Polarized $^3\text{He}^{+2}$ ions in the Alternate Gradient Synchrotron to RHIC transfer line

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The proposed electron-hadron collider (eRHIC) to be built at Brookhaven National Laboratory (BNL) will allow the collisions of 20 GeV polarized electrons with 250 GeV polarized protons, or 100 GeV/n polarized $^3\text{He}^{+2}$ ions, or other unpolarized ion species. The large value of the anomalous magnetic moment of the ^3He nucleus $G_{\text{He}} = (g - 2)/2 = -4.184$ (where g is the g -factor of the ^3He nuclear spin) combined with the peculiar layout of the transfer line which transports the beam bunches from the Alternate Gradient Synchrotron (AGS) to the Relativistic Heavy Ion Collider (RHIC) makes the transfer and injection of polarized ^3He ions from AGS to RHIC (AtR) a special case as we explain in the paper. Specifically in this paper we calculate the stable spin direction of a polarized ^3He beam at the exit of the AtR line which is also the injection point of RHIC, and we discuss a simple modifications of the AtR beam-transfer-line, to perfectly match the stable spin direction of the injected polarized ^3He beam to that of the circulating beam, at the injection point of RHIC.

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I. INTRODUCTION

The Alternate Gradient Synchrotron (AGS) in combination with the Relativistic Heavy Ion Collider (RHIC) are currently the only synchrotrons in the world to accelerate polarized protons at an energy of 250 GeV. The AGS, which serves as an injector to RHIC, accelerates proton bunches to 24 GeV, with 70% polarization. The bunches are extracted from the AGS and transferred to RHIC, via the AtR transport line, for further acceleration to 250 GeV for the experimental physics programs. Figure 1, is a schematic diagram of the accelerator complex. Currently the beam polarization of the beam extracted from the AGS is 70% and this is accomplished by the use of two partial helices, which are installed in the straight sections A20 and E20 of the AGS ring, and help eliminate the imperfection and the vertical intrinsic spin resonances [1,2]. The horizontal intrinsic spin resonances in AGS are eliminated by the use of the jump quads [3]. The interest in polarization experiments in the proposed eRHIC project [4] using ^3He ions [5,6] prompted a theoretical study of accelerating polarized ^3He ions in the AGS and transporting them to RHIC. The polarized ^3He spin resonances in AGS [6] and the stable spin direction at the extraction point of the AGS which is determined by the partial helices of the AGS, constrain the energies of the extracted beam. Due to the BMT [7] equation which describes the motion of the spin

direction in an electric and magnetic field and the fact that the AtR beam transfer line [8] includes sections with magnets that bend the beam horizontally while the beam is bent vertically as shown in Fig. 2 makes the stable spin direction at the exit of the AtR line to depend on the energy of ^3He ions extracted from the AGS. Therefore the stable spin direction at the exit of the AtR line is not always matched with the stable spin direction of the circulating beam in RHIC at the injection point of RHIC. Three locations have been spotted in AtR suitable for introducing extra dipole magnets for correcting this spin direction mismatch while the trajectory is only locally perturbed. A schematic diagram of one of the peculiar sections of the

FIG. 1. The RHIC accelerator complex: The location of the AGS partial helices are shown at the A20 and E20 straight sections of the AGS. The AtR beam transfer line shown in the figure transports the beam from AGS to RHIC.

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FIG. 2. The first section of the AtR line where the beam is bent simultaneously along the vertical (blue rectangles) and horizontal (yellow rectangle) planes by 12.5 mrad and 15° respectively. The red or green rectangles represent proposed dipole magnets which can be placed at these locations along the AtR line to rotate the stable spin direction and match it to that at the RHIC injection point.

