Prethermal memory loss in interacting quantum systems coupled to thermal baths

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We study the relaxation dynamics of an extended Fermi-Hubbard chain with a strong Wannier-Stark potential tilt coupled to a bath. When the system is subjected to dephasing noise, starting from a pure initial state the system's total von Neumann entropy is found to grow monotonously. The scenario becomes rather different when the system is coupled to a thermal bath of finite temperature. Here, for sufficiently large field gradients and initial energies, the entropy peaks in time and almost reaches its largest possible value (corresponding to the maximally mixed state), long before the system relaxes to thermal equilibrium. This entropy peak signals an effective prethermal memory loss and, relative to the time where it occurs, the system is found to exhibit a simple scaling behavior in space and time. By comparing the system's dynamics to that of a simplified model, the underlying mechanism is found to be related to the localization property of the Wannier-Stark system, which favors dissipative coupling between eigenstates that are close in energy.

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Introduction. The problem of particles moving in a tilted lattice (Wannier-Stark system) is associated with various interesting phenomena, such as Bloch oscillations [1], Stark localization [2], and Landau-Zener tunneling [3]. Despite its long history, it has remained a frontier research topic both theoretically [4] and experimentally [5–16]. Recent studies [17,18] show that an interacting Wannier-Stark system can exhibit nonergodic behavior analogous to disorder-induced many-body localization (MBL) [19–22]. Understanding both the differences and similarities between such disorder-free and conventional MBL constitutes a fundamental question, which currently attracts a lot of attention [23–36].

On the other hand, there is increased recent interest in the nonequilibrium properties of *open* many-body quantum systems [37–69]. While the coupling to an environment of finite temperature constitutes a natural situation, it is rather cumbersome to simulate. Therefore, the impact of dissipation is often treated by using dephasing noise [70] as a simpler bath model. Even though dephasing noise will eventually drive the system into an infinite-temperature state, it is assumed to qualitatively capture the effect of weak coupling to a bath on the transient evolution. Understanding under which circumstances this assumption breaks down is an important question for the simulation of open many-body systems (see also Refs. [71,72]).

Here, we report on a surprising phenomenon in the relaxation dynamics of an interacting Wannier-Stark system coupled to a thermal bath of finite temperature. It sheds light on both of the above questions, since it can neither be observed for a disorder-localized system nor for dephasing noise. In a large parameter regime, we find that long before the system reaches thermal equilibrium, it transiently approaches the maximally mixed state. This effect implies an effective prethermal memory loss. It is reminiscent of the universal dynamics recently observed in isolated quantum gases [73,74].

Model. We consider a one-dimensional extended Hubbard chain half filled with spin-polarized fermions and subjected to a linear potential gradient. It is described by the Hamiltonian

$$H = -J \sum_{i=1}^{M-1} (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + \sum_{i=1}^M W_i n_i + V \sum_{i=1}^{M-1} n_i n_{i+1}. \quad (1)$$

Here, $c_i^\dagger$ and $n_i = c_i^\dagger c_i$ are the creation and number operators for a fermion on lattice site i . Moreover, J is the tunneling parameter, $W_i = -ri$ captures the potential gradient r , and V quantifies nearest-neighbor interactions. This model is equivalent to an XXZ chain via a Jordan-Wigner transform [75]. The single-particle (bulk) eigenstates, known as Wannier-Stark states, are centered at the lattice sites, with a localization length $\sim J/r$ (with respect to the site index i) and energies that increase by r from site to site [2]. Moreover, also the interacting system shows properties akin to MBL [17,18]. Henceforth, we use J , J/k_B , and $\hbar/J$ as units for energy, temperature, and time, respectively, so that $J = \hbar = k_B = 1$.

