Collapse of spin-orbit-coupled Bose-Einstein condensates

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A finite-size quasi-two-dimensional Bose-Einstein condensate collapses if the attraction between atoms is sufficiently strong. Here we present a theory of collapse for condensates with the interatomic attraction and spin-orbit coupling. We consider two realizations of spin-orbit coupling: the axial Rashba coupling and the balanced, effectively one-dimensional Rashba-Dresselhaus one. In both cases spin-dependent "anomalous" velocity, proportional to the spin-orbit-coupling strength, plays a crucial role. For the Rashba coupling, this velocity forms a centrifugal component in the density flux opposite to that arising due to the attraction between particles and prevents the collapse at a sufficiently strong coupling. For the balanced Rashba-Dresselhaus coupling, the spin-dependent velocity can spatially split the initial state in one dimension and form spin-projected wave packets, reducing the total condensate density. Depending on the spin-orbit-coupling strength, interatomic attraction, and initial state, this splitting either prevents the collapse or modifies the collapse process. These results show that the collapse can be controlled by a spin-orbit coupling, thus extending the domain of existence of condensates of attracting atoms.

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I. INTRODUCTION

Understanding Bose-Einstein condensates (BEC) of interacting particles is one of the most interesting problems in condensed matter physics [1]. For uniform three-dimensional systems repulsion between the bosons depletes the condensate, while attraction leads to the condensate instability seen as the appearance of Bogoliubov modes with imaginary frequencies. For the finite-size condensates this instability can be seen in their collapse [2–6]. The collapse, where the size of the state goes to zero after a finite time, strongly depends on the spatial dimension D and is possible only in the $D = 2$ and $D = 3$ condensates. The physics of the collapse is related to the fundamental problems of nonlinear optics and quantum mechanics [7], plasma instability [8], self-trapping of carriers and excitons, and polaron formation [9].

The main features of the collapse of a free, not restricted by an external potential, condensate are determined by the interplay of its positive quantum kinetic and negative attraction energies dependent on the characteristic size of the condensate a. The kinetic energy is proportional to a^{-2} while the attraction contribution behaves as $-a^{-D}$. For $D = 3$ the dependence of the total energy on a is nonmonotonic and the collapse with $a \to 0$ occurs at any interaction strength since at small a the attraction dominates [10]. For $D = 2$ the interaction and kinetic energies scale as a^{-2} and the collapse occurs only at a strong enough attraction.

The BEC physics becomes much richer with synthetic gauge fields [11] and synthetic spin-orbit coupling (SOC) [12,13]. For the latter, optically produced atomic pseudospin $1/2$ is coupled to atomic momentum and to a synthetic magnetic field. The SOC can be produced in various forms, simulating the Rashba and the Dresselhaus symmetries [14,15] known in solid state physics. This coupling opens a venue to the appearance of new phases in a variety of ultracold bosonic [16–25] and fermionic [26–29] ensembles. The SOC plays a crucial role in BEC physics in uniform three-dimensional gases with interparticle repulsion and makes condensation possible only at zero temperature [30], while at a finite temperature the thermal depletion of the condensate diverges [31]. For $D = 2$ the phases of the BEC of repelling bosons trapped in a harmonic potential were found in Ref. [32].

One of the advantages of cold-atomic gases is the fact that due to a very large particle wavelength compared with the atomic radius, the interatomic interaction can be accurately described by a single parameter, the scattering length a_s, where positive (negative) a_s corresponds to repulsion (attraction) between the atoms. The attraction can be achieved by means of the Feshbach resonance [33] in a certain range of the system parameters. Here we study the joint effect of the interatomic attraction and spin-orbit on the spread and collapse of a quasi-two-dimensional spin-orbit-coupled BEC.

This paper is organized as follows. In Sec. II we show by qualitative arguments, a variational approach, and direct numerical solution of the Gross-Pitaevskii equation that the effect of the anomalous spin-dependent velocity due to the spin-orbit coupling [34] can either completely prohibit the collapse or strongly modify the collapse process. We study the condensate dynamics and analyze the conditions at which the collapse does not occur. Possible relations to experiment and conclusions will be given in Sec. III.

