Collision monochromatization in e^+e^- colliders

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The manuscript describes several monochromatization schemes starting from A. Renieri's [Report No. LNF-75/6(R)] proposal for head-on collisions based on correlation between particles transverse position and energy deviation. We briefly explain the initial proposal and expand it for crossing angle collisions. Then we discuss new monochromatization scheme for crossing angle collisions based on correlation between particles longitudinal position and energy deviation.

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I. INTRODUCTION

The energy resolution of existent colliders is very large with respect to the energy width of narrow resonances. For example FCC-ee [1,2] at beam energy 62.5 GeV to study Higgs boson at the threshold production provides energy resolution of 140 MeV while the width of Higgs boson is 4.2 MeV. In 1975, A. Renieri [3] proposed to improve energy resolution of Italian collider Adone [4,5] by introduction of dispersion function of opposite signs for colliding beams at the interaction point (IP), thus creating a correlation between particles transverse position and energy deviation (Fig. 1). During the following years physicists proposed to upgrade existing colliders or to build new ones based on the same monochromatization principle: VEPP-4 [6], Tau-Charm factory [7,8], SPEAR [9], B factory [10], LEP [11]. However, the monochromatization principle was never tested.

A recent proposal of monochromatization for FCC-ee [12] was a reason for us to return to the topic of monochromatization in head-on collisions, derive expressions for monochromatization with crossing angle and propose a new monochromatization scheme for crossing angle collisions based on a particle's longitudinal position correlation with energy deviation.

The starting point for our calculations is definition of luminosity $\mathcal{L}$ as a ratio of number of events per second $\dot{N}$ and a total cross section σ [13,14]

$$\mathcal{L} = \frac{\dot{N}}{\sigma} = f_0 [1 + \cos(2\theta)] \int n_1 n_2 dV dt, \quad (1)$$

where f_0 is bunch collision rate, n_1 and n_2 are the densities of colliding bunches, 2θ is the full crossing angle (Fig. 2), V is a volume occupied by bunches, c is speed of light and t is time.

II. INVARIANT MASS

Collider energy resolution is defined as a square root of second central moment of luminosity distribution with respect to invariant mass M . For two colliding particles with four-momenta $P_1^\mu = \{E_1, \vec{p}_1\}$ and $P_2^\mu = \{E_2, \vec{p}_2\}$, $P^\mu = P_1^\mu + P_2^\mu$ invariant mass is

$$M^2 = P^\mu P_\mu = 2m_e^2 + 2E_1 E_2 - 2\vec{p}_1 \vec{p}_2, \quad (2)$$

where θ is a half of the crossing angle (FIG. 2), E_1, E_2 , and $\vec{p}_1, \vec{p}_2$ are the energies and momenta of two colliding particles, m_e is electron mass. We will neglect electron mass in further calculations and assume the speed of light $c = 1$, therefore energy and momentum are equal $E = |\vec{p}|$. To calculate the scalar product of momenta we choose the

FIG. 1. Monochromatization scheme for head-on collisions.

FIG. 2. Collision geometry.

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accompanying first particle coordinate system with axis z directed along the azimuth of beam orbit

$$\vec{p}_1 = \{p_{1,x}, p_{1,y}, p_{1,z}\}, \quad (3)$$

$$\begin{aligned} \vec{p}_2 = \{ & -p'_{2,x} \cos(2\theta) + p'_{2,z} \sin(2\theta), \\ & p'_{2,y}, \\ & -p'_{2,x} \sin(2\theta) - p'_{2,z} \cos(2\theta) \}, \end{aligned} \quad (4)$$

where the prime variables describe coordinates of the second particle in a self-accompanying system. Introducing normalized variables $\delta_{1,2} = (E_{(1,2)} - E_0)/E_0$, $x'_{1,2} = p_{(1,2),x}/p_0$, and $y'_{1,2} = p_{(1,2),y}/p_0$ we obtain for longitudinal momentum

$$\begin{aligned} p_{(1,2),z} &= \sqrt{p_0^2(1 + \delta_{(1,2)})^2 - p_{(1,2),x}^2 - p_{(1,2),y}^2} \\ &= p_0 \left(1 + \delta_{(1,2)} - \frac{x_{(1,2)}'^2}{2} - \frac{y_{(1,2)}'^2}{2} \right) + O(3), \end{aligned} \quad (5)$$

where E_0 and $p_0 = E_0$ are the average energy and momentum for both bunches. Substituting obtained expression in (2) we obtain

$$\begin{aligned} M^2 &= 2E_0^2 \left[(1 + \delta_1 + \delta_2 + \delta_1\delta_2)(1 + \cos(2\theta)) \right. \\ &\quad + (x'_2 - x'_1 + x'_2\delta_1 - x'_1\delta_2) \sin(2\theta) - y'_1 y'_2 \\ &\quad \left. - \left(\frac{x_1'^2 + x_2'^2 + y_1'^2 + y_2'^2}{2} - x'_1 x'_2 \right) \cos(2\theta) \right] + O(3). \end{aligned} \quad (6)$$

