Method for *in situ* and *in operando* cavity loaded Q extraction in superconducting rf accelerators

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A novel method has been developed that allows extracting the loaded quality factor of particle accelerating radio-frequency cavities at any time during beam operation from acquired control system process parameters without the need of additional measurement equipment. Concurrently it assesses the attenuation in a cavities' power transmission line for any rf generator to rf cavity mismatch condition, and thus can correct calibration inaccuracies of measured forward and reflected power levels. The mathematical formalism is described. It is specifically derived for cryogenically cooled superconducting cavities for which the loaded quality factor can be equated with the external quality factor of the input power coupler. The described method has been applied for 418 superconducting cavities in operation at the Continuous Electron Beam Accelerator Facility at the Thomas Jefferson National Accelerator Facility. The validation of the method as well as its beneficial use as a diagnostic tool is demonstrated for a finite number of examples utilizing process variables archived during a period in the past when the linac beam current was ramped up multiple times. The method is in the process of being implemented in the machines' control system for an *in situ* and *in operando* evaluation of the power coupler's external quality factor.

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I. INTRODUCTION

A method is described that enables evaluating the loaded quality factor, Q_l , of particle accelerating radio-frequency (rf) cavities at any time during beam operation (henceforth: Q_l method). The unitless Q_l is an important figure of merit of energized rf cavities that characterizes the dynamic power losses ($\sim 1/Q_l$) dissipated in the entire electromagnetically oscillating system, i.e., coupled to input and output lines thus accounting for intrinsic and external losses. Only a few simplifications concerning the mathematical derivations of the Q_l method—well applicable for superconducting rf (SRF) accelerating cavities—are outlined allowing the formalism to be employed with minor penalty. This relates to the condition that the fundamental power input coupler (FPC) is largely overcoupled to the cavity in absence of a beam, which implies that the Q_l is nearly equal to the external quality factor, Q_{ext} , of the FPC. The Q_{ext} is a crucial parameter since its value should be chosen to match the impedance of the coupled rf system to the impedance of the rf generator with the particle beam being the load during operation. In this case the rf power

efficiency is maximized. Measuring and verifying the FPC's Q_{ext} during beam operation can therefore become important. It is specifically beneficial when the Q_{ext} value cannot be adjusted during operation, which requires a separate assessment with dedicated rf equipment at least once before beam operation. This for instance is the case for SRF cavities of the Continuous Electron Beam Accelerator Facility (CEBAF) operating at the Thomas Jefferson National Accelerator Facility (JLab).

Applying the new Q_l method facilitates the *in situ*, and *in operando* extraction of the FPC's Q_{ext} , while relying merely on a few, readily available rf process variables that are usually acquired by the rf control system of an accelerating machine. The functionality and validity of the Q_l method will be demonstrated by several examples. The Q_l method simultaneously provides the attenuation loss of any power transmission line from the point where a measurement is taken to the cavity entrance, thereby correcting for the “true” power levels delivered to a cavity and reflected back from a cavity. Given this capability it is shown that the Q_l method can be used simultaneously as a diagnostic tool, especially when the Q_l values are recorded as a function of the beam current.

This paper is organized as follows. Section II presents general relationships applicable to SRF cavities followed by specifics concerning CEBAF's SRF cavities and operational parameters, which are used for several examples in subsequent sections. Section II first analytically introduces the Q_{ext} parameter of the FPC as a figure of merit and then recalls an analytical expression known for the optimization

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of a coupled generator-driven rf system in the presence of beam loading and microphonics. It is commonly applied to derive power requirements for SRF cavities. This expression also serves as the starting point to derive the mathematical formalism of the Q_l method, which is summarized in Sec. III. The procedure of how to apply the Q_l method is described explicitly in Sec. IV before detailing several practical examples for its validation. As the Q_l method's first verification with realistic input parameters—and with meaningful statistics—process variables archived for 418 CEBAF SRF cavities have been surveyed and utilized. Concurrently, the beneficial use of the Q_l method as a diagnostic tool is demonstrated. Note that though examples are focusing primarily on CEBAF cavities, the Q_l method is applicable universally as long as $Q_l \approx Q_{\text{ext}}$. Section V discusses the outcome of several examples in more detail. It includes a comparison of the findings obtained with the Q_l method and those acquired by the traditional experimental method employed at CEBAF, which relies on various rf calibration procedures and factors. Prevalent calibration inaccuracies for the generator's forward and reflected power levels are revealed. It is demonstrated how the Q_l method automatically corrects false calibration factors for the FPC transmission line—quasi-instantaneously—as a function of the beam current. The analyses provide recommendations to improve CEBAF's rf efficiency. Further discussions focus on the measurement accuracy obtained for cavity accelerating fields by dedicated high power rf equipment as part of cryomodule commissioning activities. The values assessed in this way presently require an independent recalibration via a more precise beam-based energy calibration technique in order to adequately account for the actual energy gain in the machine. The level of inaccuracy of the rf measurements is discussed. A conclusion is given in Sec. VI with an outlook on how the Q_l method is conceived to be implemented in CEBAF as an *in situ*, *in operando* tool. The Appendix summarizes the relevant mathematical derivations to formulate the new Q_l method, which are separated from the main sections to provide better readability.

II. INTRODUCTIONARY RELATIONSHIPS AND ASSOCIATED EXAMPLES

SRF cavity technology has become the primary choice for high-energy electron and hadron accelerators instead of normal-conducting cavities thanks to about a millionfold smaller achievable rf surface resistance, R_s . Apart from lowering R_s , minimizing the required rf generator power fed into an rf cavity is a general requirement to maximize the rf efficiency of an accelerator machine. This requirement closely relates to the specification of the Q_{ext} value for an FPC given some desired cavity operational parameters. To illuminate the condition of maximum rf power efficiency for SRF cavities, a few general relationships shall be explored first. These address the assessment of R_s , and the

definition of Q_{ext} and Q_l , respectively. Quantitative examples related to CEBAF cavities follow before continuing in more detail with the derivation of the Q_l method.

Conventional SRF cavities are made from bulk, high purity niobium. Niobium possesses the highest critical temperature, T_c , of ~ 9.25 K among all elements, below which the material becomes superconducting. Consequently, SRF cavities are usually cooled to cryogenic temperature by immersing the structures in liquid helium, which is provided by a helium liquefier system. In today's medium to high-beta elliptical SRF cavities, the achievable electric rf accelerating field, E_{acc} , usually safely surpasses 10 MV/m in continuous wave (cw) operation. The associated rf average power dissipated in the superconducting surfaces and transferred to the helium bath, P_c , is determined by

$$P_c = \frac{V_c^2}{\frac{R}{Q} \cdot G} \cdot R_s = \frac{E_{\text{acc}}^2 \cdot L_{\text{act}}^2}{\frac{R}{Q} \cdot Q_0}. \quad (1)$$

For accelerators operating in pulsed mode, P_c is reduced according to the rf duty cycle. The parameter V_c refers to the maximum accelerating voltage that a particle can experience, when traversing along the cavity axis at a given stored cavity rf energy. Instead of V_c , the accelerating community prefers employing E_{acc} dubbed the “gradient” of a cavity. It requires the active length of the cavity, L_{act} . The losses in passive cavity-interconnecting beam tubes are comparably small. The parameters R/Q and G are the characteristic shunt impedance and geometry factor, respectively (both in Ω), and are purely determined by the cavity cell geometry. Their values can be readily computed numerically. The unloaded quality factor, Q_0 , introduced in the second step of Eq. (1) characterizes the intrinsic cavity losses. It is given by $Q_0 = G/R_s$. R_s is the only material-dependent parameter. It is the sum of the so-called BCS surfaces resistance, R_{BCS} , originating from the Bardeen-Copper-Schrieffer (BCS) theory [1] and a residual resistance due to material defects and trapped magnetic fluxes. The BCS resistance scales with the square of the frequency, while being strongly temperature-dependent according to

$$R_{\text{BCS}} = \frac{A}{T} \cdot f^2 \cdot e^{-\frac{\Delta}{k_B T_c} \frac{T_c}{T}}. \quad (2)$$

Herein Δ denotes the energy gap, f the rf frequency, T the cavity operating temperature, k_B the Boltzmann constant, and A is a material-dependent parameter. The equation well holds below $0.5 \cdot T_c$ and at angular frequencies $\omega \ll k_B \cdot T_c / \hbar \approx 1200$ GHz [2]. R_{BCS} theoretically vanishes at $T = 0$. Though lowering T can strongly decrease P_c , practical constraints related to the complexity of the liquefier system, and not least the associated costs, will impose restrictions. For most applications T is chosen either around 2 or 4.2 K. The latter is the boiling

TABLE I. Nominal cavity and performance parameters of CEBAF cavities operating at 1497 MHz (N_c = number of cells per cavity, and $R/Q = V_{\text{eff}}^2/P_c$).

Cavity	N_c	L_{act}	G	R/Q^a	E_{acc}	$Q_0/10^9$	$R_s = G/Q_0$	P_c	$Q_{\text{ext,FPC}}/10^6$	$Q_0/Q_{\text{ext,FPC}}$	P_b
CM type		m	Ω	Ω	MV/m	at 2.07 K	n Ω	W			kW
C20	5	0.50	274	482.5	5	2.4	114	5.4	6.6 ($\pm 20\%$)	364	1.2
C50	5	0.50	274	482.5	12.5	6.8	38	11.9	8.0 ($\pm 25\%$)	850	2.9
C75	5	0.49	275.6	525.4	19.1	8.0	34	20.9	2.0 ($\pm 15\%$)	400	4.3
C100	7	0.70	280.3	868.9	19.2 ^d	7.2	39	29.2	3.2 ($\pm 20\%$)	225	6.2

temperature of liquid helium at atmospheric pressure, and operating at this temperature significantly simplifies helium refrigeration, but will lead to relatively high surface losses. Around 2 K, the Q_0 value of niobium SRF cavities can be in the upper 10^9 to mid- 10^{10} range. Depending on E_{acc} , this yields wall power losses in the order of one to a few Watts per cavity cell. However, and in particular due to $P_c \sim E_{\text{acc}}^2$, overall cost rationales for large-scale cw accelerators do not favor SRF cavities operating beyond 20 MV/m [3]. This has the beneficial implication that cavities can be operated below the critical flux density, where well-prepared cavities do not suffer from premature quenches or a significant Q_0 degradation, as long as field emission concerns are mitigated at the same time.

A. CEBAF-specific cavity operational parameters

CEBAF is an electron recirculating machine dedicated to nuclear physics research achieving up to 12 GeV beam energy in up to 5.5 passes with 400 SRF cavities installed in two main linacs [4], while the beam is injected into the recirculator with an additional 18 SRF injector cavities delivering up to 123 MeV [5]. The cavities operate in cw at 1497 MHz, and are cooled to nominally 2.07 K by two central helium liquefier plants. Table I quantifies some of the above discussed parameters for different types of SRF cavities installed [6]. Cavities are distributed equally in quantity in a south and north linear accelerator, and are connected to each other by eleven isochronous, achromatic semicircular arcs for recirculation of the electrons at different energies. The two linacs comprise a total of 50 cavity cryomodules (CMs) with each CM housing a string of eight multicell cavities. There are different types of CMs dubbed “C#” accommodating different cavities, with the number # denoting the nominal beam energy gain in MeV per CM. Table I lists nominal values for E_{acc} , Q_0 , R_s , and P_c , which might or might not be achieved depending on actual performance limitations.

Generally, the Q_{ext} of the FPC accounts for the power losses, $P_{\text{ext}} \sim 1/Q_{\text{ext}}$, dissipated in the external circuit that connects the rf generator source to the cavity via a transmission line including at least one dielectric rf vacuum window. An important quantity that characterizes the strength of the coupling of the external circuit to the rf cavity is the coupling factor, β , which is defined by

$\beta \equiv P_{\text{ext}}/P_c = Q_0/Q_{\text{ext}}$. With the beam as a consumer of power, the generator should be matched to the cavity-beam rf system. The losses in the cavity system can then be described by $P_c + P_b$, where P_b is the power consumed by the beam known as beam loading. For a cavity excited on resonance, it is simply the product of the average beam current, I_{avg} , and the effective voltage experienced by the beam. The sum $P_c + P_b$ is the minimum generator power, P_{g0} , which is required for a cavity driven on resonance in absence of microphonics. In that case, the optimum coupling factor, i.e., providing zero power reflection at the cavity FPC port, is $\beta = P_{g0}/P_c = 1 + P_b/P_c$. For SRF cavities, it is typical that $\beta \gg 1$ (largely overcoupled) since $P_b \gg P_c$ even at relatively small beam currents. In contrast, other external losses, e.g., dissipated in the circuit of a diagnostic pickup probe, are (and shall be kept) small, i.e., $Q_{\text{ext,other}}$ is adjusted by design to be in the order of 10^{11} or higher (strongly undercoupled or $\beta \ll 1$). Overall this implies

$$\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext,FPC}}} + \frac{1}{Q_{\text{ext,other}}} \approx \frac{1}{Q_{\text{ext}}} \quad \text{or} \quad Q_l \approx Q_{\text{ext,FPC}}. \quad (3)$$

Since other external losses are deemed negligible, the attribute “FPC” in the subscript of $Q_{\text{ext,FPC}}$ is dropped, so that Q_{ext} shall always relate to the external Q of the FPC throughout. Each CEBAF cavity is energized by its own klystron. Table I reveals that the FPC’s coupling β is within 10^2 – 10^3 . Consequently, $Q_l = Q_{\text{ext}}$ holds true with a discrepancy of less than 0.5% nominally among all CEBAF cavities.