II. COLLAPSE IN THE PRESENCE OF SPIN-ORBIT COUPLING

A. General formulation: Hamiltonian and the collapse process

We consider a pancake-shaped condensate of pseudospin-$1/2$ particles described by a two-component wave function

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$\Psi = [\psi^{\uparrow}(\mathbf{r},t),\psi^{\downarrow}(\mathbf{r},t)]^T$, where $\mathbf{r} \equiv (x,y)$, normalized to the total number of particles $N \gg 1$. In the presence of the spin-orbit coupling, the evolution of the wave function is described by a system of coupled nonlinear partial differential equations in the Gross-Pitaevskii-Schrödinger form

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2M}\Delta + \widehat{H}_{\rm so} + \frac{1}{2}(\mathbf{B}\cdot\widehat{\boldsymbol{\sigma}}) - g_2\,|\Psi|^2\right]\Psi. \quad (1)$$

Here M is the particle mass, $\widehat{H}_{\rm so}$ is the SOC Hamiltonian, $\mathbf{B}$ is the effective magnetic field, and $\widehat{\boldsymbol{\sigma}} = (\widehat{\sigma}_x,\widehat{\sigma}_y,\widehat{\sigma}_z)$ is the spin operator. The coupling constant in Eq. (1) is given by $g_2 = -4\pi\hbar^2 a_s/Ma_z$, which we assume for simplicity to be spin-independent, where a_z is the condensate extension along the z axis, and a_s is negative [2,4,5]. Below we consider two strongly different forms of $\widehat{H}_{\rm so}$: the Rashba coupling with the spectrum axially symmetric in the momentum space and the balanced, essentially one-dimensional Rashba-Dresselhaus coupling.

Without loss of generality, we consider an initial state prepared in a parabolic potential at zero temperature as

$$\Psi(\mathbf{r},t=0) \equiv A(0)\exp\left[-\frac{r^2}{2a^2(0)}\right]\boldsymbol{\psi}(0), \quad (2)$$

where $\boldsymbol{\psi}(0)$ is the initial spinor, $A(0) = \sqrt{N/\pi}/a(0)$, and $a(0)$ is the initial width. At $t = 0$, the confining potential is switched off [1] and the spin-orbit coupling and the attraction between the atoms are switched on. The subsequent dynamics is, thus, a response of the system to the instantaneous change in the potential, interaction, and spin-orbit coupling.

In what follows we use the units $\hbar \equiv M \equiv 1$ and the dimensionless interaction $\widetilde{g}_2 \equiv -4\pi a_s/a_z$. The unit of length ℓ can be chosen arbitrarily, and the corresponding unit of time is ℓ^2.

We address first the collapse without spin-dependent effects. Here the energy of the system is

$$E = -\frac{1}{2}\int[\Psi^{\dagger}\Delta\Psi + \widetilde{g}_2\,|\Psi|^4]dxdy, \quad (3)$$

and the evolution can be described by a variational approach based on the Gaussian ansatz [4]

$$\Psi(\mathbf{r},t) = A(t)\exp\left[-\frac{r^2}{2a_{\rm v}^2(t)}[1+ib_{\rm v}(t)]\right]\boldsymbol{\psi}(0), \quad (4)$$

where the variational parameters $b_{\rm v}(t)$ and $a_{\rm v}(t)$ are the chirp and the packet width, respectively. The equation of motion for $a_{\rm v}$ becomes $\ddot{a}_{\rm v} = -\Lambda/a_{\rm v}^3$, where $\Lambda = (\widetilde{g}_2N - \lambda_{\rm v})/2$. The collapse occurs if $\widetilde{g}_2N$ exceeds the variational threshold value $\lambda_{\rm v} = 2\pi$ [35]. The solution of this equation is

$$a_{\rm v}(t) = a(0)\sqrt{1-\frac{\Lambda t^2}{a^4(0)}}. \quad (5)$$

The time scale of the evolution is the collapse time $T_c \equiv a^2(0)/\sqrt{\Lambda}$, and the characteristic collapse velocity is $v_c \equiv a(0)/T_c = \sqrt{\Lambda}/a(0)$.

