Erratum: Time-Bin and Polarization Superdense Teleportation for Space Applications [Phys. Rev. Applied 14, 014044 (2020)]

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Table I has been updated as seen below to correct several typographical errors in rows 13 and 21 where part of the last two entries in the row were switched. Furthermore, the states in the first four columns have been rearranged to make the phase convention of Appendix D consistent to allow for easier reproduction of the table. (Originally, we chose a phase convention which we thought made the table easier to reproduce by looking at Fig. 2.)

To make the phase convention consistent throughout Appendix D and correct an error during manuscript creation, M should be right after the equals sign in Eqs. (D5)–(D8) as shown below.

$$\text{Setang}_1 \equiv M \times P_r \times H_1 \times Q_1 \times \text{Interf}_{P1} \times Q_2 \times H_2 \times \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (\text{D5})$$

$$\text{Setang}_2 \equiv M \times P_r \times H_1 \times Q_1 \times \text{Interf}_{P1} \times Q_2 \times H_2 \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (\text{D6})$$

$$\text{Setang}_3 \equiv M \times P_r \times H_1 \times Q_1 \times \text{Interf}_{P2} \times Q_3 \times H_4 \times \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad (\text{D7})$$

$$\text{Setang}_4 \equiv M \times P_r \times H_1 \times Q_1 \times \text{Interf}_{P2} \times Q_3 \times H_4 \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (\text{D8})$$

There was also a typographical error, where the second and third rows of Eq. (D10) were flipped. The correct version is

$$\text{Interf}_{P2} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}. \quad (\text{D10})$$

We thank Dr. Jason Gallicchio and his group for their help in catching the above typographical errors and encouraging us to make the tables in the manuscript reproducible with the given formulas and phase convention.

Additionally, the authors noticed a small error in Fig. 12. In the original publication, the data in the top and bottom panel of the right-most graphs were mistakenly offset from one another by 0.7° .

None of the changes discussed in this erratum affect any of the results, calculations, or conclusions in this paper.

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TABLE I. Tomography states. This table details the states that each single-photon detector is projecting onto for each setting of the tomography. Additionally, for the nine-setting tomography, it shows which coincidences are used to calibrate the relative efficiencies of Bob's four detectors for each tomography measurement. For example, C_{S2B1} corresponds to the coincidences with Bob's detector 1 on measurement setting 2 with one of Alice's detectors.

