Transmission-Line Model for Materials with Spin-Momentum Locking

Shehrin Sayed,^{1,*} Seokmin Hong,² and Supriyo Datta^{1,†}

¹*School of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana 47907, USA*

²*Center for Spintronics, Korea Institute of Science and Technology, Seoul 02792, Republic of Korea*

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We provide a transmission-line representation for channels exhibiting spin-momentum locking (SML) that can be used for both time-dependent and steady-state transport analysis on a wide variety of materials with spin-orbit coupling such as topological insulators, Kondo insulators, transition metals, semimetals, oxide interfaces, and narrow band-gap semiconductors. This model is based on a time-dependent four-component diffusion equation obtained from the Boltzmann transport equation assuming linear response and elastic scattering in the channel. We classify all electronic states in the channel into four groups (U^+ , D^+ , U^- , and D^-) depending on the spin index [up (U), down (D)] and the sign of the x component of the group velocity (+, −) and assign an average electrochemical potential to each of the four groups to obtain the four-component diffusion equation. For normal metal channels, the model decouples into the well-known transmission-line model for charge and a time-dependent version of the Valet-Fert equation for spin. We first show that, in the steady-state limit, our model leads to simple expressions for charge-spin interconversion in SML channels in good agreement with existing experimental data on diverse materials. We then use the full time-dependent model to study spin-charge separation in the presence of SML, a subject that has been controversial in the past. Our model shows that the charge and spin signals travel with two distinct velocities, resulting in well-known spin-charge separation, which is expected to persist even in the presence of SML. However, our model predicts that the lower velocity signal is purely spin, while the higher velocity signal is largely charge with an additional spin component proportional to the degree of SML, which has not been noted before. Finally, we note that our model can be used within standard circuit simulators such as SPICE to obtain numerical results for complex geometries.

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I. INTRODUCTION

Transport properties in materials with spin-orbit coupling (SOC) are of great interest for potential spintronic applications, especially because of the unique spin-momentum locking (SML) observed in diverse classes of materials, e.g., topological insulators (TI) [1–9], Kondo insulators [10], transition metals [11–18], semimetals [19], oxides [20–22], and narrow band-gap semiconductors [23–26]. There has been an immense effort to model the interplay between spin and charge in such materials using time-dependent classical [27] or quantum Boltzmann equations [28,29], nonequilibrium Green's functions

[30–34], phenomenological equations coupled to magnet dynamics [35], and time-independent diffusion equations used to explain the bulk spin Hall effect [36].

In this paper, we derive a transmission-line model that can be used for both time-dependent and steady-state transport analysis on multicontact-based structures implemented with materials exhibiting SML. The model is compatible within a standard circuit simulator tool such as Simulation Program with Integrated Circuit Emphasis (SPICE), which will enable straightforward analysis of complex geometries. The proposed model is an addition to our multiphysics spin-circuit framework [37], which has been previously used to explain experiments and evaluate spin-based device proposals [38,39]. We first show that, in the steady-state limit, the transmission-line model provides simple expressions for charge-spin interconversion that have been previously used by a number of experimental groups [3–10,26] to quantify spin voltages on diverse materials measured with ferromagnetic contacts. We further derive a simple expression for a parameter that has been widely used to quantify the inverse Rashba-Edelstein effect (IREE) in two-dimensional (2D) channels, which

*ssayed@purdue.edu

†datta@purdue.edu

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shows good agreement with existing experimental data on diverse materials [20,40–42]. We then use the full time-dependent model to study spin-charge separation in the presence of SML in materials with SOC, a subject that has been controversial in the past (see, for example, [43–46]). Our model suggests that, depending on the channel cross section, the charge and spin signals travel with two distinct velocities, resulting in spin-charge separation, which is well-known for materials without SOC [47–50] based on the Luttinger liquid theory. However, our model shows that the lower-velocity signal is purely spin, while the higher-velocity signal is largely charge, with an additional spin component proportional to the degree of SML in the channel, which has not been discussed before.

The transmission-line model is based on a time-dependent four-component diffusion equation obtained from the Boltzmann transport equation assuming linear response and elastic scattering processes in the channel. The assumptions made to derive the model are discussed in detail in Sec. V. The basic approach is to assign one electrochemical potential $\mu(\vec{p}, s)$ to each of the eigenstates $(\vec{p}, s)$, where $\vec{p}$ is the momentum confined in the z - x plane and $s = \pm 1$ is the spin index with $+1$ and -1 denoting the up (U) and the down (D) spins with respect to the spin quantization axis $\hat{y} \times (\vec{p} - q\vec{A})$ ($\vec{A}$ is the vector magnetic potential). We then classify the eigenstates into four groups (U^+ , D^+ , U^- , and D^-) based on the spin index (U , D) and the sign of the x component of the group velocity ($+$, $-$) and define an average electrochemical potential corresponding to each of the four groups, resulting in a four-component diffusion equation. This procedure can be viewed as an extension of the Valet-Fert equation, which uses two electrochemical potentials for U and D states [51].

In our generalized view, U^+ (and U^-) states have the same number of modes M (and N) as D^- (and D^+) states due to the time-reversal symmetry. The degree of SML in our model is given by [52,53]

$$p_0 = \frac{M - N}{M + N}, \quad (1)$$

where M and N are evaluated at the Fermi energy for zero temperature and, in general, require thermal averaging. For normal metal (NM) channels, $p_0 = 0$, i.e., $M = N$. For a perfect topological insulator (TI) $N = 0$ leading to $p_0 = 1$, however, p_0 gets effectively lowered by the presence of parallel channels. p_0 has been quantified for different TIs by a number of groups [3–9] by measuring the charge-current-induced spin voltage using a ferromagnetic (FM) contact, motivated by a theoretical proposal [34]. For a Rashba channel with a coupling coefficient, α_R , $p_0 \approx \alpha_R k_F / (2E_F) \ll 1$ [25,34], which has been quantified

with similar spin voltage measurements [26]. Here, k_F and E_F are the Fermi wave vector and the Fermi energy, respectively. Recently, spin voltage measurements have been reported on transition metals (Ta, Pt, W, Ir, and Au) [16–18,54]; semimetals (WTe₂ [19]); and Kondo insulators (SmB₆ [10]). These experiments can also be quantified by p_0 , although the underlying mechanism of transition metals is subject to active debate [55–57] and could involve a bulk spin Hall effect [14,36,58] or interface Rashba-like channel [15,59–61].

The four-component diffusion equation in steady state reduces to our prior model that predicted a unique three-resistance state in a two-FM-contact-based multiterminal spin-valve structure on SML materials [52]. The prediction has recently been observed in InAs [26] and Pt [16,17] up to room temperature and should be observed in other materials with SML as well. We translate the full time-dependent version of the four-component diffusion equation into a transmission-line model with two-component (charge and z component of spin) voltages and currents, where the coupling between the charge and spin in a SML channel is characterized by p_0 in Eq. (1). Note that the proposed model does not take into account the effects such as spin precession involving the off-diagonal elements of the density matrix, which we assume to be negligible. An extension of this model to include x and y components of spin could possibly address such issues, as done earlier for materials without SOC (see [38] and references therein). For NM channels (i.e., $p_0 = 0$), the proposed transmission-line model decouples into the well-known model for charge that has been previously used to analyze transport in quantum wires [50,62,63] and a time-dependent version of the Valet-Fert equation [51] for spin.

The paper is organized as follows. In Sec. II, we describe the transmission-line model for SML channels and show that special cases lead to prior well-known models. In Sec. III, we derive several results on charge-spin interconversion from our transmission-line model in steady state and present comparison with SPICE simulations using the full model. We obtain a simple expression for a parameter that has been widely used to quantify IREE and show that it is in good agreement with available experiments on diverse materials. In Sec. IV, we study the spin-charge separation in terms of spin and charge signal velocities obtained from our time-dependent transmission-line equations. We show that the separation persists even in SML channels; however, in SML channels there exists an additional spin component accompanied by the high-velocity charge signal. In Sec. V, we derive the transmission-line model starting from the Boltzmann transport equation with all the assumptions clearly stated. We discuss different scattering mechanisms in the channels and their effects on charge and spin transport. Finally, in Sec. VI, we end with a brief summary.

II. TRANSMISSION-LINE MODEL

A. Model description

We consider the structure and axes in Fig. 1(a) to derive the transmission-line model. The model has two components: the charge and z component of spin, with a coupling between them characterized by p_0 in Eq. (1). The charge model is given by

$$\left(\frac{1}{C_E} + \frac{1}{C_Q}\right)^{-1} \frac{\partial}{\partial t} V_c = -\frac{\partial}{\partial x} I_c, \quad (2)$$

$$(L_K + L_M) \frac{\partial}{\partial t} I_c + R_c I_c = -\frac{\partial}{\partial x} V_c + p_0 \eta_c V_s,$$

where I_c and V_c are the charge current and voltage along the $\hat{x}$ direction, C_E and C_Q are the electrostatic and quantum capacitances per unit length respectively, L_M and L_K are the magnetic and kinetic inductances per unit length respectively, and R_c is the charge resistance per unit length.

The spin model is given by

$$\frac{C_Q}{\alpha^2} \frac{\partial}{\partial t} V_s + G_{sh} V_s + p_0 g_m V_{em} = -\frac{\partial}{\partial x} I_s + p_0 \gamma_s I_c, \quad (3)$$

$$\alpha^2 L_K \frac{\partial}{\partial t} I_s + R_s I_s - p_0 r_m I_{em} = -\frac{\partial}{\partial x} V_s + p_0 \eta_s V_c,$$

where I_s and V_s are the spin current and voltage along the $\hat{x}$ direction with spin polarization along the $\hat{z}$ direction. In this discussion, the $\hat{y}$ direction is out of plane. Here, $\alpha = 2/\pi$ is an angular averaging factor, R_s is the series spin resistance per unit length of the channel, and G_{sh} is the shunt spin conductance per unit length that captures the spin lost in the channel due to the spin relaxation. A detailed derivation of Eqs. (2) and (3) from the Boltzmann transport equation will be discussed in Sec. V with clearly stated assumptions.

Distributed circuit models for charge and spin are shown in Figs. 1(b) and 1(c), which are based on Eqs. (2) and (3), respectively. The dependent sources proportional to p_0 represent charge-spin intercoupling between the two models. The dependent source parameters in Fig. 1 are given by

$$V_m = \eta_c V_s, \quad (4a)$$

$$V_n = \eta_s V_c + r_m I_{em}, \quad (4b)$$

$$\text{and, } I_n = \gamma_s I_c - g_m V_{em}. \quad (4c)$$

The parameters of Eqs. (2) and (3) are given by

$$C_Q = \frac{2}{R_B |\langle v_x \rangle|}, \quad (5a)$$

FIG. 1. (a) Structure and corresponding axes of the channel with spin-momentum locking, for which a transmission-line model is derived. The model has two components: (b) charge [which corresponds to Eq. (2)] and (c) spin [which corresponds to Eq. (3)], with a coupling between them described by the degree of SML p_0 [see Eq. (1)]. The coupling between the charge and spin is modeled by dependent voltage and current sources with $V_m = \eta_c V_s$, $V_n = \eta_s V_c + r_m I_{em}$, and $I_n = \gamma_s I_c - g_m V_{em}$.

$$L_K = \frac{R_B}{2 |\langle v_x \rangle|}, \quad (5b)$$

$$R_c = \frac{R_B}{\lambda}, \quad (5c)$$

$$R_s = \frac{\alpha^2 R_B}{\lambda_0}, \quad (5d)$$

$$G_{sh} = \frac{4}{\alpha^2 R_B \lambda_s}, \quad (5e)$$

$$g_m = \frac{2}{\alpha R_B}, \quad (5f)$$

$$r_m = \frac{\alpha}{|\langle v_x \rangle| C_E}, \quad (5g)$$

$$\eta_s = \frac{2\alpha}{\lambda_0}, \quad (5h)$$

$$\eta_c = \frac{2}{\alpha \lambda_r}, \quad (5i)$$

$$\gamma_s = \frac{2}{\alpha \lambda_t}, \quad (5j)$$

$$\text{and } R_B = \frac{h}{q^2} \frac{1}{M + N}. \quad (5k)$$

Here, λ , λ_0 , and λ_s are the three distinct mean free paths that determine R_c , R_s , and G_{sh} , respectively. $|\langle v_x \rangle|$ is the magnitude of the thermally averaged electron velocity $\langle v_x \rangle$. η_c represents the spin-to-charge conversion coefficient and η_s, γ_s represent charge-to-spin conversion coefficients. These coefficients depend on different scattering mechanisms in the channel, which will be discussed later in Sec. V. r_m and g_m are transient charge-spin coupling coefficients, which are in the units of resistance per unit length and conductance per unit length, respectively.

R_B is the ballistic resistance of the channel, h is the Planck's constant, and q is the electron charge. R_B is inversely proportional to the total number of modes ($M + N$) in the channel, which represents a material property and does not imply ballistic transport. The models and related results discussed in this paper are valid all the way from the ballistic to the diffusive regime of operation.

