Erratum: Resource theory of coherence: Beyond states [Phys. Rev. A **95, 062327 (2017)]**

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Proposition 3 in our paper is not correct. The proof shows that any d -dimensional unitary can be implemented using a d -dimensional maximally coherent state and incoherent operations (IOs) [1,2], but the operation exhibited is not a strictly incoherent operation (SIO) as claimed. It was also pointed out to us that a general unitary cannot be implemented at all using SIO and any coherent resource state ψ .

To see this, the first observation is that a SIO completely positive and trace-preserving (CPTP) map T commutes with the completely dephasing map $\Delta: \rho \rightarrow \sum_{i=1}^d |i\rangle\langle i| \rho |i\rangle\langle i|$,

$$T \circ \Delta = \Delta \circ T. \quad (1)$$

This condition characterizes dephasing-covariant incoherent operations (DIOs) [3]. Using it, one can show that a qubit unitary cannot be realized by means of T .

Suppose by contradiction that there is a CPTP map $T: A \otimes R \rightarrow B$ satisfying Eq. (1), where $|A| = |B| = 2$ and a state Ψ on R such that for all ρ ,

$$T(\rho \otimes \Psi) = U \rho U^\dagger, \quad (2)$$

where the qubit unitary U acts as $U|0\rangle = \alpha|0\rangle + \beta|1\rangle$, $U|1\rangle = -\beta^*|0\rangle + \alpha^*|1\rangle$, and $\alpha\beta \neq 0$. Applying condition (1) to Eq. (2) we have that

$$T(\Delta(\rho) \otimes \Delta(\Psi)) = \Delta(U \rho U^\dagger), \quad (3)$$

which, for the input states $\rho = |0\rangle\langle 0|$, $\rho = |1\rangle\langle 1|$, and $\rho = U^\dagger|0\rangle\langle 0|U$ yields, respectively:

- (a) $T(|0\rangle\langle 0| \otimes \Delta(\Psi)) = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$;
- (b) $T(|1\rangle\langle 1| \otimes \Delta(\Psi)) = |\beta|^2|0\rangle\langle 0| + |\alpha|^2|1\rangle\langle 1|$;
- (c) $T((|\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|) \otimes \Delta(\Psi)) = |0\rangle\langle 0|$.

Now, by taking $|\alpha|^2$ (a) + $|\beta|^2$ (b), we get

$$T((|\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|) \otimes \Delta(\Psi)) = (|\alpha|^4 + |\beta|^4)|0\rangle\langle 0| + 2|\alpha|^2|\beta|^2|1\rangle\langle 1|,$$

which is in contradiction to (c). Hence, no strictly incoherent operation can implement any nonincoherent qubit unitary using any amount of coherence as a resource.

It is clear that one can generalize this argument to higher-dimensional unitaries as long as U itself is not incoherent.

Looking back at the argument we used, we note that all we actually need is the slightly weaker property,

$$\Delta \circ T \circ \Delta = \Delta \circ T \quad (4)$$

for the simulating operation (the so-called “coherence nonactivating condition” from Ref. [4]).

Thus, no coherent resource can be used to implement unitaries, or for that matter, general CPTP maps if the free operations are SIOs, DIOs, or even the operations satisfying (4). In other words, Proposition 3 and Theorem 5 are wrong as stated but remain true statements when “strictly incoherent operation” is replaced by “incoherent operation”. As a consequence, the remark following Theorem 5 is voided. The other results in our paper are not affected by this correction.

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