Erratum: All-creation and all-annihilation time-dependent $\mathcal{PT}$ -symmetric bosonic Hamiltonians: An infinite squeezing degree at a finite time [Phys. Rev. A **100, 012128 (2019)]**

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In our paper, we made a sign error in the definition of the time-dependent (TD) parameter χ , in Eq. (7), that spread through some of the other equations. Although this error does not compromise the central idea of reaching an infinite degree of squeezing at a finite time, some observations about the squeezing mechanism must be made in addition to correcting the affected equations. Starting with the TD parameter χ , its correct expression is

$$\chi = |\lambda|^2 - \lambda_0 = \lambda_0 \frac{4|\mu|^2 \sinh^2 \Xi - \Xi^2}{\Xi^2} = -\frac{2\Phi}{|z|} - 1. \quad (1)$$

Next, we have obtained, in Eq. (11), a nonlinear system of three coupled differential equations whose correct expressions are

$$|\dot{z}| = -|z|^2 \left\{ |\omega| \frac{\chi + \Phi^2}{\Phi} \sin \varphi_\omega - 2[|\alpha| \sin(\varphi - \varphi_\alpha) - |\beta| \chi \sin(\varphi + \varphi_\beta)] \right\} + \frac{|z|}{\Phi} \dot{\Phi}, \quad (2a)$$

$$\dot{\Phi} = \frac{2}{\chi - 1} \{ [|\omega| \Phi \sin \varphi_\omega - |\alpha| \sin(\varphi - \varphi_\alpha)](1 - \Phi^2) - |\beta| [(2\chi - 1)\Phi^2 - \chi^2] \sin(\varphi + \varphi_\beta) \}, \quad (2b)$$

$$\dot{\varphi} = \frac{2}{(1 - \chi)\Phi} [|\alpha|(1 - \Phi^2) \cos(\varphi - \varphi_\alpha) + |\beta|(\Phi^2 - \chi^2) \cos(\varphi + \varphi_\beta)] + 2|\omega| \cos \varphi_\omega. \quad (2c)$$

Equation (13) was also affected, whose correct form is

$$\epsilon = \frac{1}{2\sqrt{1 - |z|^2}} \ln \frac{(1 + \sqrt{1 - |z|^2})\Phi + |z|}{(1 - \sqrt{1 - |z|^2})\Phi + |z|}. \quad (3)$$

The correct expression for the time derivative of the squeezing factor, in Eq. (24a), is given by

$$\dot{r}(t) = -2|U(t)| \sin[\varphi_U(t) + \phi(t)]. \quad (4)$$

Another problem with our paper, regardless of the sign error in the definition of χ , concerns the pseudo-Hermitian forms of the field quadratures associated with the Hermitian counterpart $h(t)$ of our pseudo-Hermitian Hamiltonian $H(t)$. It is better to start from the Schrödinger picture and then move on to the Heisenberg one instead of starting from the latter, as done in our paper. In the Schrödinger picture, the quadratures are given by $x = [(a + a^\dagger)]/2$ and $p = (a - a^\dagger)/2i$, leading to the Heisenberg expressions $x_h(t) = [a_h(t) + a_h^\dagger(t)]/2$ and $p_h(t) = [a_h(t) - a_h^\dagger(t)]/2i$, where $a_h(t)$ follows from the Bogoliubov transformation $a_h(t) = \mathcal{U}^\dagger(t)a\mathcal{U}(t) = u(t)a + v(t)a^\dagger + w(t)$, with the TD coefficients being correctly defined in our paper. The pseudo-Hermitian forms of the field quadratures in the Schrödinger picture are then given by

$$X(t) = \eta^{-1}(t)x\eta(t) = \frac{1}{|z|\sqrt{\Phi^2 - \chi}} \left\{ [(1 - i|z| \sin \varphi)\Phi + |z|]x + i(1 - |z| \cos \varphi)\Phi p - \frac{|z|}{2} [\zeta + \chi\zeta^* - 2\Phi \operatorname{Re}(e^{-i\varphi}\zeta)] \right\}, \quad (5a)$$

$$P(t) = \eta^{-1}(t)p\eta(t) = \frac{1}{|z|\sqrt{\Phi^2 - \chi}} \left\{ [(1 + i|z| \sin \varphi)\Phi + |z|]p - i(1 + |z| \cos \varphi)\Phi x - i\frac{|z|}{2} [\zeta - \chi\zeta^* + 2i\Phi \operatorname{Im}(e^{-i\varphi}\zeta)] \right\}, \quad (5b)$$

which are the correct expressions for Eq. (32) in our paper, in which the Heisenberg operators $x_h(t)$ and $p_h(t)$ must be additionally replaced by the Schrödinger's x and p . The related expressions in Secs. VII and X of our paper must be corrected accordingly.