The key point in the understanding of the role of the spin-orbit coupling in the collapse process is the modified velocity

$$\mathbf{v} = \mathbf{k} + \nabla_{\mathbf{k}}\widehat{H}_{\rm so}, \quad (6)$$

with $\mathbf{k} = -i\partial/\partial\mathbf{r}$, including the anomalous velocity [34] term $\nabla_{\mathbf{k}}\widehat{H}_{\rm so}$ (here $\nabla_{\mathbf{k}} \equiv \partial/\partial\mathbf{k}$) directly related to the particle spin. The evolution of the probability density $\rho = \Psi^{\dagger}\Psi$ is given by the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{J}(\mathbf{r},t) = 0, \quad (7)$$

with the components of the flux density

$$\mathbf{J}(\mathbf{r},t) = \frac{i}{2}[\Psi\nabla\Psi^{\dagger} - \Psi^{\dagger}\nabla\Psi] + \Psi^{\dagger}[\nabla_{\mathbf{k}}\widehat{H}_{\rm so}]\Psi. \quad (8)$$

The spin components of the condensate are given by expectation values

$$\langle\widehat{\sigma}_i(t)\rangle = \frac{1}{N}\int\Psi^{\dagger}\widehat{\sigma}_i\Psi dxdy. \quad (9)$$

B. Rashba spin-orbit coupling

The first form of spin-orbit interaction that we consider is the Rashba coupling

$$\widehat{H}_{\rm so} \equiv \widehat{H}_R = \alpha(k_x\widehat{\sigma}_y - k_y\widehat{\sigma}_x), \quad (10)$$

with the coupling constant α and $\mathbf{k} \equiv (k_x,k_y)$. The corresponding spin-dependent terms in the velocity operators in Eq. (6) become

$$\frac{\partial\widehat{H}_R}{\partial k_x} = \alpha\widehat{\sigma}_y, \quad \frac{\partial\widehat{H}_R}{\partial k_y} = -\alpha\widehat{\sigma}_x. \quad (11)$$

The spatial scale of the SOC effects is described by the characteristic distance the particle has to move to flip the spin, $L_{\rm so} = 1/\alpha$. The corresponding spin rotation angle at the particle displacement L is of the order of $L/L_{\rm so}$. At the initial stage of the BEC evolution $t \ll T_c$ we obtain from Eq. (1) for $\Psi(\mathbf{r},t=0)$ in Eq. (2) with $\boldsymbol{\psi}(0) = [1,0]^T$

$$\frac{\partial}{\partial t}\psi^{\downarrow}(\mathbf{r},t\to 0) = i\frac{\sqrt{N}}{\sqrt{\pi}a^3(0)}\frac{x+iy}{L_{\rm so}}\exp\left[-\frac{r^2}{2a^2(0)}\right]. \quad (12)$$

As a result, the $\psi^{\downarrow}(\mathbf{r},t)$ component begins to grow at distances $r \sim a(0)$ with a rate proportional to α. At a sufficiently large α this growth can eventually lead to the collapse prevention.

At $t > 0$, the spatially nonuniform spin evolution begins. Since the spin precession angle at the displacement of $a(0)$ is of the order of $a(0)/L_{\rm so}$, starting from the fully polarized $\boldsymbol{\psi}(0) = [1,0]^T$ state, the atoms acquire the anomalous velocity of the order of $a(0)/L_{\rm so}\times\alpha \sim \alpha^2 a(0)$ for the weak SOC $a(0) \ll L_{\rm so}$, or of the order of α otherwise. The criterion of a large spin rotation in the collapse is $a(0) > L_{\rm so}$, that is $\alpha > 1/a(0)$, while the condition of a sufficiently large developed anomalous velocity is $\alpha > v_c$, that is $\alpha > \sqrt{\Lambda}/a(0)$. If the latter inequality is satisfied, the centrifugal component in the flow caused by the SOC can prevent the collapse, as we explain in detail below [36]. The condition of a weak effect of magnetic field on the collapse can be formulated as smallness of spin precession angle due to the Zeeman splitting compared to the precession angle due to the spin-orbit coupling, that is $T_cB \ll \min\{a(0)/L_{\rm so},1\}$. We will assume this condition and neglect the effects of the Zeeman splitting.