When coupled weakly to a thermal bath, which is modeled as a collection of harmonic oscillators in thermal equilibrium and couples to the on-site occupations, the system can be

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described by a Redfield master equation [37],

$$\begin{aligned} \frac{d\rho}{dt} = & -i[H, \rho] + \eta \sum_{k,q,p,l=1}^M [R_{kqlp}(L_{kq}\rho L_{pl}^\dagger - L_{pl}^\dagger L_{kq}\rho) \\ & + R_{plqk}(L_{kq}\rho L_{pl}^\dagger - \rho L_{pl}^\dagger L_{kq})], \end{aligned} \quad (2)$$

with jump operators $L_{kq} = |k\rangle\langle q|$ between many-body eigenstates $|k\rangle$ of energy ε_k [46,58]. The corresponding transition rates read $R_{kqpl} = \pi v_{kqpl} g(\varepsilon_k - \varepsilon_q)$, with $v_{kqpl} = \sum_{i=1}^M \langle k|n_i|q\rangle\langle p|n_i|l\rangle$ and bath correlation function $g(E) = J(E)/(e^{E/T} - 1)$, where we assume an Ohmic spectral density $J(E) = E$.

In the high-temperature limit, we have $g(E) \simeq T$ and the transition rate R_{kqpl} becomes independent of energy. Thus, the master equation reduces to

$$\frac{d\rho}{dt} = -i[H, \rho] + \gamma \sum_{i=1}^M \left(n_i \rho n_i - \frac{1}{2} n_i^2 \rho - \frac{1}{2} \rho n_i^2 \right), \quad (3)$$

with $\gamma = \eta T$, describing dephasing noise.

Prethermal memory loss. Figure 1 shows the time evolution of the von Neumann entropy $S = -\text{tr}\{\rho \log(\rho)\}$ of the total system, when coupled to a dephasing bath [Fig. 1(a)] or a finite-temperature bath [Fig. 1(b)]. It is calculated by numerically integrating Eqs. (3) and (2), respectively, starting from a Fock state with the left half of the chain occupied. The temperature T of the thermal bath is chosen such that the corresponding equilibrium entropy S_T (obtained for the Gibbs state $\rho_T = \mathcal{Z}_T^{-1} \sum_k e^{-\varepsilon_k/T} |k\rangle\langle k|$ with $\mathcal{Z}_T = \sum_k e^{-\varepsilon_k/T}$) is equal to half the largest possible entropy $S_\infty \equiv S_{T=\infty} = \log(\mathcal{D})$, with Hilbert space dimension $\mathcal{D} = M!/(M/2!)^2$ (see Fig. S1 of Ref. [76] for other temperatures). For dephasing noise [Fig. 1(a)], the entropy grows monotonously to the maximum value S_∞ , being insensitive to the sign of the potential gradient r . In turn, when the system is coupled to the finite-temperature bath [Fig. 1(b)], the entropy approaches its equilibrium value rather differently for negative and positive r . While in the former case the entropy grows monotonously to S_T (except for small $|r|$), in the latter case it first reaches a peak value S_{peak} well above S_T , before relaxing to equilibrium. As will become apparent in the following, this difference can be attributed to the different mean energies of the initial states.

Remarkably, we can observe in Fig. 1(b) that for large positive gradients r , the peak entropy almost reaches the largest possible entropy S_∞ (dotted line), which uniquely corresponds to the maximally mixed state $\rho_\infty \equiv \rho_{T=\infty} = \mathcal{D}^{-1} \sum_k |k\rangle\langle k|$. This effect can be observed for a wide range of initial conditions: In Fig. 1(c) we plot the peak entropies S_{peak} reached during the evolution starting from various initial Fock states, versus their mean energy E_{ini} (scaled between 0, for the ground-state energy ε_{min} , and 1, for the energy ε_{max} of the most excited state). Peak entropies close to S_∞ are found as long as E_{ini} lies well above the energy $E_\infty = \text{tr}\{\rho_\infty H\}$ of the maximally mixed state (dashed line).