B. Presence of an external contact

In the presence of an external contact on the channel [see Fig. 2(a)], the charge model in Eq. (2) is modified as

$$\left(\frac{1}{C_E} + \frac{1}{C_Q} \right)^{-1} \frac{\partial}{\partial t} V_c = -\frac{\partial}{\partial x} I_c + i^c, \quad (6)$$

$$(L_K + L_M) \frac{\partial}{\partial t} I_c + R_c I_c = -\frac{\partial}{\partial x} V_c + p_0 \eta_c V_s + \Delta v^c,$$

and the spin model in Eq. (3) is modified as

$$\begin{aligned} \frac{C_Q}{\alpha^2} \frac{\partial}{\partial t} V_s + G_{sh} V_s + p_0 g_m V_{em} &= -\frac{\partial}{\partial x} I_s + p_0 \gamma_s I_c + i^s, \\ \alpha^2 L_K \frac{\partial}{\partial t} I_s + R_s I_s - p_0 r_m I_{em} &= -\frac{\partial}{\partial x} V_s + p_0 \eta_s V_c + \Delta v^s, \end{aligned} \quad (7)$$

where i^c and i^s represent charge and spin currents entering into the channel per unit length from the external contact. Δv^c and Δv^s represent the change in channel charge and spin voltages per unit length in the region under the

external contact. They are given as

$$\begin{Bmatrix} i^c \\ i^s \end{Bmatrix} = G_0 \begin{bmatrix} 1 & \frac{p_f}{\alpha} \\ \frac{p_f}{\alpha} & \frac{1}{\alpha^2} \end{bmatrix} \begin{Bmatrix} v_c - V_c \\ v_s - V_s \end{Bmatrix}, \quad \text{and} \quad (8)$$

$$\begin{aligned} \begin{Bmatrix} \Delta v^c \\ \Delta v^s \end{Bmatrix} &= -\frac{G_0 R_B^2}{4} \begin{bmatrix} 1 & \alpha p_f \\ \alpha p_f & \alpha^2 \end{bmatrix} \begin{Bmatrix} I_c \\ I_s \end{Bmatrix} \\ &+ \frac{p_0 G_0 R_B}{2} \begin{bmatrix} p_f & \frac{1}{\alpha} \\ \alpha & p_f \end{bmatrix} \begin{Bmatrix} v_c \\ v_s \end{Bmatrix}, \end{aligned} \quad (9)$$

where $v_c = (v_u + v_d)/2$ and $v_s = \alpha(v_u - v_d)/2$ are charge and spin voltages applied at the external contact, G_0 is the contact conductance per unit length, and p_f is the contact polarization with $p_f = 0$ indicating a normal metal contact and $p_f \neq 0$ indicating a ferromagnetic contact. The derivation of the model starting from the Boltzmann transport equation is shown in Sec. V with clearly stated assumptions streamlined with subheadings.

Modified distributed circuit models for charge and spin are shown in Figs. 2(b) and 2(c), which are based on Eqs. (6) and (7), respectively. The presence of a contact with conductance G_0 adds series resistances $R_{\text{cont}}^c = G_0 R_B^2/4$ and $R_{\text{cont}}^s = \alpha^2 G_0 R_B^2/4$ in the charge and spin models as shown in Fig. 2. This effect exists even if the channel is NM. The presence of the external contact also modulates the dependent sources in Eqs. (4a) and (4b) as

$$V'_m = \eta_c V_s + \frac{G_0 R_B}{2} \left(\frac{v_s}{\alpha} + p_f v_c \right), \quad (10a)$$

$$V'_n = \eta_s V_c + r_m I_{em} + \frac{G_0 R_B}{2} (\alpha v_c + p_f v_s), \quad (10b)$$

with V_a and V_b representing the voltage drop across R_{cont}^c and R_{cont}^s in charge and spin models in Fig. 2, respectively. Note that the additional terms due to the presence of an external contact, are proportional to G_0 and negligible for potentiometric contacts where G_0 is very low.

C. Special case: Normal metals ($p_0 = 0$)

We consider a special case of Eqs. (2) and (3) for a normal metal channel, i.e., $p_0 = 0$. For a NM channel, the charge and spin model decouples to well-known models as described below.

1. Charge model: Transport model for quantum wires

For NM channels ($p_0 = 0$), the charge model in Eq. (2) reduces to the well-known transmission-line model for a

FIG. 2. (a) Structure of the spin-momentum-locked channel in the presence of an external contact. Both (b) charge and (c) spin transmission-line models are modified as compared to Fig. 1, which now correspond to Eqs. (6) and (7), respectively. The dependent sources are $V'_m = \eta_c V_s + (G_0 R_B / 2) (v_s / \alpha + p_f v_c)$, $V'_n = \eta_s V_c + r_m I_{\text{em}} + (G_0 R_B / 2) (\alpha v_c + p_f v_s)$, and $I_n = \gamma_s I_c - g_m V_{\text{em}}$. Note that the model reduces to that shown in Fig. 1 in the limit $G_0 \rightarrow 0$.

charge that has been previously used to analyze transport in quantum wires, given by

$$\left(\frac{1}{C_E} + \frac{1}{C_Q} \right)^{-1} \frac{\partial}{\partial t} V_c = -\frac{\partial}{\partial x} I_c, \quad (11)$$

$$(L_K + L_M) \frac{\partial}{\partial t} I_c + R_c I_c = -\frac{\partial}{\partial x} V_c.$$

The model in Eq. (11) was first derived from Luttinger liquid theory [50,62] and then from the Boltzmann transport equation with one electrochemical potential [63]. In the quantum wire limit, $L_K \gg L_M$ and $C_Q \ll C_E$, while, in the classical transmission-line limit, $L_K \ll L_M$ and $C_Q \gg C_E$ [50,63].

In steady state ($\partial/\partial t \rightarrow 0$), we get the diffusion equation for charge from Eq. (11), given by

$$\frac{d^2}{dx^2} V_c = 0. \quad (12)$$

2. Spin model: Valet-Fert equation

For NM channels ($p_0 = 0$), the spin model in Eq. (3) becomes a time-dependent spin-diffusion equation, given by

$$\frac{C_Q}{\alpha^2} \frac{\partial}{\partial t} V_s + G_{\text{sh}} V_s = -\frac{\partial}{\partial x} I_s, \quad (13)$$

$$\alpha^2 L_K \frac{\partial}{\partial t} I_s + R_s I_s = -\frac{\partial}{\partial x} V_s,$$

which, in steady state ($\partial/\partial t \rightarrow 0$), becomes the well-known Valet-Fert equation [51], given by

$$\frac{d^2}{dx^2} V_s = \frac{V_s}{\lambda_{\text{sf}}^2}, \quad (14)$$

with the spin diffusion length given by

$$\lambda_{\text{sf}} = \frac{1}{\sqrt{R_s G_{\text{sh}}}} = \frac{\sqrt{\lambda_0 \lambda_s}}{2}. \quad (15)$$

A spin model similar to Eq. (13) has been discussed previously based on Luttinger liquid theory [50]; however, the model did not take into account the spin relaxation processes in the channel (the shunt conductance G_{sh}).

III. STEADY-STATE TRANSPORT RESULTS

In this section, we discuss several established steady-state results on charge-spin interconversion in the potentiometric limit. We start from the steady-state ($\partial/\partial t \rightarrow 0$) form of the transmission-line model with external contact in Eqs. (6) and (7), given by

$$\frac{d}{dx} I_c = i^c, \quad (16a)$$

$$\frac{d}{dx} V_c = -R_c I_c + p_0 \eta_c V_s + \Delta v^c, \quad (16b)$$

$$\frac{d}{dx} I_s = -G_{\text{sh}} V_s + p_0 \gamma_s I_c + i^s, \quad (16c)$$

$$\text{and } \frac{d}{dx} V_s = -R_s I_s + p_0 \eta_s V_c + \Delta v^s. \quad (16d)$$

Under the steady-state condition ($\partial/\partial t \rightarrow 0$), the capacitors and inductors in Fig. 2 become open and short circuits, respectively. The steady-state form in Eq. (16) is equivalent to our prior time-independent semiclassical equations with four electrochemical potentials [52] and all our previous results can be reproduced using Eq. (16).

We first derive a resistance matrix for a three-terminal setup (two charge and one spin) with potentiometric contacts (see Fig. 3) and assuming reflection with spin flip to

be the dominant scattering mechanism in the channel. We present dc simulation results on charge-spin interconversion in the SML channel using the full model [Eqs. (6) and (7)] in SPICE and compare with the results from the resistance matrix. We then derive a simple expression for a parameter that has been widely used to quantify the IREE in 2D channels. We compare the expression with available experiments on diverse materials as well as dc SPICE simulation results using the full model.

A. Resistance matrix for potentiometric setup

We consider a structure with three NM contacts ($p_f = 0$) on top of a SML channel, as shown in Fig. 3. Contacts 1 and 2 are the charge terminals and contact 3 is the spin terminal with no charge current flowing through it (i.e., $i^c = 0$). We start from Eq. (16) and make two assumptions to drive the resistance matrix: (i) the contacts are potentiometric, i.e., the contact conductance per unit length G_0 is very low such that the following condition is satisfied:

$$\frac{1}{\lambda}, \frac{1}{\lambda_0}, \frac{1}{\lambda_r} \gg \frac{G_0 R_B}{4}, \quad (17)$$

FIG. 3. (a) Setup to observe the charge-current- I_c -induced spin voltage v_s in the SML channel. The charge terminal of contact 3 is kept open and spin terminals of contacts 1 and 2 are grounded. (b) v_s/I_c vs p_0 for $i_{\text{tot}}^s = 0$ from SPICE simulation and comparison with Eq. (19). (c) Setup to observe the spin-current- i_{tot}^s -induced charge voltage difference ΔV_c across the SML channel. The charge terminal of contact 3 is kept open and spin terminals of contacts 1 and 3 are grounded. (d) $\Delta V_c/i_{\text{tot}}^s$ vs p_0 for $I_c = 0$ from SPICE simulation and comparison with Eq. (20). The SPICE setup is shown in Fig. 6 of Appendix B. The parameters are $\alpha = 2/\pi$, $G_0 \approx 0.05G_B$, $M + N = 100$ and the scattering rate per unit mode = 0.04 per lattice points. We assume reflection with spin flip to be the dominant scattering process in the channel.

and (ii) the reflection with spin flip is the dominant scattering mechanism in the channel. The details of the derivation are given in Appendix A.

The resistance matrix is given by

$$\begin{Bmatrix} \Delta V_c \\ v_s \end{Bmatrix} = \begin{bmatrix} \frac{R_B L}{\lambda} + \frac{2}{G_0''} & -\frac{\alpha p_0 R_B}{2} \\ \frac{\alpha p_0 R_B}{2} & \frac{\alpha^2}{G_0'} \end{bmatrix} \begin{Bmatrix} I_c \\ i_{\text{tot}}^s \end{Bmatrix}, \quad (18)$$

where ΔV_c is the charge voltage difference between contacts 1 and 2, I_c is the charge current flowing in the channel with length L , v_s is the spin voltage at contact 3, $i_{\text{tot}}^s = \tilde{v}^s L$ is the spin current at contact 3, contacts 1 and 2 have equal conductance G_0'' , and $G_0' = G_0 L$ is the contact conductance of contact 3. Equation (18) is similar to that previously reported in Ref. [53] with corrections in the diagonal components. The diagonal components (1, 1) and (2, 2) in the matrix represent the charge resistance between contacts 1 and 2 and spin resistance at contact 3, respectively.

Note that Eq. (18) is derived under the assumption that reflection with spin flip is the dominant scattering mechanism. In the presence of other scattering mechanisms, the basic structure of Eq. (18) remains the same; however, additional factors related to scattering rates multiply each of the components in the matrix (see Appendix A).

B. Direct effects: Charge-current-induced spin voltage

For a charge current I_c flowing through the SML channel, the open circuit spin voltage v_s at the contact 3 (i.e., $i_{\text{tot}}^s = 0$) can be derived from Eq. (18) as

$$v_s = \frac{\alpha p_0}{2G_B} I_c. \quad (19)$$

Equation (19) was originally proposed in Ref. [34], which was later confirmed by a number of experiments [3–9] on different TI materials using potentiometric ferromagnetic contact. Recently, Eq. (19) has been used to quantify such experiments on semiconductors [26] and Kondo insulators [10]. Moreover, the charge-current-induced spin voltage has been measured on transition metals [16–18] and can be quantified by Eq. (19) as well. Note that the spin voltage measured using a contact with higher conductance will be lower than Eq. (19) due to the current-shunting effect in the contact [52]. The full model in Eqs. (6) and (7) takes into account such effects related to the contact conductances.

We simulate the structure in Fig. 3(a) by connecting the full two-component circuit models given in Figs. 1 and 2 in a distributed manner using the standard circuit rules. The details of the simulation setup are discussed in Appendix B. The simulation results of v_s/I_c as a function of p_0 are shown in Fig. 3(b), which is in good agreement with Eq. (19).

C. Inverse effects: Spin-current-induced charge voltage

The reciprocal effect of Eq. (19) is the spin-current- i_{tot}^s -induced open-circuit charge voltage difference ΔV_c across the SML channel [53]. For $I_c = 0$, we have from Eq. (18)

$$\Delta V_c = -\frac{\alpha p_0}{2G_B} i_{\text{tot}}^s. \quad (20)$$

The reciprocal relation between Eqs. (19) and (20) including the negative sign has been observed experimentally [6,17,26]. Note that Eq. (20) is exact for a potentiometric contact. For a contact with higher conductance, current shunting effect will lower ΔV_c than Eq. (20) by the same amount as that for the direct effect [in Eq. (19)] [52], which can be analyzed using the full model in Eqs. (6) and (7). We simulate the structure shown in Fig. 3(c) using the full two-component models in Figs. 1 and 2. The simulation results of $\Delta V_c/i_{\text{tot}}^s$ as a function of p_0 are shown in Fig. 3(d), which show good agreement with Eq. (20). The details of the simulation setup are discussed in Appendix B.

D. IREE length

The phenomenon described by Eq. (20) is often known as the inverse Rashba Edelstein effect for 2D channels. IREE is often quantified with a parameter called IREE length, defined as

$$\lambda_{\text{IREE}} = \frac{J_c}{J_s}, \quad (21)$$

where J_c is the longitudinal short-circuit charge current density in A/m induced by the injected transverse spin current density J_s in A/m².

We derive a simple expression for IREE length starting from the first row of Eq. (18) for a short-circuit condition between contacts 1 and 2 (i.e., $v_{c1} = v_{c2}$) and assuming large channel resistance compared to the contact resistance [i.e., $L/(G_B\lambda) \gg 2/G_0$]. The expression is given by

$$\lambda_{\text{IREE}} = \frac{\alpha p_0 \lambda}{2}. \quad (22)$$

The derivation is given in Appendix A. Note that both p_0 and λ are two completely independent parameters.

Here, α is an angular averaging factor that can vary between 0 and 1 depending on the angular variation of the spin polarization of the eigenstates and details of the scattering mechanism. We assume that the distribution is such that the angle between the z axis and the spin polarization of the eigenstates with a particular group velocity (+ or -) varies between $-\pi/2$ to $+\pi/2$, which yields $\alpha = 2/\pi$.

Thus, from Eq. (22), we have the following expression:

$$\lambda_{\text{IREE}} = \frac{p_0 \lambda}{\pi}, \quad (23)$$

which has been previously reported in Ref. [52]. We simulate the setup in Fig. 4(a) in order to estimate λ_{IREE} of diverse ranges of p_0 and λ . The setup in Fig. 4(a) is same as that in Fig. 3(c), except we observe the short-circuit charge current I_c between the charge terminals of contacts 1 and 2 induced by the injected spin current i_s through the spin terminal of contact 3.

