Comment on “Collision monochromatization in e^+e^- colliders”

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Bogomyagkov and Levichev [Phys. Rev. Accel. Beams **20**, 051001 (2017)] have recently reported on monochromatization in collision schemes with crossing angle. From their results, in particular, it may seem that: (1) horizontal dispersion at the IP can provide monochromatization factor $\Lambda \gg 1$ while retaining Piwinski angle $\phi > 1$, (2) production rate in such a scheme for FCC-ee at 62.5 GeV can be larger than that in the nominal crab waist collision, and (3) strong rf focusing can be used for monochromatization purposes. We demonstrate here that the first two statements are not correct, and the last one is very doubtful.

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I. PIWINSKI ANGLE

Crossing angle, though it can be very small, always exists in two-ring colliders since the trajectories of e^+ and e^- beams are different. The measure of whether the collision is head-on or not is the Piwinski angle

$$\phi = \frac{\sigma_z}{\sigma_x} \tan(\theta), \quad (1)$$

where θ is the half crossing angle, σ_z is the bunch length and σ_x is the horizontal beam size. “Head-on” means neither $\theta = 0$ (that is impossible) nor $\theta \ll 1$ (that is always true, but not enough), it means $\phi \ll 1$. Of course, ideally head-on should mean $\theta = 0$, but practically the criterion is the smallness of ϕ .

From the physical meaning of this term it follows that $\sigma_{x,z}$ in (1) refer to the opposite bunch, and therefore σ_x is the *full* beam size regardless of what share in this have betatron and synchrotron components.

Bogomyagkov and Levichev in [1] determine “Piwinski angle” through the betatron beam size $\sigma_{x\beta}$, while $\sigma_x \gg \sigma_{x\beta}$ in the monochromatization scheme with large horizontal dispersion at the IP. Of course, authors may enter their own designations, but do not give them names of well-established parameters, otherwise confusion arises and readers are misled.

II. HOURGLASS

The presence of both dispersions at the IP will result in a tilt of bunches in the X-Y plane and should be avoided. If

we exclude this case from consideration, then subsection B in Sec. III of [1] contains only a derivation of already known results.

Indeed, in the assumption that $\beta_x^* \gg \sigma_z$ (also adopted in [1]), σ_x does not depend on azimuth within the interaction area. Therefore we may use standard equations, just remembering that σ_x stands for the full beam size:

$$\sigma_x^2 = \sigma_{x\beta}^2 + (\eta_x \sigma_\delta)^2. \quad (2)$$

The luminosity reduction factor with account of hourglass and crossing angle can be found in Chapter 4.1 of [2]. From there it follows that

$$\mathcal{L} = f_0 \frac{N_1 N_2}{4\pi \sigma_x \sigma_y} \cdot \frac{\beta_y^*}{\sqrt{\pi} \sigma_z} \exp(a^2) K_0(a^2), \quad (3)$$

$$a = \frac{\beta_y^*}{\sqrt{2} L_i}. \quad (4)$$

Here L_i is the length of interaction area (see e.g. in [3]):

$$L_i = \frac{\sigma_z}{\sqrt{1 + \phi^2}}. \quad (5)$$

For convenience (3) can be written in the form

$$\mathcal{L} = f_0 \frac{N_1 N_2}{4\pi \sigma_x \sigma_y \sqrt{1 + \phi^2}} R_{hg}, \quad (6)$$

where

$$R_{hg} = \sqrt{\frac{2}{\pi}} a \exp(a^2) K_0(a^2) \quad (7)$$

is the hourglass factor. Note that R_{hg} is the same as for the head-on collision with σ_z replaced by L_i in (4) and dependence on θ is hidden in L_i .

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Now let us consider the case of $\eta_y \neq 0$. For $\mathcal{L}$ and R_{hg} we need only two things: the full vertical beam size σ_y at the IP and its dependence on the azimuth. The first can be obtained from (2) with the index x replaced by y . The second is given by

$$\sigma_y^2(s) = \sigma_y^2 \left(1 + \frac{s^2}{\beta_y^2} \right), \quad (8)$$

$$\beta_y^\circ = \beta_y^* \frac{\sigma_y}{\sigma_{y\beta}}. \quad (9)$$

Thus we only need to replace β_y^* by β_y° in (4). It is easy to verify that (35) and (36) of [1] conform to the above equations. It should also be noted that the horizontal axis in Fig. 5 of [1] must be marked $\sqrt{2}t$, that is β_y°/L_i .