B ₁ State	B ₂ State	B ₃ State	B ₄ State	36-sett #	9-sett #	B ₁ Eff. Cor.	B ₂ Eff. Cor.	B ₃ Eff. Cor.	B ₄ Eff. Cor.
$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle - t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle + t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle - i t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle + i t_2\rangle)$	1	1	C_{S1B1}/C_{S1B1}	C_{S2B1}/C_{S1B2}	C_{S3B1}/C_{S1B3}	C_{S4B1}/C_{S1B4}
$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle + t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle - t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle + i t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle - i t_2\rangle)$	2					
$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle - i t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle + i t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle + t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle - t_2\rangle)$	3					
$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle + i t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle - i t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle - t_2\rangle)$	$ H\rangle \frac{1}{\sqrt{2}}(t_1\rangle + t_2\rangle)$	4					
$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle - t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle + t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle - i t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle + i t_2\rangle)$	5	2	C_{S3B1}/C_{S3B1}	C_{S6B1}/C_{S5B2}	C_{S7B1}/C_{S5B3}	C_{S8B1}/C_{S5B4}
$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle + t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle - t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle + i t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle - i t_2\rangle)$	6					
$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle - i t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle + i t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle + t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle - t_2\rangle)$	7					
$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle + i t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle - i t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle - t_2\rangle)$	$ V\rangle \frac{1}{\sqrt{2}}(t_1\rangle + t_2\rangle)$	8					
$\frac{1}{\sqrt{2}}(Lt_1\rangle - Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle + Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle + Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle - Lt_2\rangle)$	9					
$\frac{1}{\sqrt{2}}(Lt_1\rangle + Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle - Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle - Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle + Lt_2\rangle)$	10					
$\frac{1}{\sqrt{2}}(Lt_1\rangle + i Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle - i Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle + i Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle - i Lt_2\rangle)$	11					
$\frac{1}{\sqrt{2}}(Lt_1\rangle - i Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle + i Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle - i Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle + i Lt_2\rangle)$	12					
$\frac{1}{\sqrt{2}}(Dt_1\rangle - At_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle + At_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle + Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle - Dt_2\rangle)$	13	3	C_{S13B1}/C_{S13B1}	C_{S14B1}/C_{S13B2}	C_{S18B1}/C_{S13B3}	C_{S17B1}/C_{S13B4}
$\frac{1}{\sqrt{2}}(Dt_1\rangle + At_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle - At_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle - Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle + Dt_2\rangle)$	14					
$\frac{1}{\sqrt{2}}(Dt_1\rangle - i At_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle + i At_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle - i Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle + i Dt_2\rangle)$	15	4	C_{S13B1}/C_{S15B1}	C_{S16B1}/C_{S15B2}	C_{S19B1}/C_{S15B3}	C_{S20B1}/C_{S15B4}
$\frac{1}{\sqrt{2}}(Dt_1\rangle + i At_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle - i At_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle + i Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle - i Dt_2\rangle)$	16					
$\frac{1}{\sqrt{2}}(At_1\rangle - Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle + Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle + At_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle - At_2\rangle)$	17					
$\frac{1}{\sqrt{2}}(At_1\rangle + Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle - Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle - At_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle + At_2\rangle)$	18					
$\frac{1}{\sqrt{2}}(At_1\rangle - i Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle + i Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle - i At_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle + i At_2\rangle)$	19					
$\frac{1}{\sqrt{2}}(At_1\rangle + i Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(At_1\rangle - i Dt_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle + i At_2\rangle)$	$\frac{1}{\sqrt{2}}(Dt_1\rangle - i At_2\rangle)$	20					
$\frac{1}{\sqrt{2}}(Rt_1\rangle - Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle + Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle + Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle - Rt_2\rangle)$	21	5	C_{S21B1}/C_{S21B1}	C_{S22B1}/C_{S21B2}	C_{S10B1}/C_{S21B3}	C_{S9B1}/C_{S21B4}
$\frac{1}{\sqrt{2}}(Rt_1\rangle + Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle - Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle - Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle + Rt_2\rangle)$	22					
$\frac{1}{\sqrt{2}}(Rt_1\rangle + i Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle - i Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle + i Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle - i Rt_2\rangle)$	23	6	C_{S23B1}/C_{S23B1}	C_{S24B1}/C_{S23B2}	C_{S11B1}/C_{S23B3}	C_{S12B1}/C_{S23B4}
$\frac{1}{\sqrt{2}}(Rt_1\rangle - i Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Rt_1\rangle + i Lt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle - i Rt_2\rangle)$	$\frac{1}{\sqrt{2}}(Lt_1\rangle + i Rt_2\rangle)$	24					
$ At_1\rangle$	$ Dt_2\rangle$	$ At_2\rangle$	$ Dt_1\rangle$	25					
$ Ht_1\rangle$	$ Vt_2\rangle$	$ Ht_2\rangle$	$ Vt_1\rangle$	26	7	C_{S26B1}/C_{S26B1}	C_{S32B1}/C_{S26B2}	C_{S34B1}/C_{S26B3}	C_{S28B1}/C_{S26B4}
$ Lt_1\rangle$	$ Rt_2\rangle$	$ Lt_2\rangle$	$ Rt_1\rangle$	27					
$ Vt_1\rangle$	$ Ht_2\rangle$	$ Vt_2\rangle$	$ Ht_1\rangle$	28					
$ Dt_1\rangle$	$ At_2\rangle$	$ Dt_2\rangle$	$ At_1\rangle$	29	8	C_{S29B1}/C_{S29B1}	C_{S35B1}/C_{S29B2}	C_{S31B1}/C_{S29B3}	C_{S25B1}/C_{S29B4}
$ Rt_1\rangle$	$ Lt_2\rangle$	$ Rt_2\rangle$	$ Lt_1\rangle$	30	9	C_{S30B1}/C_{S30B1}	C_{S36B1}/C_{S30B2}	C_{S33B1}/C_{S30B3}	C_{S27B1}/C_{S30B4}
$ Dt_2\rangle$	$ At_1\rangle$	$ Dt_1\rangle$	$ At_2\rangle$	31					
$ Vt_2\rangle$	$ Ht_1\rangle$	$ Vt_1\rangle$	$ Ht_2\rangle$	32					
$ Rt_2\rangle$	$ Lt_1\rangle$	$ Rt_1\rangle$	$ Lt_2\rangle$	33					
$ Ht_2\rangle$	$ Vt_1\rangle$	$ Ht_1\rangle$	$ Vt_2\rangle$	34					
$ At_2\rangle$	$ Dt_1\rangle$	$ At_1\rangle$	$ Dt_2\rangle$	35					
$ Lt_2\rangle$	$ Rt_1\rangle$	$ Lt_1\rangle$	$ Rt_2\rangle$	36					