We begin the analysis of the joint effect of the SOC and the interatomic attraction with numerical results obtained by

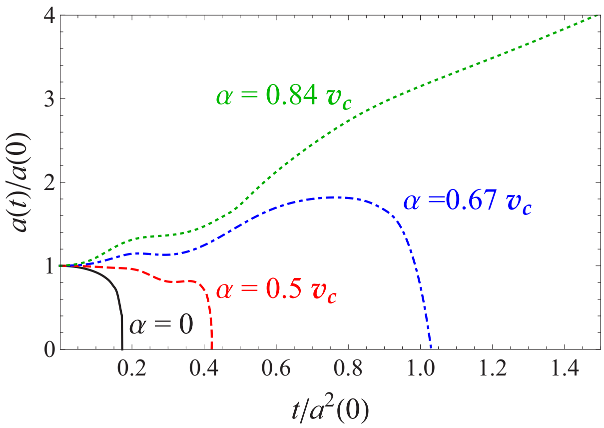

FIG. 1. (Color online) Time dependence of the condensate width for $\widetilde{g}_2 N = 16\pi$ and the values of α marked near the lines. The green short-dashed line corresponds to the absence of collapse.

direct integration of Eq. (1) for a strong attraction, $\widetilde{g}_2 N \gg 1$, where the effect is clearly seen, taking the initial spin state $\boldsymbol{\psi}(0) = [1,0]^T$. Figure 1 shows the time-dependent width of the packet defined as

$$a(t) \equiv \frac{N}{\sqrt{2\pi}} \left[\int |\Psi|^4 \, dxdy \right]^{-1/2}, \tag{13}$$

where Ψ is obtained by a direct solution of Eq. (1) for several values of α. The solid line in Fig. 1 corresponds to the collapse at $\alpha = 0$ where in the vicinity of T_c, the numerically calculated using Eqs. (1) and (13) width $a(t)$ is accurately described by the variational Eq. (5) with $a(t) \sim (T_c - t)^{1/2}$.

When spin-orbit coupling is included, the following features may be seen. (i) At short time $t \ll T_c$, the attraction-induced velocity develops linearly with t, while the anomalous velocity increases as t^2. As a result, the $a(t)$ dependencies for all values of α are the same at small t. (ii) The packet width $a(t)$ increases with time, reaches a plateau, and then decreases to zero. Thus, with the increase in α, the collapse still can occur, albeit taking a longer actual time $t_c > T_c$. (iii) Increasing further, α reaches a critical value $\alpha_{\rm cr} \approx 0.7 v_c$ such that at $\alpha > \alpha_{\rm cr}$ the anomalous velocity is large enough to prevent the collapse. The dependence of t_c on the SOC strength can be described as $t_c \sim (\alpha_{\rm cr} - \alpha)^{-1}$.

To get an insight of the effects of SOC on the collapse, we depict the density profiles in Fig. 2. At a large α the density forms a double peak with the maxima positions separating with time as a result of the centrifugal component in the flux. The resulting two-dimensional density distribution is given by a ring of radius $R(t)$ and width $w(t)$ with $a(t) \sim \sqrt{R(t)w(t)}$, responsible for the broad plateaus in $a(t)/a(0)$ seen in Fig. 1 at subcritical spin-orbit coupling. At $R(t) \gg a(t)$ the interatomic interaction energy tends to zero as $-1/R(t)w(t)$, and the conserved total energy is the sum of the kinetic and SOC terms. At $\alpha < \alpha_{\rm cr}$ [see Fig. 2(a)] the attraction is still strong enough to reverse the splitting and to restore the collapse. At $\alpha > \alpha_{\rm cr}$ [see Fig. 2(b)] the anomalous velocity takes over, the splitting continues, and the collapse does not occur [37]. This process is naturally accompanied by evolution of the condensate flux and spin presented in the Supplemental Material [38].