AtR line [8], is shown in Fig. 2 where the transferred beam undergoes a simultaneous 12.5 mrad vertical and a 15° horizontal bend. Note that the second vertical bend which brings the beam along the horizontal is 12.06 mrad and not 12.5 mrad because of the 15° horizontal bend. The other relevant section of the AtR line is shown in Fig. 3 where the simultaneous vertical and horizontal bends are 3.0 and 38.0 mrad respectively. This part of the line as shown in Fig. 3 contains additional, but smaller simultaneous vertical and horizontal bends [9] which for simplicity are not shown in Fig. 3 but are included in the calculations of the stable spin direction. In this paper we calculate the stable spin direction at the exit of the AtR line as a function of $G_{He}\gamma$, and we show that by the simple modification of the AtR line [10], shown in Figs. 2 and 3 it is possible to match the stable spin direction at the exit of the AtR line to that of the circulating beam at the injection point of RHIC.

FIG. 3. The second section of the AtR line where the beam is bent simultaneously along the vertical (blue rectangles) and horizontal (yellow rectangle) planes by 3.0 and 38.0 mrad respectively. The red rectangles represent proposed dipole magnets which can be placed at this location along the AtR line to rotate the stable spin direction and match it to that at the RHIC injection point.

II. STABLE SPIN DIRECTION IN A SYNCHROTRON OR BEAM LINE

Dealing with polarized beams is important to know the stable spin direction at any given point of the accelerator or beam transfer line, because the preservation of the beam polarization requires that the beam polarization vector is aligned along the stable spin direction of the accelerator or beam line. In the following two subsections, we will briefly describe the methods we use to calculate the stable spin direction along the trajectory of the reference particle circulating in a synchrotron or transported in a beam transfer line, and we will present the results for the stable spin direction as a function of $G\gamma$, at the extraction point of the AGS synchrotron, and at the exit point of the AtR transfer line.

A. The stable spin direction at the extraction point of the AGS

One of the methods to calculate the stable spin direction of the central particle in a synchrotron which employs partial helical magnets is the spinor method which applies to spin 1/2 particles, and we use it to calculate the stable spin direction in the AGS [11]. In the following subsections after a brief description of the AGS we present the results for the stable spin direction at the extraction point of the AGS as a function of $G\gamma$.

1. Brief description of the AGS

The AGS consists of 240 combined function magnets which are grouped into 12 superperiods each containing 20 magnets. Each superperiod is labeled with a capital letter starting with the letter “A” and ending with the letter “L”. The magnets in each superperiod are labeled with the letter of the superperiod followed by the number of the magnet in the sequence. For example the fifth magnet of the superperiod B is labeled as B05 and the straight section following the B05 magnet is labeled as SSB05. Although the 240 combined function main magnets of the AGS provide the required focusing to accelerate a stable circulating beam, in each superperiod there are two tune quadrupoles, one defocusing and one focusing at the straight sections SS03 and SS17 respectively to further control the betatron tunes. Two families of chromaticity sextupoles are placed at the straight sections SS05 and SS13. Other straight sections of the AGS are occupied by other devices like rf cavities and in particular the straight sections A20 and E20 are occupied by the cold and warm partial helical magnets respectively [2].

2. The stable spin direction of the AGS with two partial helical magnets

The stable spin direction at any point along the reference orbit of the AGS is given by the equation

$$\vec{b} = \left[\frac{i}{2 \sin(\phi/2)} \right] \text{Tr}(\vec{\sigma} M) \quad (1)$$

where $\sin(\phi/2)$ can be calculated from the expression

$$\cos\left(\frac{\phi}{2}\right) = \frac{1}{2} \text{Tr}(M). \quad (2)$$

The symbol ϕ in Eqs. (1) and (2) is the spin rotation angle about the stable spin direction $\vec{b}$ and the symbol M is the total spin rotation matrix of all the AGS main dipoles including the two partial helices of the AGS. The symbols $\vec{\sigma}$ and i in Eq. (1) represent the Pauli matrices and the imaginary unit respectively. The Eqs. (1) and (2) are derived in Ref. [12] and are also used in Ref. [11]. The components of the Pauli matrices $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ correspond to the radial, longitudinal, and vertical directions respectively. In each main magnet of the AGS the spin rotates about the vertical direction by an angle $G\gamma\theta$ where γ is the relativistic factor and θ is the angle of bend in the magnet. The spin rotation angles of the ^3He nuclei in the cold and warm helices have been calculated [13] by raytracing the ^3He nuclei in the 3D field map of the helices. These field maps were calculated by modeling the helices with the OPERA computer code [14]. The spin rotation angles δ_{ch} , and, δ_{wh} , of the cold and warm helices respectively have been calculated as a function of $|G\gamma|$ [13] and are given by:

