Resonant excitation of high order modes in the 3.9 GHz cavity of the Linac Coherent Light Source

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Construction of the Linac Coherent Light Source II (LCLS-II) is underway for the world's first hard x-ray free-electron laser. A central part of the LCLS-II project is a 4 GeV superconducting radio frequency electron linac that will operate in the continuous wave (cw) mode. The linac is segmented into four sections named as $L0$, $L1$, $L2$, and $L3$. Two 3.9 GHz cryomodules, each housing of eight third-harmonic cavities similar to the cavities developed for the European X-ray Free Electron Laser (XFEL), will be used in section $L1$ of the linac for linearizing the longitudinal beam profile. In this paper, we present a study of trapped high order modes (HOMs) excited by a cw electron beam in the third-harmonic cavities of the LCLS-II linac. A detailed comparison of the original XFEL design and the LCLS-II design with a modified end group is performed in order to estimate the effect of a reduced beam pipe aperture on the efficiency of HOM damping. Furthermore, we apply a statistical analysis of the eigenmode spectrum for the estimation of the probability of resonant HOM losses and influence of HOMs on beam dynamics.

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I. INTRODUCTION

A continuous wave (cw) operation regime of the 3.9 GHz Linac Coherent Light Source II (LCLS-II) accelerating structure at a maximum average beam current of 0.3 mA and ~ 1 MHz bunch repetition rate might result in significant coherent rf losses in the form of trapped high order modes (HOMs) [1,2]. Consequently, extra cryogenic losses, overheating of beam line components, and an unstable beam dynamic are possible deteriorating effects of a resonant HOM excitation. The initial rf design of the 3.9 GHz cavity was developed at Fermilab for the Deutsches Elektronen-Synchrotron FLASH facility, and later with minor changes it was adapted to the X-ray Free Electron Laser (XFEL) project [3–5]. Comparing to the XFEL design, the LCLS-II third-harmonic cavity has several modifications including a reduced beam pipe aperture from $\varnothing 40$ mm to $\varnothing 38$ mm and a modified HOM coupler [6]. The layout of cavities in the LCLS-II cryomodule has changed as well, resulting in longer cavity distances at 126 mm length and $\varnothing 38$ mm aperture interconnecting bellows. Nominal parameters of the bunch train are presented in Table I for third-harmonic sections of both machines. Despite different beam temporal structures, both

XFEL and LCLS-II cavities have comparable incoherent losses of 5 and 13.6 W, respectively. Thus, we decided to verify the LCLS-II cavity performance to efficiently damp HOMs and make a comparison with the original XFEL design.

Except for the operating mode, which is tuned separately, the parameters of HOMs vary from one cavity to another due to finite mechanical tolerances of cavity fabrication. In this paper, we use the method of generating a random cavity geometry with imperfections while preserving the operating mode frequency and field flatness [7]. Based on the eigenmode spectrum calculation of series of randomly generated cavities, we can accumulate the data for the evaluation HOM statistics and, thus, probabilities of resonant HOM losses. A similar procedure is described in Refs. [8,9] for the 1.3 GHz LCLS-II cavity, where only frequency variations are used for the evaluation of power loss. While such an approach allows simplifying calculations, it might result in a significant overestimation of resonant HOM losses, since all modes should be taken for analysis with a substantial overhead for shunt impedances and quality factors to reproduce the worst possible scenario. On the other hand, an analysis using the full statistical data of HOM parameters provides the most accurate results but is a time-consuming operation. For that reason, reducing the simulation time, we start the cavity eigenmode analysis with its nominal geometry to find out frequency passbands containing HOMs with the largest shunt impedances or quality factors. Next, we proceed with statistical calculations of random cavities but only for the HOM spectrum within the given passbands. Finally, we evaluate the mean

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TABLE I. Nominal beam parameters in third-harmonic sections of XFEL and LCLS-II linacs.

