Case study of a magnetic system for low-energy machines

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The extra low-energy antiproton ring (ELENA) is a CERN particle decelerator with the purpose to deliver antiprotons at lowest energies aiming to enhance the study of antimatter. The hexagonal shaped ring with a circumference of about 30 m will decelerate antiprotons from momenta of 100 to 13.7 MeV/c. In this paper, the design approach for a magnet system for such a machine is presented. Due to the extra-low beam rigidity, the design of the magnet system is especially challenging because even small fields, arising for example from residual magnetization and hysteresis, have a major impact on beam dynamics. In total, seven prototype magnets of three different magnet types have been built and tested. This paper outlines challenges, describes solutions for the design of the magnet system and discusses the results of the prototypes.

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I. INTRODUCTION

Beams with extra-low beam rigidity make the design of the magnet system especially challenging because even very small fields, arising for example from residual magnetization, hysteresis, or high-frequency field harmonics from power converters, have a major impact on beam dynamics. Therefore, the magnet design for low-energy machines has to aim for the best possible field quality, and for a reduction of the remanent field effects to allow for reliable and reproducible operation at low field. In this paper, a case study for the extra low-energy antiproton ring (ELENA) magnet system is presented, which is providing insight in the design principles of magnets and material selection for low-energy machine magnets. The ELENA ring will decelerate antiprotons delivered by the CERN antiproton decelerator (AD) [1,2] with energies of 5.3 MeV to 100 keV or, if expressed in momenta, from $p = 100$ MeV/c to $p = 13.7$ MeV/c. The AD was constructed after the completion of the exploitation of the low-energy antiproton ring (LEAR) [3,4] and currently provides 5.3 MeV energy antiprotons for experiments. For most experiments the beam is decelerated by using degrader foils or a decelerating radio frequency quadrupole to allow for capturing the antiprotons in traps, usually operated with a trapping voltage of 10 kV. Both deceleration processes produce significant antiproton losses, leading to an overall trapping efficiency of less than 1%. The ELENA project aims for constructing a small 30.4 m circumference synchrotron to improve the trapping efficiencies of existing experiments by 1 to 2 orders of magnitude. Therefore, the

deceleration is performed in a small synchrotron and the emittance is reduced with an electron cooler [5]. The increased efficiency and lower extraction energy also allows for new types of experiments. ELENA will allow for sending several (according to the baseline four) bunches with lower intensity to several experiments which leads to an increase of the available beam time for each experiment. The main parameters of ELENA are given in Table I and a sketch of the machine is shown in Fig. 1. The ring will be of hexagonal shape and consists of six C-shaped bending magnets, 12 quadrupoles, two skew quadrupoles, four sextupoles, eight two-plane correctors, and two compensation solenoids. A detailed design report on ELENA and the magnets can be found in [6–13].

II. DIPOLES

Each dipole magnet has to provide a bending angle of $\alpha = 360/6$ deg = 60 deg. An optimization of the beam dynamics [6] shows first, that for a bending radius of $\rho = 927$ mm, the resulting focusing can be handled with a combined function magnet; and second that the footprint of the machine is small enough to fit into the available space in the existing AD building [6]. The injection momentum is $p = 100$ MeV/c, leading, for the given magnetic length per dipole of $L = \frac{2\pi\rho}{6} \approx 970.8$ mm, to a required nominal magnetic field of $B = \frac{3.3356p}{\rho} \approx 0.36$ T. Having larger magnetic fields and smaller bending radii is excluded due to the resulting strong focusing. The focusing is compensated by having a combined function magnet, i.e., by having a combined dipole and quadrupole field. The parameters of the ELENA ring dipole magnet are summarized in Table II. The different design options which have been explored are summarized in Fig. 2.

It is important to note that in this paper only normal-conducting magnets are considered as they can meet the

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TABLE I. ELENA machine and beam parameters [5].

Parameter	Value	Unit
Momentum range	100–13.7	MeV/c
Kinetic energy range	5.3–0.1	MeV
Machine tunes H/V^a	2.3/1.3	
Circumference	30.4	
Repetition rate ^b	≈ 100	s
Injected beam population	3×10^7	
Ejected beam population	1.8×10^7	
Number of extracted bunches ^c	4	
$\Delta p/p$ of extracted bunches (95%) ^d	2.5×10^{-3}	
Bunch length at extraction ^d	1.3/300	m/ns
Emittance (H/V) at extraction (95%) ^d	6/4	$\pi\mu\text{m}$
Nominal (dynamic) vacuum pressure	4×10^{-10}	Pa

^aWith sufficient tuning range e.g. to avoid resonances.^bLimited by the AD repetition rate; the expected ELENA cycle length is ≈ 25 s.^cLess extracted bunches is an option leading to slightly larger emittances and momentum spreads.^dPresent best guesses based on simulations.

specifications. Superconducting magnets would potentially reduce the power consumption but are more complex [14]. For a small low-energy machine operated with normal-conducting magnets the overall power consumption is relatively small.

A. Design

In coil-dominated iron-free magnets no hysteresis effects are present. Such magnets would ease the operation of ELENA. To study if such magnets are within reach an estimate of the coil size and the power consumption is given below. Let us assume a sector coil with a coil width $w = r_{\text{out}} - r_{\text{in}}$ [14,15] (see Fig. 3). With $w = \frac{B\pi}{2\mu_0 J \sin \phi}$, $J = 5 \text{ A/mm}^2$, $\phi = 60^\circ$, and $B = 0.36 \text{ T}$ one finds a coil width of $w = 104 \text{ mm}$. The coil mass accounts to $m = \frac{1}{3}\pi l \rho ((R+w)^2 - R^2) = 375 \text{ kg}$, where the aperture radius $R = 40 \text{ mm}$, $\rho = 8950 \text{ kg/m}^3$ and the average coil

FIG. 1. Sketch of the ELENA ring.

TABLE II. Parameters of ELENA ring bending magnets.

Parameter	Value	Unit
Number	6	
Field	0.36 to 0.05	T
Pole iron gap	76	mm
Bending angle	60°	
Radius	927	mm
Magnetic length	970.8	mm
Edge angle	17°	
Ramping speed (up)	0.37	T/s
Ramping speed (down)	0.05	T/s
Field homogeneity	$\pm 2 \times 10^{-4}$	
Good field region	$66(\text{H}) \times 48(\text{V})$	mm^2

length $l = 2.2L = 2.1 \text{ m}$. The power consumption $P = \rho_{\text{Cu}} l J^2 [(R+w)^2 - R^2] = 17 \text{ kW}$, with $\rho_{\text{Cu}} = 1.7 \times 10^{-8} \Omega\text{m}$. Reducing the current density $J = 2 \text{ A/mm}^2$ (considered the most cost efficient design for normal-conducting magnets [16,17]) would yield to $w = 260 \text{ mm}$, $m = 1740 \text{ kg}$, and $P = 13 \text{ kW}$. This calculation shows that coil-dominated normal-conducting dipole magnets are within the reach. However, coil-dominated magnets cause challenges—for example—due to the required precise winding operations to reach the required field quality, and the limited access to the vacuum chamber. By using a careful design and selecting a well-suited yoke material, iron-dominated magnets operated with repetitive cycles can meet the specifications, as described in the following sections. Therefore, iron-dominated dipole magnets are chosen. However, for magnets with very small field ($\int B dl = 2.5 \times 10^{-3} \text{ Tm}$), less-stringent requirements on the field quality ($\pm 1\%$) and nonrepetitive cycles, such as the horizontal/vertical (H/V) corrector magnets, coil-dominated magnets remain an interesting option (see Sec. IV).