We compare the simulation results with Eq. (23) as well as available measurements from spin-pumping and lateral spin valve experiments on different Rashba interfaces and a TI channel: Ag/Bi [40], Cu/Bi [41], LaAlO₃/SrTiO₃ (LAO/STO) [20], and Bi₂Se₃ [42]. The comparison is shown in Fig. 4(b). We estimate λ from the reported

FIG. 4. (a) Setup similar to that in Fig. 3(c) but short-circuit charge current between contacts 1 and 2 is being observed. (b) IREE length (λ_{IREE}) vs $p_0 \lambda$ from SPICE simulation and comparison to Eq. (23) and experiments on different interfaces with Rashba SOC: Ag/Bi [40], Cu/Bi [41], LaAlO₃/SrTiO₃ (LAO/STO) [20], and a TI channel Bi₂Se₃ [42]. The backscattering length λ is estimated from measured sheet resistance or resistivity. The degree of spin-momentum locking p_0 is estimated from the Rashba coupling coefficient and the Fermi velocity using Eq. (24). The details of estimations are given in Appendix B. The SPICE simulation parameters are $\alpha = 2/\pi$, $G_0 \approx 0.05G_B$, and $M + N = 100$. We assume the reflection with spin flip to be the dominant scattering process in the channel.

TABLE I. Estimated material parameters. The estimations are discussed in detail in Appendix C.

Material	λ (nm)	p_0	λ_{IREE} (nm) [Eq. (23)]	λ_{IREE} (nm) (measured)
Ag/Bi	22.6 ^a	0.05 ^b	0.36	0.3 [40]
Cu/Bi	0.88 ^a	0.05 ^b	0.014	0.009 [41]
LAO/STO	180.8 ^a	0.0616 ^b	3.55	6.4 [20]
Bi ₂ Se ₃	3.2 ^a	0.025 ^c	0.026	0.035 [42]

^aEstimated from the measured sheet resistance or resistivity of the samples.

^bEstimated from the Rashba coupling coefficient and Fermi velocity of the materials.

^cEstimated from spin-pumping-induced voltage and Eq. (20).

sheet resistance or resistivity of the samples. p_0 for Ag/Bi, Cu/Bi, and LAO/STO are estimated using the Rashba coupling coefficient (v_0) and the Fermi velocity (v_F) quoted in the literature, using the following expression:

$$p_0 = \frac{v_0}{\sqrt{v_0^2 + v_F^2}}. \quad (24)$$

For Bi₂Se₃, we estimate p_0 from spin-pumping-induced inverse voltage and Eq. (20). Note that p_0 estimated for the sample in Ref. [42] is much lower than previous reports [3–9], which may be due to the presence of a large number of parallel channels. We expect higher λ_{IREE} for Bi₂Se₃ samples with higher p_0 .

The derivation of Eq. (24) from the Rashba Hamiltonian is shown in Appendix C. The estimations are summarized in Table I and the details are given in Appendix C. These two independent estimations of p_0 and λ when applied to Eq. (23) agree very well with experimentally reported λ_{IREE} , as shown in Fig. 4 and Table I.

IV. TIME-DEPENDENT TRANSPORT RESULTS

In this section, we use the full time-dependent model in Eqs. (2) and (3) to discuss a well-known phenomenon called the spin-charge separation. The spin-charge separation in the presence of spin-orbit coupling is a subject that has been controversial in the past. Our model shows that the charge and spin propagates with two distinct velocities that persist even in the materials with spin-orbit coupling exhibiting spin-momentum locking ($p_0 \neq 0$). However, we show that the lower-velocity signal is purely spin, while the higher-velocity signal is largely charge with an additional spin component proportional to p_0 .

A. Velocities of charge and spin signals

The velocities for charge and spin signals can be derived by finding the eigenvalues of Eqs. (2) and (3) assuming the low loss limit and constant coefficients. In addition,

we find the corresponding eigenvectors to analyze the coupling between spin and charge in the channel due to SML. The details of the derivation are given in Appendix D.

The lower-velocity eigenvalue is given by

$$v_{g,s} = \pm \frac{1}{\sqrt{L_K C_Q}} = \pm \langle v_x \rangle, \quad (25)$$

which is determined by the quantum capacitance C_Q and the kinetic inductance L_K , resulting in thermally averaged electron velocity $\langle v_x \rangle$. The corresponding eigenvector is given by

$$\begin{Bmatrix} \tilde{V}_c \\ \tilde{I}_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \tilde{V}_s \\ \tilde{I}_s \end{Bmatrix} = \begin{Bmatrix} \pm \alpha^2 \sqrt{\frac{L_K}{C_Q}} \\ 1 \end{Bmatrix}, \quad (26)$$

which shows that the lower-velocity signal is purely spin and no charge accompanies the signal even in channels with SML, i.e., $p_0 \neq 0$.

The higher-velocity eigenvalue is given by

$$v_{g,c} = \pm \frac{1}{\sqrt{L_{\text{eff}} C_{\text{eff}}}}, \quad (27)$$

where C_{eff} is a series combination of C_E and C_Q and L_{eff} is a series combination of L_M and L_K . The corresponding eigenvector is given by

$$\begin{Bmatrix} \tilde{V}_c \\ \tilde{I}_c \end{Bmatrix} = \begin{Bmatrix} \pm \sqrt{\frac{L_{\text{eff}}}{C_{\text{eff}}}} \\ 1 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \tilde{V}_s \\ \tilde{I}_s \end{Bmatrix} = \frac{p_0}{v_{g,c}^2 - v_{g,s}^2} \begin{Bmatrix} -\alpha^2 g_m \frac{L_M}{C_Q} v_{g,c}^2 + \frac{r_m}{\alpha^2} v_{g,s}^2 \\ \pm v_{g,c} v_{g,s}^2 C_Q \left(-\alpha^2 g_m \frac{L_M}{C_Q} + \frac{r_m}{\alpha^2} \right) \end{Bmatrix}, \quad (28)$$

which shows that the higher-velocity signal is largely charge that will be accompanying an additional spin signal proportional to p_0 , which has not been discussed before. This additional spin component vanishes in a NM channel where there is no SML (i.e., $p_0 = 0$) and the signal is purely charge. Further evaluation of this high-velocity spin component we leave to future work.

The quantum capacitance C_Q is proportional to the total number of modes ($M + N$) in the channel [see Eq. (5a)], while the kinetic inductance L_K is inversely proportional to $M + N$ [see Eq. (5b)]. $M + N$ is proportional to the channel width (for two dimensions) or cross-sectional area (for three dimensions) [64].

For a conductor with a very large cross section, we may have $C_E \ll C_Q$ and $L_M \gg L_K$, which is the standard transmission-line limit. In this limit, the velocity in

Eq. (27) becomes

$$c = \frac{1}{\sqrt{L_M C_E}},$$

which is the velocity predicted by standard transmission-line theory and can be as high as the speed of light.

For a conductor with a very small cross section, such as quantum wires, we may have $C_E \gg C_Q$ and $L_M \ll L_K$. In this limit, the velocity in Eq. (27) becomes the thermally averaged electron velocity, given by

$$\langle v_x \rangle = \frac{1}{\sqrt{L_K C_Q}},$$

which is the same as Eq. (25). Note that the two velocity eigenvalues are equal at this limit.

B. Spin-charge separation

From Eqs. (25) and (27), we have

$$\frac{v_{g,s}}{v_{g,c}} = \sqrt{\frac{1 + \delta \frac{C_Q}{C_E}}{1 + \frac{C_Q}{C_E}}}, \quad (29)$$

where $\delta = (L_M C_E) / (L_K C_Q) = (\langle v_x \rangle / c)^2$ [see Eq. (6) of Ref. [63]], which is usually much less than 1, making the spin signal slower than the charge signal, given by

$$v_{g,s} < v_{g,c}. \quad (30)$$

This process results in spin-charge separation, which is well established for channels without SML (i.e., $p_0 = 0$) from Luttinger liquid theory (see, for example, Refs. [47–50], and references therein). Electrons' charge excites C_E and L_M ; hence, the charge signal velocity is determined by C_{eff} and L_{eff} given by Eq. (27). However, pure spin signals do not excite C_E and L_M ; hence, its velocity is determined by C_Q and L_K only, given by Eq. (25). Similar arguments have been made in the past [50,62] in the context of carbon nanotubes without SOC.

Note that the argument in Eq. (30) is independent of p_0 (see Appendix D), which indicates that the spin-charge separation persists even in channels with SOC exhibiting SML (i.e., $p_0 \neq 0$). Similar arguments have been discussed previously by considering SOC [43–45], although there exists arguments that the presence of SOC may destroy the spin-charge separation [46].

In SML channels, an additional spin signal proportional to p_0 accompanies the charge signal at the same velocity as the charge ($v_{g,c}$) as seen from Eq. (28). This additional spin component is induced by the instantaneous voltage drop across L_M and the instantaneous current through C_E of the channel. However, the low-velocity signal [see Eq. (25)]

remains purely spin since the spin signals do not excite L_M and C_E [50,63] to induce a similar accompanying charge component.

V. TRANSMISSION-LINE MODEL FROM BOLTZMANN FORMALISM

In this section, we derive the transmission-line model in Eqs. (6) and (7) starting from the time-dependent Boltzmann transport equation under several assumptions. The assumptions allowed us to obtain the simple expressions for the model parameters stated in Eq. (5). Several of our predictions for steady-state transport [52] have already received experimental support [16,17,26] suggesting that the assumptions used here are in the reasonable limit. The assumptions can be revisited as the field evolves, which may effectively change the model parameters; however, the general structure of our model [Eqs. (6) and (7)] should remain valid.

A. Boltzmann transport equation

We assume a structure where the spatial variations and the applied fields are along the $\hat{x}$ direction. The time-dependent Boltzmann transport equation is given by

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + F_x \frac{\partial f}{\partial p_x} = \sum_{\vec{p}', s'} S(\vec{p}, s \leftrightarrow \vec{p}', s') (f - f'), \quad (31)$$

where we assume elastic scattering so that the scattering rates are the same in both directions, i.e.,

$$S(\vec{p}, s \rightarrow \vec{p}', s') = S(\vec{p}', s' \rightarrow \vec{p}, s) \equiv S(\vec{p}, s \leftrightarrow \vec{p}', s').$$

Here, $f \equiv f(x, t, \vec{p}, s)$ is the occupation factor of a state for a particular position x , time t , momentum $\vec{p}$, and spin index $s = \pm 1$; $f' \equiv f(x, t, \vec{p}', s')$ with momentum $\vec{p}'$ and spin index $s' = \pm 1$; $v_x = \partial E / \partial p_x$ is the x component of the group velocity; $F_x = -\partial E / \partial x$ is the force on electrons along the $\hat{x}$ direction; and E is the total energy. Note that the spin index $+1$ and -1 corresponds to up (U) and down (D) spin polarized states for a particular $\vec{p}$.

B. Occupation factor

We write the occupation factor f in terms of an electrochemical potential $\mu \equiv \mu(x, t, \vec{p}, s)$, in the form

$$f(x, t, \vec{p}, s) = \frac{1}{1 + \exp\{[E(x, t, \vec{p}, s) - \mu(x, t, \vec{p}, s)] / k_B T\}}, \quad (32)$$

where k_B is the Boltzmann constant and T is the temperature.

C. Linearization

We apply a variable transform $\xi = E - \mu$ on the left-hand side of Eq. (31). On the right-hand side of Eq. (31), we expand both f and f' into Taylor series around

$$f_0 = \frac{1}{1 + \exp\{[E(x, \vec{p}, s) - \mu_0]/k_B T\}}, \quad (33)$$

with constant electrochemical potential μ_0 , and we apply linear response approximation. Thus, Eq. (31) can be written as (see Appendix E for details of the derivation)

$$\begin{aligned} & \left(-\frac{\partial f_0}{\partial E}\right) \left(\frac{\partial \mu}{\partial t} - \frac{\partial E}{\partial t} + v_x \frac{\partial \mu}{\partial x} + F_x \frac{\partial \mu}{\partial p_x}\right) \\ &= - \sum_{\vec{p}', s'} S(\vec{p}, s \leftrightarrow \vec{p}', s') \left(-\frac{\partial f_0}{\partial E}\right) (\mu - \mu'). \end{aligned} \quad (34)$$

We assume that there are no internal fields in the present discussion. Hence, F_x comes from the applied voltage and the term $F_x(\partial \mu / \partial p_x)$ depends on the higher order of the applied voltage, which can be neglected in the linear response regime [64]. Thus, Eq. (34) is given by

$$\begin{aligned} & \left(-\frac{\partial f_0}{\partial E}\right) \left(\frac{\partial \mu}{\partial t} - \frac{\partial E}{\partial t} + v_x \frac{\partial \mu}{\partial x}\right) \\ &= - \sum_{\vec{p}', s'} S(\vec{p}, s \leftrightarrow \vec{p}', s') \left(-\frac{\partial f_0}{\partial E}\right) (\mu - \mu'). \end{aligned} \quad (35)$$

The term $\partial E / \partial t$ can be evaluated from the dispersion relation of a given Hamiltonian in the semiclassical approximation as discussed below.

D. Dispersion relation

We start from the following Rashba Hamiltonian:

$$\mathcal{H} = \frac{|\vec{p} - q\vec{A}|^2}{2m} I_{2 \times 2} - v_0 [\vec{\sigma} \times (\vec{p} - q\vec{A})] \cdot \hat{y} + U_E I_{2 \times 2}. \quad (36)$$

Here, $I_{2 \times 2}$ is a 2×2 identity matrix; $\vec{p}$ and $\vec{A}$ are the momentum and vector magnetic potential, respectively, in the z - x plane; $\vec{\sigma}$ is the Pauli's matrices; v_0 is the Rashba coefficient; U_E is the electrostatic potential; m is the electron mass; and q is the electron charge.

Eigenstates of Eq. (36) are given by

$$E(\vec{p}, s) = \frac{|\vec{p} - q\vec{A}|^2}{2m} - s v_0 |\vec{p} - q\vec{A}| + U_E, \quad (37)$$

where $\vec{p}$ is confined to the z - x plane.