$$\delta_{\text{ch}}(\text{deg}) = 26.831 + \frac{2.672}{|G\gamma|} - \frac{114.96}{|G\gamma|^2} + \frac{4144.0}{|G\gamma|^3} \quad (3)$$

$$\delta_{\text{wh}}(\text{deg}) = 24.839 + \frac{23.721}{|G\gamma|} - \frac{133.1}{|G\gamma|^2} + \frac{4172.7}{|G\gamma|^3}. \quad (4)$$

The directional cosine S_z of the stable spin direction with the vertical axis at the extraction point SSH10 of the AGS is calculated by using Eqs. (1) and (2) and is plotted in Fig. 4 as a function of $|G\gamma|$. The periodicity of S_z observed in Fig. 4 is related to the location of the two partial helices of the AGS ring which are separated azimuthally by 120° . Fig. 4 shows that the stable spin direction at the extraction point of AGS (SSH10) is almost along the vertical at half integer values of $|G\gamma|$. One may conclude that by extracting the polarized beam from the AGS at half integer values the beam will be injected into RHIC with vertical polarization vector. However this is not always true because of the simultaneous vertical and horizontal bends (Figs. 2 and 3) of the AtR line that causes the stable spin direction at the exit point of the AtR line to depend on the beam energy as shown in the next section. In the same section we will also describe a way of lining up the stable spin direction at the exit of the AtR line along the vertical axis.

FIG. 4. The directional cosine S_z of the stable spin direction at the extraction point of AGS (SSH10) as a function of $|G\gamma|$ in presence of two partial helices which are installed in the SSA20 and SSE20 straight sections and rotate the spin about the longitudinal axis by angles δ_{ch} , and, δ_{wh} respectively.

B. The stable spin direction at the RHIC injection point

We may use the spinor method to calculate the stable spin direction at the exit of the AtR transfer line. Unlike a synchrotron, the stable spin direction at the entrance of a beam transfer line may be not identical with that at the exit of the line, therefore we may use a slightly different approach which is described below. Knowing the stable spin direction $\vec{S}_{\text{entr}} = (S_x, S_y, S_z)_{\text{entr}}$ at the entrance of the transfer line we can calculate the spinor wave function Ψ_{entr} at this point. The spinor at the entrance of a beam line is the eigenfunction $\Psi_{\text{entr}} = \begin{pmatrix} a \\ b \end{pmatrix}$ of the equation $\vec{S}_{\text{entr}} \cdot \vec{\sigma} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ whose solution is

$$\begin{aligned} \Psi_{\text{entr}} &= \begin{pmatrix} a \\ b \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= -\frac{(S_y + iS_x)}{\sqrt{2(1 - S_z)}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{i(1 - S_z)}{\sqrt{2(1 - S_z)}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \end{aligned} \quad (5)$$

The proof of Eq. (5) appears in Appendix. Subsequently the stable spin direction at the exit of a transfer line $\vec{S}_{\text{exit}} = (S_x, S_y, S_z)_{\text{exit}}$ can be calculated by using the following relations:

$$\begin{aligned} \vec{S}_{\text{exit}} &= \Psi_{\text{exit}}^* \cdot \vec{\sigma} \cdot \Psi_{\text{exit}} \\ &= (\Psi_{\text{entr}}^* M_{2 \times 2}^T) \vec{\sigma} (M_{2 \times 2} \Psi_{\text{entr}}) \\ &= \Psi_{\text{entr}}^* (M_{2 \times 2}^T \vec{\sigma} M_{2 \times 2}) \Psi_{\text{entr}} \end{aligned} \quad (6)$$