Now, we assume that colliding bunches' population obey normal distribution with average energy and momentum $E_0 = \langle E_1 \rangle = \langle E_2 \rangle$ and $E_0 = p_0$, and standard deviation σ_E with relative value $\sigma_\delta = \sigma_E/E_0$, with angular spreads in vertical and horizontal planes $\sigma_{x'} = p_{x'}/p_0$ and $\sigma_{y'} = p_{y'}/p_0$. We also take a square root of (6) which gives us

$$\begin{aligned} M &= 2E_0 \left[\left(1 - \frac{y_1'^2 + y_2'^2}{4} - \frac{(x'_1 - x'_2)^2}{8} \right. \right. \\ &\quad \left. \left. - \frac{(\delta_1 - \delta_2)^2 - 4(\delta_1 + \delta_2)}{8} \right) \cos(\theta) + \frac{(y'_1 - y'_2)^2}{8 \cos(\theta)} \right. \\ &\quad \left. + \frac{2(x'_2 - x'_1) + (\delta_1 - \delta_2)(x'_2 + x'_1)}{4} \sin(\theta) \right] + O(3). \end{aligned} \quad (7)$$

Averaging (6) and (7) over angular spreads we derive expressions ready for further use with luminosity distribution:

$$\begin{aligned} \langle M^2 \rangle_{x',y'} &= 2E_0^2(1 + \delta_1 + \delta_2 + \delta_1\delta_2)[1 + \cos(2\theta)] \\ &\quad - 2E_0^2(\sigma_{x'}^2 + \sigma_{y'}^2) \cos(2\theta) + O(3), \end{aligned} \quad (8)$$

$$\begin{aligned} \langle M \rangle_{x',y'} &= 2E_0 \left[1 - \frac{(\delta_1 - \delta_2)^2}{8} + \frac{\delta_1 + \delta_2}{2} \right] \cos(\theta) \\ &\quad - \frac{E_0}{2} \sigma_{x'}^2 \cos(\theta) - \frac{E_0}{2} \sigma_{y'}^2 \frac{\cos(2\theta)}{\cos(\theta)} + O(3). \end{aligned} \quad (9)$$

III. MONOCHROMATIZATION WITH TRANSVERSE CORRELATION

A. Renieri [3] proposed to introduce at IP horizontal dispersion $\pm\psi_x$ of opposite signs for colliding beams in head-on collisions. In this case the nature of monochromatization is obvious—collision rate for particles with opposite energy deviation is higher than contrariwise (Fig. 1). Increasing collision angle (zero is head-on) will decrease efficiency of monochromatization but it is still possible (Fig. 3) because particles with energy deviation will meet higher density of the particles with opposite energy deviation.

For two colliding bunches (Fig. 4.) we write the bunch density n_i in own frame system

$$\begin{aligned} n_i(x_i, y_i, s_i, t, \delta_i) &= \frac{N_i}{(2\pi)^{4/2} \sigma_x \sigma_y \sigma_z \sigma_\delta} \exp \left[-\frac{(x_i \pm \psi_x \delta_i)^2}{2\sigma_x^2} \right. \\ &\quad \left. - \frac{(y_i \pm \psi_y \delta_i)^2}{2\sigma_{y,i}(s_i)^2} - \frac{(s_i \mp ct)^2}{2\sigma_z^2} - \frac{\delta_i^2}{2\sigma_\delta^2} \right], \end{aligned} \quad (10)$$

where we also introduced vertical dispersion $\pm\psi_y$ of opposite signs for colliding bunches, N_i is bunch

FIG. 3. Crossing angle collision with opposite dispersion.

FIG. 4. Crossing angle collision with opposite dispersion.

population, $\{x_i, y_i, s_i\}$ is usual accelerator basis of i th bunch with transverse and longitudinal coordinates, $\delta_i = (E_i - E_0)/E_0$ is energy deviation, and betatron beam sizes are

$$\sigma_x = \sqrt{\varepsilon_x \beta_x}, \quad (11)$$

$$\sigma_{y,i} = \sqrt{\varepsilon_y \beta_y(s_i)}, \quad (12)$$

$\varepsilon_{x,y}$, $\beta_{x,y}$ are emittance and beta function in corresponding plane at the IP, σ_z and σ_δ are standard deviations of bunch longitudinal and energy deviation distributions. Then we need to transform coordinates into laboratory frame with the following expressions

$$\begin{cases} x_i = x \cos(\pm\theta) + s \sin(\pm\theta), \\ y_i = y, \\ s_i = x \sin(\pm\theta) + s \cos(\pm\theta). \end{cases} \quad (13)$$

Using (1) we calculate luminosity according to

$$\begin{aligned} \mathcal{L} &= f_0(1 + \cos(2\theta)) \\ &\times \int n_1(x, y, s, t, \delta_1) n_2(x, y, s, t, \delta_2) dx dy ds dt d\delta_1 d\delta_2. \end{aligned} \quad (14)$$