At a fixed Q_{ext} , the optimum coupling β is only achieved for a single beam loading scenario. For the original 4 GeV CEBAF machine, the injected beam current was limited to 200 μA . While this is a relatively small beam current, the resulting beam power is a remarkable 800 kW. The administrative limit for the beam power is 1 MW. This is the maximum allowable power, for which beam dumps were designed in experimental end stations [7]. Consequently, the allowable injected beam current had to be reduced to only 93 μA for the upgraded machine. This yields 1 MW at 11 GeV accomplished in

five electron recirculating passes. The full beam loading in each cavity has to take into account the number of passes, which yields $I_{\text{avg}} = 0.465$ mA (linac current) maximally for the 12 GeV machine opposed to $I_{\text{avg}} = 1$ mA for the original 4 GeV machine. The latter explains why a comparably small Q_{ext} was chosen for the original C20 cavities. This has the implication that with the present beam current limit a stronger mismatch is experienced by the C20 cavities resulting in a rather strong power reflection from the cavities unless the Q_{ext} is readjusted. CEBAF employs so-called fixed waveguide FPCs since there is no mechanical adjustability possible to vary the Q_{ext} . Yet, some adjustability of Q_{ext} is facilitated by using waveguide stub tuners located in the power transmission line close to the cavity klystrons [8]. A stub tuner consists of movable plungers that can be more or less inserted into the transmission line. In case of CEBAF cavities up to three plungers can be moved manually to adjust the Q_{ext} if needed.

The optimum coupling β and thus Q_{ext} is affected by random, uncontrollable microphonics. In the presence of microphonics, a frequency detuning requires extra generator power for its control [9]. The typical probability density of microphonics-induced frequency changes resembles a Gaussian shape [10] though this is a simplified representation. The peak microphonics allowance is often defined as 6 times of the standard deviation of the Gaussian microphonics probability density. If the value of the peak microphonics detuning is unknown, an educated guess has to be made to allow considering its impact on rf power requirements, respectively the optimal Q_{ext} .

It is reasonable to assume that gradients vary among cavities in cryomodules. For instance, some cavities might need to operate above the nominal gradient to compensate for low-performing cavities in order to meet a certain energy gain. This imposes a power overhead to be available from the rf generators. Furthermore, though the power transmission line is a rather low loss line—whether waveguide or coaxial technology is used—the attenuation losses need to be considered as these can noticeably increase the rf source power requirement. This will be further discussed qualitatively in the following for the specific CEBAF power transmission line. A corresponding quantitative line loss analysis is presented in Sec. IV based on the Q_l method.

The *in situ* and *in operando* loaded Q extraction requires only a few process variables (PVs), which are continuously acquired and recorded by the machines' low level rf control (LLRF) systems. These are the generators' incident power, P_g , the cavity reflected power, P_r , the cavity detuning angle, ψ , the cavity voltage V_c , and the phase, ϕ_b , of the bunched beam with respect to the peak of the rf voltage. Furthermore, the average beam current, I_{avg} , is needed, which is typically recorded by beam-current monitors. Otherwise only cavity-specific data are needed, namely R/Q and L_{act} .

B. Power requirement to operate SRF cavities as a function of beam current

To derive the analytical expressions that constitute the Q_l method, a generally applicable and already known equation to assess the required generator power P_g is engaged first. It allows computing P_g in the presence of microphonics detuning accounting for off-crest and off-resonance operation. For example, in a linac the beam can be accelerated on crest ($\phi_b = 0$) as is the case in CEBAF, but off-crest operation can be required to suppress instabilities arising from phase oscillations such as encountered in electron storage rings, in which cavities are detuned off resonance. In any scenario, P_g can be derived by considering that an rf cavity can be described as an equivalent parallel LCR circuit coupled to an rf generator and with the beam represented as a current generator. This will relate the cavities' fundamental parameters to the inductance, capacitance, and shunt impedance of the circuit. The derivation has been carried out for instance in Ref. [11] resulting in

$$P_g = \frac{V_c^2}{\frac{R}{Q} \cdot Q_l} \cdot \frac{1 + \beta}{4\beta} \cdot \left\{ \left[1 + \frac{I_{\text{avg}} \frac{R}{Q} Q_l}{V_c} \cdot \cos \phi_b \right]^2 + \left[\tan \Psi - \frac{I_{\text{avg}} \frac{R}{Q} Q_l}{V_c} \cdot \sin \phi_b \right]^2 \right\}. \quad (4)$$

The factor $R/Q \cdot Q_l$ is the loaded shunt impedance of the cavity, which is twice the shunt impedance of the LCR circuit. All phases are in reference to the cavity voltage. The detuning angle relates to the amount of total cavity detuning as given by

$$\begin{aligned} \tan \Psi &\equiv -Q_l \cdot \left(\frac{f}{f_r} - \frac{f_r}{f} \right) \approx -2Q_l \cdot \frac{f - f_r}{f_r} \\ &= \frac{-2Q_0}{1 + \beta} \cdot \frac{f - f_r}{f_r} = \frac{-2}{1 + \beta} \cdot \frac{\delta f}{\Delta f_0}. \end{aligned} \quad (5)$$

The approximation applied herein is due to $f \approx f_r$. The cavity total frequency detuning, $\delta f = f - f_r$, and the intrinsic cavity bandwidth, $\Delta f_0 = f_r / Q_0$, have been introduced in the last step. Furthermore, it has been considered that $Q_l = Q_0 / (1 + \beta)$ resulting from Eq. (3). Equation (4) implies that a minimization of P_g with respect to the detuning angle necessitates

$$\tan \Psi = \frac{I_{\text{avg}} \frac{R}{Q} \cdot Q_l}{V_c} \cdot \sin \phi_b. \quad (6)$$

For on-crest operation ($\phi_b = 0$) this requires $\Psi = 0^\circ$ or $\delta f = 0$ (no microphonics). Given Eq. (4) one can readily determine the power reflected from a cavity, P_r , knowing that $P_r = P_g - P_c - P_b$ must be satisfied. This yields

$$P_r = \frac{V_c^2}{\frac{R}{Q} \cdot Q_l} \cdot \frac{1 + \beta}{4\beta} \cdot \left\{ \left[\frac{\beta - 1}{\beta + 1} - \frac{I_{\text{avg}} \frac{R}{Q} Q_l}{V_c} \cdot \cos \phi_b \right]^2 + \left[\tan \Psi - \frac{I_{\text{avg}} \frac{R}{Q} Q_l}{V_c} \cdot \sin \phi_b \right]^2 \right\}. \quad (7)$$

Hereby it was utilized that $P_c = V_c^2 / (R/Q \cdot Q_0)$ per Eq. (1), $\beta = Q_0/Q_{\text{ext}}$, and $P_b = V_c \cdot I_{\text{avg}} \cdot \cos \phi_b$. It shall be mentioned that subtracting Eq. (7) from Eq. (4) results in

$$P_g - P_r = \frac{V_c^2}{\frac{R}{Q} \cdot Q_0} + V_c \cdot I_{\text{avg}} \cos \phi_b. \quad (8)$$

This is simply $P_g - P_r = P_c + P_b$. By introducing the term

$$b \equiv \frac{P_b}{P_c} = \frac{I_{\text{avg}} \frac{R}{Q} Q_0}{V_c} \cos \phi_b, \quad (9)$$

which describes the ratio of the power transferred to the beam over the power dissipated in the cavity walls, Eqs. (4) and (7) can be rewritten as follows:

$$P_g = \frac{P_c}{4\beta} \cdot \{ [\beta + 1 + b]^2 + [(\beta + 1) \tan \Psi - b \cdot \tan \phi_b]^2 \}, \quad (10a)$$

$$P_r = \frac{P_c}{4\beta} \cdot \{ [\beta - 1 - b]^2 + [(\beta + 1) \tan \Psi - b \cdot \tan \phi_b]^2 \}. \quad (10b)$$

The optimum coupling factor, β_{opt} , which minimizes the required power, can be determined by setting the derivative of Eq. (10a) with respect to β to zero, while accounting for the dependency of $\tan \Psi$ with β per Eq. (5). Doing so leads to

$$\beta_{\text{opt}} = \sqrt{(b + 1)^2 + \left(\frac{2\delta f}{\Delta f_0} + b \tan \phi_b \right)^2}. \quad (11)$$

The optimum P_g and P_r are achieved when inserting Eq. (11) back into Eq. (10a) and Eq. (10b), respectively. This results in

$$P_g = \frac{1}{2} P_c \cdot \{ \beta_{\text{opt}} + (b + 1) \}, \quad (12a)$$

$$P_r = \frac{1}{2} P_c \cdot \{ \beta_{\text{opt}} - (b + 1) \}. \quad (12b)$$

In the absence of microphonics and for on-crest operation, Eq. (11) simplifies to $\beta_{\text{opt}} = b + 1 = I_{\text{avg}} \cdot R/Q \cdot Q_0 / V_c + 1$. This can be more readily derived when forcing the first term in the bracket of Eq. (7) to vanish since it will yield $P_r = 0$. In this case it follows that

$$P_{g,0} = P_c \cdot (b + 1) = P_b + P_c, \quad (13a)$$

$$P_{r,0} = 0. \quad (13b)$$

FIG. 1. Required forward power for CEBAF cavities as a function of the linac current for on-crest operation in the absence of microphonics. Nominal performance parameters have been assumed per Table I.

An accelerator might not always operate at full beam loading, and P_r will be finite if the coupling factor is not readjusted even without microphonics detuning present. As quantitative examples, Figs. 1 and 2 plot P_g and P_r , respectively, as a function of the linac beam current for the differing CEBAF cavities. Nominal cavity operational parameters are used per Table I, while assuming on-crest operation and $\delta f = 0$.

The required klystron power for CEBAF cavities is actually higher than implied in Fig. 1 since no microphonics has been considered yet. For C100 cavities for instance, prior cavity experience suggested that a peak frequency detuning of 25 Hz can be expected, which results from the addition of 6 times the dynamic rms microphonics detuning observed (3.5 Hz) and twice the static mechanical tuner

FIG. 2. Cavity reflected power for CEBAF cavities as a function of the linac current for on-crest operation in the absence of microphonics. Nominal performance parameters have been assumed per Table I.

FIG. 3. Required klystron power for CEBAF cavities as a function of Q_{ext} accounting for on-crest operation with 25 Hz peak microphonics detuning at full linac current of 0.465 mA. Nominal performance parameters have been assumed per Table I. The dots mark the specified Q_{ext} values for each cavity type. Note that these do not coincide with the minimally achievable P_g for this particular beam loading scenario.

resolution/error (2 Hz) [12].¹ Figure 3 depicts the required forward power per Eq. (10a) for CEBAF cavities as a function of Q_{ext} accounting for 25 Hz peak detuning in all cases, nominal performance parameters, and a maximum linac current of 0.465 mA.

When only fixed couplers are employed, it is favorable to choose the FPC's Q_{ext} value so that critical coupling is achieved at the nominal gradient and at full beam loading. This will ideally lead to a minimal P_g when the klystrons are most stressed, but tradeoffs were considered for CEBAF cavities. For C20 cavities for instance a coupling factor 0.8 times the value required for critical coupling at full beam loading (then referred to 1 mA) was chosen resulting in $Q_{\text{ext}} = 6.6 \times 10^6$. The reason was to increase the cavity bandwidth for better rf control, while somewhat less generator power is required for less than full beam current than would be for critical coupling at full beam current [12]. For C50 cavities, the Q_{ext} specification was increased to 8×10^6 , though a higher value would be needed to improve impedance matching.² The C50 cavities are

therefore relatively strongly mismatched at any beam current. For the C100 cavities it was contemplated that some cavities might be able to operate at up to 21 MV/m, while it was accounted for that the Q_{ext} value can be $\leq 30\%$ off from the optimum value [12]. The C75 cavities—contemplated after the 12 GeV program with the first CM still under construction—shall operate at almost the same gradient as C100 cavities [6]. To minimize operational trip rates experienced in CEBAF due to peak microphonics excursions—particularly a concern for C100 cavities—it was decided to choose a somewhat smaller than optimal Q_{ext} at full beam loading. Only two C75 prototypes are currently installed in an otherwise C50 CM.