We assume that U_E and $\vec{A}$ vary slowly with x and t , so that in the semiclassical approximation, we have

$$E(x, t, \vec{p}, s) = \frac{|\vec{p} - q\vec{A}(x, t)|^2}{2m} - s v_0 |\vec{p} - q\vec{A}(x, t)| + U_E(x, t). \quad (38)$$

Differentiating Eq. (38) with respect to t yields

$$\frac{\partial E}{\partial t} = \vec{v} \cdot \left(-q \frac{\partial \vec{A}}{\partial t}\right) + \frac{\partial U_E}{\partial t}, \quad (39)$$

where $\vec{v} = \nabla_{\vec{p}} E$ (see Appendix F for the derivation).

The electrostatic potential U_E and the vector magnetic potential $\vec{A}$ on the structure of interest can be evaluated from the theory of electromagnetism.

E. From potentials to charge and current

The electrostatic potential U_E is related to the total charge Q in the channel by the electrostatic capacitance C_E of the structure under consideration, given by

$$\frac{U_E}{q} = \frac{Q}{C_E}. \quad (40)$$

We assume that the charge current I_c flows along the $\hat{x}$ direction, which is uniform in the channel. The vector magnetic potential $\vec{A}$ is related to I_c by the magnetic inductance L_M of the channel, given by

$$\vec{A} \equiv \hat{x} A_x = \hat{x} L_M I_c. \quad (41)$$

Thus, Eq. (39) can be written as

$$\frac{\partial E}{\partial t} = -q v_x L_M \frac{\partial I_c}{\partial t} + \frac{q}{C_E} \frac{\partial Q}{\partial t}. \quad (42)$$

We combine Eq. (35) with Eq. (42) to get

$$\begin{aligned} & \left(-\frac{\partial f_0}{\partial E}\right) \left(\frac{\partial \mu}{\partial t} + v_x \frac{\partial \mu}{\partial x} + q v_x L_M \frac{\partial I_c}{\partial t} - \frac{q}{C_E} \frac{\partial Q}{\partial t}\right) \\ &= - \sum_{\vec{p}', s'} S(\vec{p}, s \leftrightarrow \vec{p}', s') \left(-\frac{\partial f_0}{\partial E}\right) (\mu - \mu'). \end{aligned} \quad (43)$$

F. Classification

We classify all $\vec{p}, s$ states into four groups based on the sign of v_x (+ or -) and the spin index $s = \pm 1$, given by

$$\Re : \begin{cases} U^+ \in \{\vec{p}, s | v_x > 0, s = +1\}, \\ D^- \in \{\vec{p}, s | v_x < 0, s = -1\}, \\ U^- \in \{\vec{p}, s | v_x < 0, s = +1\}, \\ D^+ \in \{\vec{p}, s | v_x > 0, s = -1\}, \end{cases} \quad \text{and} \quad (44)$$

where $s = +1$ and -1 denote up (U) and down (D) spins with respect to the spin quantization axis defined by $\hat{y} \times$

$(\vec{p} - q\vec{A})$, which is different for each direction of $\vec{p}$. Such classification can be mapped onto the two Fermi circles of a Rashba channel [see Fig. 5(a)]. The large circle corresponds to U^+ and D^- groups that share the same number of modes $n_m(U^+) = n_m(D^-) = M$, satisfying the time-reversal symmetry. Similarly, the small circle corresponds to U^- and D^+ groups sharing the same number of modes $n_m(U^-) = n_m(D^+) = N$. Note that the eigenstates belonging to each of the four half Fermi circles in Fig. 5(a) have an average spin polarization along the $\hat{z}$ direction with an averaging factor of $\alpha = 2/\pi$, which we will use later when writing spin currents and voltages.

G. Averaging

We define the thermal average of a variable $\psi \equiv \psi(\vec{p}, s)$ within each of the groups $\vec{p}, s \in \mathfrak{N}$ as

$$\langle \psi \rangle_{\vec{p}, s \in \mathfrak{N}} = \frac{\sum_{\vec{p}, s \in \mathfrak{N}} \left(-\frac{\partial f_0}{\partial E} \right) \psi(\vec{p}, s)}{\sum_{\vec{p}, s \in \mathfrak{N}} \left(-\frac{\partial f_0}{\partial E} \right)}. \quad (45)$$

We sum both sides of Eq. (43) over all $\vec{p}, s$ states within the range $\vec{p}, s \in \mathfrak{N}$ in the x - x plane as

$$\begin{aligned} & \frac{D_0(\mathfrak{N})}{2} \frac{\partial \langle \mu \rangle}{\partial t} + \frac{D_0(\mathfrak{N})}{2} \frac{\partial \langle v_x \mu \rangle}{\partial x} \\ & + \frac{qD_0(\mathfrak{N})}{2} \langle v_x \rangle L_M \frac{\partial I_c}{\partial t} - \frac{q}{C_E} \frac{D_0(\mathfrak{N})}{2} \frac{\partial Q}{\partial t} \\ & = - \sum_{\vec{p}, s \in \mathfrak{N}} \sum_{\vec{p}', s'} S(\vec{p}, s \leftrightarrow \vec{p}', s') \left(-\frac{\partial f_0}{\partial E} \right) (\mu - \mu'). \end{aligned} \quad (46)$$

Here, $D_0(\mathfrak{N})$ is the thermally averaged density of states within $\vec{p}, s \in \mathfrak{N}$, given by

$$\frac{D_0(\mathfrak{N})}{2} = \sum_{\vec{p}, s \in \mathfrak{N}} \left(-\frac{\partial f_0}{\partial E} \right), \quad (47)$$

where the factor of 2 appeared since we are summing over all s states. Note that the time-reversal symmetry requires that

$$D_0(U^+) = D_0(D^-) \text{ and } D_0(U^-) = D_0(D^+).$$

We make the following assumption in Eq. (46):

$$\langle v_x \mu \rangle \approx \langle v_x \rangle \langle \mu \rangle, \quad (48)$$

FIG. 5. (a) Two Fermi circles of a Rashba channel with spin-momentum locking having opposite spin polarizations at a given energy E_F . The left (right) half of each circle represents negative (positive) group velocity. The large circle corresponds to the large number of modes M in the channel and has a net spin polarization along $+\hat{z}$ ($-\hat{z}$) on right (left) half. The small circle corresponds to a smaller number of modes N and has net spin polarization along $-\hat{z}$ ($+\hat{z}$) on the right (left) half. We classify all electronic states into four groups based on the spin index [up (U), down (D)] and the sign of the x component of the group velocity ($+$, $-$). (b) We assign four average electrochemical potentials for these four groups: $\mu(U^+)$, $\mu(U^-)$, $\mu(D^+)$, and $\mu(D^-)$. External contact is modeled as up (v_u) and down (v_d) spin voltages applied to up and down states of the channel through up (g_u) and down (g_d) spin conductances per mode per unit length, respectively. Four different currents (i_U^+ , i_D^+ , i_U^- , and i_D^-) enter the four different groups in the channel. The contact can be either normal metal $g_u = g_d$ or ferromagnet $g_u \neq g_d$.

which yields

$$\begin{aligned} & \frac{D_0(\mathfrak{N})}{2} \frac{\partial \langle \mu \rangle}{\partial t} + \frac{D_0(\mathfrak{N})}{2} \langle v_x \rangle \frac{\partial \langle \mu \rangle}{\partial x} + \frac{qD_0(\mathfrak{N})}{2} \langle v_x \rangle L_M \frac{\partial I_c}{\partial t} \\ & - \frac{q}{C_E} \frac{D_0(\mathfrak{N})}{2} \frac{\partial Q}{\partial t} \\ & = - \sum_{\vec{p}, s \in \mathfrak{N}} \sum_{\vec{p}', s'} S(\vec{p}, s \leftrightarrow \vec{p}', s') \left(-\frac{\partial f_0}{\partial E} \right) (\mu - \mu') + \frac{i_{\text{ext}}}{q}. \end{aligned} \quad (49)$$

Note that the term i_{ext}/q on the right-hand side has been added to take into account the total current entering into the $\vec{p}, s \in \mathfrak{R}$ states from an external contact.

The number of modes within $\vec{p}, s \in \mathfrak{R}$ in the channel is given by [64]

$$n_m(\mathfrak{R}) = \frac{hD_0|\langle v_x \rangle|}{2L}, \quad (50)$$

where $|\langle v_x \rangle|$ is the magnitude of $\langle v_x \rangle$ and L is the channel length. Note that $|\langle v_x \rangle|$ is the same in all four groups for the Rashba Hamiltonian considered here (see Appendix F). Thus, Eq. (49) can be written as

$$\begin{aligned} \frac{n_m(\mathfrak{R})}{|\langle v_x \rangle|} \frac{\partial \langle \mu \rangle}{\partial t} + \text{sgn}(\langle v_x \rangle) n_m(\mathfrak{R}) \frac{\partial \langle \mu \rangle}{\partial x} - \frac{q}{C_E} \frac{n_m(\mathfrak{R})}{|\langle v_x \rangle|} \frac{\partial Q}{\partial t} \\ + \text{sgn}(\langle v_x \rangle) q n_m(\mathfrak{R}) L_M \frac{\partial I_c}{\partial t} = \tilde{S}(\mathfrak{R}) + \frac{h}{q} \frac{i_{\text{ext}}}{L}, \end{aligned} \quad (51)$$

where

$$\tilde{S}(\mathfrak{R}) = -\frac{h}{L} \sum_{\vec{p}, s \in \mathfrak{R}} \sum_{\vec{p}', s'} S(\vec{p}, s \leftrightarrow \vec{p}', s') \left(-\frac{\partial f_0}{\partial E} \right) (\mu - \mu'). \quad (52)$$

Equation (51) applies to each group in Eq. (44) with an average electrochemical potential given by $\langle \mu \rangle_{\vec{p}, s \in U^+} \equiv \mu(U^+)$, $\langle \mu \rangle_{\vec{p}, s \in D^-} \equiv \mu(D^-)$, $\langle \mu \rangle_{\vec{p}, s \in U^-} \equiv \mu(U^-)$, and $\langle \mu \rangle_{\vec{p}, s \in D^+} \equiv \mu(D^+)$, respectively.

H. Scattering matrix

We make the following assumption:

$$\langle S\mu \rangle \approx \langle S \rangle \langle \mu \rangle, \quad (53)$$

which in conjunction with Eq. (52) becomes

$$\begin{aligned} \tilde{S}(\mathfrak{R}) = & \hat{S}_{\mathfrak{R} \leftrightarrow U^+} [\langle \mu(U^+) \rangle - \langle \mu(\mathfrak{R}) \rangle] \\ & + \hat{S}_{\mathfrak{R} \leftrightarrow D^-} [\langle \mu(D^-) \rangle - \langle \mu(\mathfrak{R}) \rangle] \\ & + \hat{S}_{\mathfrak{R} \leftrightarrow U^-} [\langle \mu(U^-) \rangle - \langle \mu(\mathfrak{R}) \rangle] \\ & + \hat{S}_{\mathfrak{R} \leftrightarrow D^+} [\langle \mu(D^+) \rangle - \langle \mu(\mathfrak{R}) \rangle], \end{aligned} \quad (54)$$

with

$$\hat{S}_{\mathfrak{R} \leftrightarrow \mathfrak{R}'} = \frac{h}{L} \sum_{\vec{p}, s \in \mathfrak{R}} \sum_{\vec{p}', s' \in \mathfrak{R}'} \left(-\frac{\partial f_0}{\partial E} \right) S(\vec{p}, s \leftrightarrow \vec{p}', s'). \quad (55)$$

We can evaluate Eq. (54) for each of the groups in Eq. (44), i.e., $\mathfrak{R} \equiv U^+, D^-, U^-,$ and D^+ , which together

in the $\{\mu(U^+), \mu(D^-), \mu(U^-), \mu(D^+)\}^T$ basis become the following matrix (see Appendix G for details):

$$[S] = \begin{bmatrix} -u_1 & \hat{S}_{U^+ \leftrightarrow D^-} & \hat{S}_{U^+ \leftrightarrow U^-} & \hat{S}_{U^+ \leftrightarrow D^+} \\ \hat{S}_{U^+ \leftrightarrow D^-} & -u'_1 & \hat{S}_{D^- \leftrightarrow U^-} & \hat{S}_{D^- \leftrightarrow D^+} \\ \hat{S}_{U^+ \leftrightarrow U^-} & \hat{S}_{D^- \leftrightarrow U^-} & -u_2 & \hat{S}_{U^- \leftrightarrow D^+} \\ \hat{S}_{U^+ \leftrightarrow D^+} & \hat{S}_{D^- \leftrightarrow D^+} & \hat{S}_{U^- \leftrightarrow D^+} & -u'_2 \end{bmatrix}, \quad (56)$$

where

$$u_1 = \hat{S}_{U^+ \leftrightarrow D^-} + \hat{S}_{U^+ \leftrightarrow U^-} + \hat{S}_{U^+ \leftrightarrow D^+},$$

$$u'_1 = \hat{S}_{U^+ \leftrightarrow D^-} + \hat{S}_{D^- \leftrightarrow U^-} + \hat{S}_{D^- \leftrightarrow D^+},$$

$$u_2 = \hat{S}_{U^+ \leftrightarrow U^-} + \hat{S}_{D^- \leftrightarrow U^-} + \hat{S}_{U^- \leftrightarrow D^+},$$

$$\text{and } u'_2 = \hat{S}_{U^+ \leftrightarrow D^+} + \hat{S}_{D^- \leftrightarrow D^+} + \hat{S}_{U^- \leftrightarrow D^+}.$$

The scattering matrix is such that the sum of each column is zero, satisfying the charge conservation, and the sum of each row is zero, satisfying the zero current requirement under equal potential.