In iron-dominated magnets the power consumption at nominal field is $P = \frac{1}{\mu_0} \rho_{\text{Cu}} l J B_y g \approx 2 \text{ kW}$, with the coil length $l = 2.6 \text{ m}$, $J = 2 \text{ A/mm}^2$ and a gap $g = 76 \text{ mm}$. The field quality can be accurately controlled by the pole shape. Hysteresis effects can be minimized and handled by selecting a suitable yoke material and using repetitive cycles. Also, the design of combined function iron-dominated dipole magnets is straightforward by either modifying the pole profile [18] or by introducing an edge

FIG. 2. Design options for dipole magnets.

FIG. 3. Sketch of a sector coil generating an approximate dipole field.

angle at the extremities. Here, the second option is preferred as it allows for easy beam optics optimization, easy adaptation of the electromagnetic design and manufacture. With this option, the focusing can be efficiently handled by introducing an edge angle of 17 deg [6].

Three different design options for normal-conducting iron-dominated dipoles are considered [19]: (1) H-shaped dipoles, (2) O-shaped, also often called window-frame, dipoles or (3) C-shaped dipoles. If no access to the vacuum chamber is required, H-dipole magnets or window-frame magnets are the preferred choice, as their fourfold symmetry allows only odd multipoles ($B_1, B_3, \dots, B_{2n+1}$) compared to C-shaped dipoles with a twofold symmetry allowing even and odd multipoles ($B_1, B_2, \dots, B_n$). Window frame magnets are usually only used for corrector magnets due to their need of having either bedstead coils or a double number of ampere turns resulting in doubling the consumption in addition to a larger external field. For ELENA C-shaped dipole magnets have been chosen because they allow access to the vacuum chambers. This access is required to reach a vacuum pressure in the order of 10^{-10} Pa to limit interactions with residual gas, for minimizing losses and for reducing blowup of the beam emittance. To reach such a vacuum pressure numerous pumping ports are required, which need to be placed in the dipole section as the dipoles cover a substantial part of the machine.

A curved dipole magnet is envisaged (see Fig. 4), with the parameters given in Table II, to limit the pole and therefore the magnet size. The sagitta is $(1 - \cos \theta)\rho = 124$ mm and therefore a straight magnet would be too large. To achieve a stable and excellent field quality and to increase the overall mechanical stability of the yoke, despite an edge angle of 17 deg, a wide backleg (364 mm) is foreseen. A combined structural-magnetic finite-element analysis with the mentioned conditions confirmed that the closing of the gap due to the magnetic pressure of $p = \frac{B^2}{2\mu_0} = 0.05 \dots 0.6$ MPa has a negligible influence on the field quality.

The pole profile (see Fig. 5) is optimized in such a way that the variation of the field quality over the full dynamic range remains in the order of 10^{-5} . End shims with five 45 deg chamfers are adapted, so that the integrated field

FIG. 4. Picture of ELENA preseries magnet during magnetic measurements.

quality in the magnet is better than $\pm 2 \times 10^{-4}$. A detailed report on the design is given in [7]. Magnetic measurements [20,21] on the series magnets show that the specifications are met without the need to perform experimental trimming of the shimming configuration to meet the field quality requirements.

Furthermore, the variation of the quadrupole field component over the dynamic range—a variation which is typically observed in C-shaped dipoles—shall be minimized. In order to achieve a small variation the anisotropic properties of the steel I (see Sec. II B) are exploited by choosing the easy magnetization direction (in rolling direction) to be parallel to the pole face. By this choice the flux is more equally distributed over the full pole width and the quadrupole variation is limited to around 2×10^{-4} . Such an approach also has the advantage that the mother coil with a width of 1200 mm can be almost fully used (lamination size is 1022 mm $\times$ 774 mm).

B. Yoke material selection

The dynamic range in ELENA is $p_{\text{inj}}/p_{\text{ej}} = 7.3$. Therefore, very low field levels ($B = 0.05$ T) are required during extraction. This may cause strong nonlinearities in the transfer function and changes in the field quality due to hysteresis effects, which cannot be easily modeled with commercial software packages and make the operation of an accelerator challenging. To minimize these effects a material with low coercive force H_c is desirable. Candidate materials are: pure iron, grain-oriented (GO) and nongrain oriented (NGO) steel, Fe-Ni and Fe-Co alloys. Fe-Ni and Fe-Co alloys are excluded, despite their very large permeability and low coercive force, due to their high-cost and, in the case of Fe-Co, also due to the expected high

FIG. 5. Cross section and pole profile of ELENA dipole magnet.

activation rate of Co, which makes this material an undesired choice in radiation areas [22]. In addition, Fe-Ni and Fe-Co alloys require annealing and careful handling. The use of GO steel is investigated in [23], and for the ELENA quadrupoles in Sec. III. The reported results show that the gain of using GO instead of NGO steel is small. Moreover, GO steel is only commercially available in lamination thickness ≤ 0.35 mm, making the construction more costly due to the increased number of laminations compared to thicker laminations (see Sec. II B 1 for the selection of the lamination thickness). The study of the yoke material therefore suggests the use of a NGO steel, which has been selected for the ELENA bending magnets. A compilation of commercially available magnetic materials for normal-conducting magnets is given in [17,24].

In NGO electrical steel, the coercive force is linked to the grain size, for grain sizes $D \gtrsim 1 \mu\text{m}$ the coercive force $H_c \propto 1/D$ [25]. Therefore, a commercially available grade with a large grain size is selected. In general, a larger Si content results in larger grain sizes, and therefore smaller

coercive force values [26]. Here, Isovac 270-50 A HP (see Sec. II B 1 for the selection of the lamination thickness) from the producer VoestAlpine, Austria, is selected, which has a Si content of around 2.33%. This steel has been used for the construction of the dipole prototypes and is therefore called in the following steel I.

The grain size number of steel I was determined according to ASTM E112-13 with 3.1 (average diameter 0.12 mm) in the cross section and 4.2 (average diameter 0.08 mm) on the surface. Thanks to its large grain size, steel I shows a high permeability and a low coercive force. This steel is delivered fully annealed. However, laminations of the dipole are punched, so cold work is applied at the cutting edge of each lamination, which may alter the magnetic properties along the vicinity of the cutting edge, as shown in [27]. However, as the cutting is localized and relatively homogeneously this effect is deemed to be uncritical for the overall performance of the ELENA dipoles.

The electrical resistivity $\rho = 5.2 \times 10^{-7} \Omega\text{m}$ of this steel is around 5 times larger than the electrical resistivity of pure iron. Therefore, eddy current formation is limited. The chemical composition of the electrical steel is considered uncritical for use in radiation areas. It is worth noting that the electrical steel supplier ensures a carbon content lower than 50 ppm to minimize aging of the electrical steel BH characteristics over time [28–30]. The requirements of the dimension are tightly specified in order to be able to achieve high quality stacks (see Table III). The manufacturer guaranteed and achieved these dimensions.

1. Selection of lamination thickness

The choice of the lamination thickness is usually determined by the ramp rate of the magnet and economic considerations. Thinner laminations reduce the amount of induced eddy currents, which is usually desired because the magnetic field lags behind the applied current due to eddy currents. Moreover, higher-order harmonics from the power converter may be damped by induced eddy currents, which might be desired. The damping is larger the thicker the laminations are. In case of ELENA, the power converter's higher harmonics are multiples of 6.5 kHz. The cutoff frequency of the magnet (a frequency at which the current higher harmonics are damped considerably) is

TABLE III. Dimensions of the electrical steel I.