Beam parameters	XFEL	LCLS-II
Operating regime	Pulsed	cw
Bunch charge [nC]	1	0.3
Bunch length, rms [mm]	2	1
Bunch frequency [MHz]	5	1
Pulse length [ms]	0.65	...
Repetition rate [Hz]	10	...
Beam energy [GeV]	0.5	0.25
Beam current [mA]	5	0.3

and peak values of rf power radiated to the HOM couplers and estimate a cumulative probability of the transverse kick for a resonant HOM excitation in the LCLS-II third-harmonic cavity.

II. CALCULATION OF HOM SPECTRUM IN THE 3.9 GHz CRYOMODULE

Detailed calculations of the HOM spectrum in early versions of the third-harmonic cavity are presented in Refs. [10,11] for the single resonator model with perfect electric or magnetic boundary conditions at both ends. In spite of its simplicity, which greatly saves eigenmode computation time, the single resonator model lacks accuracy, since the HOM parameters depend on a combination of boundary conditions. Recently, new results have been published for the chain of XFEL 3.9 GHz cavities with matched boundary conditions on the coupler ports [12,13]. The authors use chains of four and eight cavities connected together, which gives a precise evaluation of HOM properties for propagating modes. In fact, the 3.9 GHz cryomodule consists of eight superconducting cavities connected by copper-plated normal conducting bellows. Consequently, as a conservative approximation we can represent the cryomodule beam line as the infinite biperiodic structure, where trapped HOM quality factors are always equal to or higher than in the single cryomodule. The logic behind this statement is that HOM signals from the end cavities are freely radiated upstream or downstream of the cryomodule and fully absorbed either by the beam line absorber or by warm parts of the beam line, while HOM radiation from the inner cavities always has reflections of adjacent cavities. Figures 1(a) and 1(b) show the scheme of the third-harmonic LCLS-II section and the layout of inner cavities in the 3.9 GHz cryomodule, respectively. Note that the power couplers of neighbor cavities are rotated by 180° to compensate for the coupler kick.

Because of mechanical errors, each cavity has a different geometry and a unique HOM spectrum, and, hence, trapped modes in the chain of identical cavities will split into modes of individual resonators in the real cryomodule. Following this approach, we can categorize the trapped HOMs in the

FIG. 1. (a) Scheme of the LCLS-II third-harmonic section. BLA is the HOM beam line absorber, and CM1 and CM2 are the first and second cryomodules, respectively. (b) Layout of 3.9 GHz cavities in the cryomodule.

cryomodule in three types depending on their quality factors. The first kind of trapped HOM is weakly coupled with the beam pipe, and most of the HOM stored energy is localized within the superconducting cavity volume, resulting in a high-quality factor. The second kind of trapped HOM has a good coupling with the beam pipe, but the signal is reflected by the neighboring cavities and a part of the stored energy fills in the volume of adjacent beam pipes, causing an additional signal dampening through far upstream and downstream coupler ports and by normal conducting bellows. Note that frequencies of the first kind of HOM can be either below or above the cutoff frequencies of corresponding propagating modes in the beam pipe. The latter is realized when detuned end cells reject the signal from the inner cells or when multiple modes in the cavity end cell cancel each other in the beam pipe [14]. Frequencies of the second kind of trapped HOM are always above the beam pipe cutoff frequency. The third type of HOM represents the case when the HOM signal can propagate through the neighboring cavities. Then the stored energy is distributed along the cryomodule beam line and damped by many coupler ports and by Ohmic losses in interconnecting cavity bellows. Such modes have low quality factors close to or below 10^4 , which corresponds to the decay constant less than $1 \mu\text{s}$, and we do not count them as resonant modes considering the sparse 1 MHz bunch frequency in the LCLS-II linac. Figure 2 illustrates typical electric field distributions for the described above three types of trapped HOM in the chain of third-harmonic cavities with random HOM spectrums. Evidently, the single-cavity model is adequate for the HOM type I analysis, while HOM types II and III require at least the chain of three cavities for accurate simulations. Further increasing the number of cavities might improve results for the low- Q HOMs but, at the same time, results in the unnecessary complication of the eigenmode analysis and a

FIG. 2. Trapped modes in the infinite chain of third-harmonic cavities with a random HOM spectrum: (a) type I high- Q HOM is localized within the cavity volume, (b) type II medium- Q HOM occupies the cavity and adjacent beam pipes, and (c) type III low- Q HOM distributed along the cavity chain. All plots show the complex magnitude of the electric field, where a blue color corresponds to zero amplitude.

longer simulation time. Finally, we conclude that the chain of three cavities is an optimum solution for the calculation of a resonant HOM spectrum in the third-harmonic LCLS-II cryomodule and longer chains give little or no impact to the overall result.