Since the maximally mixed state is unique, reaching an entropy peak with $S_{\text{peak}} \approx S_\infty$ indicates that we can expect the system dynamics to become (approximately) independent of the initial conditions near and after approaching the peak entropy. Such a behavior is confirmed in Fig. 1(d), where we

FIG. 1. (a), (b) Time evolution of the von Neumann entropy for the tilted ladder (1) initialized in a Fock state with the left half of the chain occupied and coupled to (a) a dephasing bath [Eq. (3)] or (b) a thermal bath [Eq. (2)]. The dotted line marks the largest possible entropy S_∞ . (c) Normalized peak entropy for initial Fock states of various energies E_{ini} . The vertical dashed line marks E_∞ . (d) Evolution of the mean occupation of the third lattice site relative to the time t_{peak} , where the entropy peak is reached. The line colors mark different initial Fock states corresponding to colored bullets in (c). (e) Density profile $\langle n_i \rangle$ at equidistant times during the time window marked by the shaded area in (b). The black line marks $t = t_{\text{peak}}$. The inset shows the collapse of all the curves by rotating them by an angle proportional to the corresponding time. (f) Diagonal elements of the density matrix p_k at three points in time [marked by (i)–(iii) in (b)] (solid lines) compared to effective thermal states of identical average energy (dashed lines). The parameters are $M = 8$, $V = 1$, $\gamma = \eta = 0.1$, and T so that $S_T = S_\infty/2$. The field gradient for (c)–(f) is $r = 4$.

plot the evolution of the site occupation $\langle n_{i=3} \rangle$ relative to the time t_{peak} at which the entropy peak is reached. The different curves, which correspond to different initial states [labeled by line colors corresponding to the colored bullets in Fig. 1(c)], clearly converge near $\eta(t - t_{\text{peak}}) = 0$ and subsequently show almost identical behavior. Similar behavior can also be observed for other site occupations $\langle n_i \rangle$ and for larger systems (see Figs. S3, S4 and Figs. S6, S9 of Ref. [76], respectively). Thus, the system undergoes an effective [77] prethermal memory loss, long before it reaches thermal equilibrium.

Moreover, we also find that the way the system approaches the maximally mixed state shows a simple form of scaling behavior. In Fig. 1(e) we plot the density distribution $\langle n_i \rangle$ at various times near t_{peak} [within the shaded area in Fig. 1(b)], for $r = 4$ and starting from the Fock state with the left half of the chain occupied [green curve in Fig. 1(b)]. These density

profiles collapse on top of each other when rotated by an angle proportional to the evolved time (see the inset). Note that for $r = 4$ the Wannier-Stark states are already well localized on single lattice sites, so that the plotted density profile (which can directly be measured in quantum-gas systems) approximately corresponds to the occupation of the single-particle eigenstates. In this sense, the observed behavior is somewhat reminiscent of the universal scaling behavior recently observed in the occupations of long-wavelength momentum modes during the far-from-equilibrium dynamics of isolated quantum gases [73,74], which was associated with the presence of nonthermal fixed points [78–80].

Further insight on how the system approaches the maximally mixed state is gained by looking at the probability distribution $p_k = \langle k | \rho | k \rangle$ for occupying many-body energy eigenstates $|k\rangle$. In Fig. 1(f) we plot the distribution p_k (solid lines) for $r = 4$ at three times: (i) slightly before, (ii) at, and (iii) slightly after t_{peak} [as indicated in Fig. 1(b) relative to the green curve]. Interestingly, these distributions agree rather well with those for thermal states $\rho_{T_{\text{eff}}}$ (dashed lines) with the effective temperature T_{eff} determined by the instantaneous energy, $\text{tr}\{\rho_{T_{\text{eff}}} H\} \equiv E = \text{tr}\{\rho H\}$. This observation suggests that the prethermal memory loss is due to a *dissipative form of prethermalization*, where the system rapidly approaches a Gibbs state, whose effective temperature T_{eff} then slowly relaxes to the equilibrium temperature T . This scenario immediately explains that an infinite-temperature state is approached long before the system has thermalized as long as the initial energy E_{ini} lies well above the infinite-temperature energy E_{∞} . Namely, in this case the system has enough time to approach a prethermal state $\rho_{T_{\text{eff}}}$, before $1/T_{\text{eff}}$ passes through zero from below at the time when E drops below E_{∞} .