Neglecting hour glass effect the result is

$$\mathcal{L} = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y\sqrt{1+\varphi^2}} \frac{1}{\Lambda} = \frac{\mathcal{L}_0}{\Lambda}, \quad (15)$$

$$\frac{\partial^2 \mathcal{L}}{\partial \delta_1 \partial \delta_2} = \frac{\mathcal{L}_0}{2\pi\sigma_\delta^2} \exp \left[-\frac{(\delta_1 - \delta_2)^2}{4\sigma_\delta^2} - \Lambda^2 \frac{(\delta_1 + \delta_2)^2}{4\sigma_\delta^2} \right], \quad (16)$$

$$\Lambda^2 = 1 + \frac{\psi_y^2 \sigma_\delta^2}{\sigma_y^2} + \frac{\psi_x^2 \sigma_\delta^2}{\sigma_x^2 (1 + \varphi^2)}, \quad (17)$$

where $\varphi = \sigma_z \tan \theta / \sigma_x$ is Piwinski angle, $\mathcal{L}_0$ is luminosity in absence of dispersion [15]. Substitution of zero crossing angle θ and zero vertical dispersion will give expressions obtained by A. Renieri.

To simplify analysis we write the cross section of the narrow resonance with mass M_0 as

$$\sigma(\delta_1, \delta_2) = B + A \cdot \delta(\langle M \rangle_{x',y'} - M_0), \quad (18)$$

where B is describing background and independent of energy, $\langle M \rangle_{x',y'}$ is given in (9). With the help of (1) and (16) the production rate $\dot{N}$ in resonance vicinity is

$$\begin{aligned} \dot{N} &= \int \sigma(\delta_1, \delta_2) \frac{\partial^2 \mathcal{L}}{\partial \delta_1 \partial \delta_2} d\delta_1 d\delta_2 \\ &= B \frac{\mathcal{L}_0}{\Lambda} + A \frac{\mathcal{L}_0 \exp[-\frac{m^2 \Lambda^2}{4E_0^2 \sigma_\delta^2}]}{\sqrt{2\pi} \cos(\theta) \sigma_\delta \sqrt{2E_0^2 + E_0 m \Lambda^2}}, \\ m &= \frac{E_0}{2} \sigma_{x'}^2 + \frac{E_0}{2} \sigma_{y'}^2 \frac{\cos(2\theta)}{\cos(\theta)^2} + \frac{M_0}{\cos(\theta)} - 2E_0. \end{aligned} \quad (19)$$

Calculating expected values of (8) and (9) with (16) we obtain estimation of invariant mass resolution

$$\begin{aligned} \sigma_M^2 &= \langle M^2 \rangle_{x',y',\delta_1,\delta_2} - \langle M \rangle_{x',y',\delta_1,\delta_2}^2 \\ &= 2E_0^2 \left\{ \left(\frac{\sigma_\delta \cos(\theta)}{\Lambda} \right)^2 + [\sigma_{x'} \sin(\theta)]^2 \right\}. \end{aligned} \quad (20)$$

Note that for large crossing angles the term with angular spread might be dominant.

A. Notes on obtaining

For a head-on collisions expression without vertical dispersion we may further simplify the expression for Λ (17). First, for $\Lambda \gg 1$ we expand expression (17)

$$\Lambda \approx \frac{\psi_x \sigma_\delta}{\sigma_x}. \quad (21)$$

Second, we replace horizontal dispersion in the planar ring at the IP by the expression [16]

$$\begin{aligned} \psi_x &= \frac{\sqrt{\beta_x}}{2 \sin(\pi\nu_x)} \oint \frac{\sqrt{\beta_x(\tau)}}{\rho(\tau)} \cos(\varphi(\tau) - \pi\nu_x) d\tau \\ &= \sqrt{\beta_x} F_D, \\ F_D &= \frac{1}{2 \sin(\pi\nu_x)} \oint \frac{\sqrt{\beta_x(\tau)}}{\rho(\tau)} \cos(\varphi(\tau) - \pi\nu_x) d\tau, \end{aligned} \quad (22)$$

where β_x is horizontal beta function at the IP, ρ is bending radius of the dipoles, $\varphi(\tau)$ is phase factor, F_D is a factor defined by the whole lattice. Third, we substitute expressions for σ_δ and $\sigma_x = \sqrt{\varepsilon_x \beta_x}$ with the help of synchrotron radiation integrals [17]

$$\sigma_\delta = \sqrt{C_q \gamma^2 \frac{I_3}{J_s I_2}}, \quad (23)$$

$$\sigma_x = \sqrt{C_q \gamma^2 \frac{I_5}{J_s I_2} \beta_x}, \quad (24)$$

$$I_2 = \oint \frac{d\tau}{\rho^2(\tau)}, \quad (25)$$

$$I_3 = \oint \frac{d\tau}{|\rho(\tau)|^3}, \quad (26)$$

$$I_5 = \oint \frac{\mathcal{H}(\tau)d\tau}{|\rho(\tau)|^3}, \quad (27)$$

where J_x and J_s are horizontal and longitudinal damping partition numbers, and $\mathcal{H}(s) = \beta_x(s)\psi'^2(s) + 2\alpha_x(s)\psi(s)\psi'(s) + \gamma_x(s)\psi^2(s)$. Now then, we obtain expression for Λ

$$\Lambda \approx F_D \sqrt{\frac{J_x I_3}{J_s I_5}} \approx F_D \sqrt{\frac{J_x}{J_s} \frac{1}{\langle \mathcal{H} \rangle}}, \quad (28)$$

where $\langle \rangle$ denote average value over the ring.