$M_{2 \times 2}$ and $M_{2 \times 2}^T$ are the beam line's 2×2 spin transfer matrices and its transpose respectively. However in this paper we use a different approach to calculate the spin

vector. Namely from the knowledge of the spin rotation matrix $M_{2 \times 2}$ we generate the 3×3 spin rotation matrix $M_{3 \times 3}$ and subsequently transform the spin vector $\vec{S}_{\text{entr}} = (S_x, S_y, S_z)_{\text{entr}}$ at the entrance of the beam line to the spin vector $\vec{S}_{\text{exit}} = (S_x, S_y, S_z)_{\text{exit}}$ at the exit of the beam line using the transformation $M_{3 \times 3} \vec{S}_{\text{entr}} = \vec{S}_{\text{exit}}$. Figure 5 shows a plot of the directional cosine S_z of the stable spin direction at the exit of the AtR line or RHIC injection point (red trace) together with the stable spin direction at the AGS extraction point of the AGS, which is the entrance of the AtR line (blue trace). From the plot of Fig. (5) (red trace) we observe that for $|G\gamma| > 51.5$ the stable spin direction at the exit of the AtR line is no more along the vertical direction for beam energies corresponding to half integer values of $|G\gamma|$. This considerable mismatch between the stable spin direction at the exit point of the AtR line and that at the RHIC injection point for half integer values of $|G\gamma| > 51.5$ is due to the layout of the AtR line, namely to the simultaneous bending of the beam in the horizontal and vertical planes (see Figs. 2 and 3). Reference [9] provides a detailed description of the effect of the AtR line layout on the stable spin direction and Ref. [10] describes a method using the set of the triplet-dipole magnets at the location #2 shown in Figs. 2, to match the stable spin direction at the injection point of RHIC. In the next section we will show results on optimizing the stable spin direction at the exit point of the AtR line for three cases: (a) excite the three dipoles (green rectangles in Fig. 2) at the location #1, (b) excite the three dipoles (red rectangles in Fig. 2) at the location #2, and (c) excite all six dipoles (green and red rectangles in Fig. 2) at the locations #1 and #2. The insertion of any of these sets of three dipoles does not affect the beam optics of the AtR beam line as we briefly discuss in the next section which mainly deals with the stable spin direction at the RHIC injection point for any half integer value of $|G\gamma|$ within the range of $51.5 < |G\gamma| < 64.5$ for the cases (a), (b) and (c) mentioned above.

III. MATCHING THE STABLE SPIN DIRECTION AT EXIT OF AtR AND INJECTION POINT OF RHIC

In Fig. 5 we plot the directional cosine with the vertical S_z of the stable spin direction (red trace) at the exit point of the AtR line (RHIC injection point) as a function of $|G\gamma|$. Since the injection energies into RHIC of any polarized species correspond to half integer values of $|G\gamma|$, we only consider the transfer of polarized beams at such injection energies. In the next subsections we discuss a modification of the AtR line that matches the stable spin direction at the exit point of the AtR line to that of the RHIC's circulating beam at the RHIC injection point. In brief this modification consists in inserting horizontally bending triplet-dipole magnets, otherwise known as a “chicane”, at specific

FIG. 5. The directional cosine S_z of the stable spin direction at the AGS extraction point (blue trace) and at the RHIC injection point (red trace) as a function of $|G\gamma|$. There is considerable mismatch between the stable spin direction at the exit of the AtR line and that at the RHIC injection point for $|G\gamma| > 51.5$.

locations of the AtR line where the horizontal dispersion is zero. Such a chicane will act on the beam optics almost as a drift element by locally deflecting the beam horizontally and restoring it back to its original direction without affecting significantly the optics of the beam. Any small changes in the beam optics, if any, due to any small focusing effect of these dipoles can be corrected by the main quadrupole of the line.

