Using betatron coupling to suppress horizontal intrinsic spin resonances driven by partial snakes

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(Received 29 December 2020; accepted 22 February 2021; published 3 March 2021)

A method for using betatron coupling to suppress partial snake driven resonances in a synchrotron is described. While the partial snakes permit avoidance of the strong resonances associated with the vertical betatron motion of the particles, they also excite numerous weak resonances associated with the horizontal betatron motion. These resonances occur whenever the fractional spin tune equals the fractional horizontal betatron tune. Since these are the same frequencies at which depolarizing resonances from betatron coupling occur, coupling resonances can be intentionally driven to exactly cancel the resonance driving terms of the partial snakes, thereby avoiding polarization loss due to these resonance crossings. Presented here is an explicit derivation of the partial snake driven resonances as betatron sidebands of strong imperfection resonances and a description of a practical scheme for their correction using the Brookhaven AGS as an example. Tracking results that verify the suppression via coupling are presented.

DOI: [10.1103/PhysRevAccelBeams.24.031001](https://doi.org/10.1103/PhysRevAccelBeams.24.031001)

I. INTRODUCTION

Partial Siberian snakes have been used successfully to overcome the depolarizing effects of vertical imperfection and intrinsic resonances encountered during the acceleration of polarized protons to high energy [1,2]. A partial snake provides a localized spin rotation of the spin vector around an axis in the horizontal plane of some angle less than 180° . This localized rotation allows avoidance of depolarization due to both imperfection resonances (via full spin flip at the resonance condition $G\gamma = N$) and strong vertical intrinsic resonances (by manipulating the spin tune to avoid the resonance condition $G\gamma = N \pm \nu_y$) [3]. Here G is the anomalous gyromagnetic ratio, γ the Lorentz factor, N is any integer and ν_y is the vertical betatron tune. Partial snakes provide a method to accelerate polarized beams in medium energy, GeV scale synchrotrons that may not have sufficient straight section length or aperture available for full, 180° spin flip snakes like those used in high energy rings [4].

In the presence of horizontal betatron motion, however, partial snakes can themselves drive numerous weak horizontal intrinsic resonances which collectively can lead to substantial polarization loss. This was observed, for example, at the Brookhaven AGS where partial snakes have been

used to provide polarized proton beam for the RHIC spin program [5]. A horizontal betatron tune jump system has been used at the AGS to improve polarization transmission through these resonances, but this only avoids part of the depolarization (see Sec. III).

This paper describes a method for completely suppressing these horizontal intrinsic resonances using betatron coupling. In the next section an explicit derivation of the resonance driving term from a partial snake is provided and it is shown this term can be exactly canceled by terms introduced via betatron coupling spin resonances. Subsequent sections describe a correction system using the AGS lattice as an example. The resonance correction is then verified via particle tracking.

II. THEORY

In an ideal, planar ring with a single partial snake, the spin vector of a particle on the closed orbit experiences only constant precession around the main bend field and the local spin rotation of the partial snake. If, however, the particle is also undergoing horizontal betatron oscillations, then the precessional motion is modulated at the betatron frequency as the particle moves with an angle with respect to the closed orbit. For a particle executing horizontal betatron motion (but no vertical motion) in a ring with a single snake, the spin equation of motion in the spinor formalism is

$$\frac{d\Psi}{d\theta} = -\frac{i}{2} \begin{pmatrix} F & -\xi \\ -\xi^* & -F \end{pmatrix} \Psi \quad (1)$$

with

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$$\begin{aligned}\xi &= iX_s\delta(\theta - \theta_s) \\ F &= G\gamma - (1 + G\gamma)x''(\theta)\rho\end{aligned}$$

where Ψ is the spin vector in the spinor representation, θ is the azimuthal angle around the ring, $x(\theta)$ is the particle position relative to the reference orbit, X_s is the angle of rotation produced by the snake, δ is the periodic Dirac delta, θ_s the azimuthal position of the snake and ρ is the bend radius of the local reference orbit. In this case the snake rotates the spin vector about the longitudinal direction. The prime denotes differentiation with respect to the path length coordinate, denoted by s .

Equation (1) describes motion similar in form to that of a so-called hybrid spin resonance where sidebands of a strong intrinsic resonance are driven by closed orbit deviations in the horizontal plane and the analysis proceeds similarly to that in Ref. [6]. To obtain the resonance strengths we express Eq. (1) in the spin precessing frame by making the transformation

$$\Psi = \exp\left[-\frac{i}{2}\sigma_3 \int (G\gamma - (1 + G\gamma)x''(\theta)\rho d\theta)\right] \tilde{\Psi} \quad (2)$$

where σ_i is the i th Pauli spin matrix.