III. DERIVATION OF THE LOADED Q METHOD FOR *IN SITU* AND *IN OPERANDO* APPLICATION

To determine the Q_l from measurable quantities during beam operation, Eqs. (4) and (7) are utilized as they independently can be solved for Q_l . It can be neglected that β itself is dependent on Q_l , since the terms carrying β can be assumed to be constant with minor penalty for $\beta \gg 1$ (cf. Fig. 18). There are two terms to be considered, i.e.,

$$\frac{1 + \beta}{4\beta} \approx \frac{1}{4}, \quad \frac{\beta - 1}{\beta + 1} \approx 1. \quad (14)$$

These are the only approximations made valid for $\beta \gg 1$ as is the case in most SRF cavities. At $\beta = 100$ for instance, the relative error of the approximated to the actual value is 2.5% and 2%, respectively, for the first and second expression above. For CEBAF cavities with $225 \leq \beta \leq 850$ nominally, the relative errors are merely $\sim 1\%$ for each of the expressions at the lowest value of $\beta = 225$.

As an important consequence, the Q_l value can be accurately evaluated from the measurable quantities I_{avg} , V_c , ψ , ϕ_b and P_g or P_r utilizing Eq. (4) or Eq. (7), respectively. The derivations to solve for the Q_l value have been detailed in Appendix A. The solutions are provided by Eqs. (A9a) and (A9b), respectively. Explicitly denoting the functional dependencies, these are

$$Q_l(P_g, I_{\text{avg}}, V_c, \Psi, \phi_b) = \frac{I_{\text{avg}} V_c \cdot (-\cos \phi_b + \tan \Psi \sin \phi_b) + 2P_g \pm 2P_g \sqrt{1 - \frac{I_{\text{avg}} V_c}{P_g} (\cos \phi_b - \tan \Psi \sin \phi_b) + \left(\frac{I_{\text{avg}} V_c}{2P_g}\right)^2 [(\cos \phi_b - \tan \Psi \sin \phi_b)^2 - 1 - \tan^2 \Psi]}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}, \quad (15a)$$

¹Accounting for 1 dB waveguide losses and adding 10% so that the C100 klystrons will not be operated in the saturated regime gave a requirement of 13 kW compared to 5 kW for original C20 cavity klystrons.

²The increase has been accomplished by a physical deformation of the broad wall of the FPC waveguide. A higher Q_{ext} value was not achievable as it would have resulted into intolerable material damages on the FPC waveguide [6].

$$Q_l(P_r, I_{\text{avg}}, V_c, \Psi, \phi_b)$$

$$= \frac{I_{\text{avg}} V_c \cdot (\cos \phi_b + \tan \Psi \sin \phi_b) + 2P_r \pm 2P_r \sqrt{1 + \frac{I_{\text{avg}} V_c}{P_r} \cdot (\cos \phi_b + \tan \Psi \sin \phi_b) + \left(\frac{I_{\text{avg}} V_c}{2P_r}\right)^2 [(\cos \phi_b + \tan \Psi \sin \phi_b)^2 - 1 - \tan^2 \Psi]}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}. \quad (15b)$$

In case of on-crest operation ($\phi_b = 0$)—as is nominally the case for CEBAF cavities—the equations simplify to

$$Q_l(P_g, I_{\text{avg}}, V_c, \Psi) = \frac{-I_{\text{avg}} V_c + 2P_g \pm 2P_g \sqrt{1 - \frac{I_{\text{avg}} V_c}{P_g} - \left(\frac{I_{\text{avg}} V_c}{2P_g}\right)^2 \tan^2 \Psi}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}, \quad (16a)$$

$$Q_l(P_r, I_{\text{avg}}, V_c, \Psi) = \frac{I_{\text{avg}} V_c + 2P_r \pm 2P_r \sqrt{1 + \frac{I_{\text{avg}} V_c}{P_r} - \left(\frac{I_{\text{avg}} V_c}{2P_r}\right)^2 \tan^2 \Psi}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}. \quad (16b)$$

Furthermore, if no microphonics detuning is present ($\Psi = 0^\circ$), the optimal matching condition ($P_r = 0$) is possible at a single beam loading condition, and Eq. (15b)—at the corresponding beam current (index 0), simplifies to the handy formula

$$Q_l(P_r = 0) = \frac{I_{\text{avg},0} V_c}{\frac{R}{Q} \cdot I_{\text{avg},0}^2} = \frac{V_c}{\frac{R}{Q} \cdot I_{\text{avg},0}}. \quad (17)$$

Combining Eqs. (9) and (11) and inserting Eq. (17) yields the optimal (index “opt”) coupling factor and thus external Q as follows:

$$\begin{aligned} \beta_{\text{opt}} &= \frac{Q_0}{Q_{\text{ext,opt}}} = \frac{I_{\text{avg},0} \frac{R}{Q} Q_0}{V_c} + 1 = \frac{Q_0}{Q_l(P_r = 0)} + 1 \rightarrow \frac{1}{Q_{\text{ext,opt}}} \\ &= \frac{1}{Q_l(P_r = 0)} + \frac{1}{Q_0}. \end{aligned} \quad (18)$$

The Q_l method neglects the cavity surface losses ($Q_0 \rightarrow \infty$) so that $Q_{\text{ext,opt}}$ is equal to $Q_l(P_r = 0)$. Yet, Eq. (9) does take the Q_0 into account leading to the right-hand derivation in Eq. (18) as a consequence, which is therefore not in violation to the generally valid Eq. (3).

Alternatively, Eq. (15a) simplifies to

$$Q_l(P_r = 0) = \frac{-I_{\text{avg}} V_c + 2P_g \pm 2P_g \sqrt{1 - \frac{I_{\text{avg}} V_c}{P_g}}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}, \quad (19)$$

which has to yield the same loaded Q as Eq. (17). Consequently, inserting Eq. (17) into Eq. (19) and solving for P_g delivers the corresponding forward power ($P_{g,0}$). Not surprisingly, the analytical result is $P_{g,0} = I_{\text{avg},0} \cdot V_c$, i.e., the beam loading P_b in case of an infinite Q_0 and when $P_r = 0$. In other words, all power is transferred to the beam

[cf. Eq. (8)], and this agrees as well with Eq. (13a) for $Q_0 \rightarrow \infty$.

The primary outcome is that Eqs. (15a) and (15b) allow determining the loaded/external Q accurately at any instant of time during beam operation, whether or not the generator power is matched to the cavity or not, in the presence of microphonics, and for on-crest or off-crest operation. Both equations should deliver the same result for the same process variables—elaborated further below—and can be used independently. Hereby one has to distinguish between the positive and negative sign in front of the square root terms yielding two possible solutions for each equation. Utilizing the negative square root term is valid as long as the reflected power decreases with increasing beam current, and until the optimal matching condition is achieved corresponding to the minimal reflected power at a certain beam current for a given gradient. Above this beam current threshold, the positive square root term has to be taken in both equations since the reflected power increases again with increasing beam current. This is further explained in Appendix A. Hereby one has to consider that errors in the extraction of the Q_l values will increase in the presence of microphonics, especially when P_r approaches zero since in this case the square roots will dominantly depend on the term carrying $(\tan \Psi)^2$. In Eq. (16a) for instance, the square root term will be close to zero.

The required process variables to evaluate Eqs. (15a) and (15b) are readily available from the machines' control system and usually archived. I_{avg} for instance can be determined by beam current monitors, and $E_{\text{acc}} = V_c / L_{\text{act}}$ is recorded via the LLRF cavity control system as is ϕ_b and Ψ . It shall be noted that the detuning angle Ψ is a measurable quantity, which does not require the knowledge of Q_l , but is assessed via the phase difference between the forward and transmitted power recorded at the cavity pickup antenna port. The power levels P_g and P_r are assessed with a bidirectional coupler located in the FPC

transmission line. It will be demonstrated in Sec. IV that it is important to correct these power levels if the power transmission line attenuation is not already accounted for. For this reason and as an additional benefit, the Q_I method is further enhanced to simultaneously allow determining the attenuation losses for any power transmission line. The rationale to determine these line losses analytically will be detailed in the following section.

A. Determination of the power transmission line attenuation

The attenuation of the generated rf wave, from the point where the power signal is measured (bidirectional coupler) to the cavity entrance/FPC port, is not necessarily negligible despite the relatively low losses in standard metallic rectangular waveguides or large coaxial transmission lines (e.g., <1 dB/100 m at several hundred MHz). The FPC port in contrast can be defined as the location, where rf losses become negligible compared to these attenuation losses.

For instance, the bidirectional couplers at CEBAF are located in the rf service buildings close to the klystrons. The power transmission line is a rectangular waveguide connecting the klystron to a cavity (approximately 10 m long), though the exact length depends on the position of the cavity in the cryomodule to which it connects. A transmission line consists of rigid straights, sweeps, and curved as well as flexible waveguides (WR650 standard). Each WR650 waveguide eventually tapers down to a custom, roughly half-height WR650 waveguide section in the tunnel that connects to the corresponding cavity FPC waveguide flange within the cryomodule. Overall a multitude of flange connections (>10) are required. The transmission line close to the cavity also consists of two rf vacuum ceramic windows. The space between the windows is actively pumped to establish an insulating vacuum that protects the cryomodule from being vented in case one window would break and leak. It also houses an infrared camera viewport and an electrical arc discharge detector port for diagnostics. For original CEBAF cavities, the FPC coupler by design provides doubly duty as a higher order mode (HOM) coupler. For this reason a waveguide filter was conceived with a low insertion loss (≤ 0.3 dB at 1497 ± 10 MHz) that is part of the FPC's waveguide in the tunnel external to the cryomodule [13]. It shall damp HOM signals traveling towards the klystron circulator. The filter also has the purpose to prevent parasitic third harmonic rf signals potentially excited in the klystrons to enter a cavity. For some C100 cavities, but not all, waveguide filters were installed in support of HOM damping though this was originally not foreseen by design. These details shall exemplify qualitatively that the attenuation in a power transmission line can strongly depend on the implementation of various functional components therein. Their insertion losses can add noticeably to the

dissipation of an otherwise smooth, low-loss transmission line. For CEBAF cavities, the actual transmission line dissipation is not measured. It therefore cannot be exactly taken into account by the LLRF system to calibrate either P_g or P_r . The order of magnitude of such losses has for instance been accounted for in the past by an educated guess of ~ 1 dB [12]. This was necessary to estimate the required power overhead for C100 klystrons before their acquisition. Without the knowledge of the actual attenuation loss, it has been experienced that Eqs. (15a) and (15b) do not necessarily provide the same Q_I value. The accuracy of existing calibration factors in the LLRF control system for both the forward and reflected power levels were therefore questioned once the equations were applied for the first time. In fact, inaccuracies concerning the power levels could be implied for several cavities, for which P_g was strongly deviating from P_r at zero beam current, whereas one needs to expect $P_g(I_{\text{avg}} = 0) \approx P_r(I_{\text{avg}} = 0)$. Such cases are visibly identifiable by extrapolating the fit functions for $P_g(I_{\text{avg}})$ and $P_r(I_{\text{avg}})$ to $I_{\text{avg}} = 0$. Additionally, cases with $P_r > P_g$ below a certain beam current have been observed, which cannot be true physically. Representative examples of such cases are presented in Sec. IV.

In summary, the accuracy of measured power levels depends on the quality of the calibration, and these levels do not necessarily represent the actual power levels as received at or reflected from the cavity. A proper calibration would require an experimental determination of the actual attenuation loss for each individual cavity transmission line. At CEBAF this is not carried out. Furthermore, attenuation losses depend on rf mismatch conditions. Fortunately, the Q_I method serves to evaluate the transmission line losses that can be later used to correct as-measured power levels. In the following, a prime is added to the corrected variables, i.e., P'_g denotes the corrected forward power and P'_r the corrected reflected power, respectively. Accounting for the yet unknown attenuation factor, α , from a bidirectional power coupler to the cavity entrance, one can make the ansatz:

$$P'_g = P_g \times 10^{\frac{-\alpha'}{10}}, \quad (20a)$$

$$P'_r = P_r \times 10^{\frac{\alpha''}{10}}. \quad (20b)$$

Herein the attenuation is given in units of dB and should be ≥ 0 . The differing signs of α account for the fact that P'_g is smaller than the measured P_g as the latter is attenuated towards the cavity FPC port, whereas P'_r is accordingly larger than P_r since the latter is measured after the signal attenuates towards the bidirectional coupler. It is further accounted for that the two ports at a bidirectional coupler are separated by a finite length. Though this separation is comparably small, it yields somewhat differing attenuation

factors (α' versus α''). To remedy this issue, one may assume that the (small³) attenuation loss between the two ports is given by γ (in dB). For the center location of the bidirectional coupler line—assuming the reflected power port is closest to the cavity—one then can write

$$P'_g = P_g \times 10^{\frac{-(\alpha' - \frac{\gamma}{2})}{10}} = P_g \times 10^{\frac{-\alpha}{10}}, \quad (21a)$$

$$P'_r = P_r \times 10^{\frac{\alpha'' + \frac{\gamma}{2}}{10}} = P_r \times 10^{\frac{\alpha}{10}}. \quad (21b)$$