In addition, the time-reversal symmetry requires that

$$\hat{S}_{U^+ \leftrightarrow U^-} = \hat{S}_{D^- \leftrightarrow D^+} \quad \text{and} \quad \hat{S}_{U^+ \leftrightarrow D^+} = \hat{S}_{D^- \leftrightarrow U^-}. \quad (57)$$

There are three types of scattering processes considered in the channel: (a) reflection with spin flip $r_{s1} = \hat{S}_{U^+ \leftrightarrow D^-}$ and $r_{s2} = \hat{S}_{U^- \leftrightarrow D^+}$, (b) reflection without spin flip $r = \hat{S}_{U^+ \leftrightarrow U^-} = \hat{S}_{D^- \leftrightarrow D^+}$, and (c) transmission with spin flip $t_s = \hat{S}_{U^+ \leftrightarrow D^+} = \hat{S}_{D^- \leftrightarrow U^-}$. They are given by

$$r_{s1} = \frac{h}{L} \sum_{\vec{p}, s \in U^+} \sum_{\vec{p}', s' \in D^-} \left(-\frac{\partial f_0}{\partial E} \right) S(\vec{p}, s \leftrightarrow \vec{p}', s'), \quad (58a)$$

$$r_{s2} = \frac{h}{L} \sum_{\vec{p}, s \in U^-} \sum_{\vec{p}', s' \in D^+} \left(-\frac{\partial f_0}{\partial E} \right) S(\vec{p}, s \leftrightarrow \vec{p}', s'), \quad (58b)$$

$$\begin{aligned} r &= \frac{h}{L} \sum_{\vec{p}, s \in U^+} \sum_{\vec{p}', s' \in U^-} \left(-\frac{\partial f_0}{\partial E} \right) S(\vec{p}, s \leftrightarrow \vec{p}', s'), \\ &= \frac{h}{L} \sum_{\vec{p}, s \in D^-} \sum_{\vec{p}', s' \in D^+} \left(-\frac{\partial f_0}{\partial E} \right) S(\vec{p}, s \leftrightarrow \vec{p}', s'), \end{aligned} \quad (58c)$$

and

$$\begin{aligned} t_s &= \frac{h}{L} \sum_{\vec{p}, s \in U^+} \sum_{\vec{p}', s' \in D^+} \left(-\frac{\partial f_0}{\partial E} \right) S(\vec{p}, s \leftrightarrow \vec{p}', s'), \\ &= \frac{h}{L} \sum_{\vec{p}, s \in D^-} \sum_{\vec{p}', s' \in U^-} \left(-\frac{\partial f_0}{\partial E} \right) S(\vec{p}, s \leftrightarrow \vec{p}', s'). \end{aligned} \quad (58d)$$

Equation (51) for each group in Eq. (44) is given as

$$\begin{aligned}
 & \frac{1}{|\langle v_x \rangle|} \frac{\partial}{\partial t} \begin{Bmatrix} M\tilde{\mu}(U^+) \\ M\tilde{\mu}(D^-) \\ N\tilde{\mu}(U^-) \\ N\tilde{\mu}(D^+) \end{Bmatrix} + \frac{\partial}{\partial x} \begin{Bmatrix} M\tilde{\mu}(U^+) \\ -M\tilde{\mu}(D^-) \\ -N\tilde{\mu}(U^-) \\ N\tilde{\mu}(D^+) \end{Bmatrix} \\
 &= \begin{bmatrix} -u_1 & r_{s1} & r & t_s \\ r_{s1} & -u_1 & t_s & r \\ r & t_s & -u_2 & r_{s2} \\ t_s & r & r_{s2} & -u_2 \end{bmatrix} \begin{Bmatrix} \tilde{\mu}(U^+) \\ \tilde{\mu}(D^-) \\ \tilde{\mu}(U^-) \\ \tilde{\mu}(D^+) \end{Bmatrix} \\
 & - qL_M \frac{\partial I_c}{\partial t} \begin{Bmatrix} M \\ -M \\ -N \\ N \end{Bmatrix} + \frac{q}{|\langle v_x \rangle|} \frac{\partial Q}{\partial t} \begin{Bmatrix} M \\ N \\ N \\ N \end{Bmatrix} \\
 & + \frac{h}{q} \begin{Bmatrix} i_U^+ \\ i_D^- \\ i_U^- \\ i_D^+ \end{Bmatrix}. \tag{59}
 \end{aligned}$$

Note that the electrochemical potentials are referenced with respect to the constant μ_0 , i.e., $\tilde{\mu} = \mu - \mu_0$. i_U^+ , i_D^- , i_U^- , and i_D^+ are the currents per unit length entering into the four groups from an external contact [see Fig. 5(b)], which are given as [52]

$$\begin{aligned}
 i_U^+ &= \frac{q^2}{h} M g_u \left[v_u - \frac{\tilde{\mu}(U^+)}{q} \right], \\
 i_D^+ &= \frac{q^2}{h} N g_d \left[v_d - \frac{\tilde{\mu}(D^+)}{q} \right], \\
 i_U^- &= \frac{q^2}{h} N g_u \left[v_u - \frac{\tilde{\mu}(U^-)}{q} \right], \\
 \text{and } i_D^- &= \frac{q^2}{h} M g_d \left[v_d - \frac{\tilde{\mu}(D^-)}{q} \right]. \tag{60}
 \end{aligned}$$

Here, v_u and v_d are the up and down spin voltages at the external contact, respectively. g_u and g_d are the up and down spin conductances per unit mode per unit length. The contact can be either NM ($g_u = g_d$) or FM ($g_u \neq g_d$). In steady state, Eq. (59) reduces to our prior model in Ref. [52].

I. Conversion to charge-spin basis

The charge and spin voltages and currents in the channel are defined in terms of the four average electrochemical

potentials as

$$\begin{Bmatrix} I_c R_B \\ 2V_s \\ I_s R_B \\ 2V_c \end{Bmatrix} = \frac{q}{h} R_B \begin{bmatrix} 1 & -1 & -1 & 1 \\ \alpha & -\alpha & \alpha & -\alpha \\ \frac{1}{\alpha} & \frac{1}{\alpha} & -\frac{1}{\alpha} & -\frac{1}{\alpha} \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} M\tilde{\mu}(U^+) \\ M\tilde{\mu}(D^-) \\ N\tilde{\mu}(U^-) \\ N\tilde{\mu}(D^+) \end{Bmatrix}, \tag{61}$$

where R_B is the ballistic resistance of the channel given in Eq. (5k) and α is an angular averaging factor. See Appendix H for the derivation of Eq. (61).

The net z -spin polarization (or z -spin voltage) is expected to be lower due to the angular distribution of the spin polarization of the eigenstates on the half Fermi circles indicated by U^+ , U^- , D^+ , and D^- in Fig. 5(a) and the details of the scattering mechanisms. In order to take into account this scenario, we multiply the second row of Eq. (61) with a factor $0 \leq \alpha \leq 1$, which appears in Eq. (19), to indicate a lowering of the charge-current-induced spin voltage in the channel from the ideal value. Onsager reciprocity requires that the spin-current-induced charge voltage in the channel be lowered by the same factor α as shown in Eq. (20), which has been taken into account by multiplying $1/\alpha$ to the third row of Eq. (61). In the simplest approximation, the angle between the z axis and the spin polarization of the eigenstates of a particular half Fermi circle in Fig. 5(a) varies from $-\pi/2$ to $+\pi/2$, which yields $\alpha = 2/\pi$.

Combining Eq. (59) with Eq. (61) yields

$$\begin{aligned}
 & \frac{1}{|\langle v_x \rangle|} \frac{\partial}{\partial t} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha^2 & 0 \\ 0 & \frac{1}{\alpha^2} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} I_c R_B \\ 2V_s \\ I_s R_B \\ 2V_c \end{Bmatrix} + \frac{\partial}{\partial x} \begin{Bmatrix} I_c R_B \\ 2V_s \\ I_s R_B \\ 2V_c \end{Bmatrix} \\
 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2\alpha^2}{\lambda_0} & \frac{2\alpha p_0}{\lambda_0} \\ \frac{2}{\alpha \lambda'_s} & -\frac{2}{\alpha^2 \lambda_s} & 0 & 0 \\ -\frac{2}{\lambda} & \frac{2}{\alpha \lambda'} & 0 & 0 \end{bmatrix} \begin{Bmatrix} I_c R_B \\ 2V_s \\ I_s R_B \\ 2V_c \end{Bmatrix} \\
 & - 2L_M \frac{\partial I_c}{\partial t} \begin{Bmatrix} 0 \\ 0 \\ p_0 \\ \alpha \end{Bmatrix} + \frac{1}{|\langle v_x \rangle|} \frac{2}{C_E} \frac{\partial Q}{\partial t} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \\
 & + \begin{Bmatrix} R_B i^c \\ 2\Delta v^s \\ R_B i^s \\ 2\Delta v^c \end{Bmatrix}, \tag{62}
 \end{aligned}$$

with the external contact terms given by

$$\begin{aligned} i^c &= i_U^+ + i_D^- + i_U^- + i_D^+, \\ i^s &= \frac{1}{\alpha} (i_U^+ - i_D^- + i_U^- - i_D^+), \\ \Delta v^c &= \frac{R_B}{2} (i_U^+ - i_D^- - i_U^- + i_D^+), \quad \text{and} \\ \Delta v^s &= \frac{\alpha R_B}{2} (i_U^+ + i_D^- - i_U^- - i_D^+). \end{aligned} \quad (63)$$

Equation (63) combined with Eq. (60) yields Eqs. (8) and (9).

J. Continuity equation

The term $\partial Q/\partial t$ on the right-hand side of Eq. (62) is related to the charge currents according to the continuity equation given by

$$\frac{\partial Q}{\partial t} + \frac{\partial I_c}{\partial x} = i^c. \quad (64)$$

The first row of Eq. (62) combined with Eq. (64) becomes

$$\frac{\partial}{\partial t} V_c = \left(\frac{|\langle v_x \rangle| R_B}{2} + \frac{1}{C_E} \right) \left(i^c - \frac{\partial}{\partial x} I_c \right), \quad (65)$$

which is the first equation in Eq. (6).

The second row of Eq. (62) combined with Eq. (64) becomes

$$\begin{aligned} \alpha^2 \frac{R_B}{2 |\langle v_x \rangle|} \frac{\partial}{\partial t} I_s + \frac{\partial}{\partial x} V_s \\ = -\frac{\alpha^2 R_B}{\lambda_0} I_s + \frac{2\alpha p_0}{\lambda_0} V_c \\ + \alpha p_0 \frac{1}{|\langle v_x \rangle|} \frac{1}{C_E} \left(i^c - \frac{\partial I_c}{\partial x} \right) + \Delta v^s. \end{aligned} \quad (66)$$

We replace the expression for $i^c - (\partial/\partial x)I_c$ from Eq. (65) to get the second equation in Eq. (7). For contact conductance $G_0 \rightarrow 0$, Eqs. (6) and (7) reduce to Eqs. (2) and (3), respectively.

K. Mean free paths

We have three distinct mean free paths in Eq. (62), given by

$$\begin{aligned} \frac{1}{\lambda} &= \frac{1}{2} \left(\frac{r_{s2}}{N} + \frac{r_{s1}}{M} \right) + \frac{r}{2} \left(\frac{1}{N} + \frac{1}{M} \right), \\ \frac{1}{\lambda_0} &= \frac{r + t_s}{2} \left(\frac{1}{N} + \frac{1}{M} \right), \quad \text{and} \\ \frac{1}{\lambda_s} &= \frac{1}{2} \left(\frac{r_{s2}}{N} + \frac{r_{s1}}{M} \right) + \frac{t_s}{2} \left(\frac{1}{N} + \frac{1}{M} \right), \end{aligned} \quad (67)$$

where λ , λ_0 , and λ_s determine the series charge resistance R_c [see Eq. (5c)], the series spin resistance R_s

[see Eq. (5d)], and the shunt spin conductance G_{sh} [see (5e)], respectively. Note that λ_s depends on the spin-flip processes in the channel and determines the shunt conductance G_{sh} that takes into account the spin relaxation process.

L. Charge-spin coupling coefficients

The other terms of Eq. (62) are given by

$$\begin{aligned} \frac{1}{\lambda'} &= \frac{1}{2} \left(\frac{r_{s2}}{N} - \frac{r_{s1}}{M} \right) + \frac{r}{2} \left(\frac{1}{N} - \frac{1}{M} \right) \quad \text{and} \\ \frac{1}{\lambda'_s} &= \frac{1}{2} \left(\frac{r_{s2}}{N} - \frac{r_{s1}}{M} \right) + \frac{t_s}{2} \left(\frac{1}{N} - \frac{1}{M} \right), \end{aligned} \quad (68)$$

representing coupling coefficients between charge and spin. λ'_s and λ' cause a charge-induced spin signal and spin-induced charge signal, respectively. Note that the first terms of Eq. (68) indicate a purely scattering-induced spin-charge coupling even if $M = N$ (i.e., $p_0 = 0$), since r_{s1} and r_{s2} are two independent parameters, and there could be situations where $r_{s1} \neq r_{s2}$.

In this paper, we restrict ourselves to SML caused by difference between M and N (i.e., $p_0 \neq 0$). We can eliminate the first terms in Eq. (68) by assuming either of the following:

$$r_{s1} = r_{s2} = r_s, \quad (69a)$$

$$\text{or} \quad \frac{r_{s1}}{M} = \frac{r_{s2}}{N}. \quad (69b)$$

The first assumption Eq. (69a) when applied to Eq. (68) yields

$$\begin{aligned} \frac{1}{\lambda'} &= \left(\frac{r_s + r}{2} \right) \left(\frac{1}{N} - \frac{1}{M} \right) = \frac{p_0}{\lambda} \quad \text{and} \\ \frac{1}{\lambda'_s} &= \left(\frac{r_s + t_s}{2} \right) \left(\frac{1}{N} - \frac{1}{M} \right) = \frac{p_0}{\lambda_s}, \end{aligned} \quad (70)$$

which, in turn, gives $\lambda_r = \lambda$ and $\lambda_t = \lambda_s$ in Eqs. (5i) and (5j), respectively.

The second assumption Eq. (69b) when applied in Eq. (68) yields

$$\begin{aligned} \frac{1}{\lambda'} &= \frac{r}{2} \left(\frac{1}{N} - \frac{1}{M} \right) = \frac{p_0}{\lambda_r} \quad \text{and} \\ \frac{1}{\lambda'_s} &= \frac{t_s}{2} \left(\frac{1}{N} - \frac{1}{M} \right) = \frac{p_0}{\lambda_t}. \end{aligned} \quad (71)$$

M. Comments on the assumptions

The assumptions in Eqs. (48), (53), and (69) result in an effective change in the transmission-line model parameters in Eqs. (5), but they do not change the models in

Eqs. (2) and (3) (Fig. 1) and Eqs. (6) and (7) (Fig. 2) themselves. The assumptions made to derive the model can be revisited as the field evolves. However, several predictions from our model for steady state transport [52] have already received support from experiments [16,17,26], suggesting that the assumptions are within the reasonable limits.