We can evaluate the integral by using $d\theta = ds/\rho$ and by assuming that we are integrating over a sufficiently small number of turns that γ may be considered constant so that

$$\Psi = e^{\frac{i}{2}\sigma_3(G\gamma\theta - (1+G\gamma)x'(\theta))} \tilde{\Psi} \quad (3)$$

where, for a particle with maximum angular deviation $\tilde{x}'$ we can approximate the betatron motion by

$$x'(\theta) = \tilde{x}' \sin(\nu_x \theta) \quad (4)$$

and we have set initial spin and betatron phases at $\theta = 0$ to zero. Under this transformation the spinor equation of motion, Eq. (1), becomes

$$\begin{aligned}\frac{d\tilde{\Psi}}{d\theta} &= -\frac{i}{2} \begin{pmatrix} 0 & -\tilde{\xi} \\ -\tilde{\xi}^* & 0 \end{pmatrix} \tilde{\Psi} \\ \tilde{\xi} &= iX_s\delta_n(\theta - \theta_s)e^{-iG\gamma\theta}e^{-i(1+G\gamma)x'(\theta)}\end{aligned} \quad (5)$$

The sinusoidal term $x'(\theta)$ can be moved out of the exponential via a Jacobi-Anger expansion to obtain

$$\tilde{\xi} = iX_s\delta_n(\theta - \theta_s)e^{-iG\gamma\theta} \sum_{\alpha=-\infty}^{\alpha=\infty} J_\alpha[(1 + G\gamma)\tilde{x}']e^{i\alpha\nu_x\theta} \quad (6)$$

with J_α the Bessel functions.

Keeping only linear resonance terms in the sum (i.e., terms first order in $\tilde{x}'$ and $\alpha = 0, \pm 1$) and performing the Fourier expansion we obtain the resonance strength ϵ at resonance tune K

$$\begin{aligned}\epsilon &= \frac{iX_s}{2\pi} \int \delta_n(\theta - \theta_s)e^{-iG\gamma\theta + iK\theta} \\ &\times \left[1 - \tilde{x}' \frac{(1 + G\gamma)}{2} e^{\pm i\nu_x\theta}\right] d\theta.\end{aligned} \quad (7)$$

The first term in the square brackets has a nonzero contribution to the resonance strength ϵ when $K - G\gamma = N$ for integer N , and has amplitude $\frac{X_s}{2\pi}$, which is just the imperfection resonance caused by the presence of the partial snake. The second term gives nonzero contribution for $K - G\gamma \pm \nu_x = N$, which is an intrinsic resonance term due to the horizontal betatron motion. This form for the resonance strength shows that it arises as a sideband of the imperfection resonance, caused by the modulation of the spin precession phase between passages through the snake. The modulation occurs at the horizontal betatron tune.

If we include an additional resonance term in Eq. (1), so that $\xi = iX_s\delta(\theta - \theta_s) + \epsilon_x e^{i\nu_x\theta}$ then, following through the same procedure of Eqs. (2)–(7), the resonance drive becomes (again keeping only the linear resonance terms)

$$\begin{aligned}\epsilon &= \frac{1}{2\pi} \int \delta_n(\theta - \theta_s)e^{-iG\gamma\theta + iK\theta} \\ &\times \left[1 - \left(\tilde{x}' \frac{(1 + G\gamma)}{2} + \epsilon_x\right) e^{\pm i\nu_x\theta}\right] d\theta\end{aligned} \quad (8)$$

which can be compared to Eq. (7). The complex amplitude from the coupling resonance ϵ_x adds directly with the partial snake driving term. The resonances driven by the partial snake can therefore be compensated by exciting an intrinsic resonance with a tune equal to the horizontal betatron tune. Such a resonance can be created using betatron coupling [3].

Insertion of a skew quadrupole in the accelerator lattice will produce a spin coupling resonance of an amplitude and phase determined by the strength of the skew quadrupole and its location in the lattice. Suppressing the horizontal intrinsic resonance therefore consists of producing a set of skew quadrupole fields such that the resulting coupling resonance ϵ_x causes the terms proportional to $e^{i\nu_x\theta}$ in Eq. (8) to vanish.

In the following section we describe a response matrix approach to finding the necessary skew quadrupole strengths using the Brookhaven AGS polarized proton lattice as an example.