For the HOM spectrum analysis, we created the 3D model of three third-harmonic cavities including the main and HOM couplers and field pick-up probes. Figure 3 shows the scheme of corresponding ANSYS High Frequency Structure Simulator (HFSS) project with matched boundary conditions. All coaxial TEM ports are terminated with the constant free space impedance boundary, while the free radiation to the round beam pipe is ensured by a frequency-dependent wave impedance calculated for propagating modes at the given HOM frequency.

At the beginning, we performed the eigenmode analysis of the HOM spectrum up to 10 GHz in the chain of three ideal 3.9 GHz LCLS-II cavities. The upper frequency limit is defined by the increasing number of propagating modes in the beam pipe at higher frequencies where the HOM spectrum becomes almost continuous. Figure 4 illustrates distributions of electric fields of HOMs in the second monopole band. One can see that HOM power is freely radiating through the coupler ports and to the beam pipe. Because cavity HOM and power couplers break the axial

FIG. 3. ANSYS HFSS model for the eigenmode analysis of the chain of three 3.9 GHz cavities.

symmetry, we do not categorize modes as pure monopole or dipole and calculate longitudinal and transverse shunt impedances for all modes in the spectrum. Computed HOM shunt impedances and external quality factors are presented in Fig. 5 in the left and in the right plots, accordingly. We excluded spurious modes with $Q_{\text{ext}} < 100$, which appeared in simulations due to a slight mismatch of impedance boundary conditions.

Here we use the accelerator definition of a cavity normalized longitudinal shunt impedance:

$$\left(\frac{R_{\parallel}}{Q}\right) = \frac{|\int_{-\infty}^{\infty} [E_z(r, z)]_{r=0} e^{ikz} dz|^2}{\omega_0 W}, \quad (1)$$

where $E_z(r, z)$ is the longitudinal component of the electric field along the beam trajectory, r is the trajectory radial offset, W is the electromagnetic stored energy, and ω_0 is the resonant angular frequency. Similarly, the normalized transverse shunt impedance is defined as

$$\left(\frac{R_{\perp}}{Q}\right) = \frac{|V_{\perp}|^2}{\omega_0 W}, \quad (2)$$

where $V_{\perp}$ is the transverse kick acquired by the charged particle passing through the cavity. Using the Panofsky-Wenzel theorem and the paraxial approximation for the beam trajectory, we can find the transverse kick as the transverse gradient of the longitudinal kick and, thus, calculate the transverse shunt impedance:

$$\left(\frac{R_{\perp}}{Q}\right) = \frac{|\int_{-\infty}^{\infty} \{\nabla_{\perp} [E_z(r, z)]\}_{r=r_0} e^{ikz} dz|^2}{\omega_0 W} \times \frac{1}{k^2}, \quad (3)$$

where $k = \omega_0/c$ is a wave number and c is the speed of light. Notice that we calculate $(R_{\perp}/Q)$ at the offset of $r_0 = 1$ mm and the unity is Ohm. We perform a numerical integration along the whole cavity chain for a correct calculation of shunt impedances for trapped HOMs in

FIG. 4. Complex magnitude of the electric field in the second monopole band (7.048–7.612 GHz) of the 3.9 GHz LCLS-II cavity (fields are normalized to 1 J stored energy).

the middle cavity, where fields strongly decay to the far left and right beam pipes in the simulation domain. On the other hand, substituting the infinite integration in Eqs. (1)–(3) for untrapped HOMs or for HOMs in the end cavities might lead to significant errors, and, thus, we exclude those modes from our analysis with a filter scheme described in Sec. III of this paper.