A. Excitation of the three dipoles at location #1

In this subsection we show results of the stable spin direction at the exit point of the AtR line when we excite the three horizontally bending dipoles at the location #1, shown in Fig. 2 as green rectangles. In Fig. 6 the black circles correspond to the vertical component S_z of the stable spin direction at the exit of the AtR line when the three dipoles are not excited. The red squares correspond to the stable spin direction when the three dipoles are excited. The green diamonds show the needed integrated field. Here and in the Figs. 7 and 8 the integrated field is divided by the factor 2.5, 2.5 Tm being the maximum allowed integrated field $\int B \cdot d\ell$ of the dipole. This limit causes some S_z values of the stable spin directions, like at $|G\gamma| = 56.5$ to not exceed the value of 0.85. However the S_z values for most of the stable spin directions at the exit of the AtR line is ≥ 0.98 . The excitation of the three dipoles at the location #1 shown in Fig. 2 does not alter the beam optics of the AtR line.

B. Excitation of the three dipoles at location #2

The results of this study, with the three horizontally bending dipoles at the location #2 (red rectangles in Fig. 2) excited, are shown in Fig. 7 as red squares. Again the

FIG. 6. The stable spin direction at the exit of the AtR line as a function of half integer $|G\gamma|$ values. The black circles and the red squares correspond to the S_z values when the three dipoles at the location #1 (green rectangles in Fig. 2) are not excited or excited respectively. The green diamonds is the integrated dipole field $\int B \cdot d\ell/2.5$ at the location #1.

maximum values of the excitation strengths $\int B \cdot d\ell/2.5$ of each of the dipoles is kept below the 2.5 [T.m] therefore due to this limit of the integrated field some of the values of S_z like at $|G\gamma| = 60.5$ do not exceed 0.85. However the S_z values for most of the stable spin directions is ≥ 0.98 . Again the excitation of the three dipoles at the location #2 does not affect the beam optics of the AtR line.

C. Excitation of #1 and #2 dipole triplets

By exciting both sets of dipoles at the locations #1 and #2, (green and red rectangles shown in Fig. 2) we can align

FIG. 7. The stable spin direction at the exit of the AtR line as a function of half integer $|G\gamma|$ values. The black circles correspond to the S_z values when the three dipoles at the location #2, shown in Fig. 2 as red rectangles, are set to zero and the red squares when the set of three dipoles are excited. The green diamonds is the integrated dipole field $\int B \cdot d\ell/2.5$ at the location #2.

the stable spin direction at the exit of the AtR line along the vertical, ($S_z > 0.98$) for all the beam energies in the range of $51.5 \leq |G\gamma| \leq 64.5$ except that for the energy corresponding to $|G\gamma| = 60.5$ where $S_z \approx 0.9$. Figure 8 shows the directional cosines of the spin vector with the vertical S_z (red squares) when both sets of dipoles at the locations #1 and #2 are excited. The black circles are the directional cosines (S_z) when such dipoles are not excited. The green diamonds and the blue triangles in Fig. 8 are the values of the integrated dipole field $\int B \cdot d\ell/2.5$ for the dipoles at the locations #1 and #2 respectively. To align also the spin along the vertical for the energy corresponding to $|G\gamma| = 60.5$ we have to excite also the set of the three horizontally bending dipoles shown in Fig. 3. With all three sets of three-dipoles excited, the vertical component of the stable spin direction becomes $S_z > 0.985$ for all the beam energies in the range of $51.5 \leq |G\gamma| \leq 64.5$, and the corresponding maximum strength of the dipoles at the locations #1, #2, and #3, is 2.00, 1.90, and 1.85 [T.m] respectively.