III. SIMULATING RESONANCE SUPPRESSION IN THE AGS

A. AGS overview

Partial snakes have been used in the AGS for preservation of proton polarization since 1994 [2]. Currently, the AGS uses a pair of helical dipole partial snakes, separated by 1/3 of the ring circumference, that rotate the proton spin

about the longitudinal direction by 18° and 10.8° , respectively. Polarized protons are accelerated in the AGS from $G\gamma = 4.5$ to 45.5 which requires passing through 82 horizontal intrinsic resonances where $G\gamma = N \pm \nu_x$ (N is an integer). The resonance driving terms from each of the snakes add vectorially to make a net resonance strength, so the separation of the snakes by $1/3$ of the circumference means that the spin rotations of the snakes are in phase whenever $G\gamma = 3N$ and the horizontal intrinsic resonances near those values of $G\gamma$ are therefore particularly strong. For a typical horizontal tune of 8.72 and the standard ramp rate, the horizontal resonance crossings are separated in time by 4–5 ms. Table I contains a summary of the AGS polarized proton beam parameters used in this paper.

Left uncorrected, the horizontal resonances in the AGS result in a 20% relative loss in proton beam polarization [7]. Since 2011, a fast tune jump has been used to increase the resonance crossing rate, but this only prevents about half the polarization loss, leaving a 10% drop [8–10]. The resonance suppression scheme described here yields better polarization transmission through acceleration and could replace the tune jump. The performance of the two systems will be compared in a later section.

B. Resonance correction and betatron coupling

In order to suppress the horizontal resonances, we insert a number of additional skew quadrupoles into the AGS model. We use the SPRINT [11] code to calculate both the horizontal intrinsic resonance strengths and the coupling resonance strengths due to the inserted skew quadrupole fields. SPRINT is a resonance calculation and particle tracking code that can provide spin resonance strength calculations that account for both linear betatron coupling and the non-vertical stable spin direction caused by the partial snakes.

For each resonance, the SPRINT code is run once with all skew quadrupole fields at zero to establish the horizontal resonance strength ϵ and then again for each of the skew quadrupole correctors, each powered in turn, to enable calculation of the resonance response $\frac{d\epsilon}{dk_i}$. Eliminating the partial snake drive horizontal resonance then consists of solving the linear equations

$$\frac{d\epsilon}{dk_i} k_i = -\epsilon \quad (9)$$

TABLE I. AGS polarized proton beam parameters.

Parameter	Unit	Value
Horizontal tune	...	8.72
Vertical tune	...	8.985–8.991
Horizontal emittance (rms,norm.)	μm	2
Vertical emittance (rms,norm.)	μm	2
Longitudinal emittance	eVs	0.8
Ramp rate ($dG\gamma/d\theta$)	...	4.7×10^{-5}

for the required skew quad strengths k_i . In general ϵ is a complex quantity, so $\frac{d\epsilon}{dk_i}$ is representable as a $2 \times N$ matrix, where N is the number of skew quads and the two rows represent the equations for the real and imaginary parts of the resonance.

In addition to the resonance correction, the skew quadrupole strengths are also constrained by betatron coupling considerations. Because the vertical tune for polarized proton acceleration in the AGS is close to an integer ($\nu_y > 8.985$), the tolerance for vertical tune shifts is relatively small. Further, the large tune split ($\nu_y - \nu_x \approx 0.26$) means that the tune shifts due to coupling are not dominated by single difference resonance mode. We therefore use results given by Parzen for the tune shifts produced far from the difference resonance condition [12]. The formulas are reproduced here for convenience:

$$\begin{aligned} c_n &= \frac{1}{4\pi} \oint ds \frac{B_x}{B\rho} \sqrt{\beta_x \beta_y} e^{i[(n-\nu_x)\theta_y + \nu_x \theta_x]} \\ \Delta_x &= 4\nu_x \nu_y \sum_{n=-\infty}^{\infty} \frac{|c_n|^2}{(n-\nu_x)^2 - \nu_y^2} \\ \nu_1^2 &= \nu_x^2 + \Delta_x \end{aligned} \quad (10)$$

Here ν_1 is the perturbed horizontal tune, B_x is the skew quadrupole gradient, $B\rho$ the magnetic rigidity and $\beta_{x,y}, \theta_{x,y}$ the betatron functions and normalized phase advances, respectively. The expression for the perturbed vertical tune ν_2 is obtained by swapping the subscripts x and y . For this simulation, the tune shifts were confined to be less than 0.005. This constraint insures that the vertical tune will not approach the integer too closely and the residual tune shift can be removed by the tune control quadrupoles.