For the cw regime, when multiple bunches pass through the cavity, the resonant high order mode excitation may occur if any of the HOM frequencies are in synchronism with the beam harmonics. One can show that the resonant excitation results in a sum of geometrical series of individual bunch signals:

$$V_{\parallel} = \sum_{n=1}^{\infty} V_0 e^{-(nt_b/\tau)} e^{i(n\omega_0 t_b + \varphi_0)}, \quad (4)$$

where $V_{\parallel}$ is the HOM cavity voltage in the steady state regime, $V_0 = 2k_{\parallel}q_0$ is the amplitude of the HOM signal excited by a single bunch, $k_{\parallel} = 1/4\omega(R_{\parallel}/Q)$ is the HOM loss factor, q_0 is the individual bunch charge, t_b is the bunch spacing, $\tau = 2Q_L/\omega_0$ is the HOM signal decay time, φ_0 is the bunch to rf phase, Q_L is the HOM loaded quality factor, and n is the bunch number.

For steady state operation, the resonant gain of the HOM amplitude is expressed as

FIG. 5. HOM shunt impedances (left) and external quality factors (right) in the ideal 3.9 GHz LCLS-II cavity.

FIG. 6. Resonant HOM excitation, average power (left) and transverse voltage (right), of the ideal 3.9 GHz LCLS-II cavity.

$$|V_{\parallel}|_{\max} = V_0 \frac{e^{\alpha}}{e^{\alpha} - 1}, \quad (5)$$

where $\alpha = t_b/\tau$. The average power loss due to HOM excitation can be evaluated as the energy left by the single bunch in the steady state regime over the time between consecutive bunches:

$$P = \frac{1}{t_b} (q_0 |V_{\parallel}| e^{-\alpha} + k_{\parallel} q_0^2), \quad (6)$$

where $|V_{\parallel}| e^{-\alpha}$ is the HOM voltage seen by the bunch passing through the cavity and $k_{\parallel} q_0^2$ is the energy left by the bunch itself. Finally, we can rewrite Eq. (6) for a resonant case using the normalized longitudinal shunt impedance defined in (1):

$$\langle P \rangle_{\max} = \frac{(R_{\parallel}/Q) \omega_0 q_0^2}{4 t_b} \left(\frac{e^{\alpha} + 1}{e^{\alpha} - 1} \right). \quad (7)$$

Similarly, we can derive the steady state resonant amplitude of the HOM transverse voltage if the bunch train passes the cavity off axis:

$$|V_{\perp}|_{\max}^{r=r_0} = \frac{(R_{\perp}/Q) \omega_0 q_0 r_0 k}{4} \left(\frac{e^{\alpha} + 1}{e^{\alpha} - 1} \right). \quad (8)$$

Notice that both Eqs. (7) and (8) represent a general case of HOM excitation and lead to either a nonresonant single-bunch interaction regime, $\langle P \rangle = k_{\parallel} q_0^2 f_b$ if $t_b \gg \tau$, or to a resonance multibunch cavity excitation, $\langle P \rangle = (R_{\parallel}/Q) Q_L I_0$ if $t_b \ll \tau$, where $f_b = 1/t_b$ is the bunch repetition rate and I_0 is the beam average current. We also dropped here the exponential form factor $\exp(-\omega_0^2 \sigma_t^2)$, since the bunch time length σ_t in the L1 section of the LCLS-II linac is about 3.3 ps, and, hence, the bunch spectrum is much wider than the upper frequency of the 10 GHz set for the HOM analysis.

Results of resonant HOM excitation in the ideal 3.9 GHz cavity are presented in Fig. 6 for the 0.3 mA average beam current. Of most concern is the heat load for tens of watts of peak HOM power radiating through the cavity coupler ports in the first monopole passband. In the following section, we present the detailed statistical analysis of the HOM spectrum excitation considering real cavity geometries with mechanical errors.