The prethermal relaxation to a Gibbs-like state with slowly varying effective temperature occurs in a way rather different from standard (pre)thermalization [81]. It cannot be understood as the prethermalization of the system and bath together, since the bath always remains in the same thermal state with constant temperature T . Nor can it be explained by the thermalization of the system itself on a timescale that is fast compared to slow energy dissipation by the bath, since the system is localized and nonergodic. Let us therefore investigate the underlying mechanism and its relation to Stark localization.

Underlying mechanism. Although a two-level system prepared in its excited state passes through the maximum entropy state while equilibrating with a thermal bath [76], such behavior is highly nontrivial for interacting many-body systems. To figure out the conditions for a close-to-maximum peak entropy, let us focus on the weak-coupling limit, where the secular approximation [37,38] gives a Lindblad master equation,

$$\frac{d\rho}{dt} = -i[H, \rho] + \eta \sum_{k,q=1}^M R_{kq} \left(L_{kq} \rho L_{kq}^\dagger - \frac{1}{2} \{L_{kq}^\dagger L_{kq}, \rho\} \right),$$

with rates $R_{kq} \equiv R_{kqqk}$. They obey $R_{kq}/R_{qk} = e^{-(\varepsilon_k - \varepsilon_q)/T}$, so that low-energy states are favored and the system is asymptotically driven towards the Gibbs state ρ_T . The matrix elements $\rho_{kq} \equiv \langle k | \rho | q \rangle$ follow $\dot{\rho}_{kq} = -i(\varepsilon_k - \varepsilon_q)\rho_{kq} +$

FIG. 2. (a), (b) Rate matrix for the Fermi-Hubbard chain coupled to a thermal bath, with (a) $r = 0$ and (b) $r = 4$. (c) Examples for on-site potentials W_i that were optimized over the range $[-16, 16]$ to provide a large peak entropy when evolving from the most excited state. The results are representative for 500 numerical runs. (d) Time evolution of the corresponding entropies. (e), (f) Corresponding rate matrices R_{kq} . (g) Rate matrix and (h) entropy for a tilted bosonic Hubbard chain with $r = 4$ and initially all particles occupying the leftmost site. The system sizes are $M = 8$, $N = 4$ for (a)–(f), and $M = 6$, $N = 3$ for (g), (h). Other parameters are $V = 1$, $\eta = 0.1$, and T so that $S_T = S_{\infty}/2$.

$\eta \sum_p [R_{kp} \rho_{pp} \delta_{kq} - \frac{1}{2} (R_{pk} + R_{pq}) \rho_{kq}]$, where diagonal and off-diagonal elements decouple from each other. The latter decay with rates $\Gamma_{kq} = \frac{1}{2} \eta \sum_p (R_{pk} + R_{pq})$ and have to be negligible already when the peak entropy is reached, to allow for the observed transient approach of the maximally mixed state (this is indeed the case—see Fig. S2 and discussion in Ref. [76]). The dynamics of the diagonal elements $p_k \equiv \rho_{kk}$ is determined by the rate matrix R_{kq} through the Pauli rate equation $\dot{p}_k = \eta \sum_q [R_{kq} p_q - R_{qk} p_k]$ [37].