The optimization problem to be solved is to find the minimum necessary quadrupole strengths that solve Eq. (9), subject to constraints on the coupling-induced tune shifts of Eq. (10), separately for each of the 82 resonances crossed during AGS acceleration.

C. Skew quadrupole placement

The skew quadrupoles need to be added to the lattice in locations such that the strengths required to effect the correction are minimized. In principle, since a partial snake resonance is a single complex number, only two skew quadrupoles suffice to correct any given resonance. However, in order to correct all 82 resonances crossed during acceleration (each of which has a distinct phase), and satisfy the tune shift constraint, it is necessary to include several skew quadrupole correctors. Numerical experiments indicate that for the AGS 12-15 skew quadrupole correctors usually suffice to avoid degeneracy in the resonance correction equation (9) and provide for sufficiently small residual coupling. For this demonstration, 15 skew quadrupoles were inserted into the model AGS lattice

and it is assumed that they are all powered independently. The quadrupoles are assumed short enough to fit in the short drifts between any two AGS main magnets in order to maximize the number of possible locations (physical length is 0.1 m). Upright quadrupoles of this length are already used in similar locations elsewhere in the AGS lattice [13].

This choice of quadrupole design allows 72 candidate locations in the AGS for the skew quadrupoles. Choosing an optimal 15 locations cannot practically be done by solving Eq. (9) for all 10^{15} possibilities. At the strongest resonances near energies where $G\gamma \approx 3N$, the spin phase advance is such that groups of three skew quads, equidistantly placed around the circumference, produce coupling resonances of the same phase and therefore produce degenerate resonance vectors. For the purposes of selecting quadrupoles for inclusion in the correction, they are therefore grouped into 24 sets of 3 equally spaced magnets, of which only 5 sets need be selected (the strength of each magnet is still determined independently for the purposes of calculating corrections). This reduces the number of possible configurations to $\sim 10^5$, which can be evaluated in a few hours and an optimal case selected. In order to maintain tractable calculation times, the coupling constraint is ignored during this stage.

D. Resonance correction

Once a candidate set of quadrupoles has been selected, the full correction is calculated, subject to the coupling constraint. Figure 1 shows the resonance response vectors for the 15 selected skew quadrupole correctors for the last six horizontal resonance crossings in the AGS acceleration cycle. The near degeneracy for the two resonances closest to $G\gamma = 3 \times 15 = 45$ is apparent. Also highlighted in the figure is the large change in the phase of an individual corrector from one resonance to the next. This has implications for a practical implementation since the correctors will tend to require very different strengths for adjacent resonances. They will therefore have to be able to ramp quickly in the 4–5 ms between resonance crossings.

The integrated skew quadrupole gradients needed to effect the correction for all 82 AGS resonances are shown in Fig. 2. Those strengths are added to the model and the tunes shifts calculated. The tunes are refit to their original values using the tune control quadrupoles and the new, corrected resonance strengths calculated (Fig. 3). The refitting of the tunes prior to recalculating the resonance strengths insures that the frequency of the resonance is not shifted away as a result of betatron coupling, which would reduce the efficacy of the correction.

Using these skew quadrupoles, the resonance strength can be corrected to zero for nearly all resonances. The main exception is $G\gamma = 39.28$, where it happens that full correction is not possible in this configuration without violating the coupling constraint, so there is only partial compensation.

FIG. 1. Resonance response vectors $\Delta\epsilon_i = d\epsilon/dk_i$ for the last six horizontal resonances crossed by polarized protons during acceleration in the AGS ($G\gamma$ increases left to right, top to bottom). The overall phase of each plot is determined by making the horizontal resonance vector (not shown) entirely real. The response of one skew quadrupole (the same one in each plot) is highlighted with a red circle to demonstrate that the phasing of the correction vector for any given quadrupole can change significantly for adjacent resonances.

FIG. 2. Integrated skew quadrupole gradients as a function of $G\gamma$ calculated for the correction of partial snake drive horizontal resonances in the AGS. Each trace represents one of the 15 correctors.

FIG. 3. (a) Horizontal rms resonance strengths before and after implementing the correction and (b) the resulting tune shifts due to betatron coupling (prior to refitting the tunes).

The expected polarization loss due to the resonances in the corrected and uncorrected cases can be calculated using the Froissart-Stora formula [14]. For these weak, well-separated resonances we can estimate the cumulative polarization loss after N crossings as

$$\frac{P_f}{P_i} = \prod_{n=1}^N \frac{1 - \frac{\pi|\epsilon_n|}{\alpha}}{1 + \frac{\pi|\epsilon_n|}{\alpha}} \quad (11)$$

where P_i and P_f are the initial and final polarizations, ϵ_n is the rms strength of the n th resonance, $\alpha = dG\gamma/d\theta$ is the ramp rate and we have assumed Gaussian transverse intensity profiles in both planes. The estimated polarization transmission is shown as a function of energy in Fig. 4.