III. STATISTICAL ANALYSIS OF HOM SPECTRUM IN THE 3.9 GHz CAVITY WITH MECHANICAL ERRORS

Conventional thin-walled niobium superconducting radio frequency cavities consist of multiple shell components welded together. Mechanical forming of such components and further electron-beam welding introduce significant uncertainty for the final cavity geometry. Typical maximum deviations of cavity profiles in respect to the ideal shape are about ± 200 and ± 100 μm for 1.3 and 3.9 GHz cavities, respectively [15,16]. Therefore, cavities get tuned for adjusting operating frequencies and preserving the field flatness in multicell cavities. Since each cavity has a unique geometry, the HOM spectra vary from cavity to cavity, and, then, the beam to cavity interaction has a probabilistic nature.

For obtaining accurate HOM statistics, we apply the eigenmode analysis of the HOM spectrum in the 3.9 GHz cavity with mechanical errors. The procedure is based on adding random components to the geometrical parameters of each cavity cell:

$$P_n^i = P_n^{\text{nom}} + |\Delta_{\text{tol}}| [2R\text{nd}(1) - 1], \quad (9)$$

where P_n^i is a random dimension of the individual cell, P_n^{nom} is the nominal value of the n th geometrical parameter of the half-cell, i is the cell number, $\Delta_{\text{tol}} = \pm 100$ μm is the mechanical tolerance of a cavity fabrication, and $R\text{nd}(1)$ is the uniform random function in the range of 0–1. Nominal

FIG. 7. Nominal geometry of the 3.9 GHz LCLS-II cavity (alpha is a reference parameter given in degrees; the other parameters are all in millimeters).

geometrical parameters of the 3.9 GHz LCLS-II cavity are presented in Fig. 7, where the upstream and the downstream end half-cells are identical.

The presimulation cavity tuning requires finding a frequency-dependent sensitivity for each of the seven cavity geometrical parameters. The calculated frequency-dependent sensitivities of the operating π mode are shown in Table II for the middle and the end cavity half-cells. To preserve the field flatness frequencies of the half-cells, they are tuned by adjusting the cell lengths similarly as happens with real cavities. The half-cell tuning frequency balance is described then as

$$\Delta L^i \frac{\partial f}{\partial L^i} = - \sum_{n=1}^N \left[\Delta P_n^i \frac{\partial f}{\partial P_n^i} \right], \quad (10)$$

where ΔL^i is the length compensation of the i th half-cell, $\Delta P_n^i = P_n^i - P_n^{\text{nom}}$ is the random error of the i th half-cell

TABLE II. Frequency-dependent sensitivities of the operating π mode for the geometrical parameters of the 3.9 GHz LCLS-II cavity.

Parameter name	Frequency sensitivity [MHz/mm]	
	Middle half-cell	End half-cell
A	−58.9	−44.5
B	19.7	16.7
a	−39.5	−8.2
b	10.2	2.5
R	−127.2	−115.9
r	51.6	18.2
L	26.2	13.1

dimension, and $\partial f / \partial L^i$ and $\partial f / \partial P_n^i$ are frequency-dependent sensitivities of the half-cell length and the n th geometrical parameter, respectively. We assume here that small deviations of each parameter will not influence the sensitivity of other parameters and that all mechanical tolerances are the same and uncorrelated with each other.

Finally, for a statistical analysis, we generated ten chains, each of three 3.9 GHz cavities with random mechanical errors. The electromagnetic simulations were done on Fermilab high performance computing servers with the ANSYS HFSS software [17,18]. Distributions of the electrical field in the chain of three cavities are illustrated in Fig. 8 for the operating monopole π mode. Evidently, the fields are split up between individual cavities due to finite tuning of the operating mode frequencies. The average calculated field flatness of the operating mode is 82%, which is close to the LCLS-II specification of minimal 90% field flatness for nine-cell cavities in the cryomodule. Figure 9 shows frequency standard deviations for modes in the first monopole band; here is a good agreement between calculations and measured 2 K data for the 3.9 GHz cavity developed for the XFEL project.