The new attenuation factor α is now equal for P'_g and P'_r . It corrects for the fact that P_g and P_r are picked up at two different physical locations. Equations (21a) and (21b) can eventually be inserted in the general Eqs. (15a) and (15b) to substitute P_g and P_r , respectively. With this substitution it is enforced that both equations deliver the same Q_l value, so that the condition

$$Q_l(P'_g, I_{\text{avg}}, V_c, \Psi, \phi_b) = Q_l(P'_r, I_{\text{avg}}, V_c, \Psi, \phi_b) \quad (22)$$

must be satisfied. It is this condition that allows solving for the attenuation of transmission lines. As described in Appendix B, the substitution

$$Att = 10^{-\frac{\alpha}{10}} \quad (23)$$

has been made to ease the analytical solution. Corresponding derivations to solve for Att are summarized in Appendix B. The analytically exact solution is

$$Att = \frac{2I_{\text{avg}}V_c \cos \phi_b + \sqrt{2} \sqrt{I_{\text{avg}}^2 V_c^2 [1 + \cos(2\phi_b)] + 8P_g P_r}}{4P_g},$$

$$\text{wherein } 0 < Att \leq 1. \quad (24)$$

In case of on-crest operation ($\phi_b = 0$) this simplifies to

$$Att = \frac{I_{\text{avg}}V_c + \sqrt{I_{\text{avg}}^2 V_c^2 + 4P_g P_r}}{2P_g}. \quad (25)$$

The attenuation factor α can be retrieved by

$$\alpha = -10 \cdot \text{Log}_{10}(Att) \quad \text{with } \alpha \text{ (in dB)} \geq 0. \quad (26)$$

Generally, the attenuation is a function of I_{avg} according to Eq. (24) as impedance matching conditions change depending on the beam loading $P_b = V_c \cdot I_{\text{avg}} \cdot \cos \phi_b$. A value of $Att = 1$ implies no attenuation in the transmission line, whereas the attenuation increases with decreasing Att ($Att = 0$ is not possible). One can notice that the cavity-dependent R/Q vanishes in the assessment of the transmission line attenuation factor as one should

expect (ditto for the $\tan^2 \Psi$). Note though that the actual attenuation of a traveling rf wave in a metallic waveguide from one to another location is constant at a given frequency unless affected by changes in the resistive wall losses, e.g. due to heating. It is the ratio of the forward and reflected power as measured at a single waveguide position that is a function of the beam current since depending on how the waveguide is matched at its end. Equation (25) is surprisingly simple. Without beam ($I_{\text{avg}} = 0$), the solution can be simplified to

$$Att(I_{\text{avg}} = 0) = \frac{\sqrt{4P_g P_r}}{2P_g} = \sqrt{\frac{P_r}{P_g}}. \quad (27)$$

Again, in this case the cavity is highly mismatched, and almost the entire forward power will be reflected. Inserting Eq. (27) in Eq. (26) yields

$$\alpha(I_{\text{avg}} = 0) = -10 \cdot \text{Log}_{10} \left(\sqrt{\frac{P_r}{P_g}} \right). \quad (28)$$

The commonly known return loss (RL) of a transmission line is defined by

$$RL(\text{dB}) = 10 \cdot \text{Log}_{10} \left(\left| \frac{1}{S_{11}} \right| \right) = -20 \cdot \text{Log}_{10}(|S_{11}|) \quad (29)$$

with a positive value denoting losses. Herein S_{11} is the reflection response, which by definition is the ratio $\text{Sqrt}(P_r/P_g)$ and actually equals the right-hand side of Eq. (27). Therefore, this definition is identical with

$$-20 \cdot \text{Log}_{10} \left(\left| \sqrt{\frac{P_r}{P_g}} \right| \right) = -2 \cdot \alpha(I_{\text{avg}} = 0). \quad (30)$$

This demonstrates that Eq. (24) provides the correct physical interpretation of the attenuation loss since RL is twice the one-way attenuation loss α as implied by Eqs. (21a) and (21b) at $I_{\text{avg}} = 0$. If one assumes optimal matching ($P_r = 0$) at on-crest operation in the other extreme, Eq. (24) becomes

$$Att = \frac{I_{\text{avg},0} \cdot V_c}{P_g} = \frac{P_{b,0}}{P_g} < 1. \quad (31)$$

This is true for a single beam loading condition (index 0), thus beam current $I_{\text{avg},0}$ at a given V_c . Hereby the losses only depend on the forward traveling wave, but not on the reflected wave. Note that P_g is the as-measured power level before being attenuated, which determines the value of this ratio, while no assumption has been made on the specific transmission line. Any partially reflected wave is taken into account by the general Eq. (24), hence applicable to any beam loading condition.

³The loss for the WR650 bidirectional couplers employed in CEBAF is nominally less than 0.05 dB per product data sheet.

TABLE II. CEBAF naming convention.

Linac	<linac>	<zone>	EPICS <zone>	# of rf zones	<cavity>
Injector	0	02 ^a , 03, 04	2, 3, 4	3	1, 2, ..., 8
North linac (NL)	1	02, 03, ..., 26	2, ..., 9, A, B, ..., Q	25	1, 2, ..., 8
South linac (SL)	2	02, 03, ..., 26	2, ..., 9, A, B, ..., Q	25	1, 2, ..., 8

^aThe first injector CM is a quarter CM housing only two cavities (R027 and R028).

TABLE III. Relevant CEBAF EPICS PV acronyms.

Symbol	Acronym	Description	Unit
E_{acc}	$GMES \rightarrow V_c = GMES \cdot L_{\text{act}}$	Gradient measured	MV/m
I_{avg}	$ITOT^4$	Average linac current	μA
P_g	$CRFP$	Cavity rf forward power	kW
P_r	$CRRP$	Cavity rf reflected power	kW
Ψ	$TDETA^a$ or $DETA^b$	Detuning angle	deg

^ain case of C20/C50/C75 cavities.

^bin case of C100 cavities.

IV. PRACTICAL APPLICATION OF THE LOADED Q METHOD

With the equations detailed in Sec. III all information is available to apply the Q_l method for SRF cavities ($\beta \gg 1$ presupposed) at any point of time from the measured PVs, i.e., I_{avg} , V_c , Ψ , ϕ_b and either P_g or P_r depending on which of the two alternative formulas is used. The procedure is straightforward, and can be summarized as follows: (i) compute Att per Eq. (24); (ii) evaluate the corresponding attenuation factor α per Eq. (26); (iii) correct the measured generator/forward power P_g and reflected power P_r according to Eqs. (21a) and (21b), respectively, using α computed in step (ii); and (iv) insert the corrected power levels P'_g and P'_r computed in step (iii) into Eqs. (15a) and (15b), respectively, to obtain the loaded Q , which principally equals the external Q of the FPC.

This procedure has first been applied for CEBAF cavities. This has been facilitated by extracting archived PVs that have been recorded in 2018, when the beam current was ramped up several times during a period of a few days (April 6–8). At this point it is helpful to describe the CEBAF naming convention that identifies cavities referred to in the following examples. For the EPICS control system [14] of CEBAF, the prefix “R<linac><EPICS zone><cavity>” is

employed to uniquely address cavities. Table II lists the numbers and letters used for this purpose.

For instance, the fifth cavity in the 23rd CM zone in the north linac is referred to as R1O5.

PVs for specific cavities are acquired by adding a unique acronym to the prefix [15]. The rf-related acronyms required for this report are provided in Table III. Note though that the total linac beam current does not need a cavity prefix. The R/Q values and active lengths of the differing CEBAF cavities are tabulated in Table I. These constants are also stored in the CEBAF Element Database (CED) [16] for look-up.

The Q_l method has been applied for all 418 CEBAF cavities, of which a few examples shall be discussed. Some cavities were not switched on though during the analyzed period. If not otherwise stated, the following figures plot the as-measured and/or the corrected forward and reflected power levels, the accelerating field, and the resulting loaded/external Q values per above procedure as a function of the linac beam current. The first example in Fig. 4 refers to the C25 cavity R1J4. Herein Q_l values principally do not change as a function of the beam current level, which one should expect for a fixed waveguide coupler.

Signal noise is visible and appears more dominantly at higher beam current levels. This is due to a comparably longer operational dwell time at these levels during which more data were accumulated. Consequently a larger spread of detuning angles can be more readily identified in these regimes in the presence of continuous cavity-dependent microphonics. This is true for all cavities likewise. The signal noise for power levels is inevitably transferred to the evaluated Q_l values. Cavity detuning in CEBAF is automatically controlled by adjusting the P_g in order to keep E_{acc} at its set value. Cavity R1J4 operated at $E_{\text{acc}} = 6.8$ MV/m, while being mismatched at any beam current

⁴A PV for the total beam current is currently only available for the SL with prefix R2XX. Three experimental end stations (hall A, B, and C) are situated at the SL end, and one (hall D) at the NL end. The NL current is the same as the SL current unless beam is delivered to hall D after an additional sixth recirculating pass through the NL. The deviation to the SL current is small since the beam current to hall D is limited to maximally $5 \mu A$. The total NL current can be determined by summing up the products of the number of circulating passes to each hall with their corresponding beam currents as injected into the recirculator.

FIG. 4. Cavity R1J4 (C25) corrected and as-measured forward power (squares) and reflected power (diamonds) as a function of the linac current at an accelerating field of $E_{\text{acc}} = 6.8$ MV/m (triangles), and the resulting Q_l values (crosses and dots) per Eqs. (15a) and (15b), respectively. The averaged Q_l is 4.54×10^6 ($\sigma = 7.44 \times 10^4$).

level yielding a reflected power around 1 kW. This is expected when conferring to Figs. 1 and 2 though these figures assume nominal cavity performance parameters without microphonics. The evaluated Q_l ($\approx Q_{\text{ext}}$) is only 4.54×10^6 and thus below the nominal value implying stronger reflected power levels than would be the case with a higher Q_{ext} (cf. Fig. 3). It is important to emphasize that the Q_l values derived from either P'_g and P'_r are principally identical apart from a few data points. For the latter it can be assumed that the machine was in an undefined state, e.g., immediately after a cavity trip or due to microphonics excursions so that as-measured power levels are yet unreliable. Such outliers were often easily recognizable among reliable data. The fit functions in the figures (solid lines) for the as-measured power levels not only guide the eye, but highlight the discrepancy of the forward and reflected power levels at zero beam current. A large discrepancy indicates that the as-measured power values are not compensated for a transmission line loss. It shall be mentioned though that the Q_l method implies that the attenuation loss seems to be corrected for some cavities. Thanks to the Q_l method however any calibration error is principally eliminated.

More worrisome, Fig. 5 depicts an example (R1E5) where $P_r > P_g$ is encountered at small beam current levels. Similar occurrences have been detected for several CEBAF cavities. It implies a gain ($\alpha < 0$) of the signal traveling along the transmission line, which is physically impossible. Though the condition $\alpha > 0$ has been imposed by Eq. (23), it is remarkable that the Q_l method works well for $\alpha < 0$, i.e., correcting the as-measured power levels to proper values. In addition, the proper condition $P_g(I_{\text{avg}} = 0) \approx P_r(I_{\text{avg}} = 0)$ is obtained. In contrast, a

FIG. 5. Cavity R1E5 (C25) corrected and as-measured forward power (squares) and reflected power (diamonds) at an accelerating field of ~ 8 MV/m (triangles). This plot reveals a physically impossible condition at small beam currents where $P_r \gg P_g$. The Q_l method is eventually correcting this error. The averaged Q_l is 5.04×10^6 ($\sigma = 1.52 \times 10^5$).

discrepancy of about 400 W is observed between P_g and P_r at zero beam current.

The next example (Fig. 6) is for cavity R1J8, which is interesting insofar that E_{acc} was frequently changed (not in chronological sequence).

The power levels needed to be adjusted accordingly, and fit functions for related data have been omitted for that reason. Since the plot is rather busy, data for the as-measured power levels have been omitted as well. The Q_l method on average delivers a nearly constant value

FIG. 6. Cavity R1J8 (C25) corrected and as-measured forward power (squares) and reflected power (diamonds) as a function of the linac current at variable E_{acc} (triangles), and the resulting Q_l values (crosses and dots) per Eqs. (15a) and (15b), respectively. The averaged Q_l is 3.70×10^6 ($\sigma = 3.68 \times 10^5$).

FIG. 7. Cavity R2Q2 (C100) corrected and as-measured forward power (squares) and reflected power (diamonds) as a function of the linac current at an accelerating field of $E_{\text{acc}} = 16.2$ MV/m (triangles), and the resulting Q_I values (crosses and dots) per Eqs. (15a) and (15b), respectively. The averaged Q_I is 2.39×10^7 ($\sigma = 1.21 \times 10^6$).

with increasing beam current. The slight slope of the corresponding trendline (black) can be explained by the comparably large amount of data accumulated at the higher beam current levels as already explained above. For R1J8 the condition $\alpha < 0$ has been observed as well, though the value strongly fluctuated between negative and positive values covering the entire beam current range. This overall questions the LLRF calibration “fudge” factors in use, which will be briefly addressed later. Apparently an over-compensation of correction factors is present. Irregularities of PVs might contribute to Q_I uncertainties, which are significantly larger for R1J8 than observed in above examples. This results in a rather broad Q_I range covering a range of 2.8×10^6 to 5.1×10^6 . Potential explanations for this finding have not been further explored. Yet, by sorting data to separate the differing E_{acc} levels from each other, it has been verified that there is no correlation between the Q_I values and different E_{acc} levels. Hence the Q_I method is itself not affected by the gradient changes. The Q_I value averaged among all data is 3.70×10^6 with a standard deviation of 3.68×10^5 .