Parameter	Value	Unit
Nominal thickness of steel strips	0.50	mm
Maximum deviation from nominal thickness	± 0.02	mm
Maximum thickness variation in rolling direction over 2 m length	0.004	mm
Maximum thickness variation perpendicular to rolling direction over a width of 1100 mm	0.007	mm

$$f_c = \frac{1}{\pi a^2 \mu_r \mu_0 \sigma} \approx 1 \text{ kHz}, \quad (1)$$

with the lamination thickness $d = 2a = 0.5$ mm, the relative permeability $\mu_r = 2000$, and the conductivity $\sigma = 1.9 \times 10^6$ S/m. An analytical study of due to field ripples transverse emittance blowup is presented in [31]. This study took preliminary values into account and revealed that—with the power converters used for ELENA—with attenuation by vacuum chambers the transverse blowup is probably negligible. The prototype magnet (see Sec. II C) has been used to measure the cutoff frequency. Measurements confirmed that the cutoff frequency is about 1 kHz [32].

In reality, no hard criteria for the selection of the lamination thickness for a slowly ramped machine, as ELENA, can be provided. However, as the choice is made between considerations for good damping and cheaper construction—calling both for thicker lamination thickness—and reduced eddy current effects—calling for thinner laminations—this commercially available steel represents an adequate choice.

2. Impact of variation of steel properties on magnet performance

Three mother coils were produced. The coercive force and the permeability were measured at the beginning, middle and end of each mother coil in parallel ($\parallel$) and perpendicular ($\perp$) to the rolling direction and using mixed (mix) samples. The average value and the standard deviation for different excitation levels are given in Table IV. To study whether the reported variation of the steel parameters is significant to the magnet design, we use in a first approximation the following analytical equation:

$$B_y = \mu_0 \frac{NI}{g + \frac{l_{\text{iron}}}{\mu_r}}, \quad (2)$$

with $g = 76$ mm, $NI = 25$ kA and $l_{\text{iron}} = 1.2$ m. Using these assumptions, one finds that B_y changes by several 10^{-4} as a function of $\mu_r \approx \mu_{\text{mix}}$ (see Table IV), which is not acceptable in a ring accelerator. To limit this variation, shuffling is performed, as done usually in magnet productions. To perform shuffling the steel production is separated in n batches (for ELENA $n = 3$) of relatively equal magnetic properties—based on magnetic measurements. These batches are then color coded by marking them with n different colors to be able to track the shuffling. Subsequently, about 10–20 laminations are distributed on each of the m piles for the magnet yoke production (for ELENA $m = 10$). This procedure is repeated with the color-coded lamination types 2, 3, ..., n , 1, 2, 3, ... until all laminations required for the production are placed on the m piles for the yoke production.

Moreover, the field quality is evaluated by using the BH curves with the measured minimum and maximum permeability. For the calculation of the field quality variation a 2D finite-element-method (FEM) code is used. Here, it is found that the relative field quality $b_n = B_n/B_1$ varies by $\Delta b_n < 10^{-5}$ as a result of the varying steel properties. This variation is considered negligible, as it is $< 5\%$ of the available field quality budget.

C. Prototypes

Two straight prototype yokes in 1:1 scale have been built, as shown in Fig. 6, to study the overall performance of the magnets employing the steel selected above. In one of the two prototypes, the 0.5 mm electrical steel laminations are interleaved with 1 mm thick nonmagnetic stainless steel laminations, which will be referred to as “diluted magnet.” The other prototype employs only 0.5 mm electrical steel laminations, which will be referred to as “nondiluted magnet.” Dilution allows one to increase the working magnetic induction in the iron. Taking this effect into account, one might assume the magnet becomes less sensitive to differences in the quality of the iron itself

TABLE IV. Parameters of electrical steel.

H A/m	$J_{\parallel}$ T $\pm$ mT	$J_{\perp}$ T $\pm$ mT	J_{mix} T $\pm$ mT	$\mu_{\parallel}$	$\mu_{\perp}$	μ_{mix}	$H_{c,\parallel}$ A/m	$H_{c,\perp}$ A/m	$H_{c,\text{mix}}$ A/m
20	0.378 $\pm$ 27	0.066 $\pm$ 4	0.114 $\pm$ 5	14705 $\pm$ 1059	2610 $\pm$ 163	4505 $\pm$ 216	14.6 $\pm$ 0.9	9.1 $\pm$ 1.1	10.6 $\pm$ 0.9
50	0.986 $\pm$ 11	0.405 $\pm$ 27	0.677 $\pm$ 9	15664 $\pm$ 171	6414 $\pm$ 420	10720 $\pm$ 145	20.4 $\pm$ 0.7	23.3 $\pm$ 1.0	23.1 $\pm$ 0.5
100	1.251 $\pm$ 5	0.920 $\pm$ 24	1.099 $\pm$ 11	9976 $\pm$ 37	7319 $\pm$ 193	8744 $\pm$ 86	23.0 $\pm$ 0.7	30.7 $\pm$ 0.5	27.4 $\pm$ 0.3
150	1.339 $\pm$ 5	1.134 $\pm$ 12	1.240 $\pm$ 7	7114 $\pm$ 24	6026 $\pm$ 71	6589 $\pm$ 35	23.9 $\pm$ 0.7	33.3 $\pm$ 0.6	28.7 $\pm$ 0.3
250	1.413 $\pm$ 4	1.276 $\pm$ 4	1.343 $\pm$ 4	4507 $\pm$ 11	4071 $\pm$ 15	4282 $\pm$ 14	24.5 $\pm$ 0.7	34.5 $\pm$ 0.7	29.6 $\pm$ 0.3
499	1.487 $\pm$ 4	1.379 $\pm$ 2	1.430 $\pm$ 3	2372 $\pm$ 6	2201 $\pm$ 3	2280 $\pm$ 5	24.2 $\pm$ 0.7	34.9 $\pm$ 0.8	29.7 $\pm$ 0.4
998	1.553 $\pm$ 4	1.455 $\pm$ 1	1.502 $\pm$ 3	1237 $\pm$ 2	1161 $\pm$ 1	1198 $\pm$ 2	24.5 $\pm$ 0.7	35.2 $\pm$ 0.8	30.0 $\pm$ 0.4
1498	1.592 $\pm$ 4	1.498 $\pm$ 1	1.545 $\pm$ 2	846 $\pm$ 2	796 $\pm$ 1	821 $\pm$ 1	24.8 $\pm$ 0.7	35.6 $\pm$ 0.7	30.2 $\pm$ 0.4
2496	1.648 $\pm$ 4	1.556 $\pm$ 1	1.602 $\pm$ 2	525 $\pm$ 1	496 $\pm$ 0	511 $\pm$ 1	24.7 $\pm$ 0.7	35.5 $\pm$ 0.8	30.1 $\pm$ 0.4
4991	1.735 $\pm$ 3	1.650 $\pm$ 2	1.693 $\pm$ 2	277 $\pm$ 1	263 $\pm$ 0	270 $\pm$ 0	25.0 $\pm$ 0.8	35.7 $\pm$ 0.7	30.2 $\pm$ 0.4
10034	1.841 $\pm$ 4	1.773 $\pm$ 2	1.806 $\pm$ 3	146 $\pm$ 1	140 $\pm$ 1	143 $\pm$ 0	25.1 $\pm$ 0.7	36.0 $\pm$ 0.7	30.5 $\pm$ 0.3
14966	1.902 $\pm$ 3	1.853 $\pm$ 2	1.878 $\pm$ 2	101 $\pm$ 0	99 $\pm$ 0	100 $\pm$ 0	24.9 $\pm$ 0.8	35.7 $\pm$ 0.8	30.4 $\pm$ 0.2

FIG. 6. Picture of prototype. Two geometrically identical prototype magnets have been built, one with a diluted and one with a nondiluted yoke.

and in particular to the coercive force as—for example—explored for the Large Hadron Electron Collider (LHeC) project [23]. Dilution was also applied in the Large Electron-Positron Collider (LEP) magnets [33,34] at CERN to save relatively expensive electrical steel which was replaced by low-cost mortar [35,36].