For reducing the simulation time, we selected ten frequency passbands containing HOMs with the highest shunt impedances and quality factors. This allows us to lower the overall number of analyzed HOMs to about 500 compared to 1500 modes in the full spectrum up to 10 GHz. Furthermore, we filtered the calculated HOM data and selected only modes having more than 30% of stored energy in the middle cavity including adjusting beam pipes and with quality factors larger than 10^4 , which corresponds to the amplitude attenuation constant longer than the

 FIG. 8. Operating π mode in the chain of three 3.9 GHz cavities with mechanical errors.

FIG. 9. Frequency standard deviations for the first monopole band of the 3.9 GHz cavity.

minimal bunch distance of $1 \mu\text{s}$ in the LCLS-II linac. Consequently, we excluded nonresonant modes and modes of the left and right cavities from the calculated HOM spectrum. Comparisons of HOM resonant losses and transverse kicks in the XFEL along with the LCLS-II 3.9 GHz cavities are illustrated in Fig. 10. Both versions of the cavity demonstrate a similar behavior, indicating that a

reduced beam pipe aperture does not compromise the capability of HOM damping.

The detailed analysis of resonant beam excitation of the first monopole band is presented in Fig. 11, where we compare radiations of rf power to the Fundamental Power Coupler (FPC) and HOM coupler ports. We plot here the average data for both upstream and downstream HOM couplers. The error bars correspond to a standard deviation of the parameters spread. In a worst-case scenario, there is up to 10 W of peak rf power radiating mostly through the FPC port. The HOM coupler ports see only a fraction of exited resonant rf power, less than 0.5 W maximum. Notice that the $8\pi/9$ mode leakage through the HOM port is heavily suppressed by the notch filter. We also do not count here the power leakage of the operating mode itself due to a finite accuracy of the notch filter tuning, which is separately estimated in Ref. [6].

The resonant HOM excitation happens only when the HOM frequency is a harmonic of the beam spectrum. The probability of such an event is very low especially in the LCLS-II linac, where the maximum bunch repetition rate is much larger than the bandwidth of most trapped resonances in superconducting cavities. For an accurate estimation of the probability of average HOM losses, we

FIG. 10. Resonant HOM losses (left) and transverse kick (right) in 3.9 GHz XFEL and LCLS-II cavities with mechanical errors.

FIG. 11. Resonant excitation of the first monopole band in the 3.9 GHz LCLS-II cavity with 0.3 mA average beam current.

selected the most dangerous HOM resonances having either the resonant power loss above 10 mW or the resonant transverse kick larger than 10 V. After filtering those modes out of the full cavity HOM spectrum, we calculated medium values and standard deviations for HOMs frequencies, external couplings, and shunt impedances. Results of HOM statistical parameters evaluation are presented in Fig. 12. The high- Q resonances near the 8 GHz frequency are the quadruple modes. Since we do not introduce the cavity bending along the axis, the cavity geometry keeps its fully axial symmetry and these modes are trapped inside the individual cells. In reality, the cavity banding breaks the symmetry and causes coupling between quadruple and dipole modes, which effectively decreases external quality factors of quadruple modes below the 10^8 thresholds. The absence of HOMs from the second monopole passband around 7.5 GHz is explained by very low-quality factors ($<10^3$) making their inputs to the total HOM generation negligible despite high shunt impedances.

The cutoff frequency of the lowest dipole mode in the Ø38 mm beam pipe is about 4.5 GHz, and, hence, HOM signals above 4.5 GHz can be propagated through the 3.9 GHz LCLS-II cryomodule. The relative portion of the

FIG. 13. Radiation of the middle cavity trapped HOMs through the left and right beam pipes.

HOM power radiated from the middle cavity through the two adjacent cavities to the left and right beam pipes is presented in Fig. 13 as a linear fit of all HOM statistical data. The ratio is growing with the frequency, and the average amount of radiated power for HOMs below 10 GHz is about 35%, while 65% of the HOM power is absorbed by the middle cavity adjacent coupler ports. The result is consistent with the calculation made for the broadband wakefield radiation in the 3.9 GHz cryomodule, where the prediction is that the cryomodule absorbs up to 65% of the power of untrapped HOMs below 100 GHz [19].

