Erratum: Sharing of nonlocal advantage of quantum coherence by sequential observers **[Phys. Rev. A **98**, 042311 (2018)]**

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We correct an error in the original paper in determining the conditional state on Bob's side after the sequential measurements performed on the other side. Considering the initial state between Alice¹ and Bob σ_{A_1B} after performing a particular unsharp measurement from $\{E_i^\pm\}$ ($i \in \{x, y, z\}$) by Alice¹, the conditional state on Bob's side becomes

$$\sigma_{B|E_i^\pm} = \frac{\text{Tr}_A[(\sqrt{E_i^\pm} \otimes \mathbb{1}_2)\sigma_{A_1B}(\sqrt{E_i^\pm} \otimes \mathbb{1}_2)]}{\text{Tr}[(\sqrt{E_i^\pm} \otimes \mathbb{1}_2)\sigma_{A_1B}(\sqrt{E_i^\pm} \otimes \mathbb{1}_2)]}, \quad (1)$$

where $E_i^\pm = \lambda P_i^\pm + \frac{1-\lambda}{2}\mathbb{1}_2$ and $E_i^+ + E_i^- = \mathbb{1}_2$. State $\sigma_{B|E_i^\pm}$ occurs with probability $p(\sigma_{B|E_i^\pm}) = \text{Tr}[(E_i^\pm \otimes \mathbb{1}_2)\sigma_{A_1B}]$. Using the above conditional state on Bob's side, Eq. (15) becomes

$$N_{A_1B}^{l_1} = 3\lambda_1, \quad (2)$$

which gives a violation of Eq. (5) when $\lambda_1 > \sqrt{\frac{2}{3}} \simeq 0.82$. By applying a similar argument for Alice², Eq. (17) becomes

$$\overline{N_{A_2B}^{l_1}} = \lambda_2(1 + 2\sqrt{1 - \lambda_1^2}). \quad (3)$$

So, if $\lambda_2 = 1$, $\overline{N_{A_2B}^{l_1}} > \sqrt{6}$ when $\lambda_1 < 0.69$. Hence, no region for λ_1 within $[0, 1]$ can be found where both Alice¹ and Alice² can achieve nonlocal advantage of quantum coherence (NAQC) with Bob using the l_1 norm. Or in other words, at most, one Alice can exhibit NAQC with Bob. Equations (18) and (19) are thereby rendered irrelevant.

Similarly, for relative entropy of NAQC, the right-hand side of Eq. (20) will be changed to

$$N_{A_1B}^E = 3 \log_2(e) \lambda_1 \tanh^{-1}(\lambda_1) + \frac{3}{2} \log_2(1 - \lambda_1^2), \quad (4)$$

which violates Eq. (7) when $\lambda_1 > 0.91$. Now following Eq. (21), the right-hand side of Eq. (22) becomes

$$\begin{aligned} \overline{N_{A_2B}^E} &= \log_2(e) \lambda_2 \tanh^{-1}(\lambda_2) + \frac{1}{2} \log_2(1 - \lambda_2^2) + 2 \log_2(e) \sqrt{1 - \lambda_1^2} \lambda_2 \\ &\quad \times \tanh^{-1}(\sqrt{1 - \lambda_1^2} \lambda_2) + \log_2[1 - (1 - \lambda_1^2) \lambda_2^2], \end{aligned} \quad (5)$$

which, in the range of $\lambda_1 \in (0.91, 1]$, gives the quantum mechanical (QM) maximum 1.24. Therefore, the inference that only one Alice can share relative entropy of NAQC with a single Bob remains unaffected.

Similarly, again, for skew information of NAQC, Eq. (23) turns out to be

$$N_{A_1B}^S = 3(1 - \sqrt{1 - \lambda_1^2}), \quad (6)$$

which violates Eq. (9) when $\lambda_1 > \frac{2\sqrt{2}}{3} \simeq 0.94$. Using Eq. (24), now Eq. (25) becomes

$$\overline{N_{A_2B}^S} = 3 - \sqrt{1 - \lambda_2^2} - 2\sqrt{1 - (1 - \lambda_1^2) \lambda_2^2}. \quad (7)$$

In the range of $\lambda_1 > 0.94$, the QM maximum of $\overline{N_{A_2B}^S}$ is 1.11. Here also the conclusion that at most one Alice can exhibit skew information of NAQC with a single Bob remains unaltered.

Typographical errors in Eq. (16) are corrected by replacing E_j^+ by P_j^+ and E_j^- by P_j^- .

To conclude, we now get uniform results using all the three measures of coherence, i.e., at most one Alice can reveal NAQC with Bob on the other side. This fits well with the conjecture that, as the strength of quantum correlations increases, the bound

on the number of observers exhibiting such correlations decreases, viz. from three using a three-settings steering inequality [1] to two for Bell-Clauser-Horne-Shimony-Holt violation [2] and one for NAQC in our paper.

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[1] S. Sasmal, D. Das, S. Mal, and A. S. Majumdar, [Phys. Rev. A](#) **98**, 012305 (2018).

[2] S. Mal, A. S. Majumdar, and D. Home, [Mathematics](#) **4**, 48 (2016).