1. Comparison between diluted and nondiluted yoke design

The flux density in the iron of a diluted magnet can be calculated according to (neglecting stray fields)

$$B_{\text{gap}}A_{\text{gap}} = B_{\text{yoke}}A_{\text{yoke}}. \quad (3)$$

By calculating the average flux density in the tangential direction of the lamination, one finds the effective permeability for a diluted yoke as follows:

$$\mu_r^d = \lambda\mu_r + (1 - \lambda) \approx \lambda\mu_r; \quad (4)$$

where the iron fraction $\lambda = d/(d + b)$ with the lamination thickness d and a nonmagnetic filler material with thickness b . Using this modified permeability μ_r^d the equations for nondiluted magnets can be used.

Initially, in the project the average relative permeability $\bar{\mu}_r$ over the whole working range of the material was maximized $\bar{\mu}_r = \frac{1}{J_{\text{max}} - J_{\text{min}}} \int_{J_{\text{min}}}^{J_{\text{max}}} \mu_r(J) dJ$. In order to achieve this maximizing effect, the effective surface area of the yoke, A_{yoke} , with respect to the effective area of the gap, A_{gap} , has been decreased, by spacing the laminations in the yoke with low-permeability material. Here, the maximum polarization can be expressed in terms of the minimum polarization and the dynamic factor as $J_{\text{max}} = \frac{0.36\text{T}}{0.05\text{T}} J_{\text{min}} = 7.2J_{\text{min}}$, with $J = B - \mu_0 H$. For future projects the effective permeability for a diluted yoke μ_r^d shall be used for this calculation.

FIG. 7. Average permeability of steel I for different iron fractions by using measurement results performed with mixed samples.

Figure 7 presents the average permeability in the iron over the working range as a function of the iron fraction λ , equivalent to the iron percentage. The maximum iron permeability can be found for an iron fraction of around $\lambda = 0.35$. For practical purposes an iron fraction $\lambda = 0.33$ with 0.5 mm thick electrical steel (steel I) and 1 mm thick stainless steel is chosen. The average, minimum and maximum permeability for the constructed diluted and nondiluted magnet yokes over the load cycle are summarized in Table V (values are calculated with the steel used for the construction of the prototype). These values are calculated by using measurement results performed with mixed samples. The ratio between the largest and smallest relative permeability in the course of the load cycle is around 1.8 for a diluted yoke compared to 2.2 for a nondiluted yoke and the average relative permeability of the diluted yoke is around 40% larger than the one for the nondiluted yoke. For this study the permeability of macroscopic electrical steel samples at small fields down to 0.02 T (for mixed samples) was measured with a dedicated Epstein frame measurement setup at CERN [37].

2. Finite-element simulation of dilution

Simulation techniques for nondiluted magnets are well established. For diluted magnets the following approaches have been studied: (1) increasing the current by the iron fraction, (2) diluting the BH curve $B_d = (\mu_0 H) + \lambda(B - \mu_0 H)$, and (3) modeling individual laminations.

TABLE V. Average, minimum and maximum permeability for nondiluted and diluted magnet yokes calculated by using measurement results performed with mixed samples.

Description	Nondiluted	Diluted
$\bar{\mu}_r$	5300	7500
$\mu_{r,\text{max}}$	7100	8500
$\mu_{r,\text{min}}$	3200	4700

The first two options can be performed by 2D and 3D simulations, the latter only with 3D simulations. In the case of 2D simulations, the first two methods have been exploited: (1) increasing the current or (2) diluting the BH curve. The difference between the two is small, but non-negligible: The multipole b_3 is different by around 1.5×10^{-4} . Also, the transfer function is affected. For example, at a field in the gap of 0.4 T, the efficiency $\eta = B/(\mu_0 \frac{NI}{g})$ in the prototype is reduced to 86% compared to simulated efficiencies of 97.6% and 94.7% with simulations with increased current and a diluted BH curve, respectively (discussion see later). The field quality in the prototype is affected by mechanical tolerances, which are kept larger than usual to allow for a more economic and faster construction. Therefore, no conclusion on the tiny differences between both 2D simulation methods can be drawn. However, thinning is represented by $B_d = (\mu_0 H) + \lambda(B - \mu_0 H)$, which is equivalent to a decreased permeability $\mu_r^d \approx \lambda \mu_r$ and therefore diluting the BH curve is the correct choice.

In the case of 3D simulations, all three options have been investigated and it has been found that a 3D simulation using a packing factor representing the dilution results in a fairly precise prediction of the magnetic length of the prototype magnet for fields larger than 0.3 T. This method predicts the early saturation effects as it takes the strongly reduced permeability in longitudinal direction of the yoke into account, but not the linear decrease at smaller field levels. The effective permeability normal to the lamination in a diluted magnet is reduced to $\mu_z = \frac{\mu_r}{\lambda + (1-\lambda)\mu_r}$, with μ_z being for diluted magnets typically below 2.

Modeling a full C-shaped dipole with individual laminations results, however, in very resource intense simulation models. Here, a scaled model with lamination thickness of 10 times the physical lamination thickness and single laminations (magnetic and nonmagnetic) was explored. The scaled model also could not predict the linear decrease but showed the best prediction of the longitudinal field profile. Simulating each individual lamination is performed for the quadrupole magnets (see Sec. III C 3), as they have more symmetry planes and result therefore in smaller models.

3. Remanent field

The remanent field shall be minimized to keep the transfer function $B = f(I)$ at small field reasonably predictable for operation. Ampere's law in the case of no applied current is

$$\oint H dl = \frac{B_{\text{res}} g}{\mu_0} + H_y l_{\text{Fe}} = 0, \quad (5)$$

where no fringe field is assumed. In Eq. (5) the residual field B_{res} in the gap and the magnetic field H_y are unknown

[38]. Equation (5) has to be fulfilled on the measured hysteresis curve of steel I and provides both values H_y and B_{res} . Around the point (H_y, B_{res}) , the hysteresis curve is almost vertical with a slope of $\frac{B(H)}{\mu_0} \frac{1}{H-H_c} \approx 10^5$ (Table IV) compared to the slope $\frac{l_{\text{Fe}}}{g} \approx \frac{1}{0.076} \approx 10$ [calculated with Eq. (5)]. Therefore, in magnets with gaps the residual field is driven by $H_y \approx H_c$.