The off-resonance excitation of the cavity HOM spectrum is described by the same physical process given in Eq. (4), and the HOM amplitude gain in the steady-state regime is

$$|V_{\parallel}| = V_0 \frac{e^{\alpha}}{\sqrt{e^{2\alpha} - 2e^{\alpha} \cos(\omega_0/f_b) + 1}}, \quad (11)$$

where ω_0/f_b is the phase difference of the HOM and the nearby beam harmonic frequencies. The corresponding average power loss and transverse kick caused by the off-resonance HOM excitation are, respectively,

FIG. 12. Spread of HOM parameters for high-risk trapped modes in the 3.9 GHz LCLS-II cavity.

FIG. 14. Cumulative probability of HOM power radiation through coupler ports in the single 3.9 GHz cavity (left) and in the third-harmonic section (right) of the LCLS-II linac.

$$\langle P \rangle = \frac{(R_{\parallel}/Q)\omega_0 q_0^2}{4t_b} A_{\infty}, \quad (12)$$

$$|V_{\perp}|_{x=x_0} = \frac{(R_{\perp}/Q)\omega_0 q_0 x_0 k}{4} A_{\infty}, \quad (13)$$

where $A_{\infty} = 2[e^{2\alpha} - 2e^{\alpha} \cos(\omega_0/f_b) + 1]^{-0.5} + 1$ is the resonant form factor in the steady-state regime. Finally, we seeded 10^6 random HOM spectrums and calculated the cumulative probability of HOM losses, and the transverse kick in the single cavity and in the third-harmonic section contains two cryomodules of total 16 3.9 GHz cavities. The results are summarized in Figs. 14 and 15 for 0.3 mA average beam current in the LCLS-II linac. The anticipated median value of HOM power radiating to the FPC and HOM coupler ports is below 100 and 1 mW, respectively. There is a small 10% probability that the peak HOM power larger than 1 W will be radiated to the FPC port and a low

probability ($<0.1\%$) of radiating power more than 0.1 W to the HOM ports of at least one cavity in the third-harmonic section.

The median HOM transverse kick is expected to be less than 10 V per cavity, while the peak value of a resonant transverse kick in the third-harmonic section could be as large as 10 kV with the probability of $\sim 0.01\%$. Note here that the flattop portion of the probability curve for the single cavity represents the incoherent HOM excitation and, thus, the 5 V transverse kick corresponding to the right end of the flattop in our analysis is a fraction of the total wake kick estimated elsewhere to be 25 V/mm for the LCLS-II third-harmonic cavity [20]. The result is in a reasonable agreement with the expectation, since we limited the number of resonant HOMs compared to the overall HOM spectrum of the 3.9 GHz cavity.

Thereby, the third-harmonic cavity with a reduced beam pipe aperture demonstrates an efficient damping of the resonant HOM excitation by the continuous beam in the LCLS-II linac.

IV. CONCLUSIONS

We performed the statistical analysis of the eigenmode spectrum up to 10 GHz in the 3.9 GHz LCLS-II cavities with a reduced beam pipe aperture. The analysis is done for the cavities with typical $\pm 100 \mu\text{m}$ surface deviation due to mechanical fabrication errors. Cavities are expected to have a low average HOM power loss and a transverse kick of <1 W and <100 V, respectively. The average kick is significantly less than rms transverse momentum spread in bunches entering to the third-harmonic linac section: For the beam energy of 250 MeV, the bunch emittance 0.7×10^{-6} m, and beta function of 50 m, the rms transverse momentum spread in the bunch is about 1.3 kV [21]. Therefore, the beam emittance dilution caused by HOMs is most probably small. The probabilities of high peak resonant HOM losses or kicks are below 0.1% or less.

FIG. 15. Cumulative probability of HOM transverse kick in the single 3.9 GHz cavity (blue line) and in the third-harmonic section (red line) of the LCLS-II linac.

Finally, we conclude that the modified 3.9 GHz cavity is capable of efficiently damping the resonant excitation of the HOM spectrum by the continuous beam in the LCLS-II linac. The proposed method of the resonant losses statistical evaluation can be easily generalized and applied for other high current and high duty factor superconducting machines.

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