This example demonstrates how the use of the full beam sizes, where appropriate, makes everything easier and clear.

III. MONOCHROMATIZATION WITH DISPERSION IN THE PRESENCE OF CROSSING ANGLE

In this scheme, when particles cross the opposite bunch, they collide with the counterparticles of different energies despite the correlation between transverse position and energy deviation. In order to mitigate this, the width of a strip which the particle traces within the oncoming bunch should be much smaller than σ_x . Thus, from geometric consideration we obtain the restriction:

$$\Lambda^2 < 1 + \frac{1}{\phi^2}. \quad (10)$$

This estimate is in good agreement with (17) of [1], which in the absence of vertical dispersion can be rewritten as

$$\Lambda^2 = 1 + \frac{\lambda_m^2}{1 + \phi^2(1 + \lambda_m^2)}, \quad (11)$$

where λ_m is the ratio between synchrotron and betatron horizontal beam sizes, also known as monochromatization parameter [4]. For the inequality (10) to become an equality, two conditions are required: $\lambda_m \gg 1$ and $\lambda_m \gg 1/\phi$.

Hence $\Lambda \gg 1$ is possible only in the head-on collision, in the sense that we indicated above. One of the consequences is that L_i becomes equal to σ_z and we are back to the old problem: how to avoid strong hourglass with small β_y^* and having high populated bunches which should be not very short. Table I of [1] (hereafter Table) is a good example to illustrate this.

One remark concerning the energy spread reduction in the last two columns. In [1], right before the Table, it is written: “This happens owing to beamstrahlung: dispersion at the IP increases horizontal emittance, larger emittance

means smaller beam density and weaker fields of the opposite bunch; therefore, smaller energy spread.” In fact, this happens solely due to dispersion. The beam density, on which beamstrahlung depends, is inversely proportional to the full beam size σ_x which is formed mainly by the synchrotron part. For reference: λ_m for the last two columns is 17.5 and 10, so the impact of $\sigma_{x\beta}$ on σ_x is negligible—less than half percent.

Considering the Table, one should pay attention to the following. (i) “ σ_x ” in the last two columns actually is $\sigma_{x\beta}$. The full horizontal beam size is much larger. (ii) The correct values for the Piwinski angle in the last two columns are 0.08 and 0.10 instead of 1.5 and 0.95. (iii) Luminosity in the first column should be approximately 10 times larger. The discrepancy can be easily detected from the Table itself, taking into account that $\mathcal{L} \propto \xi_y R_{hg} / \beta_y^*$. (iv) The production rate in the first column (nominal crab waist collision) is about 1.4 times larger than in the next two (monochromatization). Here and below we assume that in the vicinity of narrow resonance the production rate $\dot{N} \propto \mathcal{L} \cdot \Lambda / \sigma_\delta$, i.e. the contribution of background rate to $\dot{N}$ is very small. (v) The bunch population $N_p = 3 \times 10^{11}$ is unrealistically large for the last two columns, where the bunches are short.

It must be said that decreasing N_p in the last two columns (with a proportional increase in the number of bunches) will have a positive impact on performance. Indeed, beamstrahlung will weaken and reduce ε_x and $\sigma_{x\beta}$. As a result λ_m will rise, but the increase in Λ will not be significant since Λ is defined mainly by ϕ when $\lambda_m > 1/\phi$. What is more important, σ_y decreases owing to $\varepsilon_y \propto \varepsilon_x$. To estimate this effect, we performed quasi-strong-strong beam-beam simulations [5] with $N_p = 3 \times 10^{11}$ and 1×10^{11} . In the latter case the luminosity was the same for the second column and even higher for the third.