For the variances of the field quadratures we have the general expressions

$$\begin{aligned} [\Delta X(t)]_{\Theta}^2 &= \langle \psi(t) | X^2(t) | \psi(t) \rangle_{\Theta} - \langle \psi(t) | X(t) | \psi(t) \rangle_{\Theta}^2 \\ &= \langle \Pi(t) | x^2 | \Pi(t) \rangle - [\langle \Pi(t) | x | \Pi(t) \rangle]^2 = [\Delta x(t)]^2 \\ &= \langle \Pi(0) | x_h^2(t) | \Pi(0) \rangle - \langle \Pi(0) | x_h(t) | \Pi(0) \rangle^2 = [\Delta x_h(t)]^2 \\ &= \frac{1}{4} |u(t) + v^*(t)|^2, \end{aligned} \quad (6a)$$

$$[\Delta P(t)]_{\Theta}^2 = \frac{1}{4} |u(t) - v^*(t)|^2, \quad (6b)$$

noting that the results in Eq. (33) of our paper, for $[\Delta X(t)]_{\Theta}^2$ and $[\Delta P(t)]_{\Theta}^2$, are both correct despite the somewhat confused way in which they were presented. Towards the computation of the squeezing factor, we must make the choice $\varphi_U(t) = 3\pi/2 - \phi(t)$ [instead of $\varphi_U(t) = \pi/2 - \phi(t)$ in our paper], in order to obtain the expressions in Eq. (39): $r(t) = 2 \int_0^t |U(\tau)| d\tau$ and $\dot{\varphi}_U(t) = -\dot{\phi}(t) = 2W(t)$. Going to the all-annihilation Hamiltonian, the expression for $|z(t)|$ must be changed by a negative signal, to

$$|z(t)| = -\frac{2\Phi(t)}{\Phi^2(t) + 1 - \lambda_0(t)}, \quad (7)$$

where, as correctly written in our paper: $\Phi(t) = \Phi(0) - \Lambda \int_0^t |\alpha(\tau)| d\tau$, $\Lambda = 2[\Phi^2(0) - 1]/[1 - \Phi^2(0) + \lambda_0(0)]$, and $\lambda_0(t) = \lambda_0(0)[\Phi^2(t) - 1]/[\Phi^2(0) - 1]$. Finally, considering the expression for the squeezing factor for the all-annihilation Hamiltonian $r_{aa}(t) = \int_0^t |\Lambda \alpha(\tau)| / |\Phi^2(\tau) - 1| d\tau$, which is also correctly written in our paper, we immediately notice that in the case where the TD functions $\chi(t)$ and $\Phi(t)$ approach unity, the squeezing factor diverges. We will next show that this divergence still takes place under the corrected equations, despite some important considerations about the physical viability of the process, thus ensuring the conclusions obtained in our paper.

We first observe, from Eq. (1), that for $\chi(t)$ and $\Phi(t)$ to approach unity, $\Phi(t)$ must be negative. Then, from the above expressions for $\Phi(t)$ and Λ , we note that it is convenient to choose the negative initial condition $\Phi(0) = -(1 + |s|)$, which leads to a negative Λ when imposing $\lambda_0(0) < (2 + |s|)|s|$. We thus hope that an appropriate choice of $\alpha(t)$ will bring $\Phi(t)$ closer to -1 , from below, while $-\Lambda \int_0^t |\alpha(\tau)| d\tau$ grows from zero to unity. This choice is the same made in our paper: $\alpha(t) = \alpha_0 e^{-\gamma t}$. Next, by substituting $\lambda_0(t)$ into Eq. (7) and considering $|z(0)| < 1$, we further restrict the imposition $\lambda_0(0) < (2 + |s|)|s|$ to $\lambda_0(0) < |s|^2$; however, considering $|z(t)| < 1$, we derive the condition

$$\Phi^2(t) + 2 \frac{\Phi^2(0) - 1}{\Phi^2(0) - 1 - \lambda_0(0)} \Phi(t) + \frac{\Phi^2(0) - 1 + \lambda_0(0)}{\Phi^2(0) - 1 - \lambda_0(0)} \geq 0. \quad (8)$$