VI. SUMMARY

We propose a two-component (charge and z component of spin) transmission-line model for channels with spin-momentum locking, which is a new addition to our SPICE compatible multiphysics model library [37–39]. The model enables easy analysis of complex geometries involving materials with spin-orbit coupling observed in diverse classes of materials, e.g., topological insulators, Kondo insulators, transition metals, semimetals, oxide interfaces, and narrow band-gap semiconductors. The model is derived from a four-component diffusion equation obtained from the Boltzmann transport equation assuming linear response and elastic scattering in the channel. The four-component diffusion equation uses four average electrochemical potentials based on a classification depending on the sign of the z component of spin [up (U) or down (D)] and the sign of the x component of group velocity (+ or –). Such classification can be viewed as an extension of the Valet-Fert equation [51], which uses two electrochemical potentials for U and D states. For a normal metal channel, the time-dependent model presented here decouples into (i) the well-known transmission-line model for charge transport in quantum wires [50,62,63] and (ii) a time-dependent version of the Valet-Fert equation [51] for spin transport. We first derive several results on charge-spin interconversion, starting from our model in steady state. The steady-state results show good agreement with existing experiments on diverse materials. We then study the phenomenon of spin-charge separation using our full time-dependent model, especially in the materials with SOC exhibiting SML. Our model shows the expected spin-charge separation with two distinct velocities for charge and spin, which persist even in channels exhibiting SML. However, we show that the lower velocity signal is purely spin, while the higher velocity signal is largely charge with an additional spin component proportional to the degree of SML.

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APPENDIX A: STEADY-STATE RESULTS

This appendix provides the details of the derivation of Eqs. (18) and (23).

1. Derivation of the resistance matrix

Potentiometric NM contacts: Equations (8) and (9) for $p_f = 0$ become

$$i^c = G_0 (v_c - V_c), \quad (\text{A1a})$$

$$i^s = \frac{G_0}{\alpha^2} (v_s - V_s), \quad (\text{A1b})$$

$$\Delta v^c = \frac{G_0 R_B}{2\alpha} \left(p_0 v_s - \frac{\alpha I_c R_B}{2} \right), \quad (\text{A1c})$$

$$\text{and } \Delta v^s = \frac{\alpha G_0 R_B}{2} \left(p_0 v_c - \frac{\alpha I_s R_B}{2} \right). \quad (\text{A1d})$$

We apply Eqs. (A1a) and (A1b) to Eqs. (A1c) and (A1d), respectively, which yields

$$\begin{aligned} \Delta v^c &= \frac{\alpha p_0}{2G_B} i^s + p_0 \frac{G_0 R_B}{2\alpha} V_s - R_{\text{cont}}^c I_c, \\ \Delta v^s &= \frac{\alpha p_0}{2G_B} i^c + p_0 \frac{\alpha G_0 R_B}{2} V_c - R_{\text{cont}}^s I_s. \end{aligned} \quad (\text{A2})$$

We assume that the contact conductance per unit length G_0 is very low, such that the potentiometric condition in Eq. (17) is satisfied. Thus, we have

$$\begin{aligned} R_c &\gg R_{\text{cont}}^c, \quad R_s \gg R_{\text{cont}}^s, \\ \eta_c &\gg \frac{G_0 R_B}{2\alpha}, \quad \text{and} \quad \eta_s \gg \frac{\alpha G_0 R_B}{2}. \end{aligned}$$

Under this condition, we combine Eq. (16) with Eq. (A2) for $i^c = 0$, which yields

$$\frac{d}{dx} I_c = 0, \quad (\text{A3a})$$

$$\frac{d}{dx} V_c = -R_c I_c + p_0 \eta_c V_s + \frac{\alpha p_0}{2G_B} i^s, \quad (\text{A3b})$$

$$\frac{d}{dx} I_s = -G_{\text{sh}} V_s + p_0 \gamma_s I_c + i^s, \quad (\text{A3c})$$

$$\text{and } \frac{d}{dx} V_s = -R_s I_s + p_0 \eta_s V_c. \quad (\text{A3d})$$

Uniform spin voltage: We assume that the spin voltage V_s is uniform in the channel region of interest (from $x = 0$

to $x = L$, where L is the channel length)

$$\frac{d}{dx} V_s = 0. \quad (\text{A4})$$

Thus, from Eq. (A3d), we can write

$$I_s = \frac{2p_0 G_B}{\alpha} V_c, \quad (\text{A5})$$

where we use the definitions in Eqs. (5d) and (5h).

Differentiating both sides of Eq. (A5) with respect to x and combining with Eqs. (A3b) and (A3c) yields

$$\begin{aligned} -G_{\text{sh}} V_s + p_0 \gamma_s I_c + i^s &= \frac{2p_0 G_B}{\alpha} \\ &\times \left(-R_c I_c + p_0 \eta_c V_s + \frac{\alpha p_0}{2G_B} i^s \right), \end{aligned}$$

which in conjunction with the definitions in Eq. (5) yields

$$\begin{aligned} V_s &= \frac{\alpha p_0}{2G_B} \frac{(1/\lambda_t + 1/\lambda)}{(1/\lambda_s + p_0^2/\lambda_r)} I_c \\ &+ \frac{\alpha^2 (1 - p_0^2)}{4G_B} \frac{1}{(1/\lambda_s + p_0^2/\lambda_r)} i^s. \end{aligned} \quad (\text{A6})$$

From Eq. (A1b), we have

$$V_s = v_s - \frac{\alpha^2 i^s}{G_0}, \quad (\text{A7})$$

which, when combined with Eq. (A6), becomes

$$\begin{aligned} v_s &= \frac{\alpha p_0}{2G_B} \frac{\left(\frac{1}{\lambda_t} + \frac{1}{\lambda}\right)}{\left(\frac{1}{\lambda_s} + \frac{p_0^2}{\lambda_r}\right)} I_c \\ &+ \left[\frac{\alpha^2 (1 - p_0^2)}{4G_B} \frac{1}{\left(\frac{1}{\lambda_s} + \frac{p_0^2}{\lambda_r}\right)} + \frac{\alpha^2}{G_0} \right] i^s. \end{aligned} \quad (\text{A8})$$

Charge voltage in the channel: Let us assume that the contact 1 is located at $x = 0$ and contact 2 is located at $x = L$ as shown in Fig. 3, where L is the channel length between the two contacts. The charge voltages in the channel at $x = 0$ and $x = L$ are V_{c1} and V_{c2} , respectively. According to (A3a), I_c is constant along the channel. Since V_s and i^s are also uniform along the channel, we can integrate

Eq. (A3b) from $x = 0$ to $x = L$ as

$$V_{c1} - V_{c2} = \frac{L}{\lambda} R_B I_c - \frac{2p_0 L}{\alpha \lambda_r} V_s - \frac{\alpha p_0}{2G_B} i^s L. \quad (\text{A9})$$

From Eq. (A1a), we can write the equation for V_{c1} in terms of the terminal voltage v_{c1} of contact 1 at $x = 0$ as

$$V_{c1} = v_{c1} - \frac{I_c}{G_0'}, \quad (\text{A10})$$

noting that $i^c = I_c$. Here, G_0'' is the contact conductance of contact 1. Similarly, we can write an equation for contact 2, noting that $i^c = -I_c$, given by

$$V_{c2} = v_{c2} + \frac{I_c}{G_0''}, \quad (\text{A11})$$

where we assume that contacts 1 and 2 have the same conductance.

We combine Eq. (A9) with Eqs. (A6), (A10), and (A11) to have

$$\begin{aligned} v_{c1} - v_{c2} &= \left\{ \left[\frac{L}{\lambda} - \frac{p_0^2 L}{\lambda_r} \frac{\left(\frac{1}{\lambda_t} + \frac{1}{\lambda}\right)}{\left(\frac{1}{\lambda_s} + \frac{p_0^2}{\lambda_r}\right)} \right] R_B + \frac{2}{G_0''} \right\} I_c \\ &- \frac{\alpha p_0}{2G_B} \frac{\left(\frac{1}{\lambda_r} + \frac{1}{\lambda_s}\right)}{\left(\frac{1}{\lambda_s} + \frac{p_0^2}{\lambda_r}\right)} i^s L. \end{aligned} \quad (\text{A12})$$

Scattering condition: We assume that the reflection with a spin-flip scattering mechanism is dominant in the channel

$$r_{s1,2} \gg r, \quad t_s. \quad (\text{A13})$$

This condition in Eqs. (67), (70), and (68) yields

$$\frac{1}{\lambda} \approx \frac{1}{\lambda_s}, \quad \text{and} \quad \frac{1}{\lambda}, \frac{1}{\lambda_s} \gg \frac{1}{\lambda_0}, \frac{1}{\lambda_r}, \frac{1}{\lambda_t}. \quad (\text{A14})$$

Applying Eq. (A14) to Eqs. (A12) and (A8), we have the first and second rows of Eq. (18), respectively, by setting $\Delta V_c = v_{c1} - v_{c2}$, $G_0' = G_0 L$, and $i_{\text{tot}}^s = i^s L$.

2. Derivation of the IREE Length

We start from the first row of Eq. (18), given by

$$\Delta V_c = \left(\frac{L}{G_B \lambda} + \frac{2}{G_0'} \right) I_c - \frac{\alpha p_0}{2G_B} i_{\text{tot}}^s.$$

We assume that the channel resistance is much larger than the contact resistances, i.e., $L/(G_B \lambda) \gg 2/G_0'$. This

FIG. 6. SPICE setup for dc simulation. The setup connects the transmission-line circuit models in a distributed manner. Block 1 corresponds to the model in Fig. 1 and Block 2 corresponds to the model in Fig. 2. All the external contacts have $p_f = 0$. All the terminals are two-component terminals: charge (c) and spin (s). All indicated spin terminals are grounded except the spin terminal of contact 3.

assumption yields

$$\Delta V_c = \frac{L}{G_B \lambda} I_c - \frac{\alpha p_0}{2 G_B} i_{\text{tot}}^s. \quad (\text{A15})$$

We apply the short-circuit condition $v_{c1} = v_{c2}$ at contacts 1 and 2, i.e., $\Delta V_c = 0$. Thus, we get

$$I_c = \frac{\alpha p_0 \lambda}{2} \frac{i_{\text{tot}}^s}{L}. \quad (\text{A16})$$

The IREE length is given by Eq. (21) as

$$\lambda_{\text{IREE}} = \frac{J_c}{J_s} = \frac{I_c/w}{i_{\text{tot}}^s/(wL)} = \frac{\alpha p_0 \lambda}{2}, \quad (\text{A17})$$

which is the expression in Eq. (22). Applying $\alpha = 2/\pi$ yields Eq. (23).

APPENDIX B: SIMULATION SETUP

This appendix provides the details of the simulation setup in SPICE that is used to analyze steady-state results of charge-spin interconversion in Sec. III.

We discretize the structure in Figs. 3(a), 3(c), and 4(a) into 100 small sections and represent each of the small sections with the corresponding circuit model. For example, block 1 and block 2 indicated in Fig. 6 are represented with the models in Figs. 1 and 2, respectively. Note that each of the nodes in Fig. 6 is a two-component node: charge (c) and z component of spin (s). We connect the charge and spin terminals of the models for all the small sections in a modular fashion using standard circuit rules as shown in Fig. 6. We perform a dc simulation in SPICE. Note that during dc simulation in SPICE, capacitors and inductors automatically become open and short circuit, respectively, and correspond to the steady-state ($\partial/\partial t \rightarrow 0$) form in Eq. (16).

The contacts (1, 2, and 3) in this discussion are point contacts. The contact polarizations $p_f = 0$ and conductances are in the potentiometric limit with $G_0 \approx 0.05 G_B$. We set the total number of modes $M + N$ in the channel

to be 100. We assume that the reflection with the spin-flip scattering mechanism is dominant in the channel, i.e., $r_{s1,2} \gg r, t_s$. The scattering rate per unit mode was set to 0.04 per lattice point.

We apply the charge open and spin ground boundary condition at the two boundaries given by

$$\begin{Bmatrix} i_c \\ v_s \end{Bmatrix}_L = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} i_c \\ v_s \end{Bmatrix}_R = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \quad (\text{B1})$$

Here, i_c and v_s indicate the boundary charge current and boundary spin voltage, respectively. Indices L and R indicate the left and right boundaries, respectively.

Setup in Fig. 3(a): For the setup in Fig. 3(a), we apply a current I_c at the charge terminals of contacts 1 and 2, given by

$$\begin{Bmatrix} i_c \\ v_s \end{Bmatrix}_1 = \begin{Bmatrix} I_c \\ 0 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} i_c \\ v_s \end{Bmatrix}_2 = \begin{Bmatrix} -I_c \\ 0 \end{Bmatrix}, \quad (\text{B2})$$

where indices 1 and 2 represent contacts 1 and 2, respectively. The spin terminals of contacts 1 and 2 are grounded to take into account the spin relaxation within the contact. Both charge and spin terminals of contact 3 are open and we observe open circuit spin voltage at the spin terminal. The boundary condition at contact 3 is

$$\begin{Bmatrix} i_c \\ i_s \end{Bmatrix}_3 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \quad (\text{B3})$$

where the index 3 indicates contact 3.

Setup in Fig. 3(c): For the setup in Fig. 3(c), the charge terminal of contact 3 is open and we apply a current i_{tot}^s at the spin terminal, given by

$$\begin{Bmatrix} i_c \\ i_s \end{Bmatrix}_3 = \begin{Bmatrix} 0 \\ i_{\text{tot}}^s \end{Bmatrix}. \quad (\text{B4})$$

The spin terminals of contacts 1 and 2 are grounded to take into account the spin relaxation within the contact and charge terminals are kept open. Here, we observe the open circuit voltage difference between the charge terminals of contacts 1 and 2. The boundary conditions are

$$\begin{Bmatrix} i_c \\ v_s \end{Bmatrix}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} i_c \\ v_s \end{Bmatrix}_2 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \quad (\text{B5})$$

Setup in Fig. 4(a): For the setup in Fig. 4(a), the charge terminal of contact 3 is open and we inject a current i_{tot}^s through the spin terminal. The boundary condition at contact 3 is

$$\begin{Bmatrix} i_c \\ i_s \end{Bmatrix}_3 = \begin{Bmatrix} 0 \\ i_{\text{tot}}^s \end{Bmatrix}. \quad (\text{B6})$$

We short circuit the charge terminals of contacts 1 and 2

TABLE II. Estimation of k_F .