For Fig. 7, a C100 cavity (R2Q2) has been chosen. At $E_{\text{acc}} = 16.2$ MV/m the cavity operated 3 MV/m below the design value. The average Q_I is 2.39×10^7 ($\sigma = 1.21 \times 10^6$) and hence below the specification (cf. Table I), but at this value the cavity is well matched at the highest beam loading.

The corresponding attenuation loss is examined in Fig. 8. Per Eq. (24), Att is a function of the beam current. Equation (24) accounts for any scenario from principally full reflection at $I_{\text{avg}} = 0$ to full matching at higher beam currents. At nearly full matching for instance (here

FIG. 8. Transmission line attenuation as evaluated as part of the Q_I method for the case of Fig. 8.

426 μA), the attenuation loss per Fig. 8 is 0.79 dB on average. This is on the order of what has been estimated for the specific CEBAF transmission line loss. One can see though that the value on average increases with the beam current as indicated by the trendline, which is not readily explainable. Such a trend was not uncommon to observe among cavities. The underlying reason has not been investigated in the frame of this work. However it can be conceived that rf losses may rise with increasing forward power as a consequence of waveguide heating—evidenced at rf windows with infrared cameras—that consequently increases localized losses. Moreover with stub tuners in place, a resonant wave component is created between the stub tuner plungers and the cavity even when the generator impedance is matched to the cavity impedance with beam. The resulting standing wave pattern can play a role in how localized and severe the losses are manifested. Note that a reflected rf wave component will lead to a standing wave with twice the peak field amplitudes of the forward traveling rf wave, which quadruples the localized peak power losses. This is of concern especially for original CEBAF cavities employing a cold rf window situated at ~ 2.1 K. Experimental evidence has shown that waveguide losses—primarily suspected to arise in the normal-conducting metallization of cold rf windows otherwise embedded in a superconducting Nb eyelet welded to a Nb flange—can lead to noticeable rf power losses (in the order of Watts) that dissipate in the helium bath. This in turn can significantly reduce a cavities’ Q_0 value, which strongly depends on the actual standing wave positions [6]. These losses are indistinguishable from cavity surface losses, when assessing the Q_0 during cavity commissioning before beam operation. Calibration factors in use by the LLRF control system can eventually alter the actual magnitude of the attenuation losses assessed by the Q_I method.

FIG. 9. Cavity R1C2 (C50) corrected and as-measured forward power (squares) and reflected power (diamonds) as a function of the linac current at an accelerating field of $E_{\text{acc}} = 10.9$ MV/m (triangles), and the resulting Q_I values (crosses and dots) per Eqs. (15a) and (15b), respectively. The averaged Q_I is 9.44×10^6 ($\sigma = 1.90 \times 10^5$).

The last two examples shall cover C50 cavities. In Sec. II it was explained that these cavities will nominally exhibit the largest power reflections among CEBAF cavities. Comparably strong standing wave amplitudes are therefore expected. Figure 9 plots the results for cavity R1C2 operating at $E_{\text{acc}} = 10.9$ MV/m. The average Q_I value of 9.44×10^6 ($\sigma = 1.90 \times 10^5$) is actually higher than specified for a C50 cavity, but it still delivers a mismatch at all operational beam currents.

A stronger mismatch is apparent for cavity R1C6 in Fig. 10 as the corresponding Q_I value is very low. It averages to 4.50×10^6 ($\sigma = 6.62 \times 10^4$). The required generator power levels are close to those for R1C2 though R1C6 operated at only $E_{\text{acc}} = 7.9$ MV/m.

V. FURTHER ANALYSES AND DISCUSSIONS

The representative examples above illustrate how the Q_I method can be employed as a quality assurance diagnostic tool, particularly when results are plotted as a function of the beam current. After surveying the findings for CEBAF cavities in this manner, several cavities with surprisingly low Q_{ext} values, clearly below specifications, have been identified. Recommendations provided to the operational group members were to increase the Q_{ext} values with the stub tuner if practically feasible. This not only can improve the rf efficiency and reduce stressing the klystrons for improved longevity, but can reduce standing waves in the transmission line, which in turn might mitigate waveguide heating and potentially allows for such cavities to reach higher accelerating fields and/or to minimize trip rates.

FIG. 10. Cavity R1C6 (C50) corrected and as-measured forward power (squares) and reflected power (diamonds) as a function of the linac current at an accelerating field of $E_{\text{acc}} = 7.9$ MV/m (triangles), and the resulting Q_I values (crosses and dots) per Eqs. (15a) and (15b), respectively. The averaged Q_I is 4.50×10^6 ($\sigma = 6.62 \times 10^4$).

Note also that original CEBAF cavities are less vulnerable to microphonics than C100 cavities, which should make a Q_{ext} increase for such cavities a lesser operational concern. The range of adjustability for the Q_{ext} by stub tuner plungers is limited though, but higher values are easier to obtain than lower values. Fabrication tolerances are a source for the cavity's Q_{ext} value to deviate from the desired/design value (even significantly). The actual value is revealed only after cryomodule installation in the tunnel. It is generally unfavorable to require stub tuning because plungers need to be inserted into the power transmission line. This will create a resonant wave component in the power transmission line. Moreover, for CEBAF it has been evidenced that individual plungers cannot be moved too far into the WR650 waveguide since arcing is observed beyond a certain protrusion. Not all transmission lines in CEBAF are equipped with stub tuners. Presently 107/140 out of 200 cavities can make use of stub tuners in the north/south linac, but stub tuners are in place for all C100 cavities. The latter are overall well matched for full beam loading. Based on the recommendations provided, it is planned to acquire more stub tuners for original CEBAF cavities, which would benefit most from the improved impedance matching.⁵ In case of remotely adjustable power couplers—more readily facilitated with coaxial couplers via a movable inner conductor—the generator impedance

⁵A motorization of the stub tuner plungers is technically feasible but might not be realized on short or long term in CEBAF. The Q_I method could otherwise be utilized to adjust the Q_I during beam operation without dedicated rf equipment.

can be matched to the cavity impedance at any beam current level. This is not the case for the fixed FPC waveguide coupler employed for CEBAF cavities. In both cases though, the Q_l method will be of benefit and comes at principally no additional costs once implemented in the control system for on-line use.

New CMs installed in the CEBAF tunnel or CMs being reinstalled after refurbishment activities are often commissioned with stub tuners “as found” [17], i.e., the stub tuners are not necessarily and specifically adjusted to meet the Q_{ext} specification. The FPC waveguide body of a cavity is however mechanically tuned during cavity postprocessing and before cavity-string assembly. The measurement procedure requires a vector network analyzer, and is performed at ambient temperature and low power levels on a cavity tuning bench. Hereby the FPC waveguide body is mechanically squeezed or stretched at the broad wall close to the cavity to obtain the specified Q_{ext} value within tolerances. Several iterations are usually required to eventually achieve this goal, while at the same time obtaining and preserving the required field flatness of the π -mode and the cavity straightness. This is usually well achieved within tolerated limits [6]. Ideally, a cavities’ Q_{ext} value should then be reproduced in the machine, when the stub tuner plungers are positioned flush with the waveguide wall.⁶ This however is not well documented. It is only after the commissioning that the Q_l values of the cavities are measured again, at least once before beam operation. This is done with a vector network analyzer in the rf service buildings with direct access to the stub tuners.⁷ The results of these measurements before beam operation have not been electronically recorded in the past and cannot be retrieved [19]. The CED previously stored the Q_{ext} value of cavities, which were measured during the high-power cavity commissioning without beam. Results date partially back more than a decade. Note that during commissioning, cavities in a CM are fully equipped, i.e., hooked up to the power transmission line and with field probes attached for signal transmission measurements, while the temperature of the cavities is ≤ 2.15 K. The loaded Q of a cavity-waveguide system is measured at a low field level utilizing the decay time method with the klystron driven in pulsed mode [20]. Together with the forward power, this provides E_{acc} .

⁶This presupposes that rf reflections from the circulator are reasonably small at 1497 MHz, though this might not be always guaranteed [18].

⁷Required adjustments are made when cavities are cooled down to their nominal operating temperature. A vector network analyzer records the Q_l in resonance utilizing the standard -3.01 dB method in transmission (S21), i.e., measuring the loaded bandwidth Δf_l below the resonant peak at f_r , which yields $Q_l = f_r / \Delta f_l \approx Q_{\text{ext}}$. For this purpose the signal is launched into the reflected power port of the bidirectional coupler, transmitted through the cavity and picked up at the high Q_{ext} field probe port of the cavity. A signal amplifier is required.

FIG. 11. External Q_{ext} values of the FPC for CEBAF cavities as determined with the Q_l method (dots) from archived PVs (April 2018) during beam operation versus Q_{ext} values measured during cryomodule commissioning before beam operation as stored previously in the CED.

Knowing E_{acc} is a prerequisite to determine the Q_0 value of a cavity. The Q_0 value is measured applying a dedicated calorimetric method [20],⁸ and can be assumed to be accurate within about $\pm 10\%$ [17]. Given the Q_0 , the external Q can then be determined per Eq. (3). Cavity CM commissioning in the tunnel requires dedicated hardware and software and is an elaborate labor effort. This might have been a reason why it was by-passed to record the Q_{ext} values for all C50 cavities but eight belonging to CM 1L12 and six in CM 1L13. The latter is the latest refurbished CM also housing two prototype C75 cavities [6]. The new Q_l method has provided the opportunity to acquire the previously unknown values and to update existing Q_{ext} values. This has been done once so far utilizing archived PV from April 2018. These Q_{ext} values have been meanwhile stored in the CED.

Figure 11 plots these Q_{ext} values (symbols) versus the previously stored data in the CED (crosses) as measured during commissioning. Some cavities were not operating in April 2018 so that the Q_l method could not be applied. This includes all cavities of CM 2L21. It can be observed that the Q_{ext} values of C20 cavities are in majority below the specified value of 6.6×10^6 . The lowest value observed is

⁸First the entire CM is isolated from the cryogenic system closing supply and return valves. Then the change in the helium pressure rise is measured due to the static heat load only, when the klystron is turned off. This is done over a finite time (~ 1 min) before and after a CM internal heater is turned applying a well-known heating power. Eventually, the klystron is turned on to measure the pressure rise at an established field level, from which the rf heat load in the cavity is determined. Applying Eq. (1) then yields the Q_0 of a cavity. Usually the Q_0 is measured in steps of 1 MV/m from a low field level (few MV/m) to the highest sustainable field.

FIG. 12. Cavity R1M2 (C100) corrected forward power (squares) and reflected power (diamonds) as a function of the linac current at an accelerating field of $E_{\text{acc}} = 16.0$ MV/m (triangles) a measured (*GMES*). The curved lines are the analytically computed power values per Eqs. (10a) and (10b) inserting the performance parameters denoted explicitly in the figures' lower legend for either $Q_{\text{ext}} = 1.70 \times 10^7$ per Q_l method (solid curves) or $Q_{\text{ext}} = 3.42 \times 10^7$ per commissioning result (dashed curves).

around 2×10^6 per Q_l method. For many C50 cavities a similar conclusion can be made. Lower than nominal Q_{ext} values for C100 cavities—around 2×10^7 —can actually be beneficial for better impedance matching at full beam loading at nominal performance parameters. However, at first glance of Fig. 12 there seems to be a visible discrepancy particularly for C100 cavities concerning the external Q s being overall smaller as determined by the Q_l method compared to the values measured during commissioning. For instance, Q_{ext} for cavity R1M2 is 1.70×10^7 on average ($\sigma = 1.03 \times 10^6$) compared to 3.42×10^7 measured during commissioning. This constitutes a difference as high as a factor of 2. The question immediately raised is which of the two differing methods is actually more accurate.

To investigate this question, Eqs. (10a) and (10b) are employed to analytically compute the forward and reflected power levels as a function of I_{avg} . Hereby a Q_0 of 7.04×10^9 , as measured during CM commissioning for R1M2, has been inserted to calculate the required P_c . The operating field of the cavity (R1M 2 *GMES*) used as an input was 16.0 MV/m. Additionally, the recorded detuning angle during beam operation was within ± 4 degrees throughout, which is relatively small and thus has a minor impact on the results. Nevertheless, for each computed data point a random value has been evaluated for $\tan \Psi$ based on this detuning range. Eventually a Q_{ext} of either 1.70×10^7 or 3.42×10^7 has been inserted into the equations to compute P_g and P_r . The resulting curves were finally

FIG. 13. Similar to Fig. 13 but for cavity R2P7 (C100) using either $Q_{\text{ext}} = 1.82 \times 10^7$ per Q_l method (solid curves) or $Q_{\text{ext}} = 4.15 \times 10^7$ per commissioning result (dashed curves).

compared with the data obtained by the Q_l method for P'_g and P'_r . The findings are plotted in Fig. 12.

