Erratum: Measurement-induced criticality in (2 + 1)-dimensional hybrid quantum circuits [Phys. Rev. B **102**, 014315 (2020)]

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In our paper, we incorrectly reported that singular value decomposition (SVD) directly evaluates the entanglement entropy of a stabilizer state generated by the dynamics we are interested in. In fact, SVD only provides a rigorous upper bound to the entropy.¹ We are grateful to Y. Li and M. P. A. Fisher for correspondence that elucidated this aspect.

Specifically, in the published version of the paper we compute entanglement of the stabilizer state by means of the Hamma-Ionicioiu-Zanardi theorem. This requires the computation of the matrix rank. SVD computes the value of the rank in the field of real numbers $\mathbb{R}$, while due to the algebraic structure of the stabilizer group, we should have computed the rank for the field $\mathbb{F}_2$.

We observe that any matrix A with binary elements $a_{ij} = 0, 1$ satisfies

$$\text{rank}_{\mathbb{F}_2} A \leq \text{rank}_{\mathbb{Q}} A = \text{rank}_{\mathbb{R}} A. \quad (1)$$

In the previous equation, $\mathbb{F}_2$ is the finite field over $\mathbb{Z}_2$, while $\mathbb{Q}$ is the field of rational numbers.

Thus, in order to obtain the correct results for the models described in the published version of the paper, we need an exact computation of the rank over $\mathbb{F}_2$. This computation is done by Gaussian elimination, which works for any field $\mathbb{K}$. The algorithm scales as $\mathcal{O}(N_s^3)$, with $N_s \propto L^2$ the number of spins.

We have repeated all simulations and extracted the entanglement entropy using Gaussian elimination. All the corresponding results are presented below (Fig. 1). The three main findings of our work are *confirmed*, namely:

- (1) There is a universal critical behavior for 2 + 1D hybrid quantum circuits (Figs. 2 and 3);
- (2) The critical point shows a multiplicative logarithmic correction to the area law entanglement entropy (Fig. 4);
- (3) The universality class is different from that of 3D percolation.

The critical values of p_c and ν were instead not correct for both rank-1 and rank-2 measurement schemes. The correct values are $p_c = 0.54(1)$, $\nu = 0.67(1)$ and $p_c = 0.84(1)$, $\nu =$

0.68(1) for rank-1 and rank-2 measurement schemes, respectively. Thus, compared to the previous results, the critical exponent ν for both models is 20% different from the values reported previously, while the critical points p_c are offset 20% for rank-1 measurements and 6% for rank-2 measurements.

FIG. 1. Bare data for the hybrid random circuits entanglement entropy computed through Gaussian elimination. The two upper panels pertain to the rank-1 measurements, while the lower ones the rank-2 measurements. The red line characterizes the critical line in both models.

¹In 1D, results obtained via analyzing such upper bounds up in chains with up to $L = 128$ spins return critical properties that are the same (within error bars) of those obtained via Gaussian elimination.

FIG. 2. Finite-size scaling results for both models. The procedure is the one presented in the published version of the paper. The updated results show a different estimate for both models considered with respect to the published results. In particular, the rank-1 model (left panel) gives $p_c = 0.54(1)$, $\nu = 0.67(1)$. The rank-2 model (right panel) gives $p_c = 0.84(1)$, $\nu = 0.68(1)$. For convenience, the respective critical values are indicated through a star in the figures.

FIG. 3. Data collapse on the results of the finite-size scaling in Fig. 2. Rank-1 HRC results are presented in the upper panel, while the rank-2 HRC are in the lower panel.

FIG. 4. Entanglement entropy for values around the criticality at $L = 64$ for rank-1 (top panel) and rank-2 (bottom panel) HRC. The blue line is a fit for a logarithmic behavior, while the green ones are a fit for an area law.

The compatibility of the critical exponent for both the considered hybrid random circuit models within 2% of error suggests universality holds for the corrected data. This exponent is $>30\%$ different from that of 3D percolation [$\nu_{\text{perc}}^{3D} = 0.87(1)$], suggesting the $2 + 1D$ HRC belongs to a different universality class.

We conclude by pointing out that, despite that the SVD gives an upper bound of our data, it presents a hint of universality (see published version of the paper). It is possible that the SVD is capturing different features of the model of interest or some specific limit of it. We leave further investigation of its relationship to Clifford hybrid random circuits for future work.