Furthermore, N_p in the first column also is not optimal and needs to be reduced several times. Of course, the luminosity will drop, but together with the energy spread. In our range of parameters, where σ_δ is defined mainly by the beamstrahlung, it scales as $\sigma_\delta \propto \sqrt{N_p}$. Given that $\sigma_z \propto \sigma_\delta$ and $\mathcal{L} \propto \xi_y \propto N_p / \sigma_z$ we come to a conclusion that $\mathcal{L} / \sigma_\delta$ does not depend on N_p . Therefore, its reduction within certain limits does not affect the production rate and reduces the background rate. This is exactly what the authors of [1] wanted to achieve with the help of monochromatization. Simulations also confirmed that reducing N_p from 3×10^{11} to 1×10^{11} results in a decrease of $\mathcal{L}$ and σ_δ by the same factor $\sqrt{3}$.

IV. MONOCHROMATIZATION WITH STRONG RF FOCUSING

Bogomyagkov and Levichev considered also monochromatization by means of strong rf focusing. They referred in

particular to [6], but apparently did not pay enough attention to Chapter 3 where a growth of the energy spread is discussed.

Equation (44) of [1] gives us the upper limit for the monochromatization factor:

$$\Lambda^2 < 1 + \alpha_z^2, \quad (12)$$

where α_z is the longitudinal Twiss parameter. For a simplified isomagnetic lattice (see also (7) in [6]) the following inequality holds:

$$|\alpha_z| \leq \frac{1 - \cos \mu}{\sin \mu}, \quad (13)$$

where μ is the one turn synchrotron phase advance. Consequently, in order to achieve large Λ , μ should be close to π . But this also increases the energy spread, which can be estimated as

$$\sigma_\delta^2 \approx \frac{2}{3} \sigma_{\delta 0}^2 \cdot \frac{2 + \cos \mu}{1 + \cos \mu}, \quad (14)$$

where $\sigma_{\delta 0}$ is the “conventional” value—see (12) and (14) in [6]. With the help of (12)–(14) we obtain

$$\frac{\Lambda^2}{\sigma_\delta^2} < \frac{1}{\sigma_{\delta 0}^2} \cdot \frac{3}{2 + \cos \mu}. \quad (15)$$

It follows that the maximum possible improvement in the invariant mass resolution is less than $\sqrt{3}$.

Of course, we realize that this limitation refers to the case of simple isomagnetic lattice and one rf cavity. The authors of [1] considered a more complicated lattice with many cavities, but even there the energy resolution was improved about $\sqrt{3}$ times in the best case (60/35, see Table II in [1]). In any case, the increase in Λ is closely linked to the increase in σ_δ , but the authors did not consider it necessary to draw the readers’ attention to this, and did not explain how it can be overcome.

Even more doubtful is the assertion that “production rate of the resonance is increased by Λ ” (top right of page 6 in [1]). The question is: increased compared to what? Apparently, the authors considered the bunches of the same shape and size, with and without monochromatization. However, such a comparison is not entirely correct.

First of all, obtaining the same beam dimensions without monochromatization is a nontrivial task. If we lower rf voltage or move cavities to other places to reduce α_z at the IP, σ_z also changes and this affects $\xi_{x,y}$ and $\mathcal{L}$. We can only guess what is the ratio between production rates in Table II,

but certainly it is not Λ . Second, any monochromatization requires some lattice changes. This means that comparing cases with and without monochromatization is in fact a comparison between different lattices and different beam sizes.

Now we recall what is the alternative to monochromatization in our case with the large Piwinski angle. Evidently, it should be a crab waist scheme where very high luminosity can be obtained. One of the important conditions for this is small transverse emittances, which usually implies small momentum compaction factor α . It is with this that we should compare the production rate which is achieved with monochromatization.

On the other hand, strong rf focusing requires very large α (e.g., 0.5 in Table II). This means large dispersion in the arcs and, accordingly, larger emittances. Of course, they can be then reduced with the help of damping wigglers located in dispersion-free straight sections, but at the expense of significant increase in σ_δ , which is unacceptable in our case. It follows that transition to a lattice with so large α will make the luminosity much lower than in the ordinary crab waist collision (the exact numbers can only be discussed on specific examples). After that, an additional factor $\sqrt{3}$ will have little impact.

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