Material	Electron density	k_F (nm ⁻¹)
Ag/Bi ^a	$5.86 \times 10^{28} \text{ m}^{-3}$ [65]	12 [Eq. (C1a)]
Cu/Bi ^a	$8.49 \times 10^{28} \text{ m}^{-3}$ [65]	13.6 [Eq. (C1a)]
LAO/STO	$2.6 \times 10^{17} \text{ m}^{-2}$ [20]	1.278 [Eq. (C1b)]
Bi ₂ Se ₃	$4 \times 10^{17} \text{ m}^{-2}$ [42]	1.59 [Eq. (C1b)]

^aWe use the electron densities of Ag and Cu, respectively, as they are the most conductive layer in the corresponding bilayer.

and observe the short-circuit charge current I_c flowing in the channel induced by i_{tot}^s . The spin terminals of contacts 1 and 2 are grounded to take into account the spin relaxation within the contact. The boundary conditions at contacts 1 and 2 are

$$\begin{Bmatrix} v_c \\ v_s \end{Bmatrix}_1 = \begin{Bmatrix} v_c \\ 0 \end{Bmatrix}_2 \quad \text{and} \quad \begin{Bmatrix} v_c \\ v_s \end{Bmatrix}_2 = \begin{Bmatrix} v_c \\ 0 \end{Bmatrix}_1. \quad (\text{B7})$$

APPENDIX C: PARAMETER ESTIMATIONS

This appendix provides the details of the estimations made for the IREE lengths on diverse materials using Eq. (23).

1. Estimations

2. Estimation of Fermi wave vector (k_F)

We estimate the Fermi wave vector k_F of the channel from the electron density n_{3D} (units of m⁻³) or n_{2D} (units of m⁻²), using the following expressions:

$$k_F = \sqrt[3]{3\pi^2 n_{3D}}, \quad (\text{C1a})$$

$$k_F = \sqrt{2\pi n_{2D}}. \quad (\text{C1b})$$

Estimations are summarized in Table II.

3. Estimation of ballistic conductance (G_B)

We estimate the total number of modes in the three-dimensional (3D) channel using the following expression:

$$M + N = \frac{k_F^2 w t}{2\pi}, \quad (\text{C2})$$

TABLE III. Estimation of G_B .

Material	w (μm)	t (nm)	$M + N$	G_B [Eq. (C4)]
Ag/Bi ^a	400 [40]	5 [40]	4.6×10^7 [Eq. (C2)]	1.77 kS
Cu/Bi ^a	0.15 [41]	20 [41]	8.8×10^4 [Eq. (C2)]	3.4 S
LAO/STO	400 [20]	...	3.3×10^5 [Eq. (C3)]	12.6 S
Bi ₂ Se ₃	1000 [42]	9 ^b [42]	3.6×10^6 (Eq. (C2))	140 S

^aWe use the thicknesses of the most conductive layer (Ag and Cu, respectively) to estimate G_B considering bulk conduction.

^bWe considered six quintuple-layer (QL) samples in Ref. [42]. 1 QL $\approx$ 1.5 nm.

TABLE IV. Estimation of λ .

Material	R_S ($\Omega/\square$)	ρ ($\mu\Omega \text{ cm}$)	λ (nm)
Ag/Bi	10 ^a	...	22.6 [Eq. (C5)]
Cu/Bi	...	100 ^c	0.88 [Eq. (C6)]
LAO/STO	176 ^b	...	180.8 [Eq. (C5)]
Bi ₂ Se ₃	...	2000 ^d	3.2 [Eq. (C6)]

^aTaken from Fig. 3(a) of Ref. [40] for Ag/Bi samples with 5-nm Ag.

^bTaken from Fig. 1(d) of Ref. [20] for LAO/STO at 7 K.

^c ρ of Bi layer is used, which is taken from Ref. [41].

^dTaken from Fig. 2(b) of Ref. [42] at approximately 300 K.

where w is the width and t is the thickness. For a 2D channel, the expression is

$$M + N = \frac{2k_F w}{\pi}. \quad (\text{C3})$$

Note that, in a strong TI 2D channel with limited bulk conduction, $M + N$ will be lower by a factor of 2 than Eq. (C3). We estimate the ballistic conductance of the channel using the following expression:

$$G_B = \frac{q^2}{h} (M + N). \quad (\text{C4})$$

Estimations are summarized in Table III.

4. Estimation of mean free path (λ)

The mean free path (λ) is estimated from the measured sheet resistance R_S of the sample using the expression

$$\frac{R_S}{w} = \frac{1}{G_B \lambda}, \quad (\text{C5})$$

or from the resistivity ρ of the sample using the expression

$$\frac{\rho}{wt} = \frac{1}{G_B \lambda}. \quad (\text{C6})$$

Estimations are summarized in Table IV.

TABLE V. Estimation of p_0 .

Material	α_R (eV Å)	v_F ($\times 10^6$ m s $^{-1}$)	p_0
Ag/Bi	0.56 [40]	1.39 (Ag) [65] 1.87 (Bi) [65]	0.05
Cu/Bi	0.56 [41]	1.57 (Cu) [65] 1.87 (Bi) [65]	0.05
LAO/STO	0.03 [20]	0.074 ^a	0.0616
Material	ΔV_c (μ V)	J_s (A m $^{-2}$)	p_0
Bi ₂ Se ₃	40 ^b [42]	1.39×10^5 ^c	0.025
(Eq. (20)).			

^aWe estimate the Fermi velocity using $v_F = \hbar k_F / m^*$. $m^* \approx 2 \times 9.1 \times 10^{-31}$ kg as reported in Ref. [20].

^bInverse-effect-induced voltage due to spin pumping at 3 GHz.

^cTaken from table 1 in supplementary information of Ref. [42] for 6 QL. The sample dimension is 1 mm $\times$ 5 mm [42], which yields $i_{\text{tot}}^s \approx 0.695$ A.

5. Estimation of degree of spin-momentum locking (p_0)

For Ag/Bi, Cu/Bi, and LAO/STO, the degree of SML p_0 is estimated using the Rashba coupling coefficient ($v_0 = \alpha_R / \hbar$) and the Fermi velocity (v_F) of the materials, using Eq. (24). For Bi₂Se₃, p_0 is estimated using spin current density reported from spin pumping (J_s), the measured inverse effect voltage ΔV_c , and Eq. (20). The estimations are summarized in Table V.

6. Derivation of Eq. (24)

We start from the eigenstates in Eq. (37) of the Rashba Hamiltonian in Eq. (36), given by

$$E = \frac{p'^2}{2m} - s v_0 p' + U_E,$$

with $p' = |\vec{p} - q\vec{A}|$. Solutions for p' are given by

$$p'_1(s) = s m v_0 + \sqrt{m^2 v_0^2 + 2m(E - U_E)},$$

$$p'_2(s) = s m v_0 - \sqrt{m^2 v_0^2 + 2m(E - U_E)},$$

noting that $s^2 = 1$. Here, p'_1 ($s = +1$) and p'_1 ($s = -1$) correspond to M and N , respectively. Similarly, p'_2 ($s = -1$) and p'_2 ($s = +1$) correspond to M and N , respectively. Thus, the degree of SML p_0 is given by

$$\begin{aligned} p_0(E_F) &= \frac{p'_1(s = +1) - p'_1(s = -1)}{p'_1(s = +1) + p'_1(s = -1)} \\ &= \frac{v_0}{\sqrt{v_0^2 + \frac{2(E_F - U_E)}{m}}}. \end{aligned} \quad (\text{C7})$$

Assuming $U_E = 0$ and applying $E_F = \frac{1}{2} m v_F^2$, we have the expression in Eq. (24).

APPENDIX D: CHARGE AND SPIN VELOCITIES

This appendix provides the derivation of eigenvalues and eigenvectors of Eqs. (2) and (3) to find the charge and spin velocities and their coupling during propagation.

The matrix form of Eqs. (2) and (3) is given by

$$\begin{aligned} & \begin{bmatrix} C_{\text{eff}} & 0 & 0 & 0 \\ 0 & \frac{C_Q}{\alpha^2} & p_0 g_m L_M & 0 \\ 0 & 0 & L_{\text{eff}} & 0 \\ -p_0 r_m C_{\text{eff}} & 0 & 0 & \alpha^2 L_K \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} V_c \\ V_s \\ I_c \\ I_s \end{bmatrix} \\ & + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & G_{\text{sh}} & -p_0 \gamma_s & 0 \\ 0 & -p_0 \eta_c & R_c & 0 \\ -p_0 \eta_s & 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} V_c \\ V_s \\ I_c \\ I_s \end{bmatrix} \\ & = - \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \frac{\partial}{\partial x} \begin{bmatrix} V_c \\ V_s \\ I_c \\ I_s \end{bmatrix}. \end{aligned} \quad (\text{D1})$$

In the low-loss limit, we assume a solution of the form

$$\begin{bmatrix} V_c \\ V_s \\ I_c \\ I_s \end{bmatrix} \equiv \begin{bmatrix} \tilde{V}_c \\ \tilde{V}_s \\ \tilde{I}_c \\ \tilde{I}_s \end{bmatrix} e^{j(kx - \omega t)}, \quad (\text{D2})$$

which results in

$$\begin{aligned} & -j\omega \begin{bmatrix} C_{\text{eff}} & 0 & 0 & 0 \\ 0 & \frac{C_Q}{\alpha^2} & p_0 g_m L_M & 0 \\ 0 & 0 & L_{\text{eff}} & 0 \\ -p_0 r_m C_{\text{eff}} & 0 & 0 & \alpha^2 L_K \end{bmatrix} \begin{bmatrix} \tilde{V}_c \\ \tilde{V}_s \\ \tilde{I}_c \\ \tilde{I}_s \end{bmatrix} \\ & + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & G_{\text{sh}} & -p_0 \gamma_s & 0 \\ 0 & -p_0 \eta_c & R_c & 0 \\ -p_0 \eta_s & 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} \tilde{V}_c \\ \tilde{V}_s \\ \tilde{I}_c \\ \tilde{I}_s \end{bmatrix} \\ & = -jk \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_c \\ \tilde{V}_s \\ \tilde{I}_c \\ \tilde{I}_s \end{bmatrix}. \end{aligned} \quad (\text{D3})$$

We assume that the coefficients of the transmission-line model [Eqs. (2) and (3)] are constant with frequency ω and propagation vector k . We differentiate both sides of Eq. (D3) with respect to k and the following matrix

equation:

$$v_g \begin{Bmatrix} \tilde{V}_c \\ \tilde{V}_s \\ \tilde{I}_c \\ \tilde{I}_s \end{Bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{C_{\text{eff}}} & 0 \\ -\frac{\alpha^2 p_0 g_m L_M}{C_Q L_{\text{eff}}} & 0 & 0 & \frac{\alpha^2}{C_Q} \\ \frac{1}{L_{\text{eff}}} & 0 & 0 & 0 \\ 0 & \frac{1}{\alpha^2 L_K} & \frac{p_0 r_m}{\alpha^2 L_K} & 0 \end{bmatrix} \begin{Bmatrix} \tilde{V}_c \\ \tilde{V}_s \\ \tilde{I}_c \\ \tilde{I}_s \end{Bmatrix}, \quad (\text{D4})$$

where v_g is the group velocity given by

$$v_g = \frac{\partial \omega}{\partial k}. \quad (\text{D5})$$

Note that the eigenvalues of Eq. (D4) give the velocities in Eqs. (25) and (27). For a particular eigenvalue v_g , we can write the following equation from Eq. (D4):

$$\begin{bmatrix} -v_g & \frac{1}{C_{\text{eff}}} \\ \frac{1}{L_{\text{eff}}} & -v_g \end{bmatrix} \begin{Bmatrix} \tilde{V}_c \\ \tilde{I}_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \quad (\text{D6})$$

$$\begin{bmatrix} -v_g & \frac{\alpha^2}{C_Q} \\ \frac{1}{\alpha^2 L_K} & -v_g \end{bmatrix} \begin{Bmatrix} \tilde{V}_s \\ \tilde{I}_s \end{Bmatrix} = p_0 \begin{Bmatrix} \frac{\alpha^2 g_m L_M}{C_Q L_{\text{eff}}} \tilde{V}_c \\ -\frac{r_m}{\alpha^2 L_K} \tilde{I}_c \end{Bmatrix}.$$

For the eigenvalue $v_g = v_{g,s} = \pm 1/\sqrt{L_K C_Q}$ in Eq. (D6), we have $\tilde{V}_c = \tilde{I}_c = 0$ and, assuming $\tilde{I}_s = 1$, we get Eq. (26). Again, for the eigenvalue $v_g = v_{g,c} = \pm 1/\sqrt{L_{\text{eff}} C_{\text{eff}}}$, we get Eq. (28), assuming $\tilde{I}_c = 1$.

APPENDIX E: DERIVATION OF SEMICLASSICAL MODEL

This appendix provides the derivation of Eq. (35), starting from the Boltzmann transport equation in Eq. (31).

We apply a variable transformation $\xi(x, t, \vec{p}, \vec{s}) \equiv E(x, t, \vec{p}, \vec{s}) - \mu(x, t, \vec{p}, \vec{s})$ on the left hand side of Eq. (31), which yields

$$\begin{aligned} \frac{\partial f}{\partial t} &= \frac{\partial f}{\partial \xi} \left(\frac{\partial E}{\partial t} - \frac{\partial \mu}{\partial t} \right), \\ v_x \frac{\partial f}{\partial x} &= v_x \frac{\partial f}{\partial \xi} \left(\frac{\partial E}{\partial x} - \frac{\partial \mu}{\partial x} \right), \text{ and} \\ F_x \frac{\partial f}{\partial p_x} &= F_x \frac{\partial f}{\partial \xi} \left(\frac{\partial E}{\partial p_x} - \frac{\partial \mu}{\partial p_x} \right). \end{aligned} \quad (\text{E1})$$

We first substitute $F_x = -\partial E/\partial x$ and $v_x = \partial E/\partial p_x$. Finally, we set $\partial f/\partial \xi = \partial f_0/\partial E$ to get the left-hand side of Eq. (34).