Indeed, the computed power curves (solid) for $Q_{\text{ext}} = 1.70 \times 10^7$ well fit with the data points (symbols) for P'_g and P'_r . Note that the curves (colored) are not fit functions of the data, but the analytically computed results. In case of $Q_{\text{ext}} = 3.42 \times 10^7$ however a strong deviation of the computed curves (dashed) from the data points is obvious. It would take $E_{\text{acc}} \approx 22.7$ MV/m to obtain the same forward and reflected power at zero beam current at this high Q_{ext} than at the lower Q_{ext} value, which is an unrealistic operating field for this cavity. Moreover, these curves do not fit well with the as-measured power values. A similar example is provided as another demonstration in Fig. 13 for C100 cavity R2P7. This cavity operated at only $E_{\text{acc}} = 12.6$ MV/m, and the commissioning data suggest a $Q_{\text{ext}} = 4.15 \times 10^7$ compared to $Q_{\text{ext}} = 1.82 \times 10^7$ evaluated with the Q_l method. As in case of Fig. 13, the Q_l method provides a good agreement of measured and analytically calculated power values.

Consequently, the major source of the discrepancies between the two methods is principally explained. It is a very convincing argument to trust the results of the Q_l method in favor of commissioning results. In contrary, commissioning results referring to Q_{ext} values can be questioned.⁹

Yet, while the Q_l method provides consistent results, it has to rely on accurately measured PVs. The accuracy of these PVs depends on proper calibration methods. In CEBAF for instance, corrective fudge factors are being

⁹It should be kept in mind that it is not documented whether the cavities' stub tuners were adjusted before or after cryomodule commissioning (and for which cavities), which will alter the measurement results.

employed in the LLRF system control that are separately introduced to eliminate the deviation between the beam energy implied by the measured E_{acc} values (*GMES*) and the beam energy actually measured via a beam-based calibration method. This calibration is done for each cavity by measuring the change in the energy/momentum due to the cavity in the nearest downstream dispersive beam transport arc [21].¹⁰ This means that the initial calibration for E_{acc} is overwritten by a beam-based calibration since the latter more precisely determines the actual energy gain of the linac cryomodules. This calibration has been important to ease matching the optical lattice in the arcs to design values. Otherwise discrepancies between the energy gain relying on initial E_{acc} values and the beam-based calibration method are apparent. This in turn implies the prevalence of inaccurate rf calibration factors.

Fudge factors are corrective measures nominally in the order of 1%–2% but higher values in the order of 5%–10% [22] and even up to 30% [23] have been communicated to the author from operational group members. Furthermore, a mix of rf calibration factors and a beam-based calibration factor still might be used simultaneously [22]. Also, for rather poorly performing cavities in a cryomodule, the operational group might by-pass the beam-based calibration since the error with respect to the beam energy is small.

To quantify the actual correction factors, C-program based scripts implemented into the LLRF control system have been investigated. The following findings are related to C100 cavities only since calibration factors are readily accessible from the digital LLRF control system.¹¹ Note that *GMES* is determined by the forward power and the power transmitted to the high- Q_{ext} field probe antenna resulting into an rf power loss denoted as *CTTW_ATTS* (PV suffix *GMES.L* in W). The field probe transmission

line is calibrated according to the linear coefficient *PROBE* (PV suffix *GGPRB*). Four attenuation factors (in units of dB) are used as follows: (i) *CABLE*, typically around 3 dB—PV suffix *GMES.A*; (ii) *JMP* (jumper cable) typically around 0.2 dB—PV suffix *GMES.B*; (iii) *ATTEN*, a correction factor due to a signal attenuator, which for most cavities is not in place (0 dB), and otherwise is 1, 2, 3, or 4 dB—PV suffix *GMES.C*; (iv) *BEAM*—PV suffix *GGBBC*.

BEAM refers to the cavity-specific correction factor due to the beam-based calibration. Its value is evaluated from the momentum measurement described above the desired cavity gradient set point (*GSET*) according to

$$GGBBC = 20 \log_{10} \left(\frac{\text{momentum}}{GSET \cdot L_{\text{act}}} \right). \quad (32)$$

For C100 cavities, *GGBBC* varies from -3.5 to 0.84 dB. This correction principally should not be in place if other rf calibration factors were accurately determinable. The calibrated *GMES* (in V/m) is eventually given by

$$GMES = \sqrt{CTT_Watts \cdot 10^{\frac{CABLE+JMP+ATTEN+BEAM}{10}}} \cdot PROBE. \quad (33)$$

Consider that E_{acc} is proportional to the square root of P_c per Eq. (1). Isolating the factor carrying *BEAM* in Eq. (33) leads to

$$GMES = \sqrt{10^{\frac{BEAM}{10}}} \cdot \sqrt{CTT_Watts \cdot 10^{\frac{CABLE+JMP+ATTEN}{10}}} \cdot PROBE. \quad (34)$$

Thus, the first factor determines how strong E_{acc} is corrected by the beam-based calibration alone. This factor is plotted in Fig. 14 for C100 cavities. The symbols differentiate between C100 cavities installed in the north and south linac.

The findings principally matches the verbally reported uncertainty regimes of *GMES* discussed above accounting to minimally -33% (R2M4) and maximally $+10\%$ (R2N3).¹² For R2M4 a negative attenuation factor has been observed when applying the Q_l method, which indicates an overcompensation of the transmission line loss, and thus can explain the large value of *GGBBC* to correct *GMES*.

For a few C100 cavities, the required correction for E_{acc} is close to zero as can be seen in Fig. 14. This indicates a trustworthy initial rf calibration for E_{acc} . Among those cavities is R2N7. Applying the Q_l method resulted in $Q_{\text{ext}} = Q_l \approx 1.9 \times 10^7$ compared to $Q_{\text{ext}} = 2.65 \times 10^7$ as measured during commissioning. The discrepancy is less

¹⁰Per Ref. [21] a momentum balance method was applied in CEBAF from 1995–2002 to calibrate *GMES* for a cavity under test. The highest energy-gain cavity in each linac—used as a reference cavity—was calibrated against the magnets in the downstream dispersive beam transport arc (~ 6 m). Hereby the reference cavity was set to produce a 1.5 MeV/ c change in the momentum in the downstream arc, and the gradient for the cavity under test was changed to compensate the 1.5 MeV/ c change or turned off. The reference cavity was then adjusted to restore the original orbit. The change in the gradient of the reference cavity was used to calibrate the gradient of the cavity under test. The repeatability of these measurements erred by 5% (rms) and 9% (rms) for measurements in the first arc and second arc, respectively. The error is higher in the first arc since the beam momentum is higher, while the momentum change due to a single cavity is not. Errors were later reduced by a factor of 2.5 by replacing the energy balance method with a phase-shift method. It utilizes the phase control of a cavity to alter its gradient allowing also for full deceleration so that this method nearly doubled the allowable momentum shift acceptable by the arcs. For individual C100 cavities, specifically in the SL, the errors are yet larger than for original cavities using the same phase shift method.

¹¹This was not readily possible for original CEBAF cavities controlled with an analog LLRF system.

¹²In both of these extreme cases *ATTEN*=0dB, *JMP* = 0.15 dB, and *CABLE* ≈ 3.2 dB.

FIG. 14. Beam-based correction factors applied to *GMES* for individual C100 cavities.

than for above examples, but still significant (factor ~ 1.4). Figure 15 plots the outcome similarly to Figs. 13 and 14. The importance here is that E_{acc} has not been “fudged” significantly by a beam-based calibration factor, and that the Q_l method provides properly corrected power levels in good agreement with analytical results as a function of the beam current and consistent with E_{acc} .

The application of the Q_l method led to yet another revelation for a few (5) C100 cavities. For these cavities the Q_l seemed to decline strongly as a function of the beam current. Such a case is depicted in Fig. 16 for cavity R2N3 with the Q_l spanning an unreasonably large range (values not shown for small beam currents).

Apparently though, the as-measured P_r values are close to zero throughout implying faulty data, particularly leading to $P_f - P_r > 2$ kW, when data are extrapolated to

FIG. 15. Similar to Figs. 13 and 14, but for cavity R2N7 (C100) using either $Q_{\text{ext}} = 1.90 \times 10^7$ per Q_l method (solid curves) or $Q_{\text{ext}} = 2.65 \times 10^7$ per commissioning result (dashed curves).

FIG. 16. Cavity R2N3 (C100) corrected (after first iteration) and as-measured forward power (squares) and reflected power (diamonds) as a function of the linac current at an accelerating field of $E_{\text{acc}} = 16$ MV/m (triangles). The resulting Q_l values (crosses and dots) per Eqs. (15a) and (15b), respectively, exhibit a strong decline due to faulty P_r data (gray diamonds), which are close to zero throughout.

$I_{\text{avg}} = 0$. The assessment of the attenuation factor generally has to rely on trustworthy forward and reflected power levels per Eq. (24). Since this is not the case for cavity R2N3, the Q_l method cannot succeed. It might indicate though a measurement issue, and thus functions as a diagnostic tool. Moreover, if one assumes that at least P_g is trustworthy for cavity R2N3, one can utilize the Q_l method to reconstruct P_r data—in hindsight—to reasonable values and as best as practicably possible. This excludes though an *in situ*, *in operando* application of the Q_l method. As can be seen in Fig. 17, the corrected P_r

FIG. 17. Similar to Fig. 16 after final correction recovering faulty, as-measured P_r values by applying an iterative procedure utilizing the Q_l method as detailed in the text. This results in a Q_{ext} of 1.41×10^7 , while the Q_l decline is eliminated.

values are slightly higher than the original, faulty data, thus cannot be reasonable yet (nor can be P'_g). The subsequent procedure is to insert P'_r in lieu of the original P_r values. This yields a newly corrected dataset for P'_r , and this process can be repeated iteratively to improve P'_r and P'_g simultaneously. While applying this procedure successively for R2N3, it has been evidenced that the Q_l decline diminished noticeably after each iteration. Since the Q_l method does consistently yield $P_r \approx P_f$ at zero beam current, this requirement cannot be used to halt the iterative process. Instead the forward and reflected power levels have been computed analytically as detailed before, and the iteration process was stopped when both the corrected and analytically computed power values were in sufficient agreement corresponding to a final Q_l value. The findings are documented in Fig. 17. The resulting average Q_l is 1.41×10^7 ($\sigma = 7.26 \times 10^6$), and the Q_l decline is principally eliminated. Note that $P'_g(I_{\text{avg}} = 0)$ does not deviate too much from the value of $P_g(I_{\text{avg}} = 0)$ as was the case for other examples. The commissioning data suggests $Q_{\text{ext}} = 2.93 \times 10^7$ for this cavity, but this Q_{ext} would yield analytical values for P_g not in well agreement with as-measured or corrected values (not shown here), and particularly not at $I_{\text{avg}} = 0$.

VI. CONCLUSION AND OUTLOOK

A new method has been introduced, which facilitates the *in situ*, and *in operando* extraction of the loaded Q factor of an SRF cavity and thus the external Q of the fundamental power coupler under the condition that the coupling factor $\beta \gg 0$. In this case, the loaded Q is principally equal to the external Q of the power coupler, while cavity-intrinsic surface losses can be deemed negligible. The method merely relies on a few rf process variables that are usually acquired and continuously recorded by the rf control system of the accelerating machine, and are thus readily available. Though the application of the Q_l method focused specifically on cavities operating in CEBAF as a first paradigm—providing the advantage of a large statistical ensemble—the Q_l method in its present form is applicable universally for rf accelerating cavities as long as $\beta \gg 1$. The functionality and validity of the Q_l method has been demonstrated by several examples. It simultaneously provides the attenuation loss of the (any) power transmission line from the point where a measurement is taken to the cavity entrance, thereby correcting for the true power levels delivered to a cavity and reflected back from a cavity. Hereby the transmission line attenuation is assessed for any impedance matching condition depending on the beam loading, i.e., covering the range from full power reflection to full impedance matching.

The Q_l method has concurrently served as a diagnostic tool for many CEBAF cavities to identify inaccurate rf calibration factors, specifically when plotting the

as-measured forward and reflected power levels as a function of the beam current. It revealed that the rf calibration embedded in the LLRF control system can be surprisingly imprecise. Several rf calibration factors are in place to quantify the acquired cavity gradient, the forward and the reflected power. Particularly, the unphysical conditions of observing $P_r > P_g$ and negative attenuation losses in the power transmission line have been evidenced for several cavities. Discussions concerning the usage of calibration factors have been triggered by the author with individual experts at CEBAF. Differing responses have been received for example whether or not the transmission line waveguide losses are taken into account to correct power levels. In hindsight, the Q_l method showed that both scenarios are the case, and that an overcompensation and an undercompensating of such losses are observed in many cases. It has been recommended that such issues should be addressed with the aim to eliminate calibration fudge factors. Not least it has been recommended to adjust the Q_{ext} values of strongly mismatched cavities via stub tuners as best as practically possible to provide optimal cavity performances thereby addressing operational complications/limitations as elaborated in the previous section. This could be particularly valuable to maintain the peak energy reach in the machine. The assessment of the attenuation loss of the power transmission line at any beam loading condition—whether or not wrong calibration factors are embedded in the LLRF system—could be the key to achieve a more efficient, and reliable, machine operation. Furthermore, in combination with analytical assessments, it has been revealed that the Q_{ext} values of cavities extracted by the Q_l method can be more reliable than the measured values acquired with dedicated high power rf equipment during cavity cryomodule rf commissioning. Due to the numerous findings and implications, the reader shall refer to the specific discussions already provided in the above sections for further details.