So, in order to obtain large monochromatization factor Λ it is necessary to have lattice with small horizontal emittance, i.e., small $\langle \mathcal{H} \rangle$, horizontal damping partition number greater than longitudinal $J_x > J_s$, and large lattice factor F_D (22).

B. Hour-glass

In the beginning of the chapter we neglected s dependence of beta functions. It is justified for horizontal plane because β_x changes very little over the bunches interaction region. For the vertical plane we need to write explicit dependence on azimuth in the laboratory frame (13)

$$\sigma_{y,1}(s_1)^2 = \sigma_y^2 \left(1 + \frac{(-x \sin \theta + s \cos \theta)^2}{\beta_y^2} \right), \quad (29)$$

$$\sigma_{y,2}(s_2)^2 = \sigma_y^2 \left(1 + \frac{(x \sin \theta + s \cos \theta)^2}{\beta_y^2} \right). \quad (30)$$

Straightforward integration of (14) over $dyd\tau$ gives

$$\begin{aligned} \mathcal{L} = f_0 \frac{(1 + \cos(2\theta))N_1 N_2}{\sqrt{2}(2\pi)^3} & \times \int \frac{dx ds d\delta_1 d\delta_2}{\sigma_x^2 \sigma_z \sigma_\delta^2 \sqrt{\sigma_{y,1}(s_1)^2 + \sigma_{y,2}(s_2)^2}} \\ & \times \exp \left[-\frac{(x \cos \theta + s \sin \theta - \psi_x \delta_1)^2}{2\sigma_x^2} - \frac{\delta_1^2 + \delta_2^2}{2\sigma_\delta^2} \right. \\ & \left. - \frac{(x \cos \theta - s \sin \theta + \psi_x \delta_2)^2}{2\sigma_x^2} - \frac{s^2 \cos^2 \theta}{\sigma_z^2} \right. \\ & \left. - \frac{\psi_y^2 (\delta_1 + \delta_2)^2}{2(\sigma_{y,1}^2 + \sigma_{y,2}^2)} \right]. \end{aligned} \quad (31)$$

Now, to proceed with integration over dx we need to simplify expressions for vertical beam sizes (29), (30) because their dependence on x prevents further analytical calculations. We will assume that

$$\sigma_z \cos \theta \gg \sigma_x \sin \theta, \quad (32)$$

which allows us to neglect $x \sin \theta$ terms in (29), (30). Integration over dx results in

$$\begin{aligned} \mathcal{L} = f_0 \frac{2 \cos(\theta) N_1 N_2 \sqrt{\pi}}{\sqrt{2}(2\pi)^3} & \times \int \frac{ds d\delta_1 d\delta_2}{\sigma_x \sigma_z \sigma_\delta^2 \sqrt{\sigma_{y,1}(s)^2 + \sigma_{y,2}(s)^2}} \\ & \times \exp \left[-\frac{s^2 \cos^2 \theta}{\sigma_z^2} - \frac{s^2 \sin^2 \theta - \psi_x (\delta_1 + \delta_2) s \sin \theta}{\sigma_x^2} \right. \\ & \left. - \frac{\delta_1^2 + \delta_2^2}{2\sigma_\delta^2} - \frac{\psi_x^2 (\delta_1^2 + \delta_2^2)}{2\sigma_x^2} - \frac{\psi_y^2 (\delta_1 + \delta_2)^2}{2(\sigma_{y,1}^2 + \sigma_{y,2}^2)} + \frac{\psi_x^2 (\delta_1 - \delta_2)^2}{4\sigma_x^2} \right]. \end{aligned} \quad (33)$$

The following integration over $d\delta_1 d\delta_2$ is performed with the help of new variables $V = E_0(\delta_1 - \delta_2)$ and $W = E_0(\delta_1 + \delta_2 + 2)$, $r = (s/\sigma_z) \cos \theta$ and gives

$$\begin{aligned} \mathcal{L} = f_0 \frac{N_1 N_2}{4\pi \sigma_x \sigma_y \sqrt{\pi}} & \times \int \frac{dr}{\sqrt{(1 + r^2 \frac{\sigma_z^2}{\beta_y^2})(1 + \frac{\psi_x^2 \sigma_\delta^2}{\sigma_x^2}) + \frac{\psi_y^2 \sigma_\delta^2}{\sigma_y^2}}} \\ & \times \exp \left[-r^2 \left(1 + \varphi^2 - \frac{\psi_x^2 \sigma_\delta^2 \varphi^2 (1 + r^2 \frac{\sigma_z^2}{\beta_y^2})}{\sigma_x^2 ((1 + r^2 \frac{\sigma_z^2}{\beta_y^2})(1 + \frac{\psi_x^2 \sigma_\delta^2}{\sigma_x^2}) + \frac{\psi_y^2 \sigma_\delta^2}{\sigma_y^2})} \right) \right]. \end{aligned} \quad (34)$$

The obtained expression could be further calculated analytically for particular cases.