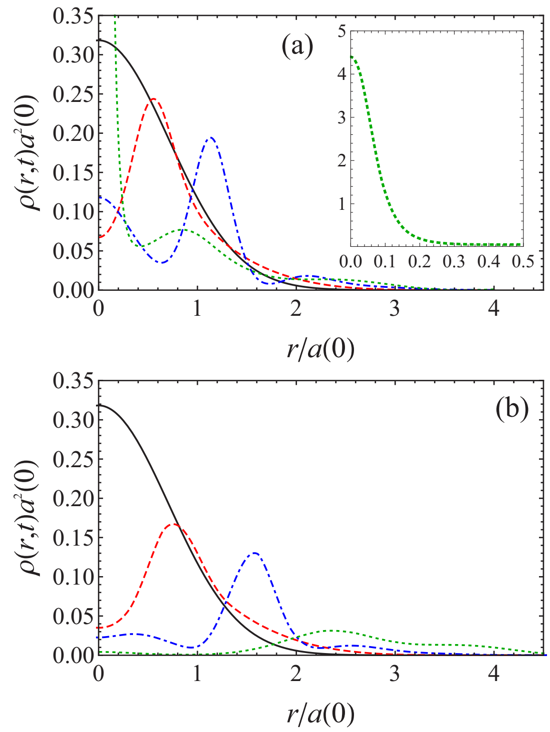

FIG. 2. (Color online) Density profile $\rho(r,t)$ for $\widetilde{g}_2 N = 16\pi$. Black solid line is for $t = 0$, red dashed line is for $t = 0.2a^2(0)$, blue dot-dashed line is for $t = 0.4a^2(0)$, and green dotted line is for $t = a^2(0)$. (a) Here $\alpha = 0.67 v_c$, and the dotted green line shown in detail in the inset clearly demonstrates the collapse. (b) Here $\alpha = 0.84 v_c$; the condensate is robust against attraction and can spread without collapsing.

C. Balanced Rashba and Dresselhaus couplings

In this subsection we consider a one-dimensional coupling

$$\widehat{H}_{\rm so} \equiv \widehat{H}_{RD} = \alpha k_x \widehat{\sigma}_z, \tag{14}$$

which is equivalent to the balanced Rashba and Dresselhaus contributions and gauged out by an x-dependent spin rotation [39] $\mathbf{U} = \exp[i\widehat{\sigma}_z x / L_{\rm so}]$.

For simplicity we consider the initial state corresponding to the spin oriented along the x axis with $\boldsymbol{\psi}(0) = [1,1]^T/\sqrt{2}$. Due to the anomalous velocity $\partial \widehat{H}_{RD}/\partial k_x = \alpha \widehat{\sigma}_z$ [cf. Eq. (11)], the initial state splits into two spin-projected wave packets moving in the absence of interactions with velocities $\pm\alpha$. As a result, the effective interaction decreases, and the collapse can be prohibited by this decrease. This happens, however, only at certain conditions, which we establish here. For qualitative analysis we use the ansatz

$$\psi^{\uparrow,\downarrow}(\mathbf{r}_{\mp}, t) = \widetilde{A}(t) \exp\left[-\frac{r_{\mp}^2}{2\widetilde{a}_{\rm v}^2(t)} (1 + i\widetilde{b}_{\rm v}(t)) \mp i\widetilde{c}_{\rm v}(t) x \right], \tag{15}$$

where the upper (lower) sign corresponds to spin up (down) and position $\mathbf{r}_{\mp} \equiv (x \mp \widetilde{d}_{\rm v}(t), y)$, and in addition to the variational chirp $\widetilde{b}_{\rm v}(t)$ and width $\widetilde{a}_{\rm v}(t)$, we introduced the variational momentum $\widetilde{c}_{\rm v}(t)$. From this ansatz we obtain, using an approach similar to that of Ref. [4], equations of motion for