In Figs. 2(a) and 2(b) we compare rate matrices for $r = 0$ and $r = 4$. Without a potential gradient, one finds long-range coupling with respect to energy. In contrast, at a large field gradient ($r = 4$) the transition rates predominantly couple states that are close by in energy. The latter is a consequence of Stark localization, where eigenstates that are close in space, so that they are coupled by the bath via the densities n_i , are close also with respect to energy. Note that for a disorder-

localized Fermi-Hubbard chain without this spatioenergetic correlation we find nonlocal rate matrices and no prethermal memory loss (see Fig. S10 of Ref. [76]). This constitutes an even more drastic difference between the relaxation dynamics of Stark and disorder-induced MBL than the one observed for dephasing noise [59].

To check whether a rate matrix with energy-local coupling is crucial for the appearance of a close-to-maximum entropy ($S_{\text{peak}} \simeq S_{\infty}$), we investigate the rate matrices for rather different on-site potentials W_i [see Fig. 2(c)] that (were optimized to) equally give rise to large peak entropies [see Fig. 2(d)]. It turns out that, indeed, they also show near-neighbor coupling [see Figs. 2(e) and 2(f)]. Moreover, also the tilted bosonic Hubbard chain [given by Eq. (1) with bosonic annihilation operators c_i and the last term replaced by on-site interactions $\frac{1}{2}V \sum_i n_i(n_i - 1)$] shows $S_{\text{peak}} \approx S_{\infty}$ together with an energy local rate matrix [see Figs. 2(g) and 2(h)]. This, together with results for simplified models shown below (and in Fig. S13 of Ref. [76]), indicates clearly that the prethermal memory loss discussed here is a very robust phenomenon.

Simplified rate model. To address the question why the appearance of a close-to-maximum entropy is associated with local coupling between energy states, let us now consider a simplified rate model. Here, the energy eigenstates have equally spaced nondegenerate energies $\varepsilon_k = rk$, with $k = 0, 1, \dots, D-1$, and are coupled by thermal rates that are homogeneous and local with respect to energy. We define $R_{k+n,k} \equiv R_n$, where $R_n = 0$ for $|n| > n_m$ and $R_n = g(nr)$ for $|n| \leq n_m$, so that (as a property of the bath correlation function g) $R_n/R_{-n} = (R_+/R_-)^n = e^{-nr/T}$, with $R_{\pm} \equiv R_{\pm 1}$.

Let us first study nearest-neighbor coupling, $n_m = 1$. By defining $\nabla^2 p_k = p_{k+1} + p_{k-1} - 2p_k$ and $\nabla p_k = (p_{k+1} - p_{k-1})/2$, we can write the Pauli rate equation as a discrete drift-diffusion equation, $\dot{p}_k = \eta(\bar{R}\nabla^2 p_k + \delta R\nabla p_k)$ [82], where diffusion and drift are quantified by $\bar{R} \equiv (R_+ + R_-)/2$ and $\delta R \equiv R_- - R_+$, respectively. Scaling time with $(\eta\bar{R})^{-1}$, the model is completely characterized by the ratio $\alpha = \delta R/\bar{R}$ and the system size D . Starting from the highest excited state, $p_k(0) = \delta_{k,k_{\text{ini}}}$ with $k_{\text{ini}} = D-1$, and setting $D = 50$, in Figs. 3(a) and 3(b) we plot p_k for $\alpha = 0.1$ and $\alpha = 1$ for different times (solid lines, fat red lines indicate t_{peak}). While for $\alpha = 0.1$, a rather uniform distribution is found at a time t_{peak} , approximating the maximum entropy state with $p_k = 1/D$, this is not the case for larger drift, $\alpha = 1$. Before reaching thermal equilibrium, we find the distribution well approximated by a Gaussian of standard deviation $\sigma = \sqrt{2\bar{R}t}$ centered at $k_0 = k_{\text{ini}} - \delta R t$ [82] (dashed lines). The condition for reaching an almost flat distribution is thus given by the intuitive requirement that the drift time needed to reach $k_0 = D/2$, $\tau_F = (k_{\text{ini}} - D/2)/\delta R$ is larger than the diffusion time giving rise to $\sigma = D/2$, $\tau_D = D^2/(8\bar{R})$. Thus, for $k_{\text{ini}} = D-1$ we expect $S_{\text{peak}} \approx S_{\infty}$ as long as $\alpha \lesssim 4D^{-1}$, which is confirmed in Fig. 3(c) [see also Fig. S12(a) of Ref. [76]].