In the case of $\psi_y = 0$ luminosity is

$$\begin{aligned} \mathcal{L}_1 = f_0 \frac{N_1 N_2}{4\pi \sigma_x \sigma_y \sqrt{\pi} \sigma_z} & \times \frac{1}{\sqrt{1 + \frac{\psi_x^2 \sigma_\delta^2}{\sigma_x^2}}} \\ & \times \exp \left[\frac{\beta_y^2}{2\sigma_z^2} \left(1 + \frac{\sigma_x^2 \varphi^2}{\sigma_x^2 + \psi_x^2 \sigma_\delta^2} \right) \right] \\ & \times K_0 \left[\frac{\beta_y^2}{2\sigma_z^2} \left(1 + \frac{\sigma_x^2 \varphi^2}{\sigma_x^2 + \psi_x^2 \sigma_\delta^2} \right) \right], \end{aligned} \quad (35)$$

where $K_0(x)$ is modified bessel function of the second kind or Macdonald function.

In the case of $\psi_x = 0$ luminosity is

$$\begin{aligned} \mathcal{L}_2 = f_0 \frac{N_1 N_2}{4\pi \sigma_x \sigma_y \sqrt{\pi} \sigma_z} & \times \exp \left[\frac{\beta_y^2}{2\sigma_z^2} (1 + \varphi^2) \left(1 + \frac{\psi_y^2 \sigma_\delta^2}{\sigma_y^2} \right) \right] \\ & \times K_0 \left[\frac{\beta_y^2}{2\sigma_z^2} (1 + \varphi^2) \left(1 + \frac{\psi_y^2 \sigma_\delta^2}{\sigma_y^2} \right) \right]. \end{aligned} \quad (36)$$

We introduce the hour-glass reduction factor (Fig. 5)

$$R_{hg}(t_i) = \frac{\mathcal{L}_i}{\mathcal{L}}, \quad (37)$$

where $\mathcal{L}_i$ is luminosity for cases (35) and (36), and $\mathcal{L}$ is luminosity without hour-glass (15), and

FIG. 5. Hour-glass reduction factor.

$$t_1 = \frac{\beta_y^2}{2\sigma_z^2} \left(1 + \frac{\sigma_x^2 \varphi^2}{\sigma_x^2 + \psi_x^2 \sigma_\delta^2} \right), \quad (38)$$

$$t_2 = \frac{\beta_y^2}{2\sigma_z^2} (1 + \varphi^2) \left(1 + \frac{\psi_y^2 \sigma_\delta^2}{\sigma_y^2} \right). \quad (39)$$

C. Example of FCC-ee

Future circular collider (FCC) is a project in CERN of the next accelerator after LHC [1,2,18]. The ultimate goal is 100 km proton-proton machine with 100 TeV central mass energy. The first possible step is e^+e^- factory—FCC-ee with central mass energy range from 80 GeV to 350 GeV and two IPs. We apply derived expressions of transverse monochromatization scheme to see what luminosity and energy resolution could be achieved at Higgs boson threshold [$\sigma(ee \rightarrow H) = 1.6$ fb, $\Gamma = 4.2$ MeV, beam energy is $E_0 = 62.5$ GeV].

Influence of synchrotron radiation of the particles in a strong electromagnetic field of the opposite bunch (*beamstrahlung*) modifies beam emittance, energy spread, and length [19]. Presence of dispersion at the IP will change beam emittance even more; therefore calculations were performed in steps, on each step new parameters were calculated for a test bunch and then interchanged with parameters of the oncoming bunch until equilibrium is reached [19]. Table I presents parameters at 62.5 GeV beam energy for baseline (no monochromatization), monochromatization with $\psi_x = 1$ m and $\psi_x = 0.5$ m. Notice with $\Lambda = 10$ energy resolution improves more than by a factor of ten this is because of energy spread reduction. This happens owing to beamstrahlung: dispersion at the IP increases horizontal emittance (24) and (27), larger emittance means smaller beam density and weaker fields of the opposite bunch; therefore, smaller energy spread. In the last column we increased rf voltage and vertical beta function in order to match the length of interaction area with vertical beta function.

TABLE I. Parameters of FCC-ee with and without monochromatization for two IPs.

Energy, GeV	62.5	62.5	62.5
ψ_x , m	0	1	0.5
β_x/β_y , mm	500/1	500/1	500/1.4
$\varepsilon_x/\varepsilon_y$, nm/pm	0.26/2	2.8/21	3.8/2.9
θ , mrad		15	
σ_x (no BS), mm	0.01	0.01	0.01
σ_y (no BS), μ m	0.045	0.045	0.053
σ_z (no BS), mm	3.8	3.8	2.6
σ_x , mm	0.01	0.04	0.04
σ_y , μ m	0.04	0.15	0.2
σ_z , mm	8.3	3.8	2.8
σ_δ , 10^{-4}	16	7	8
Piwiniski angle φ	11	1.5	0.95
V_{RF} , GeV	0.75	0.75	1.5
N_p		3×10^{11}	
N_{bunches}		2615	
Luminosity, 10^{34} cm $^{-2}$ s $^{-1}$	9	2.8	5
R_{hg}	0.9	0.54	0.71
Λ	1	10	6
σ_M , MeV	141	6	11
ξ_x/ξ_y	0.04/0.2	0.001/0.01	0.004/0.02

