Erratum: Universal simulation of Markovian open quantum systems [Phys. Rev. A **91**, 062308 (2015)]

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In Sec. VI and Appendix A it was claimed that $T(t) = e^{t\mathcal{L}}$ could be approximated via a finite sequence of quantum channels $T^{(j)}(t')$ through the use of higher-order Suzuki-Lie-Trotter integrators of the form $[S_{2k}(t/r)]^{rL_1}$ for arbitrary $k \in \mathbb{N}$. However, for $k > 1$ it is in fact the case that $(1 - 4p_k) < 0$, and therefore applying the recursion rule in Eq. (A4) allows us to see that for all $k > 1$ implementation of $S_{2k}(\lambda)$ requires the simulation of multiple propagators $T^{(j)}(\tilde{t})$ with $\tilde{t} < 0$ [1]. As such propagators are *not* quantum channels (in particular they may violate complete positivity, or even positivity) [2], it is in fact only possible to exploit first-order integrators ($k = 1$), and as such, Theorem 3 is only valid for $k = 1$ as opposed to all $k \in \mathbb{N}$. In light of these considerations, Theorem 3 and the subsequent conclusions of Appendix A should be replaced with the following theorem, which is an optimized version of Theorem 3 when restricted to the case of $k = 1$.

Theorem 1. Let $1 \geq \epsilon > 0$ be such that $(9/2)L_2mt \geq \epsilon$, then for

$$r \geq \frac{\sqrt{2L_2}(mt)^{3/2}}{\epsilon^{1/2}}, \quad (1)$$

we have that

$$\left\| \exp\left(t \sum_{j=1}^m \mathcal{L}_j\right) - [S_2(\hat{\mathcal{L}}_1, \dots, \hat{\mathcal{L}}_m, t/r)]^{rL_1} \right\|_{1 \rightarrow 1} \leq \epsilon, \quad (2)$$

and the number of exponentials required is bounded by

$$N_{\text{exp}} \leq (2m - 1) \frac{\sqrt{2L_2}L_1(mt)^{3/2}}{\epsilon^{1/2}}. \quad (3)$$

In light of this necessary restriction to the case of $k = 1$, steps (3) and (4) of the algorithm presented in Sec. VI should be modified as follows:

(3) Given $\epsilon > 0$ and $t > 0$, construct, as described in Appendix A, the Suzuki first-order integrator $S_2(\hat{\mathcal{L}}_1, \dots, \hat{\mathcal{L}}_m, t/r)$ with

$$r = \frac{\sqrt{2L_2}(mt)^{3/2}}{\epsilon^{1/2}}, \quad (4)$$

where $L_2 := \|\mathcal{L}_2\|_{1 \rightarrow 1}$.

(4) Given $\rho(0)$, implement $S_2(\hat{\mathcal{L}}_1, \dots, \hat{\mathcal{L}}_m, t/r)$ consecutively rL_1 times in order to recombine the linear combination in Eq. (100) through sequential implementations of $T_k(\tilde{t}) = e^{\tilde{t}\mathcal{L}_k}$. Each implementation of $T_k(\tilde{t})$ is achieved via

$$T_k(\tilde{t}) = \mathcal{U}_k^\dagger [T_{A^{(k)}}(\tilde{t})] \mathcal{U}_k, \quad (5)$$

where $\mathcal{U}_k(\rho) = U^{(k)}(\rho)U^{(k)\dagger}$ and $T_{A^{(k)}}(t) = \exp(t\mathcal{L}_{A^{(k)}})$.

As per Theorem 1 above, the modified algorithm now requires at most

$$N = (2m - 1) \frac{\sqrt{2L_2}L_1(mt)^{3/2}}{\epsilon^{1/2}}, \quad (6)$$

implementations of channels $T_k(\tilde{t})$ as opposed to the original claim in Eq. (105). This does not effect any of the subsequent analysis concerning the efficiency of the algorithm.

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[1] M. Suzuki, *Phys. Lett. A* **146**, 319 (1990).

[2] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, New York, 2002).