To take the anisotropy of the coercive force H_c into account the nominal path through the iron is discretized and Eq. (5) is rewritten:

$$B_{\text{res}} = -\frac{\mu_0}{g} \sum_{i=1}^N H_c(\phi_i, B_i) l_i. \quad (6)$$

To fit the angular dependence of the coercive force the generalized superellipse in polar coordinates [39] with fourfold symmetry is used (see Appendix). For an angle of $\phi = 0$ the field direction is oriented in the rolling direction:

$$H_c(\phi, B) = \left[\left(\frac{\cos \phi}{a} \right)^n + \left(\frac{\sin \phi}{b} \right)^n \right]^{-1/n}, \quad (7)$$

with $a = H_{c,\parallel}(B)$ and $b = H_{c,\perp}(B)$. For NGO steel the parameter $n = 2.2$ provides here a good fit for $B \gtrsim 0.1$ T (see the Appendix). For this calculation, the path is separated into around 25 segments along the equipotential line going through the center of the magnet. The field amplitude $B = B(x, y)$ along this path is calculated by using a 2D FEM code. Then, the $H_c(\phi, B)$ values are calculated with linear interpolation using the measured values from samples from the same batch of steel as used for the prototype (similar to values in Table IV). Another method to calculate the remanent field is to use a commercial available software package, Cobham OPERA 2D TR/DM [40]. This software uses the descending branch of the major isotropic hysteresis cycle [41] to estimate the hysteresis behavior of the material. It is worth noting that anisotropic material properties cannot be implemented in the current release of this software. The results are summarized in Table VI.

All presented simulation methods slightly underestimate the remanent field effects in the magnet. Closest is the

TABLE VI. Remanent field of the diluted and nondiluted magnet calculated with different methods.

Description	Diluted (mT)	Nondiluted (mT)
Rolling direction $\parallel$ to pole	0.80	0.42
Mixed (isotropic)	0.78	0.42
OPERA 2D TR/DM (isotropic)	0.73	0.19
Prototype	0.98	0.55

model taking the anisotropy of the material into account, providing a good estimate of the remanent field (underestimates the remanent field by 18% and 24%, for the diluted and nondiluted case, respectively). It is also worth pointing out that the remanent field in the magnet gap for a diluted magnet is higher than for a nondiluted magnet, which may be found counterintuitive at first glance because—in the case of a diluted magnet—less magnetized material is available to drive the field. However, Ampere’s law implies that the same magnetic field H will be found in magnetic and nonmagnetic laminations throughout the yoke. The field is therefore purely driven by the soft-magnetic coercive field H_c , which depends on the maximum previous excitation field. Therefore, the remanent field in the gap of the magnet, independent on whether the yoke is diluted or not, scales with the coercive force H_c . This scaling can be observed very well in Table VI with comparison to Table IV. For all models (except for OPERA 2D TR/DM) and the magnetic measurement the remanent field in the diluted magnet is around 2 times larger than for the nondiluted magnet.

4. Transfer function

The normalized transfer function $TF = B(I)/I \times I_{\min}/B(I_{\min})$ of the nondiluted magnet prototype shows—as expected—constant characteristics. However, the diluted magnets show an unexpected decrease of the normalized transfer function, as shown for the LHeC, LEP and ELENA magnets in Figs. 8 and 9. For comparison, the field in the aperture is scaled to the pole field in the iron $B_{\text{pole}} = B_{\text{gap}}/\lambda$. Four effects in the normalized transfer function can be identified: (i) remanent field; (ii) linear decrease of the

transfer function with increasing field; (iii) saturation; and (iv) slotting effect.

(i) *Remanent field effects.*—Remanent field effects are not considered in any of the magnetostatic models. Precise modeling of hysteresis and remanent fields is very challenging and requires a demanding material measurement campaign and time-consuming simulations, which are the subject of research at CERN [43] and elsewhere [44,45]. A rough estimate on the impact of the remanent field on the magnet performance can be done by recalling the remanent field in the diluted prototype gap, which is $B \approx 1$ mT (see Table VI) and therefore around 2% of the field of $B_{\text{gap}} = 0.05$ T, as visible in Fig. 9.

(ii) *Linear decrease with increasing field.*—An unexpected almost linear decrease of the transfer function with increasing field was observed (see Figs. 8 and 9). This effect could not be explained with any of the models. Also a 3D sliced model with a laminated yoke with scaled lamination thickness (lamination thickness 5 mm instead of 0.5 mm) was not predicting the decrease. In Sec. III C 3 a fully diluted quadrupole magnet model simulation without scaling is presented, which is able to predict this effect.

(iii) *Saturation effects.*—Saturation effects have been quite well predicted with the models (see Fig. 9 for fields on the pole > 1.2 T). Figure 9 shows the simulation results of single laminations. However, 2D and 3D simulation methods as described above result in very similar results (not shown here).

(iv) *Slotting effect.*—Another, but much smaller effect is the slotting effect, accountable for a small reduction in the transfer function, also visible in the 3D sliced simulations as shown in Fig. 9. This slotting effect is well known in the stator of rotating machines where a so-called Carter

FIG. 8. Normalized center transfer function of LHeC dipole prototypes ($\lambda = 0.33$) and LEP dipoles ($\lambda = 0.27$) over iron field $B_{\text{pole}} = B_{\text{gap}}/\lambda$ [42].

FIG. 9. Normalized center transfer function of ELENA diluted prototype ($\lambda = 0.33$) over iron field $B_{\text{pole}} = B_{\text{gap}}/\lambda$. Measurements from [42].

coefficient is used to adjust the efficiency of the device depending the ratio between the width of the magnetic material (tooth) and nonmagnetic material (slot) [46].

In the diluted magnet presented here, which was initially designed without a magnetic end plate, but only with small solid magnetic shims, a variation of the magnetic length by around 4% is found. A detailed analysis showed that a solid iron end plate with a thickness of around 40 mm balances the variation of the magnetic length between injection and extraction to almost zero. In case no magnetic end plate is present in the diluted magnet, the magnet extremities start to saturate even at low fields, due to the reduced longitudinal permeability. The simulation result has been confirmed with replacing one of the nonmagnetic end plates with a magnetic end plate.

5. Low-frequency eddy current effects

The eddy current effects in the diluted and nondiluted magnets are small and mainly arising from the solid end plates and shims. In principle, it is expected that the eddy current effects in a lamination in a diluted yoke are larger by the iron fraction as the eddy currents are induced with dB/dt . However, the visible effect in the gap is smaller by the iron fraction, so that the eddy current effects should be similar (as long as the yoke is not saturated). In the case of the preseries, a 40 mm thick laminated end plate is used (lamination thickness 10 mm). Here, the eddy current effects are for ramp rates of 0.25 T/s within 10^{-3} , measured with the preseries magnet and calculated as a fraction of the nominal current.

6. Conclusion

Comparing the performance for nondiluted and diluted magnets one finds that the transfer function of the nondiluted magnet shows a better linearity with respect to current, and the early-saturation phenomenon is not present. Also, the magnetic length shows a smaller change with respect to field level in a nondiluted magnet. This change is 0.7% compared to 3.9% between injection and extraction field. The remanent field effects are 0.56 times smaller in the nondiluted dipole magnet. The multipoles are stable with respect to different field levels within $\pm 10^{-4}$ compared to around $\pm 5 \times 10^{-4}$ (especially arising from a change of the quadrupole component in the diluted magnet), due to the early unexpected saturation effects described above. These results lead to the decision to build nondiluted dipoles for ELENA.