IV. MONOCHROMATIZATION WITH LONGITUDINAL CORRELATION

Another approach to enhance collider energy resolution is to use correlation between a particle's longitudinal position and energy (Fig. 6). Such longitudinal correlation may take place in case of strong focusing in the longitudinal plane [20–22]. Figure 6 illustrates idea in the phase diagram, where particles belonging to two different ellipses collide when they reach the same z but possess opposite sign of energy deviation. Having $\beta_z, \alpha_z, \gamma_z, \varepsilon_z$ Twiss parameters and emittance in longitudinal plane with $\{z = s - ct, \delta\}$ conjugate variables, beam distribution in $\{z, \delta\}$ plane is [23]

$$f(z, \delta) = \frac{1}{2\pi\varepsilon_z} \exp \left[-\frac{\gamma_z z^2 + 2\alpha_z z\delta + \beta_z \delta^2}{2\varepsilon_z} \right]. \quad (40)$$

In order for monochromatization to happen, the bunches should possess opposite sign of α_z (Fig. 7), but because time s flows in opposite direction for the bunches, in further calculations α_z has the same sign for both bunches. The bunch density is

$$n_i(x_i, y_i, z_i, \delta_i) = \frac{N_i}{(2\pi)^{4/2} \sigma_x \sigma_y \varepsilon_z} \exp \left[-\frac{x_i^2}{2\sigma_x^2} - \frac{y_i^2}{2\sigma_y^2} - \frac{\gamma_z z^2 + 2\alpha_z z\delta + \beta_z \delta^2}{2\varepsilon_z} \right], \quad (41)$$

where $z_i = s_i \mp ct$. Calculations of luminosity by (14) with coordinate transformation (13) gives similar to (15), (16),

FIG. 6. Monochromatization for crossing angle collision with correlation between particle's energy and longitudinal position.

FIG. 7. Colliding beam ellipses in longitudinal plain, showing correlation between collision energy deviation δ and longitudinal position z .

(17) results but with different definition of Λ and total luminosity is not decreased by Λ

$$\mathcal{L} = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y\sqrt{1+\varphi^2}} = \mathcal{L}_0, \quad (42)$$

$$\frac{\partial^2 \mathcal{L}}{\partial \delta_1 \partial \delta_2} = \frac{\mathcal{L}_0 \Lambda}{2\pi\sigma_\delta^2} \exp \left[-\frac{(\delta_1 - \delta_2)^2}{4\sigma_\delta^2} - \Lambda^2 \frac{(\delta_1 + \delta_2)^2}{4\sigma_\delta^2} \right], \quad (43)$$

$$\Lambda^2 = \frac{1 + \varphi^2}{1 + \frac{1}{1+\alpha_z^2}\varphi^2} \approx 1 + \alpha_z^2, \quad \varphi \gg 1, \quad (44)$$

where $\sqrt{\varepsilon_z \beta_z} = \sigma_z$, $\sqrt{\varepsilon_z \gamma_z} = \sigma_\delta$, $\varphi = \sqrt{\varepsilon_z \beta_z} \tan(\theta) / \sigma_x$ is Piwinski angle. Note that in case of low α_z , monochromatization vanishes $\Lambda = 1$, which corresponds to the absence of the phase ellipses tilt (Fig. 7) and is a usual state of operation for modern colliders (bunch energy spread and length are independent of s).

Production rate at the resonance vicinity with cross section (18) is

$$\begin{aligned} \dot{N} &= \int \sigma(\delta_1, \delta_2) \frac{\partial^2 \mathcal{L}}{\partial \delta_1 \partial \delta_2} d\delta_1 d\delta_2 \\ &= B\mathcal{L}_0 + A \frac{\Lambda \mathcal{L}_0 \exp \left[-\frac{m^2 \Lambda^2}{4E_0^2 \sigma_\delta^2} \right]}{\sqrt{2\pi} \cos(\theta) \sigma_\delta \sqrt{2E_0^2 + E_0 m \Lambda^2}}, \\ m &= \frac{E_0}{2} \sigma_x^2 + \frac{E_0}{2} \sigma_y^2 \frac{\cos(2\theta)}{\cos(\theta)^2} + \frac{M_0}{\cos(\theta)} - 2E_0. \end{aligned} \quad (45)$$

Comparing production rates for monochromatization with transverse correlation (19) and longitudinal (45) we notice that longitudinal monochromatization is contrary to transverse—background rate is not reduced by Λ , and production rate of the resonance is increased by Λ .