$\widetilde{d}_{\rm v}$ and $\widetilde{a}_{\rm v}$:

$$\ddot{\widetilde{d}}_{\rm v} = -\frac{\widetilde{g}_2 N}{\pi}\frac{\widetilde{d}_{\rm v}}{\widetilde{a}_{\rm v}^4}\exp\left(-\frac{2\widetilde{d}_{\rm v}^2}{\widetilde{a}_{\rm v}^2}\right),$$
$$\ddot{\widetilde{a}}_{\rm v} = \frac{1}{\widetilde{a}_{\rm v}^3}\left\{\pi - \frac{\widetilde{g}_2 N}{4}\left[1 + \left(1 - \frac{2\widetilde{d}_{\rm v}^2}{\widetilde{a}_{\rm v}^2}\right)\exp\left(-\frac{2\widetilde{d}_{\rm v}^2}{\widetilde{a}_{\rm v}^2}\right)\right]\right\}, \tag{16}$$

for given $\widetilde{a}_{\rm v}(0) = a(0)$ and other initial conditions

$$\widetilde{d}_{\rm v}(0) = 0, \quad \dot{\widetilde{d}}_{\rm v}(0) = \alpha, \quad \dot{\widetilde{a}}_{\rm v}(0) = 0, \tag{17}$$

where $\dot{\widetilde{d}}_{\rm v}(0)$ is due to the anomalous velocity term leading to the spin-dependent splitting. These equations show that the collapse disappears if the coupling is strong enough to sufficiently separate the spin components; that is, at a certain time $\ddot{\widetilde{a}}_{\rm v}$ changes sign from negative to positive.

Qualitative conditions of the collapse in the presence of spin-orbit coupling in Eq. (14), which can be found from Eq. (16), are as follows. If $\widetilde{g}_2 N > 4\pi$, the collapse always occurs since even if the spin states are well separated, each of them still has the sufficient number of atoms. Depending on the interatomic interaction and SOC, one can either obtain the collapse at the origin, producing a spin-nonpolarized condensate, or two spatially symmetric ones producing z-axis polarized condensates. If $\widetilde{g}_2 N < 4\pi$, the collapse occurrence depends on the SOC strength.

At a sufficiently strong SOC, the spin splitting of the initial state and possible collapse happen on different time scales. The splitting occurs fast, on the time scale of $a(0)/\alpha$, and the interatomic attraction starts to play a role after the splitting. The condition of time scale separation, which allows one to treat the splitting and the collapse independently, is formulated as $a(0) < T_c\alpha$ or, in other words, as $\alpha > \sqrt{\Lambda}/a(0)$. Although this inequality looks similar to the above condition for the critical Rashba coupling, they are qualitatively different. For the Rashba coupling, the density decreases to zero and the collapse disappears completely at any SOC stronger than the critical one. For the balanced Rashba-Dresselhaus coupling the maximum density decreases at most by a factor of 2, and therefore the collapse can occur even at a very strong SOC, when spin-up and spin-down states are already well separated in space.

Figure 3 shows the time dependence of the packet width in Eq. (13) obtained by solution of Eq. (1) with spin-orbit-coupling Hamiltonian (14) for $\widetilde{g}_2 N = 3\pi$. The behavior at small $t \ll T_c$ here depends on α since the peaks in the spin-projected densities split by $2\alpha t$ due to the anomalous velocity. The numerically obtained critical value of α here is approximately $0.83 v_c$ and $a(t) \sim (t_c - t)$ shows a linear rather than a square-root behavior near the collapse time.

III. RELATION TO EXPERIMENT AND CONCLUSIONS

To make connections to possible BEC experiments, we return to the physical units and estimate the constant $\widetilde{g}_2$ as 0.05 for $-a_s \sim 100 a_B \sim 5\times 10^{-3}$ μm and $a_z \sim 1$ μm. The condition $\widetilde{g}_2 N > 2\pi$ can be satisfied already for ensembles with $N \sim 100$ particles. The velocity of the collapse is $v_c \sim \hbar\sqrt{\widetilde{g}_2 N}/Ma(0)$. At $a(0) \sim 10$ μm and $N \sim 10^3$ this estimate yields $v_c \sim 0.03$ cm/s and the corresponding time scale $T_c = a(0)/v_c \sim 0.3$ s. Such a small value of v_c demonstrates that even a relatively weak experimentally achievable coupling [40] can prevent the BEC from collapsing. At these conditions, the characteristic distance between the particles $[a^2(0)a_z/N]^{1/3} \sim 0.5$ μm is much larger than $-4\pi a_s \leqslant 0.1$ μm, still preventing a strong depletion of the condensate.