Given the potential of the Q_l method, it is currently in the process of being implemented into CEBAF's EPICS control system as an *in situ*, *in operando* tool. This would make measurements with dedicated rf equipment obsolete, which require switching off the beam current.¹³ One hope is that the on-line tool improves the reliability of the recorded PVs as compared to archived values. This is important since the Q_l method relies on proper PVs feeding into the relevant formulas. For this reason a synchronization of the required PVs is envisaged at any recorded time stamp, while the signal noise such as imposed by microphonics has to be taken into account. One has to conceive that—while a cavity is detuned by microphonics—the tuner's stepping motor is engaged to bring the frequency back on resonance.

¹³A single low power measurement before beam operation would yet be required in order to verify the Q_l of a cavity in its installed condition or after a stub tuner is adjusted.

This is a comparably slow process since no faster tuning, i.e., by piezoelectric sensors, is presently in place for that purpose. At the same time, the generator power is adjusted to keep the cavity at a constant accelerating field. Archived PVs are stored at a 1 Hz rate at CEBAF, and a PV is not updated unless its value changes. A reliable synchronization of the PVs is therefore not guaranteed, and can affect the accuracy of the extracted Q_l values. A high synchronization of the PVs *in operando* could be achieved with a fast enough sampling rate so that any variable can be considered relatively constant during the sampling period. Given that microphonics detuning is driven by mechanical resonances in the order of a few tens of Hz, the required data sampling should be conducted faster than 10 ms. Alternatively, it is envisaged acquiring a sufficiently large number of samples for each required PV to evaluate an rms value. This would ideally cancel out noise effects. The first option is not easy to implement given the rather old analog LLRF control system in place for original CEBAF cavities as opposed to the dramatically faster digital LLRF control system installed for C100 cavities. Sampling rates achievable for the former might be limited to about 5 Hz (200 ms). Yet, five samples acquired at a 1 Hz repetition rate could still provide reasonable rms values. The necessary procedural computations for the Q_l method are desired to be carried out as fast as practically possible, while the LLRF control system is supporting a data acquisition rate up to 1 Hz. Currently it is estimated that the computations of rms values would take one to two seconds. This would still be satisfying to allow sampling the PVs at a reasonable rate, particularly to cover the relatively fast beam current ramp-up in CEBAF that might take only 30 seconds [23]. Once the Q_l method is on-line, loaded/external Q values, transmission line attenuation losses, and corrected power levels are posted in the EPICS control system for each cavity. Acronyms have already been assigned for these new PVs.

Eventually it shall be emphasized that the utilization of the Q_l method has already revealed a series of potential issues in CEBAF as an off-line tool, and raised further questions that cannot be fully addressed here, but are planned to be addressed in future work. One issue is that the inaccuracy of the extracted Q_l values can increase significantly, when approaching the minimum of the reflected power curve as a function of beam current (best match), which has been elaborated on to some extent in the Appendix A (Fig. 20). This strongly depends on the microphonics level. Related inaccuracies have become obvious for C100 cavities, which are matched prematurely, i.e., at much less than the presently envisaged full linac beam current (465 μ A) implying that these cavities operate below the nominal gradient. Fortunately however, a simple analytical formula has been provided to assess the loaded Q with reasonable accuracy in such a scenario only requiring the beam current at the point of minimal power reflection, the cavity gradient, and the R/Q value of the cavity.

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APPENDIX A: ANALYTICAL DERIVATIONS FOR SOLVING THE LOADED Q

Equations (4) and (7) are solved for Q_l leaving only measurable quantities. This yields two independent expressions for Q_l , one based on the generator/forward power, and one on the reflected power. To simplify the equations, the following abbreviations are introduced:

$$\begin{aligned} A &= \frac{V_c^2}{\frac{R}{Q} \cdot P_g}, & A' &= \frac{V_c^2}{\frac{R}{Q} \cdot P_r}, & B &= \frac{I_{\text{avg}} R / Q}{V_c}, \\ C &= \tan \Psi, & D &= \frac{1 + \beta}{4\beta}, & E &= \frac{\beta - 1}{\beta + 1}. \end{aligned} \quad (\text{A1})$$

The terms D and E explicitly depend on β , which in turn is depending on Q_l . Yet, both terms can be regarded as constants for $\beta \gg 1$, which will be taken into account later. Keeping this in mind, Eqs. (4) and (7) can be rewritten as

$$Q_l = A \cdot D \cdot \{[1 + BQ_l \cdot \cos \phi_b]^2 + [C - BQ_l \cdot \sin \phi_b]^2\}, \quad (\text{A2a})$$

$$Q_l = A' \cdot D \cdot \{[E - BQ_l \cdot \cos \phi_b]^2 + [C - BQ_l \cdot \sin \phi_b]^2\}. \quad (\text{A2b})$$

After expanding the expressions and sorting in terms of Q_l , quadratic equations are obtained as follows:

$$Q_l^2 + Q_l \cdot \frac{2ABD \cdot (\cos \phi_b - C \sin \phi_b) - 1}{AB^2D} + \frac{(1 + C^2)}{B^2} = 0, \quad (\text{A3a})$$

$$Q_l^2 - Q_l \cdot \frac{2A'BD \cdot (E \cos \phi_b + C \sin \phi_b) + 1}{A'B^2D} + \frac{(E^2 + C^2)}{B^2} = 0. \quad (\text{A3b})$$

The identity $\sin^2\phi_b + \cos^2\phi_b = 1$ could be utilized once in each case. Solving for Q_l yields

$$Q_l = -\frac{2AB \cdot (\cos\phi_b - C \sin\phi_b) - 1}{2AB^2} \pm \sqrt{\left(\frac{2AB \cdot (\cos\phi_b - C \sin\phi_b) - 1}{2AB^2}\right)^2 - \frac{(2AB)^2(1 + C^2)}{(2AB^2)^2}}, \quad (\text{A4a})$$

$$Q_l = \frac{2A'B \cdot (E \cos\phi_b + C \sin\phi_b) + 1}{2A'B^2} \pm \sqrt{\left(\frac{2A'B \cdot (E \cos\phi_b + C \sin\phi_b) + 1}{2A'B^2}\right)^2 - \frac{(2A'B)^2(E^2 + C^2)}{(2A'B^2)^2}}. \quad (\text{A4b})$$

The individual terms herein have already been arranged to combine the fractions, which allows simplifying the solutions according to

$$Q_l = \frac{2ABD \cdot (-\cos\phi_b + C \sin\phi_b) + 1 \pm \sqrt{(2ABD \cdot (\cos\phi_b - C \sin\phi_b) - 1)^2 - (2ABD)^2(1 + C^2)}}{2AB^2D}, \quad (\text{A5a})$$

$$Q_l = \frac{2A'BD \cdot (E \cos\phi_b + C \sin\phi_b) + 1 \pm \sqrt{(2A'BD \cdot (E \cos\phi_b + C \sin\phi_b) + 1)^2 - (2A'BD)^2(E^2 + C^2)}}{2AB^2D}. \quad (\text{A5b})$$

Expanding the terms in the square roots produces

$$Q_l = \frac{2ABD \cdot (-\cos\phi_b + C \sin\phi_b) + 1 \pm \sqrt{1 - 4ABD \cdot (\cos\phi_b - C \sin\phi_b) + (2ABD)^2[(\cos\phi_b - C \sin\phi_b)^2 - 1 - C^2]}}{2AB^2D}, \quad (\text{A6a})$$

$$Q_l = \frac{2A'BD \cdot (E \cos\phi_b + C \sin\phi_b) + 1 \pm \sqrt{1 + 4A'BD \cdot (E \cos\phi_b + C \sin\phi_b) - (2A'BD)^2[(E \cos\phi_b + C \sin\phi_b)^2 - E^2 - C^2]}}{2A'B^2D}. \quad (\text{A6b})$$

Reintroducing the physical quantities from Eq. (A1) and considering that

$$AB = \frac{I_{\text{avg}} V_c}{P_g}, \quad AB^2 = \frac{\frac{R}{Q} \cdot I_{\text{avg}}^2}{P_g}, \quad A'B = \frac{I_{\text{avg}} V_c}{P_r}, \quad A'B^2 = \frac{\frac{R}{Q} \cdot I_{\text{avg}}^2}{P_r} \quad (\text{A7})$$

eventually leads to

$$Q_l = \frac{I_{\text{avg}} V_c (-\cos\phi_b + \tan\Psi \sin\phi_b) + \frac{P_g}{2(\frac{1+\beta}{4\beta})} \pm \frac{P_g}{2(\frac{1+\beta}{4\beta})} \sqrt{1 - 4 \frac{I_{\text{avg}} V_c}{P_g} \left(\frac{1+\beta}{4\beta}\right) \cdot (\cos\phi_b - \tan\Psi \sin\phi_b) + [2 \frac{I_{\text{avg}} V_c}{P_g} \left(\frac{1+\beta}{4\beta}\right)]^2 [(\cos\phi_b - \tan\Psi \sin\phi_b)^2 - 1 - \tan^2\Psi]}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}, \quad (\text{A8a})$$

$$Q_l = \frac{I_{\text{avg}} V_c \left(\frac{\beta-1}{\beta+1} \cos\phi_b + \tan\Psi \sin\phi_b\right) + \frac{P_r}{2(\frac{1+\beta}{4\beta})} \pm \frac{P_r}{2(\frac{1+\beta}{4\beta})} \sqrt{1 + 4 \frac{I_{\text{avg}} V_c}{P_r} \left(\frac{1+\beta}{4\beta}\right) \cdot \left(\frac{\beta-1}{\beta+1} \cos\phi_b + \tan\Psi \sin\phi_b\right) + [2 \frac{I_{\text{avg}} V_c}{P_r} \left(\frac{1+\beta}{4\beta}\right)]^2 [(\frac{\beta-1}{\beta+1} \cos\phi_b + \tan\Psi \sin\phi_b)^2 - (\frac{\beta-1}{\beta+1})^2 - \tan^2\Psi]}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}. \quad (\text{A8b})$$

At this point one can take advantage that for SRF cavities typically $\beta \gg 1$. This is well fulfilled in CEBAF cavities with $\beta > 225$ nominally (cf. Table I). Figure 18 clarifies that the β -dependent terms D and E are quickly converging with increasing β to $1/4$ and 1 , respectively.

The following approximations therefore well hold true:

$$Q_l \approx \frac{I_{\text{avg}} V_c (-\cos \phi_b + \tan \Psi \sin \phi_b) + 2P_g \pm 2P_g \sqrt{1 - \frac{I_{\text{avg}} V_c}{P_g} (\cos \phi_b - \tan \Psi \sin \phi_b) + \left(\frac{I_{\text{avg}} V_c}{2P_g}\right)^2 [(\cos \phi_b - \tan \Psi \sin \phi_b)^2 - 1 - \tan^2 \Psi]}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}, \quad (\text{A9a})$$

$$Q_l \approx \frac{I_{\text{avg}} V_c (\cos \phi_b + \tan \Psi \sin \phi_b) + 2P_r \pm 2P_r \sqrt{1 + \frac{I_{\text{avg}} V_c}{P_r} (\cos \phi_b + \tan \Psi \sin \phi_b) + \left(\frac{I_{\text{avg}} V_c}{2P_r}\right)^2 [(\cos \phi_b + \tan \Psi \sin \phi_b)^2 - 1 - \tan^2 \Psi]}}{\frac{R}{Q} \cdot I_{\text{avg}}^2}. \quad (\text{A9b})$$

Equations (A9a) and (A9b) form the basis of the Q_l method, and each can be utilized independently to determine Q_l . Each of the equations depends on five process variables measurable during beam operation, namely I_{avg} , V_c , Ψ , ϕ_b and either P_g or P_r .

Two solutions are feasible for each equation due to the positive and negative sign in front of the square root. The proper usage depends on whether the reflected power exhibits a negative slope (use negative square root term) or positive slope (use positive square root term) as a function of the beam current. The turning point of the slope and thus local minimum of P_r corresponds to the optimal matching condition of the generator impedance to the beam impedance. This is demonstrated in Fig. 19 utilizing realistic performance parameters for a CEBAF C100 cavity. Herein the forward and reflected power levels have been analytically computed as a function of the beam current based on Eqs. (10a) and (10b). It requires choosing an unloaded Q_0 as well as a cavity R/Q to compute the intrinsic surface losses. Both ϕ_b and Ψ have been set to zero for simplicity.