III. QUADRUPOLES

The quadrupole magnetic parameters and their position in the ELENA ring have been defined based on an optimization of the beam dynamics to be $K_1 = \frac{1}{B\rho} \frac{\partial B_y}{\partial x} = (2.3, -1.2, 0.72) \text{ m}^{-2}$ for the three different quadrupole

TABLE VII. Parameters of ELENA quadrupole magnets.

Parameter	Value	Unit
Number	12	
Gradient	0.02 to 1.45	T/m
Pole field	12.4×10^{-3} to 0.09	T
Total aperture	124	mm
Magnetic length	250	mm
Ramping speed	1.45	(T/m)/s
Field homogeneity	$\pm 5 \times 10^{-4}$	
Reference diameter of good field region	54	mm

families at a working point of $(Q_x, Q_y) = (2.3, 1.3)$ [6]. From these values one finds required gradients for the quadrupoles of $G \approx 0.03 \dots 0.76 \text{ T/m}$ at injection. Here, the maximum gradient is chosen to be 1.45 T/m to be able to use this magnet in the transfer line between AD and ELENA. Therefore, the requested dynamic range over the quadrupole magnet family is 0.02 to 1.45 T/m. The length of the quadrupole magnet is chosen to be as long as possible, as requested by beam dynamics to minimize the influence of the fringe field. The gap is chosen large enough to incorporate beam-position monitors. The quadrupole parameters are summarized in Table VII.

A. Design

For the quadrupoles three different design options have been considered [19]: (1) Collins quadrupoles (similar to Figure-of-8 quadrupoles), (2) standard quadrupoles either with tapered or parallel poles, (3) Panofsky quadrupoles. The choice between these three options is driven by the fact that, in ELENA, quadrupoles must have the capability to be easily assembled around the vacuum chambers, so that neither *in situ* welding of flanges after installing the vacuum chamber nor breaking the vacuum in case of magnet failure is required. These requirements were set to achieve an ultralow vacuum by nonevaporable getter (NEG) coating all chambers. If NEG is vented around 10 times, the pumping speed is reduced to about 50% of the initial pumping speed. To regain the original pumping speed, the activation temperature $T_a = [180, 200, 220, 250, \dots]^\circ\text{C}$ has to be increased with ten venting/activation cycles at each temperature level [47], with requirements to insulate installed equipment from such temperatures.

Based on the above reasoning, standard quadrupoles, built from four quadrants, have been selected which allow for an easy construction and splitting with respect to other designs. Parallel poles with a width of 85.6 mm and standard flat-racetrack coils are chosen, as shown in Fig. 10, fulfilling all specifications. The good field quality is relatively easy to achieve, thanks to the favorable pole width and the small ratio between aperture and good-field region of 0.4. The design values for the allowed multipole values are $b_6, b_{10}, b_{14}, b_{18} \lesssim \pm 10^{-5}$ at a gradient of 1.45 T/m.

FIG. 10. Sketch of quadrupole magnet.

B. Yoke material selection

For quadrupoles, nonlinearities in the transfer function and changes in the field quality due to hysteresis effects are expected to be even larger than for the dipole magnets because the dynamic range for the quadrupole family is larger and the pole field smaller. To minimize these effects a GO and a NGO steel are explored. For the selection of the lamination thickness the same criteria as outlined in Sec. II B 1 are followed. However, GO steel is not commercially available in a thickness larger than 0.35 mm. Therefore, both NGO (M250-35 A) and GO (M140-35 S) steel with a thickness of 0.35 mm are compared. The chemical composition of the steel, measured with a PMI spectrometer, is summarized in Table VIII. The carbon values are not shown in the table as they are too small to be detected by the method used here.

For the prototypes, GO steel M140-35 S from supplier A (ArcelorMittal, Czech Republic) is used, and for the series production steel M140-35 S (internal designation C140-35) from supplier B (ThyssenKrupp, Germany) is considered. It is worth noting that, despite the same designation and specified loss values, the magnetic characteristic of these two GO steels is different, as evident from the hysteresis

TABLE VIII. Chemical composition in % of M250-35 A and M140-35 S from supplier A and B.

	M250-35 A	M140-35 S, A	M140-35 S, B
Si	1.96	3.27	3.31
Mn	0.32	0.30	0.17
Al	1.22	0.003	0.003
Cu	0.009	0.58	0.20

FIG. 11. Hysteresis cycle for M140-35 S from two different suppliers A and B.

cycle for M140-35 S, shown in Fig. 11, and the chemical composition provided in Table VIII. The permeability for the GO steel from supplier B is on average around 7 times larger than the one from supplier A and the coercive force is around 1.7 times smaller. This difference comes from a different annealing process. After magnetic annealing at 800 °C for 1 h (heating with 2–4 °C/h), decreasing the temperature with 2–4 °C/h to 300 °C and cooling down by free convection to room temperature, the BH curves resembled each other. This comparison shows that the selection of an adequate steel, independent of its official denomination, with large (initial) permeability [41] can be very efficient with respect to the increase of the permeability over the working range.

C. Prototypes

To study the overall performance of the quadrupoles employing the steel selected above, four yokes in 1 : 1 scale

FIG. 12. Picture of quadrupole prototype during magnetic measurement.

TABLE IX. Average, minimum and maximum permeability for nondiluted and diluted quadrupole magnet yokes calculated with measurement results from samples cut in the rolling direction.

Description	GO nondiluted, A	GO diluted, A	GO nondiluted, B	GO diluted, B	NGO nondiluted	NGO diluted
$\bar{\mu}_r$	3,400	4,050	22,900	30,500	2,700	4,600
$\mu_{r,\max}$	3,900	4,250	24,150	39,600	3,300	5,400
$\mu_{r,\min}$	2,500	2,750	20,600	21,150	2,100	2,150

have been built, as shown in Fig. 12: nondiluted NGO steel, diluted NGO steel, nondiluted GO steel, and diluted GO steel magnet yokes.

1. Comparison between diluted and nondiluted yoke design

To select the iron fraction, the average permeability $\bar{\mu}_r$ is again calculated as described in Sec. II C 1 for the entire operating range given in Table VII. The maximum average iron permeability $\bar{\mu}_r$ over the operating range for the three different steel grades is presented in Table IX. The maximum average permeability $\bar{\mu}_r$ is found at around an iron fraction $\lambda \approx 0.1$. To limit saturation effects and to be able to use commercially available laminations, 0.35 mm thick electrical steel laminations interleaved with 2 mm thick stainless steel laminations are chosen for the diluted yokes. This yields an iron fraction $\lambda = 0.15$. Using these values the average, minimum and maximum permeability for the diluted and nondiluted yokes is calculated, see Table IX. For the diluted yokes the effective relative permeability is lower by the dilution factor $\lambda = 0.15$. Here, the gain in terms of average permeability $\bar{\mu}_r$ for the nondiluted yokes is small. For this calculation, the magnetic hysteresis curves have been extrapolated by using polynomials, as described in [48], to be able to estimate the permeability of the nondiluted magnets. The field is too low to be measured by using the existing Epstein frame measurement setup. For the performed analysis, the initial permeability is found to be $\mu_{\text{ini}} = [2470, 20590, 2090]$ for the steel GO (supplier A), GO (supplier B), and NGO steel, respectively.