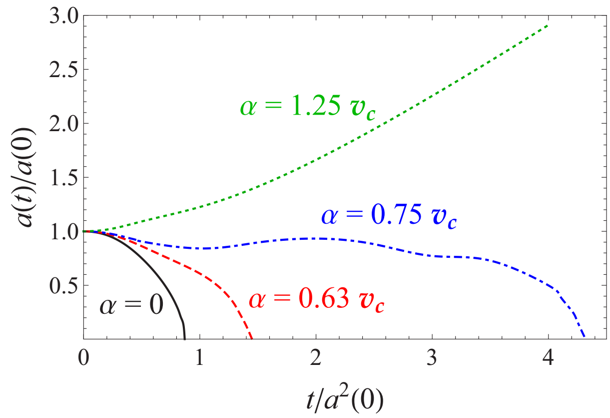

FIG. 3. (Color online) Time dependence of the condensate width for $\widetilde{g}_2 N = 3\pi$ in the presence of balanced Rashba-Dresselhaus SOC and different values of α as marked near the lines.

To conclude, we have demonstrated that the anomalous spin-dependent velocity determined by the spin-orbit-coupling strength can prevent collapse of a nonuniform quasi-two-dimensional BEC [41,42]. For the Rashba coupling with the spectrum axially symmetric in the momentum space, this velocity leads to a centrifugal component in the two-dimensional density flux. As a result, spin-orbit coupling can prevent collapse of the two-dimensional BEC if this flux is sufficiently strong to overcome the effect of interatomic attraction. In this case, the attraction between the bosons cannot squeeze the initial wave packet and force it to collapse. In the case of effectively one-dimensional balanced Rashba-Dresselhaus couplings, the anomalous velocity splits the initial state into spin-polarized wave packets, decreases the condensate density, and thus can prevent the collapse. Our approach can be generalized in a straightforward way for the intermediate case, where Dresselhaus and Rashba couplings have different strength. These results show that one can gain control over the BEC collapse process by using the experimentally available synthetic spin-orbit-coupling fields and, thus, extend the experimental abilities to study various nontrivial dynamical regimes in Bose-Einstein condensates of attracting particles.

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[37] The density can decrease to zero with infinite $R(t \to \infty)$ only in the absence of a strong confinement. If the confinement potential $\omega_0^2(x^2 + y^2)/2$, where ω_0 is the corresponding frequency, is taken into account, the energy conservation limits the value of $R(t)$ and naturally increases the critical value of SOC. However, as our simulations show, in the case of a strong spin-orbit coupling, even if the density returns to the vicinity of the origin, where the confinement potential is weak, the centrifugal flux takes over and the condensate begins to spread again rather than to collapse.
[38] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.91.043604 for additional graphic material on the flux and spin evolution and for the analysis of the BEC collapse near the threshold value of the interatomic attraction.
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[41] In terms of collective effects in solids [9], this result implies that spin-orbit coupling can prevent self-trapping of carriers and excitons.
[42] Collapse of two-dimensional BEC of polaritons in optical microcavities was, supposedly, observed in M. Vladimirova, S. Cronenberger, D. Scalbert, K. V. Kavokin, A. Miard, A. Lemaître, J. Bloch, D. Solnyshkov, G. Malpuech, and A. V. Kavokin, Phys. Rev. B **82**, 075301 (2010). It will be of interest to experimentally study the influence of spin-orbit coupling of polaritons [e.g., O. A. Egorov, A. Werner, T. C. H. Liew, E. A. Ostrovskaya, and F. Lederer, Phys. Rev. B **89**, 235302 (2014)] on the BEC collapse process in these systems.