FIG. 4. Cumulative polarization loss due to horizontal intrinsic resonances before and after correction. Estimated using the Froissart-Stora and the calculated resonance strengths.

The resonance suppression increases the polarization transmission efficiency through horizontal resonances to $>99.9\%$. The residual polarization loss is dominated by the 0.07% loss from the partial correction of the $G\gamma = 39.28$ resonance.

E. Tracking

In order to verify the resonance suppression effect, tracking simulations were performed using a Zgoubi model of the AGS and realistic beam conditions over the energy range $G\gamma = 42.5$ – 45.5 . This energy range covers the last six resonance crossings in the AGS acceleration range including two final, strongest resonances. Beam emittances and ramp rates for the tracking are the same as those listed in Table I. Here we use 1000 particles in the tracking. Tests with larger numbers of particles showed no appreciable difference in results.

Three tracking runs were performed. In the first case, no resonance amelioration of any kind is applied to calculate polarization loss in a baseline case. In the second run, the skew quadrupole resonance correction is applied. The correction is implemented by ramping the skew quadrupole strengths over 1 ms, holding the correction current for 1 ms, centered on the resonance crossing and then ramping down the correction to zero in 1 ms. Since we would like to measure the performance of the resonance correction against the highest achieved performance, we also track a third case with resonance crossing rates increased via tune jump (and no skew quad resonance suppression). As in the real AGS, the tune jump is implemented in the model as a fast ramp of two quadrupoles. The modeled jump is an idealized linear ramp of the horizontal tune from 8.70 to 8.74 over $100 \mu s$ (≈ 30 turns). The jump is exactly centered so that the resonance crossings happen at $G\gamma = N \pm 0.72$, as in the constant-tune baseline and corrected cases.

The resulting polarization transmission is shown in Fig. 5. Even in the uncorrected case the depolarization from the six resonance crossings is small owing to the weakness of each individual resonance. These small resonance strength pose a central difficulty in addressing these resonances empirically since the impact of individual resonances is difficult or impossible to measure, while the cumulative effect over many crossings is operationally significant (Fig. 4). Differences in the tracking results for the three difference cases are nevertheless discernible, despite the small depolarization. At the times of the resonance crossings, we observe that the tune jump case leaves small, but measurable residual polarization loss. Relative to the tune jump case, the coupling-based correction leaves still smaller residual loss. All three cases show loss at the strong vertical intrinsic resonance crossing which occurs near $G\gamma = 45$, which neither the tune jump nor the skew quadrupoles are meant to address.

FIG. 5. Polarization loss from Zgoubi tracking simulation in the uncorrected case (blue), the case corrected only via coupling resonance (red) and with only increase resonance crossing from the tune jump (green). Vertical dashed lines mark the locations of partial snake resonances. The excursions at integer values of $G\gamma$ are due to spin flips.

In addition to any improved polarization transmission, the resonance suppression has an additional operational benefit over the tune jump scheme. A tune jump requires precise determination of the resonance time in order to provide maximum benefit. This requires precise knowledge of the energy during acceleration. For the AGS tune jump system, the timing tolerance is $\pm 200 \mu\text{s}$, which corresponds to a relative energy error $d\gamma/\gamma = 5 \times 10^{-4}$. This is possible but challenging to maintain operationally for the RHIC program [15,16]. The resonance suppression skew quadrupole strengths can be maintained for at least $\pm 500 \mu\text{s}$ around a given resonance, increasing the tolerance for energy measurement error and drift over time.

IV. CONCLUSION

A resonance suppression scheme has been developed that can eliminate polarization loss from partial snake induced horizontal resonances using betatron coupling. Resonance strength calculations using SPRINT and particle tracking simulations in Zgoubi verify that introduction of coupling into a lattice can cancel the horizontal resonances driven by partial snakes. Since partial snakes are an important tool for accelerating polarized beams in medium energy synchrotrons, this resonance correction technique could be a complimentary tool for accelerating highly polarized beams to high energy.

ACKNOWLEDGMENTS

The author would like to thank Georg Hoffstaetter for helpful discussion and help with modifications to the SPRINT code. Work supported by Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy. This research used

resources of the National Energy Research Scientific Computing Center (NERSC), which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231.

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