Invariant mass resolution is now s dependent and independent of α_z

$$\begin{aligned} \sigma_M^2 &= \langle M^2 \rangle_{x',y',\delta_1,\delta_2} - \langle M \rangle_{x',y',\delta_1,\delta_2}^2 \\ &= 2E_0^2 \left\{ \left(\frac{\sigma_\delta \cos(\theta)}{\Lambda} \right)^2 + [\sigma_{x'} \sin(\theta)]^2 \right\} \\ &\approx 2E_0^2 \left\{ \left(\sqrt{\frac{\varepsilon_z}{\beta_z(s)}} \cos(\theta) \right)^2 + [\sigma_{x'} \sin(\theta)]^2 \right\}. \end{aligned} \quad (46)$$

In order to enhance invariant mass resolution one needs to decrease longitudinal emittance and increase beta function at the observation point.

Examples of rf focusing

Introduction of Twiss functions in longitudinal plane allowed the authors of [21,24] to estimate longitudinal emittance

$$\varepsilon_z = \sigma_{0\delta}^2 \frac{\oint \frac{\beta(\tau)}{[\rho(\tau)]^3} d\tau}{\oint \frac{1}{[\rho(\tau)]^3} d\tau}, \quad (47)$$

where $\sigma_{0\delta}^2 = C_q \gamma^2 I_3 / (2I_2)$ is a usual expression for energy spread, $\rho(\tau)$ is bending radius. Obtained expression is similar and could be obtained in the same manner as lower fundamental limit on vertical emittance. For a planar isomagnetic ring with the help of (46) we estimate invariant mass resolution

$$\sigma_M(s) \approx \sqrt{2} E_0 \sqrt{\sigma_{0\delta}^2 \frac{\langle \beta_z \rangle}{\beta_z(s)} \cos^2(\theta) + \sigma_{x'}^2 \sin^2(\theta)} \quad (48)$$

where $\langle \beta_z \rangle$ is average value of longitudinal beta function in the ring. In order to enhance energy resolution we need to minimize $\langle \beta_z \rangle$ and increase β_z , and minimize $\sigma_{x'} \sin(\theta)$.

The authors of [21] studied the case of one cavity and reported a need for high voltage and short wave length rf. We decided to compare one cavity lattice with several cavities lattice. The latter is similar to using more complex cells than FODO in order to achieve lower emittance in transverse plain. For that reason we developed a toy lattice for $2\theta = 90$ degree collision angle and with four rf cavities (Fig. 8, Table II), where we chose $\sigma_{x'} = 0$ to distinguish effect of rf focusing. Introduction of $\sigma_{x'} \neq 0$ will worsen invariant mass resolution. We tried to apply longitudinal monochromatization for low energy e^+e^- collider with central mass energy 200 MeV to produce and study true muonium (bound state of $\mu^+\mu^-$) [25,26].

FIG. 8. Sketch of rf cavities of the toy ring.

The focusing strength of the cavity is [20]

$$\frac{1}{F} = \frac{V_{RF}}{E_0} \frac{2\pi}{\lambda_{rf}}, \quad (49)$$

where V_{RF} is rf voltage amplitude in the cavity, λ_{rf} is wave length of rf field. Since rf amplitude is limited; therefore lower beam energy and shorter wave length are necessary to increase focusing strength of the cavity. We chose cavity pF2 (Fig. 8) with harmonic factor q_2 as the one to compensate synchrotron radiation energy loss. The energy spread σ_δ , bunch length σ_z and invariant mass resolution σ_M now depend on the azimuth, the index $\{0, 1, 2\}$ denotes positions at the beginning of the lattice, at the first and second cavities. Optimizing voltage and harmonic factors of other three cavities we achieved energy resolution almost twice better (Test 2) than in the initial state (Test 0) and better than with one cavity (Test 1). The voltage of the additional cavities are 3 MV with frequency of 3 GHz lower than in one cavity case (Test 1). Higher beam energy will require higher voltage and shorter wave length, which now looks unrealistic. However, using more cavities and creating multicell lattice with longitudinal final focus (like in transverse plane) might produce better monochromatization given that contribution from the term with $\sigma_{x'}$ is smaller.

TABLE II. Parameters of the toy ring with longitudinal monochromatization.

	Test 0	Test 1	Test 2
Energy, MeV		150	
Π , m		21	
2θ , rad		$\pi/2$	
α		0.5	
U_0 , keV		2	
f_0 , MHz		14.3	
$q_1/q_2/q_3$,		130/13/130	
σ_x , μm		100	
$\sigma_{x'}$, 10^{-3}		0	
σ_{δ} , 10^{-4}		4	
$V_{rf,1}/V_{rf,2}/V_{rf,3}$, MV	0/0.01/0	0/9/0	-2.9/0.01/2.9
ε_z , 10^{-6} m	33	1	3.3
$\sigma_{\delta,0}/\sigma_{\delta,1}/\sigma_{\delta,2}$, 10^{-4}	4/4/4	4.9/4.9/4.9	5.3/5.3/5.3
$\sigma_{z,0}/\sigma_{z,1}/\sigma_{z,2}$, mm	81/81/81	2.1/2.4/3.3	7/14/0.6
$\sigma_{M,0}/\sigma_{M,1}/\sigma_{M,2}$, keV	60/60/60	71/63/46	70/35/800

V. CONCLUSION

We derived expressions for energy resolution and luminosity for crossing angle collisions with monochromatization based on transverse and longitudinal correlations between particle position and energy deviation. Monochromatization based on transverse correlation is feasible at the present level of accelerator technology; the longitudinal correlation requires high voltage and high frequency rf system, special longitudinal lattice—hard to build at the present state of rf technology. Introduction of multicavity lattice reduces requirements for a rf system.