However, $S_{\text{peak}} \approx S_{\infty}$ is a nontrivial result only as long as the thermal entropy S_T , plotted in Fig. 3(d), lies well below S_{∞} . For $D \gg T$ we can neglect the upper bound of the spectrum and S_T approaches the value for an harmonic oscillator with frequency r [83], $S_T \simeq \frac{x}{x-1} \log(x) - \log(1-x)$ with $x = e^{-r/T} = R_+/R_- = (2-\alpha)/(2+\alpha)$, so that for $\alpha \ll 1$ one

FIG. 3. Simplified rate model with nearest-neighbor coupling $n_m = 1$ evolving from the most excited state. (a), (b) Probability distribution p_k at different times for $D = 50$ and (a) $\alpha = 0.1$ and (b) $\alpha = 1$. (c), (d) Normalized peak and thermal entropies (c) $S_{\text{peak}}/S_{\infty}$ and (d) S_T/S_{∞} vs α and D .

has $S_T \simeq \log(1/\alpha) + 1 + \mathcal{O}(\alpha)$ [Fig. 3(d)]. Thus, $S_T/S_{\infty} < s$ as long as $\alpha \gtrsim eD^{-s}$. While for $D \rightarrow \infty$ this requirement is incompatible with the one for large peak entropies, $\alpha \lesssim 4D^{-1}$, it turns out that the different prefactors appearing in both conditions (whose values can deviate from our estimates e and 4) still give rise to a large nontrivial regime for finite D , as can be inferred from Figs. 3(c) and 3(d).

The simplified rate model with $n_m = 1$ roughly corresponds to the case of a single particle in a tilted lattice. While it describes peak entropies $S_{\text{peak}} \approx S_{\infty}$ and prethermal memory loss, it gives rise to a Gaussian rather than an exponential prethermal distribution. This suggests that the formation of a prethermal Gibbs state requires more complex rate matrices as they are found for the many-particle case. In Fig. 4 we investigate what happens when increasing the coupling range n_m , and thus the complexity, of the simplified rate model (using $D = 50$ and $\eta = 0.1$). We observe that S_{peak} first increases with n_m before, and after reaching a maximum at $n_m = 11$, it decreases again. The first increase with n_m might be explained by the prethermal distribution becoming more Gibbs-like and thus flatter at $t = t_{\text{peak}}$ than the Gaussian (which always retains a finite σ). This is confirmed in Fig. 4(b), where we plot the

FIG. 4. Evolution of the simplified rate model starting from the most excited state for $D = 50$, $\eta = 0.1$, and T so that $S_T = S_{\infty}/2$. (a) Entropy evolution for different n_m . Inset: Normalized peak entropies vs n_m . (b) Distribution p_k vs k for $n_m = 11$ at the times marked in the inset compared to effective Gibbs states (dashed lines).

distribution p_k at different times for $n_m = 11$ and find rather good agreement with an effective Gibbs state. The subsequent decrease of S_{peak} can, in turn, be attributed to an increase of the drift velocity with n_m , which is clearly visible also in Fig. 4(a) and which reduces the time available for reaching a prethermal distribution. This mechanism explains why large transient peak entropies $S_{\text{peak}} \approx S_\infty$ are found for rate matrices that are local in energy.

Conclusion. We have shown that the nonequilibrium relaxation dynamics of interacting Wannier-Stark ladders

coupled to a finite-temperature environment can feature effective prethermal memory loss. The effect is found to rely on a dissipative form of prethermalization. In an experiment with ultracold atoms, a thermal environment could be provided, for instance, by the coupling to a second atomic species [84].

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