TABLE II. Tomography settings. This table details the angles of the wave plates and positions of the polarizers for each setting of the tomography. $\alpha_1, \alpha_2, \alpha_3$ are the angles of half-wave plates 1–3; similarly $\beta_1, \beta_2, \beta_3$ are the angles of quarter-wave plates 1–3. T_H and T_V are transmission settings of the removable polarizer.

α_1	β_1	α_2	β_2	α_3	β_3	T_H	T_V	36 settings #	9 settings #
22.5	90	−22.5	0	−22.5	135	1	0	1	1
22.5	90	22.5	0	22.5	135	1	0	2	
22.5	90	22.5	45	−22.5	90	1	0	3	
22.5	90	−22.5	45	22.5	90	1	0	4	
67.5	90	−22.5	0	−22.5	135	0	1	5	2
67.5	90	22.5	0	22.5	135	0	1	6	
67.5	90	22.5	45	−22.5	90	0	1	7	
67.5	90	−22.5	45	22.5	90	0	1	8	
0	45	−22.5	0	−22.5	90	1	1	9	
0	45	22.5	0	22.5	90	1	1	10	
0	45	−22.5	45	22.5	135	1	1	11	
0	45	22.5	45	−22.5	135	1	1	12	
22.5	90	−22.5	0	−22.5	90	1	1	13	3
22.5	90	22.5	0	22.5	90	1	1	14	
22.5	90	22.5	45	−22.5	135	1	1	15	4
22.5	90	−22.5	45	22.5	135	1	1	16	
−22.5	90	22.5	0	22.5	90	1	1	17	
−22.5	90	−22.5	0	−22.5	90	1	1	18	
−22.5	90	−22.5	45	22.5	135	1	1	19	
−22.5	90	22.5	45	−22.5	135	1	1	20	
0	135	22.5	0	22.5	90	1	1	21	5
0	135	−22.5	0	−22.5	90	1	1	22	
0	135	22.5	45	−22.5	135	1	1	23	6
0	135	−22.5	45	22.5	135	1	1	24	
−22.5	90	0	0	0	90	1	1	25	
0	90	0	0	0	90	1	1	26	7
0	45	0	0	0	90	1	1	27	
45	90	0	0	0	90	1	1	28	
22.5	90	0	0	0	90	1	1	29	8
0	135	0	0	0	90	1	1	30	9
−22.5	90	45	0	45	90	1	1	31	
0	90	45	0	45	90	1	1	32	
0	45	45	0	45	90	1	1	33	
45	90	45	0	45	90	1	1	34	
22.5	90	45	0	45	90	1	1	35	
0	135	45	0	45	90	1	1	36

FIG. 12. Two-sample KS tests of very closely spaced distributions in phase space. (Top row) Empirical CDF of phase space data along horizontal axis. (Bottom row) Measured points in phase space. Crosses (dots) are measurements of the (un)displaced distributions. The distributions of crosses (dots) are centered over 45° (39°), 147° (140°), 256° (249°) for ϕ_1 , ϕ_2 , and ϕ_3 , respectively.