2. Remanent field

From Sec. II C 3 one finds for the remanent field in quadrupole magnets:

$$G_{\text{res}} = -\frac{2\mu_0}{R^2} \sum_{i=1}^N H_c(\phi_i, B_i) l_i, \quad (8)$$

where R is the aperture radius. The integrated remanent field is measured in the prototype with a rotating coil measurement setup, scaled with the nominal length $L = 0.25$ m and compared to the remanent field values calculated with the 1D model provided in Eq. (8). For information, the pole fields $B = GR$ are calculated and also listed in Table X. The Epstein measurements to obtain the coercive force values as a function of the angle and the field have been performed at CERN with a dedicated measurement setup [37]. The reader is referred to the Appendix for information on the angle dependence of the coercive force and possible fitting techniques for $H_c(\phi, B)$.

The agreement between the remanent field values calculated with Eq. (8) and the measurements on the quadrupole prototypes is remarkably good, especially when considering that only the integrated remanent gradients in the quadrupole magnets are measured and these values are scaled with the nominal quadrupole length $L = 0.25$ m to be able to compare them to the values calculated with the 1D model given above. It is also worth noting that the remanent field values calculated with the anisotropic model and with mixed samples are in good agreement, due to the fact that the angle ϕ_i varies along the integration path between around 0 and 90 deg. The discrepancy of around 30% between calculated and measured remanent field values for the NGO prototype is believed to come from induced stress in the Epstein frame samples. Samples measured by the producer taken from the same batch show around 30% lower coercive force values, and therefore match the NGO prototype measurements.

3. Test results and discussion

The magnetic measurements have been performed by using a rotating coil [49] and they are discussed on the basis of: (1) the normalized transfer function (2) the variation of the field quality, and (3) the variation of the magnetic length.

TABLE X. Remanent pole field B_{res} and residual gradient G_{res} of the nondiluted GO (#1), diluted GO (#2), nondiluted NGO (#3), and diluted NGO (#4) prototype.

Description	$B_{\text{res,an}}$ (mT)	$G_{\text{res,an}}$ (mT/m)	$B_{\text{res,mix}}$ (mT)	$G_{\text{res,mix}}$ (mT/m)	$B_{\text{res,mes}}$ (mT)	$G_{\text{res,mes}}$ (mT/m)
GO, diluted	0.64	10.26	0.69	11.05	0.62	10.0
GO, nondiluted	0.26	4.22	0.30	4.89	0.25	4.0
NGO, diluted	1.23	19.77	1.30	20.94	0.84	13.6
NGO, nondiluted	0.34	5.53	0.39	6.26	0.42	6.8

Comparing the nondiluted and diluted magnets one finds a quite linear transfer function (a variation $< 1\%$ of the normalized TF compared to around 8%), a smaller variation of the field quality (a variation $< 10^{-5}$ compared to a variation of $0.5\text{--}3 \times 10^4$ in b_6), and a smaller variation of the magnetic length (1.5% compared to $2\%\text{--}3\%$) for the nondiluted magnets. Therefore, nondiluted magnets have been considered the better choice. The difference between the nondiluted magnets constructed with GO and NGO steel is small but GO steel is consistently better.

To compare if the strong decrease in magnetic length and transfer function, as observed for the diluted dipole magnet without an end plate, is also present in the quadrupole diluted magnet without an end plate, a full-scale sliced 3D FEM model representing each magnetic lamination has been prepared and solved. The results of the measurement and the simulation are compared in Fig. 13 and agree very well.

For all magnets, except the magnet with the diluted, GO yoke, the dynamic effects at 50 A/s are within 0.1% at nominal gradient. The magnet yoke with diluted, GO steel shows eddy current effects which are around 6 times larger. As the production process is the same for all four magnets, no explanation for this abnormal behavior could be found so far.

4. Conclusion

In total four different quadrupole magnets with GO diluted, GO nondiluted, NGO diluted, and NGO nondiluted yokes were built and tested. Comparing the performance for nondiluted and diluted quadrupole magnets one finds again that the performance of nondiluted magnet is better in all investigated aspects—linearity of transfer function, variation of field quality and magnetic length over the

FIG. 13. Comparison of measurement and simulation of the normalized transfer function of a diluted quadrupole with no end plates. Measurements are from [42].

TABLE XI. Parameters of ELENA H/V corrector magnets.

Parameter	Value	Unit
Number	10	
Nominal integrated field	2.5×10^{-3}	Tm
Free inner aperture	124	mm
Magnetic length	200	mm
Field homogeneity	$\pm 1 \times 10^{-2}$	
Reference diameter of good field region	44	mm

dynamic range. The difference between the nondiluted magnets constructed with GO and NGO steel is small but GO steel is consistently better. Therefore, nondiluted quadrupole magnets employing GO steel were chosen for ELENA.

IV. H/V CORRECTORS

A major concern with respect to H/V corrector magnets is the fact that they are operated with nonrepetitive cycles. Under these operation conditions and at the very low fields requested in ELENA, the magnetic field is strongly affected by hysteresis. Therefore, besides standard iron-dominated H/V corrector magnets, coil-dominated iron-free corrector magnets have been considered. The parameters of the H/V corrector magnets are summarized in Table XI.

A. Iron-dominated normal-conducting H/V corrector

To test if standard, low-cost, normal-conducting iron-dominated window frame dipole correctors show an acceptable performance for typical operation scenarios of ELENA, a conventional magnet with a magnetic length $l = 423.9\text{ mm}$ and an aperture of $a = 134.4\text{ mm}$ has been extensively magnetically measured by [50]. This H/V corrector magnet is shown in Fig. 14. The following aspects were studied: (1) the hysteresis effects on the

FIG. 14. Picture of a conventional H/V corrector magnet used to measure remanent field and field quality at very low fields.

transfer function; (2) the cross talk (see later for an explanation); and (3) the field quality at extra-low field.

1. Hysteresis effects

The test results show that, if the operating cycle is changed, the remanent field is changed by more than 10% after the first cycle, by 2.8% after the second cycle and by 1.2% after the third cycle compared to the fifth cycle. For this measurement, cycles with an integrated field amplitude of $\int B dl \approx \pm 7.4 \times 10^{-3}$ Tm have been performed resulting in a remanent field in the horizontal plane of $\int B_{\text{rem}} dl = 7.3 \times 10^{-5}$ Tm and in the vertical plane of $\int B_{\text{rem}} dl = 3.3 \times 10^{-5}$ Tm. Thus, in case the cycle is changed, the transfer function can only be predicted with a precision in the percent range for an integrated field being in the order of the remanent field. This also shows that remanent effects at low field are very pronounced. At an integrated gradient of 2.2×10^{-3} Tm, for example, the proportion of the remanent field is around 3% in the vertical plane of the main field.

2. Cross talk

To analyze the “cross talk” the horizontal plane has been excited up to $\int B dl \approx \pm 7.4 \times 10^{-3}$ Tm. The field angles at different excitation levels have been measured and taken as horizontal reference field angles for the respective field values. Then, the horizontal plane has been switched off and the vertical plane has been excited up to $\int B dl \approx \pm 7.4 \times 10^{-3}$ Tm and the field angles have been measured and compared to the previously measured horizontal reference field angles. The difference to 90 deg (the theoretical expected angle) is calculated for each of the angles and defined as cross talk. For fields $\int B dl \approx \pm 2.2 \times 10^{-3}$ Tm the angle is in the order of 1.6 deg.

3. Field quality at extra-low field

The multipoles are becoming worse at low field. For instance, the sextupole b_3 increases by several 10^{-3} starting from around $\int B dl \leq \pm 1.6 \times 10^{-3}$ Tm, a field level at which the ELENA H/V corrector magnets will operate.