Furthermore, values for the effective voltage V_c and Q_{ext} have been presumed. For the latter a Q_{ext} of 2.70×10^7 was chosen, which determines the coupling factor $\beta = Q_0/Q_{\text{ext}}$ required for Eqs. (10a) and (10b). After applying the Q_l method per Eq. (A9a) or Eq. (A9b)—inserting all given

FIG. 18. Convergence of the terms D and E with increasing coupling factor.

process variables—a Q_l of 2.71×10^7 is obtained as averaged over the full average beam current range. According to Eq. (18), regarding for a Q_0 of 6.4×10^9 , this in fact yields $Q_{\text{ext}} = 2.70 \times 10^7$ in agreement with the presumption. The vertical dashed line in Fig. 19 corresponds to the beam current at which the reflected power vanishes ($I_{\text{avg}} = 267 \mu\text{A}$). The light and dark green lines indicate the two proper solutions for Q_l for $I_{\text{avg}} \leq 267 \mu\text{A}$ (-sqrt) and $I_{\text{avg}} \geq 267 \mu\text{A}$ (+sqrt), respectively. The current at the optimal matching condition, where P_r is minimized, defines the threshold above or below which each equation yields swiftly diverging Q_l values following the grayed-out curves. These solutions are physically invalid and have to be disregarded.

In practice, particularly around the discussed threshold current, the error for the extracted Q_l values depends strongly on the cavity detuning in presence of microphonics. For Fig. 20 for instance, random microphonics

FIG. 19. Demonstration of the Q_l method applied to analytically computed power levels per Eqs. (10a) and (10b). The following process values have been assumed: $E_{\text{acc}} = 9 \text{ MV/m}$, $R/Q = 868.9 \Omega$, $L_{\text{act}} = 0.7 \text{ m}$, $Q_0 = 6.4 \times 10^9$, $Q_{\text{ext}} = 2.70 \times 10^7$, $\phi_b = 0$ and $\Psi = 0$. The light and dark green lines correspond to the loaded Q values evaluated by the Q_l method averaging to 2.71×10^7 . Refer to text for further explanations.

FIG. 20. Similar to Fig. 19, but allowing for a random microphonics detuning assuming Ψ to vary within ± 6 degrees.

detuning angles have been added for the above example with Ψ varying within the limits of ± 6 degrees.

The determination of the power transmission line attenuation as a function of the beam current, which also corrects

the measured reflected and forward power levels in the presence of microphonics, is important to minimize the errors of the extracted Q values. The derivation of this attenuation factor is described in the following.

APPENDIX B: SOLVING FOR THE POWER TRANSMISSION LINE ATTENUATION

As mentioned in Sec. III, the attenuation factor, α , describes the losses in a power transmission line from the point of the measurement to the cavity entrance/FPC port, and can be evaluated by setting

$$Q_l(P'_g, I_{\text{avg}}, V_c, \Psi, \phi_b) = Q_l(P'_r, I_{\text{avg}}, V_c, \Psi, \phi_b). \quad (\text{B1})$$

Herein the measured power levels P_g and P_r are substituted with the corrected power levels P'_g and P'_r accounting for α according to Eqs. (15a) and (15b). Explicitly writing out Eq. (B1) yields

$$\begin{aligned} & I_{\text{avg}} V_c \cdot (-\cos \phi_b + \tan \Psi \sin \phi_b) + 2P_g \text{Att} \\ & - 2P_g \text{Att} \sqrt{1 - \frac{I_{\text{avg}} V_c}{P_g \text{Att}} \cdot (\cos \phi_b - \tan \Psi \sin \phi_b) + \left(\frac{I_{\text{avg}} V_c}{2P_g \text{Att}}\right)^2 [(\cos \phi_b - \tan \Psi \sin \phi_b)^2 - 1 - \tan^2 \Psi]} \\ & = I_{\text{avg}} V_c \cdot (\cos \phi_b + \tan \Psi \sin \phi_b) + \frac{2P_r}{\text{Att}} \\ & - \frac{2P_r}{\text{Att}} \sqrt{1 + \frac{\text{Att} I_{\text{avg}} V_c}{P_r} \cdot (\cos \phi_b + \tan \Psi \sin \phi_b) + \left(\frac{\text{Att} I_{\text{avg}} V_c}{2P_r}\right)^2 [(\cos \phi_b + \tan \Psi \sin \phi_b)^2 - 1 - \tan^2 \Psi]}. \end{aligned} \quad (\text{B2})$$

To ease the solution the following substitution has been utilized:

$$\text{Att} = 10^{-\frac{\alpha}{10}}, \quad \text{wherein } 0 < \text{Att} \leq 1. \quad (\text{B3})$$

The aim is to solve for Att analytically. Once Att is known, one can in turn retrieve α according to

$$\alpha = -10 \cdot \text{Log}_{10}(\text{Att}) \quad \text{with } \alpha \text{ (in dB)} \geq 0. \quad (\text{B4})$$

After multiplying Eq. (B2) with Att and sorting similar terms together, one gets

$$\begin{aligned} & -\text{Att} \cdot I_{\text{avg}} V_c \cos \phi_b + \text{Att}^2 P_g \left\{ 1 - \sqrt{1 - \frac{I_{\text{avg}} V_c}{P_g \text{Att}} \cdot (\cos \phi_b - \tan \Psi \sin \phi_b) + \left(\frac{I_{\text{avg}} V_c}{2P_g \text{Att}}\right)^2 [(\cos \phi_b - \tan \Psi \sin \phi_b)^2 - 1 - \tan^2 \Psi]} \right\} \\ & - P_r \left\{ 1 - \sqrt{1 + \frac{\text{Att} I_{\text{avg}} V_c}{P_r} \cdot (\cos \phi_b + \tan \Psi \sin \phi_b) + \left(\frac{\text{Att} I_{\text{avg}} V_c}{2P_r}\right)^2 [(\cos \phi_b + \tan \Psi \sin \phi_b)^2 - 1 - \tan^2 \Psi]} \right\} = 0. \end{aligned} \quad (\text{B5})$$

The analytical solution process is relatively tedious, but only engages rudimental algebra. The derivation shall be detailed here only for the simplified case when $\phi_b = 0$, i.e., assuming on-crest operation as this applies for CEBAF cavities ideally. The general solution for $\phi_b \neq 0$ is nevertheless provided eventually. First, to shorten the rather long expression, the substitution $K = I_{\text{avg}} V_c$ shall be used. This yields

$$-Att \cdot K + Att^2 P_g - Att^2 P_g \sqrt{1 - \frac{K}{P_g Att} - \left(\frac{K}{2P_g Att}\right)^2 \tan^2 \Psi} - P_r + P_r \sqrt{1 + \frac{Att \cdot K}{P_r} - \left(\frac{Att \cdot K}{2P_r}\right)^2 \tan^2 \Psi} = 0. \quad (\text{B6})$$

Combining the fractions leads to

$$-Att \cdot K + Att^2 P_g - Att^2 P_g \sqrt{\frac{4P_g^2 Att^2 - 4P_g Att \cdot K - K^2 \tan^2 \Psi}{4P_g^2 Att^2}} - P_r + P_r \sqrt{\frac{4P_r^2 + 4P_r Att \cdot K - (Att \cdot K)^2 \tan^2 \Psi}{4P_r^2}} = 0. \quad (\text{B7})$$

One therefore can rewrite after multiplying by 2:

$$-Att^2 P_g \sqrt{\frac{4P_g^2 Att^2 - 4P_g Att \cdot K - K^2 \tan^2 \Psi}{P_g^2 Att^2}} + P_r \sqrt{\frac{4P_r^2 + 4P_r Att \cdot K - (Att \cdot K)^2 \tan^2 \Psi}{P_r^2}} = -2Att^2 P_g + 2Att \cdot K + 2P_r. \quad (\text{B8})$$

This immediately simplifies to

$$-Att \sqrt{4P_g^2 Att^2 - 4P_g Att \cdot K - K^2 \tan^2 \Psi} + \sqrt{4P_r^2 + 4P_r Att \cdot K - (Att \cdot K)^2 \tan^2 \Psi} = -2Att^2 P_g + 2Att \cdot K + 2P_r. \quad (\text{B9})$$

Since Att appears within square roots, both sides are raised by the power of 2, which produces

$$\begin{aligned} & Att^2 (4P_g^2 Att^2 - 4P_g Att \cdot K - K^2 \tan^2 \Psi) + (4P_r^2 + 4P_r Att \cdot K - Att^2 K^2 \tan^2 \Psi) \\ & - 2Att \sqrt{4P_g^2 Att^2 - 4P_g Att \cdot K - K^2 \tan^2 \Psi} \sqrt{4P_r^2 + 4P_r Att \cdot K - Att^2 K^2 \tan^2 \Psi} = (-2Att^2 P_g + 2Att \cdot K + 2P_r)^2. \end{aligned} \quad (\text{B10})$$

Expanding all expressions, dividing both sides by the factor 2, and sorting in terms of Att yields

$$\begin{aligned} & -Att \sqrt{4P_g^2 Att^2 - 4P_g Att \cdot K - K^2 \tan^2 \Psi} \sqrt{4P_r^2 + 4P_r Att \cdot K - (Att \cdot K)^2 \tan^2 \Psi} \\ & = -2Att^3 P_g K + Att^2 (2K^2 - 4P_g P_r + K^2 \tan^2 \Psi) + 2Att P_r \cdot K. \end{aligned} \quad (\text{B11})$$

With the product of square roots now isolated on one side of the equation, both sides are raised again by the power of 2, which produces

$$\begin{aligned} & Att^6 (-4P_g^2 K^2 \tan^2 \Psi - 4P_g^2 K^2) + Att^5 (8P_g K^3 \tan^2 \Psi + 8P_g K^3) + Att^4 (8P_g P_r K^2 - 4K^4 - 4K^4 \tan^2 \Psi + 8P_g P_r K^2 \tan^2 \Psi) \\ & + Att^3 (-8P_r K^3 \tan^2 \Psi - 8P_r K^3) + Att^2 (-4P_r^2 K^2 \tan^2 \Psi - 4P_r^2 K^2) = 0. \end{aligned} \quad (\text{B12})$$

When expanding all expressions and sorting in terms of Att , several terms cancel each other out or can be combined to obtain Eq. (B12). Herein it is observed that the term $Att^2 \cdot K^2 (1 + \tan^2 \Psi)$ can be factored out from each summand. Doing so and dividing the equation by the factor 4 leads to

$$-Att^2 K^2 (1 + \tan^2 \Psi) \{Att^4 P_g^2 - Att^3 2P_g K + Att^2 (K^2 - 2P_g P_r) + Att 2P_r K + P_r^2\} = 0. \quad (\text{B13})$$

One now can divide by $-K^2 \cdot (1 + \tan^2 \Psi)$ to obtain

$$Att^2 \cdot \{Att^4 P_g^2 - Att^3 2P_g K + Att^2 (K^2 - 2P_g P_r) + Att 2P_r K + P_r^2\} = 0. \quad (B14)$$

This equation is a product of factors, which allows splitting Eq. (B14) into two individual equations. The first equation ($Att^2 = 0$) merely delivers the trivial solution $Att = 0$, which can be disregarded. The second equation can be solved by identifying that

$$Att^4 P_g^2 - Att^3 2P_g K + Att^2 (K^2 - 2P_g P_r) + Att 2P_r K + P_r^2 = (Att^2 P_g - Att K - P_r)^2. \quad (B15)$$

Taking the square root thus produces the equation

$$Att^2 P_g - Att K - P_r = 0, \quad (B16)$$

which can be readily solved. It yields the special solution:

$$Att = \frac{I_{\text{avg}} V_c \pm \sqrt{I_{\text{avg}}^2 V_c^2 + 4P_g P_r}}{2P_g}. \quad (B17)$$

Herein we have reinserted the initial abbreviation $K = I_{\text{avg}} V_c$. It shall be noted that only the positive root produces the physically correct value for Att since $Att < 0$ is excluded.

Finally, the restriction of $\phi_b = 0$ is lifted to obtain the general solution. The solution steps are similar to what has been described above, but more involved and therefore not detailed here. The finding is

$$Att = \frac{2I_{\text{avg}} V_c \cos b \pm \sqrt{2I_{\text{avg}}^2 V_c^2 [1 + \cos(2b)] + 8P_g P_r}}{4P_g}. \quad (B18)$$

Again only the positive root shall be used. If $\phi_b = 0$ is inserted in Eq. (B18) it follows that

$$\begin{aligned} Att &= \frac{2I_{\text{avg}} V_c \pm \sqrt{2I_{\text{avg}}^2 V_c^2 + 8P_g P_r}}{4P_g} \\ &= \frac{I_{\text{avg}} V_c \pm \sqrt{I_{\text{avg}}^2 V_c^2 + 4P_g P_r}}{2P_g}. \end{aligned} \quad (B19)$$

This expression is identical to (B17). Hence the general solution contains the special solution for $\phi_b = 0$. This general solution has been verified utilizing a mathematical software code.

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