] and three-loop (see Fig. 1) diagrams are of opposite sign when the fermion vertex is accompanied by the matrix $\mathbb{1}_{4 \times 4}$ (GN theory) and γ_0 (potential disorder), and (ii) two-loop diagrams that renormalize either GN (g) or disorder (Δ) coupling are of the same sign for these two theories. Thus we can arrive at the flow equation for (Δ) to three-loop order from Eq. (3) by (i) first setting $n = 0$, since any contribution proportional to n involves a fermion bubble, which vanishes in the vanishing replica limit, and (ii) taking $-g = \Delta$, leading to

$$\beta_\Delta = -\epsilon \Delta + 2\Delta^2 + 2\Delta^3 + \frac{7}{2}\Delta^4, \quad (4)$$

where $\epsilon = d - 2$, and d represents the *spatial* dimensions. The exact locations of the DSM-CDM QCP obtained from one-, two-, and three-loop flow equations are, respectively,

$$\Delta_{(1)}^* = \epsilon/2, \quad \Delta_{(2)}^* = (-1 + \sqrt{1 + 2\epsilon})/2, \quad \text{and} \quad \Delta_{(3)}^* = -\frac{4}{21} + \sqrt{\frac{272}{441}} \sinh \left[\frac{1}{3} \sinh^{-1} \left(\frac{1323\epsilon + 440}{136\sqrt{7}} \right) \right].$$

When we expand the last two quantities in powers of ϵ , the radius of convergence is $1/2$ and <0.35 (extracted numerically). Thus, setting $\epsilon = 1$ to extract the CLE in $d = 3$ is not a well-defined procedure, since this expansion is not asymptotically convergent. Therefore, higher-order terms from the ϵ -expansion about two spatial dimensions may only produce unreliable results for $\epsilon = 1$. Similar situation also occurs for the GN model (with $n = 4, 8$ for example), for which the critical exponents beyond the leading order can reliably be extracted only when combined with the $1/N$ expansion [4].

FIG. 1. Three-loop diagrams that can contribute to the flow of Δ . We do not show various combinations of each class of diagrams, obtained by exchanging the disorder (dashed) lines. Notice that the three-loop diagram (f) and its various combinations, and the two-loop diagrams (*viii*), (*ix*), (*xi*), and (*xii*) from our paper, do not renormalize Δ or g .

Even if we choose to extract CLE from the powers series of Δ^* in terms of ϵ to one-, two-, and three-loop order, we find, respectively,

$$v_{(1)}^{-1} = \epsilon + O(\epsilon^2) \quad \text{and} \quad v_{(2)}^{-1} = \epsilon + \frac{\epsilon^2}{2} + O(\epsilon^3), \quad v_{(3)}^{-1} = \epsilon + \frac{\epsilon^2}{2} + \frac{3}{8}\epsilon^3 + O(\epsilon^4).$$

Therefore, in three spatial dimensions ($\epsilon = 1$), we obtain $v_{(1)} = 1$, $v_{(2)} \approx 0.67$, and $v_{(3)} \approx 0.53$. The fact that the inclusion of higher-order corrections pushes the CLE close to the mean-field value ($v = 1/2$) is alarming, possibly indicating the failure of ϵ -expansion to correctly capture the critical properties. It may be worthwhile to mention that the higher-loop results for the CLE either do not ($v_{(3)}$) or barely ($v_{(2)}$) satisfy the Chayes-Chayes-Fisher-Spencer inequality ($v > 2/d$) [6], while the one-loop result ($v = 1$) satisfies the inequality, possibly indicating that going beyond one-loop order may not be meaningful in this problem.

Furthermore, one may wish to capture the asymptotic behavior of CLE using the Padé approximation [7,8]. For $v_{(2)}^{-1}$, a Padé [1|1] approximation yields

$$[v_{(2)}^{-1}]^{[1|1]} = \frac{\epsilon}{1 - \frac{\epsilon}{2}}. \quad (5)$$

Consequently, we obtain a mean-field value of CLE, $v_{(2)} = 1/2$, for $\epsilon = 1$. On the other hand, to estimate the asymptotic behavior of $v_{(3)}^{-1}$, we can employ either Padé [2|1] or Padé [1|2] approximations, respectively, yielding

$$[v_{(3)}^{-1}]^{[2|1]} = \frac{\epsilon - \frac{\epsilon^2}{4}}{1 - \frac{3}{4}\epsilon}, \quad [v_{(3)}^{-1}]^{[1|2]} = \frac{\epsilon}{1 - \frac{\epsilon}{2} - \frac{\epsilon^2}{8}}, \quad (6)$$

in turn giving $v_{(3)} \approx 0.33$ and 0.375 for $\epsilon = 1$, both being smaller than the mean-field value of CLE, strongly suggesting the inapplicability of the ϵ -expansion for $d = 3$.

We do not discuss the corrections to the DSE (z) beyond the leading order in ϵ . Given that the radius of convergence of $\Delta_{(2/3)}^* \ll 1$, and the CLE rapidly comes closer (or goes below) its mean-field value we believe that higher-order corrections to z are not meaningful when $\epsilon = 1$ (since extraction of z also requires the notion of the DSM-CDM QPT).

Therefore, within the ϵ -expansion scheme, where $\epsilon = d - 2$, only reliable values of the critical exponents near DSM-CDM QCP are $v = \epsilon^{-1}$ and $z = 1 + \epsilon/2$, respectively, giving $v = 1$ and $z = 3/2$ in three spatial dimensions ($\epsilon = 1$) as shown in our original work and also in Ref. [10], which were first obtained from one-loop Wilsonian renormalization-group analysis [9]. Going to higher-loop calculations in ϵ for $d = 3$ can only produce progressively worse results for the critical exponents.

We point out that recent numerical calculations for the DSE (z) agree with the result from the one-loop ϵ -expansion within numerical error bars [11–15], whereas the one-loop result for CLE (v) agrees reasonably well (within error bar) with the numerically calculated exponents in Refs. [11,13–15], but it disagrees with Ref. [12]. Whether the agreement between the numerical results and the one-loop calculation is merely a coincidence or implies something deep about the problem is unknown at this stage.

Note added. During preparation of this Erratum, we became aware of a preprint [16] in which a two-loop calculation was carried out explicitly. Our flow equation in Eq. (4) to two-loop order agrees with that reported in Ref. [16]. Some of the relevant diagrams, which were missed in our paper, was also pointed out in Ref. [16].

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