On the right-hand side of Eq. (31), we expand both f and f' into Taylor series around f_0 [Eq. (33)] with constant electrochemical potential μ_0 . We set $\xi'(x, t, \vec{p}', s') \equiv E(x, t, \vec{p}', s') - \mu(x, t, \vec{p}', s')$ and $\xi_0(x, \vec{p}, s) \equiv E(x, \vec{p}, s) - \mu_0$ and assume that ξ and ξ' are close to ξ_0 , which gives

$$\begin{aligned} f &\approx f_0 + \left(-\frac{\partial f}{\partial \xi} \right)_{\xi=\xi_0} (\xi - \xi_0) \quad \text{and} \\ f' &\approx f_0 + \left(-\frac{\partial f}{\partial \xi'} \right)_{\xi'=\xi_0} (\xi' - \xi_0). \end{aligned} \quad (\text{E2})$$

We set $(\partial f/\partial \xi)_{\xi=\xi_0} = (\partial f/\partial \xi')_{\xi'=\xi_0} = \partial f_0/\partial E$. Thus, the right-hand side of Eq. (31) becomes

$$f - f' = \left(-\frac{\partial f_0}{\partial E} \right) (\xi - \xi') = \left(\frac{\partial f_0}{\partial E} \right) (\mu - \mu'), \quad (\text{E3})$$

noting that $E(x, t, \vec{p}', s') = E(x, t, \vec{p}, s)$ in the elastic scattering limit.

APPENDIX F: E-P RELATION

This appendix provides the derivation of Eq. (39), starting from the E - p relation in Eq. (38).

Differentiating Eq. (38) with respect to time t yields

$$\frac{\partial E}{\partial t} = \left(\frac{\vec{p} - q\vec{A}}{m} - s v_0 \frac{\vec{p} - q\vec{A}}{|\vec{p} - q\vec{A}|} \right) \cdot \left(-q \frac{\partial \vec{A}}{\partial t} \right) + \frac{\partial U_E}{\partial t}. \quad (\text{F1})$$

The velocity $\vec{v}(E) = \nabla_{\vec{p}} E$ is derived from Eq. (38) as

$$\vec{v}(E) = \frac{\vec{p} - q\vec{A}}{m} - s v_0 \frac{\vec{p} - q\vec{A}}{|\vec{p} - q\vec{A}|}. \quad (\text{F2})$$

Combining Eqs. (F1) and (F2) yields Eq. (39).

Note that Eq. (F2) can be written as

$$\vec{v}(E) = \left(\frac{|\vec{p} - q\vec{A}|}{m} + s v_0 \right) \frac{\vec{p} - q\vec{A}}{|\vec{p} - q\vec{A}|}, \quad (\text{F3})$$

where $(\vec{p} - q\vec{A})/|\vec{p} - q\vec{A}|$ is a unit vector along $\vec{p} - q\vec{A}$. From Eq. (38), we get

$$|\vec{p} - q\vec{A}| = -s m v_0 \pm m \sqrt{v_0^2 + \frac{2(E - U_E)}{m}}, \quad (\text{F4})$$

noting that $s^2 = 1$. Thus, from Eq. (F3), we get

$$\vec{v}(E) = \pm \sqrt{v_0^2 + \frac{2(E - U_E)}{m}} \frac{\vec{p} - q\vec{A}}{|\vec{p} - q\vec{A}|}. \quad (\text{F5})$$

Note that the magnitude of the velocity for a particular energy of interest is the same for all four groups in Eq. (44) for the Rashba Hamiltonian in Eq. (36) considered here.

APPENDIX G: SCATTERING RATES

This appendix provides the derivation of the scattering matrix in Eq. (56).

For the group $\mathfrak{R} \equiv U^+, D^-, U^-,$ and D^+ , we have from Eq. (54)

$$\begin{aligned} \tilde{S}(U^+) &= \hat{S}_{U^+ \leftrightarrow D^-} [\langle \mu(D^-) \rangle - \langle \mu(U^+) \rangle] \\ &+ \hat{S}_{U^+ \leftrightarrow U^-} [\langle \mu(U^-) \rangle - \langle \mu(U^+) \rangle] \\ &+ \hat{S}_{U^+ \leftrightarrow D^+} [\langle \mu(D^+) \rangle - \langle \mu(U^+) \rangle], \end{aligned} \quad (\text{G1})$$

$$\begin{aligned} \tilde{S}(D^-) &= \hat{S}_{D^- \leftrightarrow U^+} [\langle \mu(U^+) \rangle - \langle \mu(D^-) \rangle] \\ &+ \hat{S}_{D^- \leftrightarrow U^-} [\langle \mu(U^-) \rangle - \langle \mu(D^-) \rangle] \\ &+ \hat{S}_{D^- \leftrightarrow D^+} [\langle \mu(D^+) \rangle - \langle \mu(D^-) \rangle], \end{aligned} \quad (\text{G2})$$

$$\begin{aligned} \tilde{S}(U^-) &= \hat{S}_{U^- \leftrightarrow U^+} [\langle \mu(U^+) \rangle - \langle \mu(U^-) \rangle] \\ &+ \hat{S}_{U^- \leftrightarrow D^-} [\langle \mu(D^-) \rangle - \langle \mu(U^-) \rangle] \\ &+ \hat{S}_{U^- \leftrightarrow D^+} [\langle \mu(D^+) \rangle - \langle \mu(U^-) \rangle], \end{aligned} \quad (\text{G3})$$

and

$$\begin{aligned} \tilde{S}(D^+) &= \hat{S}_{D^+ \leftrightarrow U^+} [\langle \mu(U^+) \rangle - \langle \mu(D^+) \rangle] \\ &+ \hat{S}_{D^+ \leftrightarrow D^-} [\langle \mu(D^-) \rangle - \langle \mu(D^+) \rangle] \\ &+ \hat{S}_{D^+ \leftrightarrow U^-} [\langle \mu(U^-) \rangle - \langle \mu(D^+) \rangle]. \end{aligned} \quad (\text{G4})$$

Equations (G1), (G2), (G3), and (G4) together yield the scattering matrix in Eq. (56) in the $\{\mu(U^+), \mu(D^-), \mu(U^-), \mu(D^+)\}^T$ basis.

APPENDIX H: CHARGE AND SPIN CURRENTS AND VOLTAGES

This appendix provides the derivation of Eq. (61).

The current in any group is given by

$$I(\mathfrak{R}) = \frac{q}{L} \sum_{\vec{p}, s \in \mathfrak{R}} v_x f(x, t, \vec{p}, s), \quad (\text{H1})$$

where f is given by Eq. (32). Under the linear response approximation, we can write

$$f(x, t, \vec{p}, s) \approx f_0 + \left(-\frac{\partial f_0}{\partial E} \right) [\mu(x, t, \vec{p}, s) - \mu_0], \quad (\text{H2})$$

where f_0 is given by Eq. (33) with constant electrochemical potential μ_0 . Thus, from Eq. (H1), we can write

$$I(\mathfrak{R}) = \frac{q}{L} \left[f_0 \sum_{\vec{p}, s \in \mathfrak{R}} v_x + \frac{D_0(\mathfrak{R})}{2} (\langle v_x \mu \rangle_{\vec{p}, s \in \mathfrak{R}} - \mu_0 \langle v_x \rangle) \right], \quad (\text{H3})$$

where $D_0(\mathfrak{R})$ is given by Eq. (47) and the averaging is defined by Eq. (45).

We use the assumption in Eq. (48), given by

$$\langle v_x \mu \rangle_{\vec{p}, s \in \mathfrak{R}} \approx \langle v_x \rangle \langle \mu \rangle_{\vec{p}, s \in \mathfrak{R}},$$

which results in

$$\begin{aligned} I(\mathfrak{R}) &= \text{sgn}(\langle v_x \rangle) \frac{q}{h} n_m(\mathfrak{R}) (\langle \mu \rangle_{\vec{p}, s \in \mathfrak{R}} - \mu_0) \\ &+ \text{sgn}(|v_x|) \frac{q}{L} f_0 \sum_{\vec{p}, s \in \mathfrak{R}} |v_x|, \end{aligned} \quad (\text{H4})$$

where $n_m(\mathfrak{R})$ is given by Eq. (50).

1. Charge current

The charge current in the channel is given by

$$I_c = I(U^+) + I(D^+) + I(U^-) + I(D^-), \quad (\text{H5})$$

which in conjunction with Eq. (H4) yields

$$\begin{aligned} I_c &= \frac{q}{h} \left\{ \frac{h}{L} f_0 \sum_{\vec{p}, s \in U^+} |v_x| + M [\mu(U^+) - \mu_0] \right\} \\ &+ \frac{q}{h} \left\{ \frac{h}{L} f_0 \sum_{\vec{p}, s \in D^+} |v_x| + N [\mu(D^+) - \mu_0] \right\} \\ &- \frac{q}{h} \left\{ \frac{h}{L} f_0 \sum_{\vec{p}, s \in U^-} |v_x| + N [\mu(U^-) - \mu_0] \right\} \\ &- \frac{q}{h} \left\{ \frac{h}{L} f_0 \sum_{\vec{p}, s \in D^-} |v_x| + M [\mu(D^-) - \mu_0] \right\}, \end{aligned} \quad (\text{H6})$$

where $n_m(U^+) = n_m(D^-) = M$ and $n_m(U^-) = n_m(D^+) = N$.

Note that (U^+, D^-) and (U^-, D^+) are time-reversal symmetric pairs; hence,

$$\begin{aligned} \sum_{\vec{p}, s \in U^+} |v_x| &= \sum_{\vec{p}, s \in D^-} |v_x| \\ \text{and} \quad \sum_{\vec{p}, s \in U^-} |v_x| &= \sum_{\vec{p}, s \in D^+} |v_x|, \end{aligned} \quad (\text{H7})$$

which, when applied to Eq. (H6), give the expression for the charge current

$$I_c = \frac{q}{h} [M \tilde{\mu}(U^+) - N \tilde{\mu}(U^-) + N \tilde{\mu}(D^+) - M \tilde{\mu}(D^-)], \quad (\text{H8})$$

where we defined $\tilde{\mu} = \mu - \mu_0$.

2. Spin current

The spin current in the channel is given by

$$\tilde{I}_s = \frac{1}{\alpha} [I(U^+) + I(U^-) - I(D^+) - I(D^-)], \quad (\text{H9})$$

where $\alpha = 2/\pi$ is an angular averaging factor to take into account the average $\hat{z} \cdot \vec{s}$ on a half Fermi circle [see Fig. 5(a)]. Equation (H9) in conjunction with Eq. (H4) yields

$$\begin{aligned} \tilde{I}_s = & \frac{1}{\alpha} \frac{q}{h} \left\{ \frac{h}{L} f_0 \sum_{\vec{p}, s \in U^+} |v_x| + M [\mu(U^+) - \mu_0] \right\} \\ & - \frac{1}{\alpha} \frac{q}{h} \left\{ \frac{h}{L} f_0 \sum_{\vec{p}, s \in U^-} |v_x| + N [\mu(U^-) - \mu_0] \right\} \\ & - \frac{1}{\alpha} \frac{q}{h} \left\{ \frac{h}{L} f_0 \sum_{\vec{p}, s \in D^+} |v_x| + N [\mu(D^+) - \mu_0] \right\} \\ & + \frac{1}{\alpha} \frac{q}{h} \left\{ \frac{h}{L} f_0 \sum_{\vec{p}, s \in D^-} |v_x| + M [\mu(D^-) - \mu_0] \right\}. \end{aligned} \quad (\text{H10})$$

Applying the condition in Eq. (H7), we have the expression for channel spin current as

$$\begin{aligned} \tilde{I}_s = & \frac{1}{\alpha} \frac{q}{h} [M\tilde{\mu}(U^+) - N\tilde{\mu}(U^-) - N\tilde{\mu}(D^+) + M\tilde{\mu}(D^-)] \\ & + \frac{2}{\alpha} \frac{q}{L} f_0 \left(\sum_{\vec{p}, s \in U^+} |v_x| - \sum_{\vec{p}, s \in U^-} |v_x| \right). \end{aligned} \quad (\text{H11})$$

Note that the first term is zero at the equilibrium condition $\mu(U^+) = \mu(U^-) = \mu(D^+) = \mu(D^-) = \mu_0$.

However, the second term is nonzero even at equilibrium since

$$\sum_{\vec{p}, s \in U^+} |v_x| \neq \sum_{\vec{p}, s \in U^-} |v_x|$$

and represents the equilibrium spin current in the channel. We subtract the equilibrium part from our definition of spin current, given by

$$I_s = \frac{1}{\alpha} \frac{q}{h} [M\tilde{\mu}(U^+) - N\tilde{\mu}(U^-) - N\tilde{\mu}(D^+) + M\tilde{\mu}(D^-)]. \quad (\text{H12})$$

3. Spin voltage

The spin voltage in the channel is given by

$$qV_s = \alpha \frac{M\mu(U^+) + N\mu(U^-) - N\mu(D^+) - M\mu(D^-)}{2(M+N)}. \quad (\text{H13})$$

Subtracting each electrochemical potential by μ_0 gives

$$qV_s = \alpha \frac{M\tilde{\mu}(U^+) + N\tilde{\mu}(U^-) - N\tilde{\mu}(D^+) - M\tilde{\mu}(D^-)}{2(M+N)}. \quad (\text{H14})$$

4. Charge voltage

The charge voltage in the channel is given by

$$q\tilde{V}^c = \frac{M\mu(U^+) + N\mu(U^-) + N\mu(D^+) + M\mu(D^-)}{2(M+N)}. \quad (\text{H15})$$

We subtract each electrochemical potential by μ_0 according to

$$qV^c = \frac{M\tilde{\mu}(U^+) + N\tilde{\mu}(U^-) + N\tilde{\mu}(D^+) + M\tilde{\mu}(D^-)}{2(M+N)}, \quad (\text{H16})$$

where $V_c = \tilde{V}_c - \mu_0/q$.

Equations (H8), (H12), (H14), and (H16) together can be written in a matrix form given by Eq. (61).

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