D. Calculations of the stable spin direction using the ZGOUBI computer code

All the calculations on the stable spin direction mentioned in this paper have also been performed using the stepwise ray-tracing computer code ZGOUBI [15] which integrates numerically the Lorentz and BMT [7] equations. In these calculations we have also included the quadrupole component of all the combined-function magnets of the AtR line [8] as well as the fringe fields of all the magnets included in the calculations. The results of the stable spin

FIG. 8. The stable spin direction (red squares) at the exit of the AtR line as a function of half integer $|G\gamma|$ values when both sets of dipoles at the locations #1 and #2, (green and red rectangles shown in Fig. 2) are excited. The black circles correspond to the S_z values when both sets of the three dipoles at the locations #1 and #2 shown in Fig. 2 are set to zero. The green diamonds and the blue triangles are the values of the integrated dipole field $\int B \cdot d\ell/2.5$ for the dipoles at the locations #1 and #2 respectively.

direction as derived from the ZGOUBI computer code were compared to those using the spinor method and the differences were well within the experimental error of measuring the polarization vector. This indicates that the quadrupole components of the combined-function magnets and the fringe fields of the AtR magnets do not affect significantly the stable beam direction at the exit of the AtR line.

IV. CONCLUSION

By employing in the AtR beam transfer line one set of three dipole magnets we can align the stable spin direction of $^3\text{He}^{+2}$ ions at the exit of the AtR line (RHIC injection point) along the vertical ($S_z > 0.95$) in the energy range $51.5 \leq |G\gamma| \leq 64.5$. The excitation of additional sets of three-dipoles placed along the AtR line provides an almost complete matching (99%) of the stable spin direction for any possible beam injection energy into RHIC. The results from similar calculations using the computer code ZGOUBI [15] which uses the BMT equation [7] and includes the fringe field of the magnets are in good agreement with the spinor method we use in this paper. The agreement indicates that fringe fields do not affect significantly the stable spin direction at the exit of the AtR line.

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APPENDIX: PROOF OF EQUATION (5)

Assume the spinor

$$\Psi = \begin{pmatrix} a \\ b \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (\text{A1})$$

with the normalization condition

$$aa^* + bb^* = 1 \quad (\text{A2})$$

We want to express the coefficients a and b of the spinor Ψ in terms of the spin directional cosines S_x , S_y , S_z .

Let the components of the Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

correspond to the radial, longitudinal, and vertical directions respectively. The spin directional cosines are given by:

$$\begin{aligned} \vec{S} = S_x \hat{x} + S_y \hat{y} + S_z \hat{z} &= (a^*, b^*) (\sigma_x \hat{x} + \sigma_y \hat{y} + \sigma_z \hat{z}) \begin{pmatrix} a \\ b \end{pmatrix} \\ &= (a^*, b^*) \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \hat{x} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \hat{y} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \hat{z} \right] \begin{pmatrix} a \\ b \end{pmatrix} \\ &= (a^*b + b^*a) \hat{x} - i(a^*b - b^*a) \hat{y} + (a^*a - b^*b) \hat{z} \quad (\text{A3}) \end{aligned}$$

From Eq. (A3) we obtain:

$$S_x = (a^*b + b^*a) \quad (\text{A4})$$

$$S_y = -i(a^*b - b^*a) \quad (\text{A5})$$

$$S_z = (a^*a - b^*b). \quad (\text{A6})$$

With no loss of generality we assume b to be imaginary. Thus with $b^* = -b$ and Eqs. (A2) and (A6) becomes:

$$a^*a = 1 + b^2 \quad (\text{A7})$$

$$S_z = (a^*a + b^2). \quad (\text{A8})$$

From Eqs. (A7) and (A8) we obtain Eq. (A9).

$$b = i\sqrt{\frac{1 - S_z}{2}} = i\frac{1 - S_z}{\sqrt{2(1 - S_z)}}. \quad (\text{A9})$$

From Eqs. (A4) and (A5) we obtain Eq. (A10)

$$S_y + iS_x = 2ib^*a. \quad (\text{A10})$$

and with Eq. (A9) we obtain

$$a = \frac{S_y + iS_x}{\sqrt{2(1 - S_z)}}. \quad (\text{A11})$$

Thus Eq. (A1) is written as:

$$\Psi = \frac{S_y + iS_x}{\sqrt{2(1 - S_z)}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i\frac{1 - S_z}{\sqrt{2(1 - S_z)}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (\text{A12})$$

and the coefficients a , b of the spinor in Eq. (A1) are expressed by the spin directional cosines.

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