4. Discussion

The adjustment of the working range to the range in Table XI could reduce the remanent field levels due to the dependence of the coercive force on the previous maximum excitation field. However, as evident from the magnetic measurement results, hysteresis effects make it difficult to predict the magnetic field as a function of—nonrepetitive—current cycles.

B. Coil-dominated normal-conducting H/V corrector

To overcome the previously discussed limitations of hysteresis (see Sec. IVA) a coil-dominated

normal-conducting H/V corrector magnet design has been studied. Here, the investigated design options are: (1) Helmholtz-coil designs (round and flat-racetrack coils), (2) nested helices (also called canted-cos-theta coils), and (3) layered cos-theta coils. The layered cos-theta approach is found to be an adequate design option, as it satisfies best the specifications. The required dimensions and power consumption are reasonably small and the design can be easily split and built around the vacuum chamber, as before discussed for the quadrupoles (see Sec. III). The result of this study is described in [51].

A fully fledged design has been worked out based on a precisely machined winding mandrel on which three superimposed coils for each plane are wound (see Fig. 15). The layering is required in order to obtain the integrated field strength within the available space. To illustrate the layering a cross section of the magnet is shown in Fig. 16. Due to the short length, the field, and field quality is strongly affected by fringe field effects. Therefore, the design has been performed using Biot-Savart equations for 3D (here OPERA 3D postprocessor). The influence between the H/V corrector magnets and adjacent magnets with iron yokes has been analyzed by using a 3D OPERA model. The influence is negligible. The current density of the solid conductor is chosen as a compromise between the cost optimum and the available space to be around $J = 2.5$ A/mm², calling for indirect cooling of the conductor through cooling the winding mandrel [52]. To enhance this cooling, a winding mandrel with relatively large thermal conductivity was chosen, here the choice was made between aluminum, copper, bronze

FIG. 15. Sketch of H/V corrector magnet.

FIG. 16. Cross section of H/V corrector magnet. The inner aperture has a diameter of 124 mm.

and 3D printed titanium. The prototype was built using a machined copper winding mandrel, as this was the preferred choice of the workshop. The series magnets feature aluminum mandrels, as they have been the most cost efficient choice. For the series, the indirect cooling channels cannot be directly machined into the aluminum winding mandrel due to the risk of corrosion. Therefore, the use of stainless steel tubes for indirect cooling, placed into the winding mandrel and glued into the aluminum winding mandrel by using thermal paste, is being explored for the series production.

Another advantage of using a metallic winding mandrel is that the high-frequency harmonics from the power converter (50 kHz), which may cause considerable emittance blowup [31], are very well damped. The skin depth is well below 0.5 mm in the winding mandrel, which has an average thickness of more than 20 mm. A 2D OPERA simulation has confirmed that the expected field is around zero in the aperture with an applied frequency of $f = 50$ kHz [53].

To test the desired design concept, a prototype of the inner shell has been manufactured and measured at CERN. The prototype is shown in Fig. 17 and has met the specification in all points [54]. However, the coil was less compact than initially designed resulting in slightly larger multipoles and a less steep transfer function (both still within specification). Therefore, the design of the winding mandrel for the series has been adjusted. Dynamic low-frequency effects have been measured in pulsed mode [54]. On the current plateau at nominal field after ramping within one second (in ELENA ramping time will be around 5–8 s), the measured dynamic effects have been within 0.4%.

FIG. 17. Picture of the coil-dominated corrector prototype.

C. Conclusion

The proposed H/V corrector is iron-free and therefore the low-frequency field follows the current without any hysteresis effect. The induced eddy currents at typical ramp rates [$<2.5^{-3}$ Tm/(s)] are small ($<0.4\%$). The high-frequency (50 kHz) harmonics of the power converter are sufficiently damped to avoid emittance blowup. The machine will not be shielded and subject to external magnetic perturbations considered large for the extra-low-energy particles. By choosing iron-free corrector magnets—with very limited eddy currents—the field follows directly the applied current and nonrepetitive cycles can be efficiently performed to correct for these perturbations.

V. CONCLUSION

In this work the performance of different magnet types at low field has been studied. It could be shown that the hysteresis effects can be well handled, if a careful selection of the yoke material is performed and the magnets are operated with repetitive cycles. Interleaving the magnetic material in the yokes with nonmagnetic material, so-called dilution, has proven to worsen the performance for the dipole and quadrupole prototypes presented here. For H/V corrector magnets, operated with nonrepetitive cycles and very small fields, the hysteresis effects cannot be easily

handled. Therefore, an iron-free magnet was designed. Due to the absence of ferromagnetic material no hysteresis effects are present; therefore, the transfer function is linear and the field quality is equal at all field levels. This choice eases the operation of the machine. The design methods for such a magnet system have been described and the governing equations have been derived and verified with a total of seven prototypes. The design methods outlined here can be used for the design of a magnet system operated at low energy and over wide dynamic range.

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APPENDIX: COERCIVE FORCE AS A FUNCTION OF EXCITATION ANGLE AND FIELD

Samples for magnetic tests with an Epstein frame are usually cut in rolling (0 deg) and perpendicular (90 deg) to rolling direction [55]. Producing and measuring Epstein samples with a size of 280...320 mm times 30 mm [55] is time consuming and a minimum of at least eight samples are required in order to minimize the effects of the eddy currents in the overlapping region [56]. Therefore, the coercive force as a function of the angle is not always readily available from the steel manufacturer. To avoid having to perform these measurements, a fit is presented, which can be used to roughly estimate the coercive force as a function of the angle. Here, to obtain these values, samples with an angle of 0, 5, 10, 15, ..., 90 deg to the rolling direction are laser cut and measured in an Epstein

FIG. 18. Measurement and fit of the coercive force versus angle in polar coordinates at $J \approx 1.6$ T for two NGO steel grades.

FIG. 19. Measurement and fit of the coercive force versus angle in polar coordinates at $J \approx 1.4$ T for two GO steel grades.

FIG. 20. Coefficient of determination R^2 for different steel grades over excitation values.

frame. It is found that the measurement results can be fit well by using the superellipse equation [39]:

$$H_c(\phi, B) = \left[\left(\frac{\cos \phi}{a} \right)^{n_1} + \left(\frac{\sin \phi}{b} \right)^{n_2} \right]^{-\frac{1}{n_3}}, \quad (\text{A1})$$

with $a = H_{c,\parallel}(B)$ and $b = H_{c,\perp}(B)$. When setting $n = n_1 = n_2 = n_3$, one finds for the here investigated NGO steels $n = 2.2$, and for the here investigated GO steels $n = 4$. For the NGO the chosen fit with $n = 2.2$ follows well the measured shape (see Fig. 18). For GO steel the fit does not represent the characteristic increase of the coercive force H_c at around 30 deg and for $n = n_1 = n_2 = n_3$ the fit is not following the measurement as well as seen for NGO steel (see Fig. 19). However, choosing for each given GO steel the parameters n_1 , n_2 and n_3 would yield to a better fit, but requires to repeat the above described time-consuming measurement campaign. Therefore, the fitting parameter $n = 4$ is suggested to be able to roughly estimate the coercive force H_c in case only measurements in the rolling direction (0 deg) and perpendicular (90 deg) to the rolling direction are performed. Figure 20 shows the coefficient of determination R^2 over the field level for the suggested fits. For field values $B \lesssim 0.1$ T the fit is poor, probably because of the large measurement noise for such low field levels.

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