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- [1] M. Benedikt (2nd FCC week, Rome, 2016).
- [2] FCC-ee design study, <http://tlep.web.cern.ch>.
- [3] A. Renieri, Laboratori Nazionali di Frascati Report No. LNF-75/6(R).
- [4] F. Amman, National Laboratory for High Energy Physics Report No. KEK-76-3, 1976.
- [5] M. Bassetti *et al.*, in *Proceedings of the 9th International Conference on High-Energy Accelerators, Stanford, California, 1974* (A.E.C., Washington, DC, 1974), pp. 104–107.
- [6] A. A. Avdienko, G. A. Kornukhin, I. Y. Protopopov, A. N. Skrinsky, A. B. Temnykh, G. M. Tumaikin, and A. A. Zholents, *Proceedings of the 12th International Conference on High-Energy Accelerators, HEACC 1983* (Fermilab, Batavia, IL, 1983), p. 186.
- [7] A. A. Zholents, Technical Report No. CERN-SL-92-27-AP, 1992.
- [8] A. Faus-Golfe and J. Le Duff, Technical Report No. LAL-RT-92-01, 1992.
- [9] K. Wille and A. W. Chao, Technical Report No. SLAC/AP-032, SLAC/AP-32, 1984.
- [10] A. N. Dubrovin and A. A. Zholents, *Conference Record of the 1991 IEEE Particle Accelerator Conference (PAC 1991): Accelerator Science and Technology, San Francisco, California, 1991* (IEEE, Piscataway, NJ, 1991), p. 2835.
- [11] M. Bassetti and J. M. Jowett, *Proceedings of the 1987 IEEE Particle Accelerator Conference (PAC 1987): Accelerator Engineering and Technology, Washington, D.C., 1987* (IEEE, Piscataway, NJ, 1987), p. 115.
- [12] M. A. V. Garca, A. Faus-Golfe, and F. Zimmermann, in *Proceedings of the 7th International Particle Accelerator Conference (IPAC 2016): Busan, Korea, 2016* (JACoW, Geneva, Switzerland, 2016), p. WEPMW009.

- [13] C. Møller, in *Proceedings of the International Conference on Fundamental Particles and Low Temperatures, Cambridge, UK, 1946* (Physical Society, London, 1947), Vol. 1, p. 194.
- [14] T. Suzuki, National Laboratory for High Energy Physics Report No. KEK-76-3, 1976.
- [15] L. Smith, Technical Report No. PEP-0020, No. LBL-ERAN-191, 1972.
- [16] E. D. Courant and H. S. Snyder, Theory of the alternating-gradient synchrotron, *Ann. Phys. (N.Y.)* **3**, 1 (1958); **281**, 360 (2000).
- [17] R. H. Helm, M. J. Lee, P. L. Morton, and M. Sands, Evaluation of Synchrotron Radiation Integrals, *IEEE Trans. Nucl. Sci.* **20**, 900 (1973).
- [18] K. Oide *et al.*, Design of beam optics for the future circular collider e^+e^- collider rings, *Phys. Rev. Accel. Beams* **19**, 111005 (2016).
- [19] A. Bogomyagkov, E. Levichev, and D. Shatilov, Beam-beam effects investigation and parameters optimization for a circular e^+e^- collider at very high energies, *Phys. Rev. ST Accel. Beams* **17**, 041004 (2014).
- [20] V. N. Litvinenko, Nonlinear and collective phenomena in beam physics, in *Proceedings of the Workshop of the International Committee on Future Accelerators, Arcidosso, Italy, 1996* (AIP, New York, 1997), Vol. 395, pp. 275–283.
- [21] A. Gallo, P. Raimondi, and M. Zobov, in *30th Advanced ICFA Beam Dynamics Workshop on High Luminosity e^+e^- Collisions (e^+e^- Factories 2003) Stanford, California, 2003*, Vol. C0309101, p. THWA003, [arXiv: physics/0404020](https://arxiv.org/abs/physics/0404020).
- [22] A. Gallo, P. Raimondi, and M. Zobov, Technical Report No. DAFNE-TECHNICAL-NOTE:-G-60, 2003.
- [23] I. Kapchinsky and V. Vladimirovsky, in *Proceedings of the II International Conference on High Energy Accelerators, Geneva, Cern, 1959*, pp. 274–288.
- [24] A. W. Chao, Evaluation of beam distribution parameters in an electron storage ring, *J. Appl. Phys.* **50**, 595 (1979).
- [25] A. Bogomyagkov, V. Kiselev, E. Levichev, A. Milstein, D. Shatilov, and S. Sinytakin, eeFACT2016 58th ICFA Advanced Beam Dynamics Workshop on High Luminosity Circular e^+e^- Colliders, 2016.
- [26] S. J. Brodsky and R. F. Lebed, Evaluation of Beam Distribution Parameters in an Electron Storage Ring, *Phys. Rev. Lett.* **102**, 213401 (2009).