By computing the roots of the above polynomial we obtain the results $\Phi_1 = -[\Phi^2(0) - 1 + \lambda_0(0)]/[\Phi^2(0) - 1 - \lambda_0(0)] < -1$ and $\Phi_2 = -1$, with Φ_1 getting closer to Φ_2 the bigger $\Phi(0)$ and the smaller $\lambda_0(0)$. It is important to note that the initial condition $\lambda_0(0) = 0$ is not an option here, due to the divergence of $\epsilon(t)$ as can be verified from Eq. (3). Moreover, with the choice made for $\Phi(t)$, approaching -1 from below, the numerator of the logarithm argument of Eq. (3) is negative, and this demands an equally negative denominator, which imposes the restriction $|z(t)| > -2\Phi(t)/[\Phi^2(t) + 1]$, which is automatically satisfied by Eq. (7) when $\lambda_0(0) > 0$. From its initial value $\Phi(0) = -(1 + |s|)$, $\Phi(t)$ then evolves towards the smallest root Φ_1 , reached at time t_1 , after which the polynomial in Eq. (8) becomes negative and $|z(t)|$ becomes greater than the unit. Consequently, we have physical plausible solutions for the pseudo-Hermitian $H(t)$ from $t = 0$ to t_1 .

Finally, noting that the squeezing factor $r_{aa}(t)$ for the all-annihilation Hamiltonian is given exactly by Eq. (60) of our paper, in Figs. 1(a)–1(c) we plot $r_{aa}(t)$ against γt considering $\alpha_0/\gamma = 1$ and $|s| = 1.51$ for all three figures, while $\lambda_0(0) = 10^{-1}$, 10^{-2} , and 10^{-3} for (a), (b), and (c), respectively. We also plot in Fig. 1 the squeezing factor $r_h(t)$ for the Hermitian Hamiltonian, apart from $|z(t)|$ which, as verified above, becomes greater than unity for $t > t_1$, when the results are no longer physically meaningful. For $\lambda_0(0) = 10^{-1}$, both squeezing factors present approximately the same value at t_1 , while $r_{aa}(t_1)$ becomes increasingly larger than $r_h(t_1)$ with the decrease of $\lambda_0(0)$. Moreover, the lower $\lambda_0(0)$, the more t_1 approaches t_c .

We note that the states $|\psi(t)\rangle$ and $|\Pi(t)\rangle$ of the pseudo-Hermitian and the Hermitian systems governed by $H(t)$ and $h(t)$, respectively, are related as $|\psi(t)\rangle = \eta^{-1}(t)|\Pi(t)\rangle$. Therefore, the initial state $|\psi(0)\rangle$ will in general be a squeezed state due to the action of the nonunitary Dyson map on $|\Pi(0)\rangle$. However, focusing on the divergence of the squeezing factor, in Fig. 1 we are not considering a squeezed initial state $|\psi(0)\rangle$, so that we start from $r(0) = 0$. Regarding Fig. 1 of our paper, we obtain exactly the same curves choosing $\varphi_- = 3\pi/2$, $z(0) = 1$, $\lambda_0(0) = [1 + \Phi(0)]^2$, and $\Phi(0) = -1.5, -1.1, -1.01$, and $-1 + 10^{-10}$, leading to the solid, dashed, dashed-dotted, and dotted curves, respectively. The choice of φ_- and $\Phi(0)$ (the negative of the values in our paper) were considered in order to obtain a positive value for the right-hand side of Eq. (7).

FIG. 1. Plot of $r_{aa}(t)$, $r_h(t)$, and $|z(t)|$ against γt considering $\alpha_0/\gamma = 1$ and $|s| = 1.51$ for all three cases (a), (b), and (c), while $\lambda_0(0) = 10^{-1}$, 10^{-2} , and 10^{-3} for (a), (b), and (c), respectively.

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Correction: The Editors have agreed to allow the addition of R. A. Dourado as the first author in recognition of the extensive role he played in the creation of the Erratum. Correspondingly, a statement of thanks to R. A. Dourado has been removed from the Acknowledgment section.